Optimal Quantity of Money with Two-Sided Altruism

Keuk-Soo Kim [1]

This paper examines optimal monetary policy in an overlapping generations economy where agents exhibit altruism toward both their parents and offspring. It has shown that inflation is optimal for an equilibrium in which only gift motive is operative. We note this as an interesting exception to the Friedman rule.

Keywords: Optimal quantity of money, Two-sided altruism

JEL Classification: O42

I. Introduction

Milton Friedman (1969) presented his famous rule for optimal monetary policy, namely the Friedman rule, that an optimal monetary policy requires deflation at a rate equal to the real rate of interest rate. Not surprisingly, there has been considerable controversy on this issue.[1] For example, Brock (1974), Grandmont and Younes (1973), McCallum (1983), and Kimbrough (1986) have confirmed the rule using a variety of monetary growth models.[2] In contrast, some authors like Weiss (1980), Drazen (1981), and Freeman (1993) showed that the optimal rate of money growth may result in a positive rate of inflation.

The purpose of this paper is to reexamine the optimum quantity

*Trade Research Institute Korea International Trade Association Samsung-dong, Gangnam-gu, Seoul, Korea, (Tel) +82-2-6000-5173 (Fax) +82-2-6000-5170, (E-mail) kimksoo@kita.or.kr

[**Seoul Journal of Economics** 2001, Vol. 14, No. 4]

[1]See, among others, Woodford (1990) and Mulligan *et. al.* (1997).

[2]See also Abel (1985), Palivos and Yip (1995), and Townsend (1980).

of money in an overlapping generations model in an environment where agents exhibit two-sided altruism. The approach in this paper is similar in spirit to Freeman (1993) who employed an overlapping generations model with parental altruism as in Barro (1974). It is, however, different from Freeman in that in order to describe the behavior of altruistic agents, the current study assumes two-sided, full, altruistic preferences.

The next section provides a basic framework and characterizes the steady-state equilibrium conditions regarding the optimum quantity of money. Section III shows suboptimality of the Friedman rule when only gift motive is operative in the agents' preference. Final remarks follow.

II. The Model

The model analyzed in this section, with the exception of the agents' full altruistic preference, is identical to Freeman (1993), and thus will be briefly described. In each period, a fixed number (N) of two-period lived agents are born with endowment y units of the economy's sole consumption good.[3] For simplicity, it is assumed that agents do not consume in the first period.

There are two assets, capital and fiat money, in the economy. In the first period of life, agents can create any amount of capital to store goods. Capital lasts one period, producing $f(k_t)$ goods at period $t+1$ from k_t units of capital created at period t. The function $f(\cdot)$ is twice-continuously differentiable, increasing, concave, and assumed to satisfy the standard Inada conditions $\lim_{t\to 0} f'(k$ $\lim_{t\to\infty} f'(k$ -$\lim_{t\to\infty} f'(k)=0$. In addition to capital, there is fiat money. Let M_t be the nominal supply of fiat money at the beginning of period t. Then the supply of fiat money at t can be expressed as $M_t=zM_{t-1}$ where z is the gross money growth rate.

We assume that changes in the stock of fiat money at time t are equal to the amount of lump-sum monetary transfers from the government. Then, government budget constraint is given by $p_t a_t = M_t - M_{t-1}$ where a_t and p_t are a lump-sum subsidy and the price in units of fiat money at t of a specific good at t, respectively

[3]We normalize N to 1.

Agents have two-sided altruistic preferences as suggested by Kimball (1987) in which intergenerational altruism acts in both directions, from children to parents and from parents to children. Therefore agents utility may be affected not only by the utility of their children but also by the utility of their parents. The two-sided altruistic utility of generation t comparable to Barro's (1974) one-sided utility can be written as[4]

$$V_t = U_t + \psi V_{t-1} + \beta V_{t+1} \qquad \psi > 0, \ \ \beta > 0 \tag{1}$$

where $V_t = \sum_{j=-\infty}^{\infty} \gamma_j U_{t+j}$ is the full altruistic utility of an agent of generation t, while $V_t = \sum_{j=-\infty}^{\infty} \gamma_j U_{t+j}$ is the direct utility of that agent. If equation (1) has a solution, it can be rewritten as[5]

$$V_t = \sum_{j=-\infty}^{\infty} \gamma_j U_{t+j} \tag{2}$$

Kimball (1987) analyzed γ_j's relation to ψ and β. Instead of trying to expand equation (1) to get something in the form of equation (2), he imposes economically reasonable restrictions on γ_j, $j \in (-\infty, \infty)$ such as non-negativity, dynamic consistency in family behavior, and bounded altruism.[6]

Kimball showed that these restrictions on γ_j imply $\psi\beta < 1/4$ and $\psi + \beta < 1$.[7] Also, if theses conditions, $\psi\beta < 1/4$ and $\psi + \beta < 1$, are

[4]Note that, following Barro (1974), Freeman (1993) limited $\psi = 0$, which does not allow for gift motive.

[5]Buiter and Camichael (1984) and Burbidge (1984) also used a different type of full altruistic utility that has a form:

$$U_t = W_t + \psi U_{t-1} + \sum_{j=-\infty}^{\infty} \beta^j U_{t+j}$$

But this utility function arbitrarily assumes the bequest motives differ from the gift motives.

[6]Non-negativity ($\gamma_j > 0$, $\forall j$): People do not wish ill of their ancestors and descendants. Kimball argued that this non-negative restriction on γ_j, *i.e.* non-negative concern for ancestors and descendants, is a reasonable requirement to maintain the meaning of two-sided altruism. Furthermore, he showed that this restriction can only be satisfied if the product of ψ and β from the difference equation (1) is not too large. Dynamic consistency in family behavior: To insure dynamic consistency from generation to generation, it is required that the forward tail of weights, *i.e.* $\{\gamma_j\}$, $j \geq 0$ is a geometric series. Bounded altruism: People do not care much about very distant ancestors and descendants. Formally, $\lim_{j\to\infty}\gamma_j = 0$ and $\lim_{j\to-\infty}\gamma_j = 0$

satisfied, then the γ_j can be written as

$$\gamma_j = \frac{1}{\sqrt{1-4\phi\beta}}\mu^J \qquad \text{for } j \geq 0$$
$$= \frac{1}{\sqrt{1-4\phi\beta}}\lambda^j \qquad \text{for } j \leq 0 \tag{3}$$

where $\mu = 1/2\phi[1-\sqrt{1-4\phi\beta}]$, $\mu < 1$ and $\lambda = 1/2\phi[1+\sqrt{1-4\phi\beta}]$, $\lambda > 1$.

Next, we assume that each agent born at time t derives utility from his own consumption $(j \geq 0)$ when old, and also from the real money balances $(h_{t-1}[p_{t-1}/p_t] + a_t)$ he has when old. The real money balances generating utility equals the real money balances hold when an agents is young, $p_t/p_{t+1} = 1/z$, plus those from the subsidy, *i.e.* $p_t/p_{t+1} = 1/z$. Further, we assume that $\phi\beta < 1/4$, $\phi + \beta < 1$ since it is reasonable to suppose that an agent's concern for his ancestor and descendant is diminished with generational distance.

The full altruistic lifetime utility of each agent born at t can be expressed as

$$V_t = \sum_{j=-\infty}^{\infty} \gamma_j [V(c_{t+1+j}) + W(h_{t+1+j})] \tag{4}$$

The function $f(\cdot)$, $V(\cdot)$ is concave, increasing, continuously differentiable with $\lim_{c\to 0} \nu'(c) = \infty$ and $\lim_{c\to 0} \nu'(c) = \infty$. The function $W(\cdot)$, on the other hand, is concave, continuously differentiable with $\lim_{c\to 0} \nu'(c) = \infty$ and reaches a unique maximum at $\lim_{c\to 0} \nu'(c) = \infty$, where $\lim_{c\to 0} \nu'(c) = \infty$ is a finite, positive level of $\lim_{c\to 0} \nu'(c) = \infty$.

Let $b_t g_t$, $b_t g_t$ denote bequests from parents to children at t and gifts made by children to parents in period t, respectively. Then, the budget constraints faced by each agent in the first and second period of life are given by

[7]The first inequality implies that altruism is not too strong. If it is, a "Hall of Mirror" effect leads all attempts to solve equation (1) to infinities. The second inequality means the sum of an agent's concern for the utility of his parents and children must be less than the concern for his own utility. If this condition is violated, altruism increases with distance.

$$y + b_t = k_t + h_t + g_t \tag{5}$$

$$f(k_t) + h_t\left[\frac{p_t}{p_{t+1}}\right] + a_{t+1} + g_{t+1} = c_{t+1} + b_{t+1} \tag{6}$$

The first constraint states that each agent allocates his wealth which consists of initial endowment (y) and bequests from his parents (b_t,g_t) among capital (k_t), real money balances (h_{1t}), and gifts (g_t) to his parents at period t. The second constraint implies that his second period wealth which consists of goods produced at $t+1$ from capital invested at period t, real money balance holding, and gifts (g_{t+1}) from his children must equal his consumption and bequests to his children at $t+1$.

Each agent maximizes his lifetime utility, given in equation (4), subject to the budget constraints (5) and (6) with respect to k_t,g_t, h_{1t}, g_t and b_{t+1}. Making the appropriate substitutions and performing straightforward differentiation, one obtains the first order conditions for this maximization problem.

$$\gamma_0 V'(c_{t+1}) f'(k_t) - \gamma_0 V'(c_{t+1})\left[\frac{p_t}{p_{t+1}}\right] - \gamma_0 W'(h_{t+1})\left[\frac{p_t}{p_{t+1}}\right] = 0 \tag{7}$$

$$\gamma_0 V'(c_{t+1}) f'(k_t) - \gamma_0 V'(c_{t+1})\left[\frac{p_t}{p_{t+1}}\right] - \gamma_0 W'(h_{t+1})\left[\frac{p_t}{p_{t+1}}\right] = 0 \tag{8}$$

(zero gift if inequality is strict)

$$\gamma_1 V'(c_{t+2})\left[\frac{p_{t+1}}{p_{t+2}}\right] - \gamma_0 V'(c_{t+1}) + \gamma_0 W'(h_{t+1})\left[\frac{p_t}{p_{t+1}}\right] \leq 0 \tag{9}$$

(zero bequest if inequality is strict),

The period t market clearing condition for money can be written as $p_t h_t = M_t$, implying $p_t h_t = M_t$ in the steady-state.[8] Let consumption, capital, real money balances, gifts, and bequests in the steady-state be denoted c, k, h, g, and b. We can rewrite the conditions as

[8]By definition, in the steady-state, consumption, capital, real money balances, gifts and bequests are all constant over time.

$$f'(k_t) = \frac{1}{V'(c)}\left[\frac{1}{z}\right]\{V'(c)+L'(h)\} \tag{10}$$

$$\frac{\gamma_{-1}}{\gamma_0} \leq \frac{1}{V'(c)}\left[\frac{1}{z}\right]\{V'(c)+L'(h)\} \tag{11}$$

$$\frac{\gamma_0}{\gamma_1} \geq \frac{1}{V'(c)}\left[\frac{1}{z}\right]\{V'(c)+L'(h)\} \tag{12}$$

These three first order conditions imply the following inequalities.

$$\frac{\gamma_{-1}}{\gamma_0} \leq f'(k*) \leq \frac{\gamma_0}{\gamma_1} \tag{13}$$

Equation (13) implies that the interest rate can fall within an interval of positive length. Using the definition of γ_j in equation (3), the inequalities in equation (13) can be rewritten as

$$\frac{1}{\lambda} \leq f'(k*) \leq \frac{1}{\mu} \tag{14}$$

where $1/\lambda \leq f'(k*) \leq 1/\mu$

Equation (10) can also be rewritten as

$$V'(c*)\left[\frac{1}{z} - f'(k*)\right] + W'(h*)\frac{1}{z} = 0 \tag{15}$$

Using inequality (14), equation (15), and budget constraint, we can classify three types of steady-state equilibria characterized by (a) positive bequest and zero gift (b) zero bequest and positive gift, and (c) zero bequest and zero gift. However, the equilibria characterized by (a) and (c) were already derived or implied by earlier works such as in Freeman (1993) and Woodford (1990). Hence, to conserve space, we now focus on the equilibrium when only gift motive is operative.

III. The Steady-State Equilibrium with Zero Bequest and Positive Gifts

The steady-state equilibrium when only gift motive is operative can be expressed as a vector (c^*,k^*,h^*,g^*,b^*) satisfying the following set of equations:

$$\frac{1}{\lambda} \leq f'(k^*) \leq \frac{1}{\mu} \tag{16}$$

$$V'(c^*)\left[\frac{1}{z} - f'(k^*)\right] + W'(h^*)\frac{1}{z} = 0 \tag{17}$$

$$y + f(\bar{k}) = \bar{c} + \bar{k} \tag{18}$$

Consider the optimal monetary policy. Following Weiss (1980), Freeman (1993), and Palivos (1997), we employ the steady-state utility of an agent as the criterion of optimality.[9] A central planner maximizes the steady-state utility

$$U = \sum_{j=-\infty}^{\infty} \gamma_j [V(c) + L(h)] \tag{19}$$

subject to the feasibility constraint:

$$c_{t+1} = y + f(k_t) - k_{t+1} \tag{20}$$

Immediate differentiation implies that the optimal value of $z = \lambda$ requires the satiation level of money balance, *i.e.* $W'(h^*) = 0$.

Accordingly, using equation (17), we can see that this optimum quantity of money can be achieved if $z = \lambda$ which is greater than 1.[10] At $z = \lambda$, the real rate of return of capital equals the rate of return of money; agents are satiated with real money balances. Therefore, in this case, the optimum quantity of money results in a positive rate of inflation even though a satiation level of money

[9]See Woodford (1990) and Freeman (1993) for justification.

[10]We restrict our analysis to a second-best framework in the sense that the only policy instrument available to the government is lump-sum transfer of money.

balances exists and money is superneutral. Note that, in this equilibrium, money is also superneutral since $f'(k^*) = \gamma_{-1}/\gamma_0 = 1/\lambda$.

IV. Concluding Remarks

This paper examined optimal monetary policy in an overlapping generations model with two-sided altruism. In a steady-state equilibrium characterized by zero bequest and positive gifts, the optimal monetary policy requires deflation, although a satiation level of money balance exists and money is superneutral. We note that this is an interesting exception to the Friedman rule.

(*Received ; Revised*)

References

Abel, Andrew B. "Dynamic Behavior of Capital Accumulation in a Cash-in-Advance Model." *Journal of Monetary Economics* 16 (1985): 55-71.

Barro, Robert J. "Are Government Bonds Net Wealth?" *Journal of Political Economy* 82 (1974): 1095-117.

Brock, William A. "Money and Growth: The Case of Long Run Perfect Foresight." *International Economic Review* 15 (1974): 750-77.

Buiter, Willem H., and Carmichael, Jeffrey. "Government Debt: Comment." *American Economic Review* 74 (1984): 762-5.

Burbidge, John B. "Government Debt: Reply." *American Economic Review* 74 (1984): 766-7.

Drazen, Allan. "Inflation and Capital Accumulation under a Finite Horizon." *Journal of Monetary Economics* 8 (1981): 247-60.

Freeman, Scott. "Resolving Differences over the Optimal Quantity of Money." *Journal of Money, Credit, and Banking* 25 (1993): 801-11.

Friedman, Milton. *The Optimum Quantity of Money and Other Essays*. Chicago: Aldine, 1969.

Grandmont, Jean M. and Younes, Yves. "On the Efficiency of Monetary Equilibrium." *Review of Economic Studies* 40 (1973): 149-65.

Kimball, Miles S. "Making Sense of Two-Sided Altruism." *Journal of

Monetary Economics 20 (1987): 301-26.

Kimbrough, Kent P. "The Optimum Quantity of Money Rule in the Theory of Public Finance." *Journal of Monetary Economics* 18 (1996): 277-84.

McCallum, Bennett T. "The Role of Overlapping Generations Models in Monetary Economics." *Carnegie-Rochester Conference Series on Public Policy* 18 (1983): 9-44.

Mulligan, Casey B., and Sala-i-Martin, Xavier X. The Optimum Quantity of Money: Theory and Evidence. NBER Working Paper 5954, 1997.

Palivos, Theodore. Optimal Monetary Policy with Heterogeneous Agents: Is there a Case for Inflation? Unpublished Manuscript, Louisiana State University, 1997.

Palivos, Theodore, and Yip, Chong K. "Government Expenditure Financing in an Endogenous Growth Model: A Comparison." *Journal of Money, Credit, and Banking* 27 (1995): 1159-78

Townsend, Robert M. "Models of Money with Spatially Separated Agents." In John Kareken and Neil Wallace (eds.), *Models of Monetary Economies*. Minneapolis: Federal Reserve Bank of Minneapolis, 1980.

Weiss, Laurence. "The Effects of Money Supply on Economic Welfare in the Steady State." *Econometrica* 48 (1980): 565-76.

Woodford, Michael. "The Optimum Quantity of Money." In Benjamin M. Friedman and Frank H. Hahn (eds.), *Handbook of Monetary Economics*, Vol. II, 1067-152. Amsterdam: North-Holland, 1990.

Acknowledgements: The author gratefully acknowledges Ted Palivos and two anonymous referees for helpful comments and suggestions.