The Incentive to Take Care and the Deterrence of a Nuisance Suit

Gyu Ho Wang [1]

This paper investigates three important issues in law and economics: the incentive to take care in the presence of nuisance suits, the incentive to bring about a nuisance suit by a plaintiff, and how to resolve a legal dispute. For this, we consider a three-stage game between a plaintiff and a defendant. We identify two types of equilibria one of which prevails, depending on the parameter values. The main results show that: generally the equilibrium level of care differs from the socially optimal care level; nuisance suits are not fully deterred; in some cases, litigation cannot be avoided because of the informational asymmetry.

Keywords: Incentive to take care, Nuisance suits, Litigation, Informational asymmetry

JEL Classification: C72, D82, K41

I. Introduction

Following are important issues in law and economics literature. (1) The incentive to take care in the presence of nuisance suits. (2) The incentive to bring about a nuisance suit by a plaintiff. (3) How to resolve a legal dispute. In the existing literature, these issues have been analyzed separately. For the first issue, starting with P'ng (1983), many authors have analyzed one-period model under incomplete information and have shown that a trial many result as an equilibrium behavior.[1] However, these models are criticized on

*Associate Professor, Department of Economics, Sogang University, Shinsoo Dong 1, Mapo-Ku, Seoul, 121-742, Korea. (Tel) +82-2-705-8699, (Fax) +82-2-705-8180, (E-mail) ghwang@sogang.ac.kr. I would like to thank the editor and anonymous referees. Usual disclaimer applies.
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the ground that it lacks a dynamic structure. Cheung (1988) extends Rubinstein's (1982, 1985) sequential bargaining model to the context of pre-trial negotiation. He shows that in a unique subgame perfect equilibrium, the parties will settle immediately. With complete information, he cannot explain going to court as an equilibrium behavior. Wang, Kim, and Yi (1994) extend his model to the incomplete information case and show that a trial may arise in equilibrium. Spier (1992) also considers a sequential bargaining model of pre-trial negotiation. She shows that for the fixed trial date, a deadline effect emerges where much settlement occurs just prior to the trial. With endogenous trial date, even under the complete information, multiple equilibria exist and that agreement may be delayed in equilibrium and the case may go to court. This line of literature mainly focuses on the resolution of legal disputes and the remaining two issues have not been properly addressed.

Recently, in law and economics so-called the nuisance suits or frivolous suits receive many attentions.[2] Broadly speaking, a nuisance suit is defined to be a non-meritorious suit that is filed only in the hope of obtaining a favorable out-of-court settlement. Although the plaintiff's threat to go to court is not credible in a nuisance suit, it is quite possible when the plaintiff has private information about the damage. In other words, the defendant rarely has complete information about the merit of a claim against him. As noted by Katz (1990), nuisance suits may cause serious problems with both efficiency and fairness. In the presence of nuisance suits, the problem is, on one hand, to protect the genuine claims and, on the other hand, to deter nuisance suits. In the existing literature, static models prevail. The analysis needs to be extended to a model with a dynamic structure.

The third line of literature focuses on whether the court can give people an incentive to treat each other fairly or, in other words, an incentive to take due care. The central theme of the economic theory of liability is how to induce an injurer to take the socially optimal level of care.[3] This line of literature does not consider the

[1]For example, Bebchuk (1984) and Nalebuff (1987) consider a model where an uninformed plaintiff makes an offer. Salant (1984) and Reinganum and Wilde (1986) examine cases where an informed plaintiff makes an offer. P'ng (1987) considers a cases where an informed defendant makes an offer.

[2]For example, see Rosenberg and Shavell (1985), Bebchuk (1988) and Katz (1990).

possibility of pre-trial negotiation by simply assuming that whenever there is a damage, the victims automatically go to court. Since litigation is a costly process, the possibility of settlement out of court may be an important feature. In particular, in connection with nuisance suits, the possibility of pre-trial negotiation should be explicitly considered. Furthermore, a rich model with dynamic structure is preferable.

These issues are clearly interrelated with one another and should be desirably addressed together. This paper provides an integrated model which analyzes three issues together; (1) What is the incentive to take care in the presence of nuisance suits when pre-trial negotiation is possible? (2) What is the incentive for the plaintiff to bring about a nuisance suit? and (3) Once a suit is filed so that the defendant and the plaintiff are engaged in a legal dispute, how do they resolve the conflict, either by settling out of court, or by going to court?

For this, we consider a three-stage game between a defendant and a plaintiff. In the first stage, called the **care-taking stage**, the defendant chooses a level of care which affects the probability distribution of damage to the plaintiff. In the second stage, called the **filing stage**, once the defendant takes the care, the actual amount of damage is realized. This is private information to the plaintiff which cannot be observed by the defendant. Knowing the actual damage, she decides whether to file or not. If she does not file, the game ends. However, if she does file a suit by paying a filing cost, the game moves to the third stage called the **bargaining stage**. In this stage, the plaintiff and the defendant decide whether to settle or to resort to the court. We assume that the plaintiff and the defendant will play a variant of extensive form game studied by Wang, Kim, and Yi (1994).

By solving the game backward, we characterize a unique sequential equilibrium of the entire game. The bargaining stage is basically the same model examined by Wang, Kim, Yi (1994). Although Wang, Kim, Yi (1994) do not consider the possibility of nuisance suits explicitly, it can easily be incorporated into their model. For the bargaining stage, we adopt their result which shows that in equilibrium, when the defendant makes an offer, the

[3]For example, see Posner (1986), and Polinsky and Rubinfeld (1988a, 1988b).

plaintiff whose net recovery from the litigation is smaller than the offer accepts it. The plaintiff whose net recovery from the litigation is greater than the offer rejects it and goes to court next period.

In the filing stage, we identify a critical level of care with the following properties. If care level less than that, all the plaintiffs file a suit so that no nuisance suit is deterred at all. It is called the first type of equilibrium. For care level higher than that, some portion of nuisance suits are filed and some portion are not. All the plaintiffs with merit of claim file. In the bargaining stage, the defendant makes an offer equal to the filing cost. In this equilibrium, some nuisance suits are deterred. It is called the second type equilibrium.

Finally, in the care-taking stage, as in the filing stage, we identify two types of equilibria in the entire game one of which prevails, depending on the relative magnitudes between care-taking cost and bargaining cost. In our model, the defendant has a trade-off between a level of care and a settlement offer. In equilibrium, these two factors are balanced. The first type of equilibrium is more likely when the cost of taking high care is enormous. In this case, the defendant chooses a low level of care which induces the first type equilibrium in the filing stage. The second type of equilibrium is more likely to arise when the cost for taking high care is small. In this case, the defendant chooses a high level of care which leads to the second type equilibrium in the filing stage.

Finally, some comparative statics and welfare results are given. For the first type of equilibrium, the equilibrium level of care is increasing with respect to the discount factor and the litigation costs. Also there exists a threshold level of discount factor such that for all discount factors less than that, the equilibrium level of care is less than the first best level of care, and for all discount factors greater than that, the equilibrium level of care is greater than the first best level of care.

The paper is organized as follows. In section II, a formal model is presented. In section III, we analyze the bargaining stage. As mentioned earlier, the main result is adopted from Wang, Kim, Yi (1994). Following section III, we consider the incentive of the plaintiff whether to file or not in section IV. In section V, we examine the incentive to take care and characterize the equilibrium of the entire game. In section VI, we report some comparative static results and welfare analysis which compares the equilibrium level of care with the first best level of care. Concluding remarks follow.

II. The Model

We consider the following three-stage game between two economic agents. In the first stage called **the care-taking stage**, an economic agent called 'Defendant,' denoted by D is engaged in some economic activity which may cause damage to the other economic agent called 'Plaintiff,' denoted by P. The amount of damage is private information to the plaintiff; it cannot be observed by the defendant, who only knows the distribution of the damage.

Although the defendant cannot observe the amount of damage, he anticipates that in the later stage the plaintiff may file a lawsuit against him, and he may therefore have an incentive to take 'care' in order to reduce the amount of damage in a probabilistic way. We denote by a the level of care taken by the defendant. We assume $a \in (0, \infty)$ and that the level of care is observable to the plaintiff, but may not be verifiable in court. We denote by $V(a)$ the cost to the defendant when he takes care level a.

In the second stage called **the filing stage**, once the defendant takes the care level a, then according to the probability density function $f(x,a)$, the amount of damage denoted by x is realized. x is private information to the plaintiff. We identify the type of plaintiff with the amount of damage, thereby, denote by P_x the plaintiff with the amount of damage x. In this stage, each plaintiff has to decide to whether to file a suit or not. If she does not file a suit, the game ends and $-V(a)$ and zero are the payoffs for the defendant and the plaintiff, respectively. If the plaintiff decides to file a suit, she has to pay c as a filing cost. Once the plaintiff files, it will initiate the third stage.

In the third stage called **the bargaining stage**, the defendant and the plaintiff decide whether to settle the suit or to go to court. We assume that the defendant and the plaintiff will play the following extensive form game.

D moves first. He offers some settlement amount. P either accepts it so that the game ends, or rejects it. If she rejects the offer, in the next period she makes a counteroffer or goes to court. If she makes a counteroffer, and D accepts it, the game ends. If D

rejects, he can make a counteroffer in the next period, and so on. If P goes to court, the court decides the amount that D should pay to P. We assume the amount of damage is verifiable at court so that whenever P_x goes to court, the court always decides that D should pay x to P_x. In this case, however, each party incurs the litigation costs. Let c_p and c_d be the litigation costs incurred by P and D, respectively. Let δ be the common discount factor for both parties. When P and D settle at period t at the amount s, $\delta^{t-1}s-c$ and $-(V(a)+\delta^{t-1}s)$ are the payoffs to P_x and D, respectively. Instead of settling outside the court, if P_x goes to court at period t, $\delta^{t-1}(x-c_p)-c$ and $-(V(a)+\delta^{t-1}(x+c_d))$ are the payoffs to P_x and D, respectively. If both parties do not settle forever and P does not go to court, $-c$ and $-V(a)$ are the payoffs to P and D, respectively.

We now make the following assumptions about $V(a)$, the cumulative distribution, $F(x,a)$ and its density function $f(x,a)$. For the distribution and density functions, $'$ and subscript a mean the differentiation with respect to x and a, respectively. For example, $f'=\partial f/\partial x$, $f_a=\partial f/\partial a$.

***Assumption* 1** $V(a)$ is twice continuously differentiable with $V_a(a)>0$ and $V_{aa}(a)\geq 0$.

Assumption 1 says that the higher care the defendant takes, the more it costs, and the rate of increase is non-decreasing.

***Assumption* 2** For all $a\in(0,\infty)$, $f(x,a)>0$ over $[0,\infty)$, and $f(x,a)$ is twice continuously differentiable with respect to x and a.

This is an assumption for the support of the distribution. We assume that regardless of a, the support is the non-negative real line.

***Assumption* 3** For all $a\in(0,\infty)$, $E(x:a)=\int_0^\infty xf(x)dx<\infty$ and $\lim_{a\to 0} E(x:a)=\infty$.

If $E(x:a)=\infty$, in the bargaining stage, the expected cost for the defendant is infinite and the problem becomes trivial. In order to avoid this, we assume that for all a, $E(x;a)<\infty$. $\lim_{a\to 0}E(x:a)=\infty$ guarantees that the defendant should choose strictly positive a.

***Assumption* 4** For all $a \in (0,\infty)$, $f(x,a)/F(x,a)$ is decreasing in x, *i.e.*, $\partial(f(x,a)/F(x,a))/\partial x < 0$.

This is an assumption for the behavior of density-distribution ratio. A sufficient condition for this assumption is that the hazard rate $f(x,a)/(1-F(x,a))$ is decreasing in x.

***Assumption* 5** The family of densities $\{f(x,a), a \in (0,\infty)\}$ satisfies *Monotone Likelihood Ratio Property* (MLRP), *i.e.*, whenever $a' > a$, $f(x,a')/f(x,a)$ is decreasing in x. In other words, for all $a' > a$, $\partial(f(x,a')/f(x,a))/\partial x < 0$.

What MLRP says is that under higher level of care, smaller damage is more likely. It is well-known that MLRP implies the first-order stochastic dominance; for all $x \in (0,\infty)$, $F(x,a)$ is increasing in a. Actually, what we need is that for all $x \in (0,\infty)$, $f(x,a)/F(x,a)$ is decreasing in a. Later, we will show that MLRP implies this.

***Assumption* 6** For all $x \in (0,\infty)$, $F_a(x,a) > 0$, and $F_{aa}(x,a) < 0$.

The first part of this assumption is already implied by MLRP. The second part says that the higher level of care increases the distribution of x at a decreasing rate.

The readers may wonder at this point whether there exists a family of distributions that satisfies assumptions 2 through 6. Here is an example of a family of distributions, the so-called the family of exponential distribution. For the exponential distribution,

$$F(x,a) = 1 - e^{-ax} \text{ and } f(x,a) = ae^{-ax},\ a > 0,\ x \geq 0.$$

Note that for the exponential distribution, we have:

$$E(x,a) = \frac{1}{a},\quad \frac{f(x,a)}{F(x,a)} = \frac{ae^{-ax}}{1-e^{-ax}},\quad \text{and}\quad \frac{f(x,a')}{f(x,a)} = \frac{a'}{a}\, e^{-(a'-a)x}$$

Clearly, Assumption 2 is satisfied. For all $a > 0$, $E(x;a$ $\lim_{a \to 0} E(x:a) = \infty$, which satisfies Assumption 3. Since $f(x,a)$ is decreasing in x, Assumption 4 is satisfied. Whenever $a' > a$, $f(x,a')/f(x,a)$ is decreasing in x. Therefore, MLRP is satisfied. Finally, for all $a>0$, $F_a(x,a)=xe^{-ax}$ and $F_{aa}(x,a)=-x^2e^{-ax}<0$. Hence Assumption 6 is also satisfied.

Finally, we define what a 'nuisance suit' is. There is some controversy over the definition of a nuisance suit. In this paper, we adopt the following definition: A nuisance suit is a non-meritorious suit brought by the plaintiff solely in order to extract a positive settlement offer. In the nuisance suit, although the plaintiff files a suit against the defendant, she does not want to go to court because by resorting to the court, she gains nothing, in fact she loses her money by paying the filing cost, the litigation cost and so on. The only reason that she brings the suit is to take advantage of the informational asymmetry, by extracting a positive settlement offer from the defendant.

In our model, the plaintiff must pay the filing cost c in order to file a suit. Since the defendant makes an offer first, if P_x goes to court, she can get $\delta(x-c_p)$. Once we adopt above definition of a nuisance suit, only the suits brought by P_x with $\delta(x-c_p)-c>0$ are meritorious. All the suits by P_x with $x\leq c/\delta+c_p$ are nuisance suit s.[4] Hence, in this paper, we consider suits by P_x with $x\leq c/\delta+c_p$ as nuisance suits.

In the next section, we will characterize the sequential equilibrium of the entire game by solving backward. First, we will characterize the unique equilibrium in the bargaining stage. Given the equilibrium in the bargaining stage, we will analyze the incentive of the plaintiff whether to file a suit or not. Then, finally we will characterize the equilibrium level of care taken by the defendant.

III. The Bargaining Stage

In this stage, the defendant has already chosen a, and the plaintiff has filed a suit. Hence $V(a)$, the cost for taking care a and the filing cost c are sunk. Since these costs do not affect the incentives of the plaintiff and the defendant in the bargaining stage, we will omit these costs from the payoffs.

The bargaining stage is a special case of the model studied by Wang, Kim, Yi (1994). except that the possibility of a nuisance suit

[4]Bebchuk (1988) calls such suits negative-expected-value (NEV) suits.

is not explicitly considered. Since it can easily be incorporated into their analysis, their result is adopted without proof.

Given the care level a, let $F(x,a)$ and $f(x,a)$ be the distribution and the density functions of x. The main observation of their analysis is that in any sequential equilibrium, given a settlement offer s by D, the plaintiff shows the following dichotomous behavior: For all x with $\delta(x-c_p)>s$, P_x rejects s and goes to court next period. For all x with $\delta(x-c_p)<s$, P_x accepts s (See Propositions 4 and 5 in Wang, Kim, and Yi (1994)).

Given $s\geq 0$, let $h(s)$ be defined as $s=\delta(h(s)-c_p)$. Given the dichotomous behavior by the plaintiff, the problem for the defendant reduces to choosing s which minimizes his expected cost:

$$\min_{s} \int_0^{h(s)} sf(t,a)dt+\int_{h(s)}^{\infty}\delta\,(t+c_d)f(t,a)dt.$$

Equivalently instead of choosing s, he can choose the threshold level of x such that if $x'>x$, $P_{x'}$ rejects the offer and go to court so that it costs $\delta(x'+c_d)$ to D, and if $x'\leq x$, $P_{x'}$ accepts the offer $\delta(x-c_p)$:

$$\begin{aligned}\min_{x}\ W(x,a) &= \int_0^x \delta(x-c_p)f(t,a)dt+\int_x^{\infty}\delta\,(t+c_d)f(t,a)dt\\ &= \delta\{\int_x^{\infty}(1-F(t,a))dt-(c_p+c_d)F(x,a)+x+c_d\}.\end{aligned}$$

The first order condition gives Equation (1).

$$F(x,a)-(c_p+c_d)f(x,a)=0,\quad \text{or}\quad 1=(c_p+c_d)\frac{f(x,a)}{F(x,a)}. \tag{1}$$

Assumption 4 guarantees that there exists a unique x which satisfies Equation (1), and the second order condition is also satisfied. Denote this unique solution by $x^0(a)$ and define $s^0(a)$ as follows:

$$s^0(a)=\begin{cases}\delta(x^0(a)-c_p) & \text{if } x^0(a)\geq c_p,\\ 0 & \text{if } x^0(a)<c_p.\end{cases}$$

The unique sequential equilibrium in the bargaining stage is completely characterized by $s^0(a)$ and $x^0(a)$.

***Proposition* 1**
Given distribution and density functions $F(x,a)$ and $f(x,a)$ that satisfy Assumptions 2 through 6, in a unique equilibrium, in the first period D offer $s^0(a)$. If $s^0(a)>0$, P_x with $x \le x^0(a)$ accepts the offer, and P_x with $x > x^0(a)$ rejects the offer and goes to court next period. If $s^0(a)=0$, P_x with $x \le c_p$ accepts the offer, and P_x with $x > c_p$ rejects the offer and goes to court next period.

***Proof*:** See Theorem in Wang, Kim, and Yi (1994).

IV. The Filing Stage

In this section, based on the analysis of the bargaining stage, we examine the incentives of the plaintiff to file a suit. Note that in this stage, a is given. One simple but useful observation is that if P_x files a suit, $P_{x'}$ with $x'>x$ also does.

***Lemma* 1** Whenever P_x files a suit, for all $x'>x$, $P_{x'}$ also files a suit.

***Proof*:** P_x files a suit because the continuation payoff is no less than the filing cost. If $P_{x'}$ does not file a suit, she gets 0. Suppose $P_{x'}$ files a suit and mimics P_x. With the dispute settled outside the court, $P_{x'}$ gets the same amount as P_x. If the dispute goes to the court, $P_{x'}$ gets $x'-c_p$ (current value) which is greater than $x-c_p$. Hence the continuation payoff for $P_{x'}$ cannot be less than that for P_x, which, in turn, is no less than the filing cost. Therefore, whenever P_x files a suit, for all $x'>x$, $P_{x'}$ also files a suit.

Q.E.D.

Due to Lemma 1, an equilibrium in the filing stage can be identified with a cutoff level z which is the least type of plaintiff which files a suit. For this, consider the conditional distribution of x given $x \ge z$. Let $G(x|z,a)$ and $g(x|z,a)$ be the conditional distribution and the density function of x given a and $x \ge z$.

$$G(x|z,a)=\frac{F(x,a)-F(z,a)}{1-F(z,a)}, \text{ and } g(x|z,a)=\frac{f(x,a)}{1-F(z,a)}, \; x \ge z(\ge 0).$$

Note that $g(x|z,a)$ satisfies MLRP. $g(x|z,a)/G(x|z,a)$ inherits the properties of $f(x,a)/F(x,a)$.

Lemma 2 For all a and $x > z \geq 0$, $g(x|z,a)/G(x|z,a)$ is increasing in z and decreasing in a and x.

Proof: Note that $g(x|z,a)/G(x|z,a) = f(x,a)/(F(x,a) - F(z,a))$. With $x > z' > z$, $F(x,a) > F(z',a) > F(z,a)$. Hence $f(x,a)/(F(x,a) - F(z',a)) > f(x,a)/(F(x,a) - F(z,a))$. This shows that $g(x|z,a)/G(x|z,a)$ is increasing in z.

Let $x' > x > z$. By Assumption 4, $f(x',a)/(F(x',a) < f(x,a)/F(x,a)$. With $x' > x > z$, $1/(F(x',a) - F(z,a)) < 1/(F(x,a) - F(z,a))$. By multiplying each side by $f(x,a)$, we have $f(x,a)/(F(x,a) - F(z,a)) > f(x',a)/(F(x',a) - F(z,a))$. This shows that $g(x|z,a)/G(x|z,a)$ is decreasing in x.

For all $a' > a$, and $z < t < x$, $f(t,a')/f(t,a) > f(x,a')/f(x,a)$ by Assumption 5. By rearranging the terms, we have $f(t,a')f(x,a) > f(t,a)f(x,a')$. By integrating both sides with respect to t, we obtain

$$
\begin{aligned}
(F(x,a') - F(z,a'))f(x,a) &= \int_z^x f(t,a')dt\, f(x,a) \\
&> \int_z^x f(t,a)dt\, f(x,a') = (F(x,a) - F(x,a))f(x,a'),
\end{aligned}
$$

which is equivalent to $(f(x,a)/(F(x,a) - F(z,a))) > (f(x,a')/(F(x,a') - F(z,a')))$. This shows that $g(x|z,a)/G(x|z,a)$ is decreasing in a.

Q.E.D.

Suppose that only P_x with $x \geq z$ has filed a suit. Then, the posterior distribution of x is the conditional distribution of x given $x \geq z$, $G(x|z,a)$. Hence in the bargaining stage, D chooses x which satisfies Equation (1) with $F(x,a)$ and $f(x,a)$ replaced by $G(x|z,a)$ and $g(x|z,a)$. We denote by $x^*(a,z)$ the solution of the following equation:

$$G(x|z,a) - (c_p + c_d)g(x|z,a) = 0, \quad \text{or} \quad 1 = (c_p + c_d)\frac{f(x|z,a)}{F(x,a) - F(z,a)}. \tag{2}$$

By Lemma 2, $f(x,a)/(F(x,a) - F(z,a))$ is decreasing in x and $\lim_{x \to z} f(x,a)/(F(x,a) - F(z,a)) = \infty$ and $\lim_{x \to \infty} f(x,a)/(F(x,a) - F(z,a)) = 0$. Hence given a and z, there exists a unique solution to Equation (2). So $x^*(a,z)$ is well-defined. By Lemma 2, $f(x,a)/(F(x,a) - F(z,a))$ is decreasing in x, *i.e.*, $\partial(f(x,a)/(F(x,a) - F(z,a)))/\partial x < 0$, which is equivalent to that $f^2(x,a) - f'(x,a)(F(x,a) - F(z,a)) > 0$. Since $x^*(a,z)$ satisfies Equation (2), we have $f(x^*(a,z),a) - (c_p + c_d)f'(x^*(a,z),a) > 0$. Hence the second order condition for minimization is satisfied as well.

***Lemma* 3** $x^*(a,z)$ is decreasing in a, increasing in z and for all $a, x^*(a,z)>z$.

***Proof*:** By differentiating Equation (2) with respect to a, we have

$$\frac{\partial}{\partial x}\left(\frac{f(x^*(a,z),a)}{F(x^*(a,z),a)-F(z,a)}\right)\frac{\partial x^*(a,z)}{\partial a}+\frac{\partial}{\partial a}\left(\frac{f(x^*(a,z),a)}{F(x^*(a,z),a)-F(z,a)}\right)=0.$$

Since $\partial(f(x^*(a,z),a)/(F(x^*(a,z),a)-F(z,a)))/\partial x<0$ and $\partial(f(x^*(a,z),a)/(F(x^*(a,z),a)-F(z,a)))/\partial a<0$ by Lemma 2, $\partial x^*(a,z)/\partial a<0$. Similarly, $\partial x^*(a,z)/\partial z>0$. Finally, since $\lim_{x\to z} f(x,a)/(F(x,a)-F(z,a))=\infty$, for all a, $x^*(a,z)>z$.

Q.E.D.

As z increases, only the plaintiffs with higher damage remain in the bargaining stage. Accordingly, the defendant chooses the higher x. In the last section, we defined $x^0(a)$ as a unique solution to Equation (1). Note that $x^0(a)=x^*(a,0)$. Define $s^*(a,z)$ as follows:

$$s^*(a,z)=\begin{cases}\delta(x^*(a,z)-c_p) & \text{if } x^*(a,z)\geq c_p,\\ 0 & \text{if } x^*(a,z)<c_p.\end{cases}$$

Note that $s^0(a)$ in the last section is equal to $s^*(a,0)$. Depending upon the relative magnitude between $s^*(a,0)$ $(=s^0(a))$ and c, two types of equilibria prevail. For this, let's define a^* as the level of a such that $s^0(a)=c$. Equivalently, a^* is given by $x^0(a)=c_p+c/\delta$. Since $x^0(a)$ is decreasing in a, a^* is unique. We assume $a^*\in(0,\infty)$. If $a^*=0$ (*i.e.*, $x^0(0)\leq c_p+c/\delta$) or $a^*=\infty$ (*i.e.*, $x^0(\infty)\geq c_p+c/\delta$), as seen in Proposition 2, regardless of effort level a, only one type of equilibrium prevails.

***Proposition* 2**

(1) Type 1 equilibrium, $a\leq a^*$; In a unique equilibrium, all the plaintiffs file a suit so that the prior distribution is preserved. In the bargaining stage, D offers $s^*(a,0)$, and P_x with $x>x^*(a,0)$ rejects the offer and goes to court next period. P_x with $x\leq x^*(a,0)$ accepts the offer.

(2) Type 2 equilibrium, $a>a^*$; In a unique equilibrium, there exists $z^*(a)$ (>0) such that every plaintiff with $x \leq z^*(a)$ does not file a suit, and every plaintiff with $x>z^*(a)$ file a suit. In the bargaining stage, D offers c, P_x with $z^*(a)<x \leq x^*=c/\delta+c_p$ accepts the offer, and P_x with $x>x^*$ rejects and goes to court next period.

Proof: Two cases are considered separately.

***Case* 1** $a \leq a^*$; For $a \leq a^*$, $x^*(a,0)=x^0(a) \geq c_p+c/\delta$ and $s^*(a,0)=s^0(a) \geq c$. Let z^0 the cutoff level for filing a suit. Suppose $z^0>0$. Since $x^*(a,z)$ is strictly increasing in z, $x^*(a,z^0)>x^*(a,0) \geq c_p+c/\delta$. Hence, $s^*(a,z^0)>s^*(a,0) \geq c$. Namely, D offers more than c. Then, for all $0 \leq z < z^0$, P_z should have filed a suit. It is a contradiction. Hence z^0 should be zero, *i.e.*, all plaintiffs file a suit, and in the bargaining stage D offers $s^*(a,0)$.

***Case* 2** $a>a^*$; For $a>a^*$, $x^*(a,0)=x^0(a)<c_p+c/\delta$ and $s^*(a,0)=s^0(a) < c$. If z^0 equals 0 so that all plaintiffs file a suit, in the bargaining stage D offers $s^*(a,0)$ which is strictly less than c. Then, at least for z sufficiently close to 0, P_z loses by filing a suit. They should have not filed a suit. Hence, $z^0>0$. In order for z^0 to be the cutoff level, P_{z^0} should be indifferent between filing and not. Namely, $s^*(a,z^0)=c$ should hold. As $x^*(a,z)$ is continuos and increasing is z, so is $s^*(a,z)$. Since $x^*(a,z)>z$, as $z \to \infty$, $s^*(a,z) \to \infty$. With $s^*(a,0)<c$, by intermediate value theorem, there exists a unique $z^0>0$ such that $s^*(a,z^0)=c$. In order to emphasize the dependence of z^0 on a, it is denoted by $z^*(a)$. Then, $z^*(a)$ is determined implicitly by Equation (3).

$$s^*(a,z^*(a))=c, \quad \text{or} \quad x^*(a,z^*(a))=\frac{c}{\delta}+c_p. \tag{3}$$

$x^*(a,z^*(a))=(c/\delta)+c_p$ is the cutoff level for going to court, denoted by x^*, which is independent of a. Since $x^*(a,z)>z$, $x^*=(c/\delta)+c_p= x^*(a,z^*(a))>z^*(a)$. Hence, if $a>a^*$, every plaintiff with $x \leq z^*(a)$ does not file a suit, and every plaintiff with $x>z^*(a)$ file a suit. In the bargaining stage, D offers c, P_x with $z^*(a)<x \leq x^*$ accepts the offer, and P_x with $x>x^*$ rejects and goes to court next period.

Q.E.D.

V. The Care-Taking Stage

In this section, we will consider the incentive of the defendant to take care in the first stage. As seen above, depending upon whether a is greater or less than a^*, the equilibrium in filing stage takes different form. If $a \leq a^*$, type 1 equilibrium prevails in the filing stage. Hence the expected cost for D after the filing stage is given by

$$W(a) = \delta\{\int_{x^0(a)}^{\infty}(1-F(t,a))dt-(c_p+c_d)F(x^0(a),a)+x^0(a)+c_d\}.$$

If $a > a^*$, type 2 equilibrium is a relevant one in the filing stage. Hence, the expected cost for D after the filing stage is given by

$$W(a) = \delta\{\int_{x^*}^{\infty}(1-G(t\,|\,z^*(a),a))dt-(c_p+c_d)G(x^*\,|\,z^*(a),a)+x^*+c_d\}.$$

In the care-taking stage, the total cost for the defendant is $C(a) = V(a) + W(a)$ and the defendant chooses a minimizing it, denoted by a^e. Since $\lim_{a\to\infty} V(a) = \infty$ by Assumption 1, and $\lim_{a\to 0} W(a) = \infty$ by Assumption 3, existence of a^e is guaranteed. Unfortunately, however, our Assumptions do no guarantee, in general, that $W(a)$ is convex in a.[5] In order to locate a^e, we assume that $V(a)$ is sufficiently convex so that $C(a)$ is indeed convex in a. Then, a^e is completely determined by the first order condition.

In applying the first order condition, we should be careful because $W(a)$ is kinked at $a = a^*$. The left hand derivative of $W(a)$ at $a = a^*$ differs from the right hand derivative. We now calculate both left and right derivatives of $W(a)$ at $a = a^*$. With $a < a^*$,

$$W(a) = \delta\{\int_{x^0(a)}^{\infty}(1-F(t,a)dt-(c_p+c_d)F(x^0(a),a)+x^0(a)+c_d\}.$$

Note that $x^0(a^*) = c_p + c/\delta = x^*$. Since $x^0(a)$ minimizes $W(x,a)$ and $W(a) = W(x^0(a),a)$, by envelope theorem, $dW(a)/da = \partial W(x^0(a),a)/\partial a$ holds. Hence, the left hand derivative of $W(a)$ at $a = a^*$, denoted by $W'(a^*_-)$ is given by as follows;

[5]For exponential distributions, if $a \leq a^*$, $W(a) = \delta\{((1/a \quad .(ka+1))/a)-c_p\}$, and if $a > a^*$, $W(a) = \delta\{(1/a)+(c/\delta)\}$, where $k = c_p + c_d$. In either case, $W(a)$ is convex in a.

$$W'(a^*_-) = -\ \delta\{\int_{x^*}^{\infty} F_a(t,a^*)dt + (c_p + c_d)F_a(x^*,a^*)\}.$$

We now consider the right hand derivative, denoted by $W'(a^*_+)$. With $a > a^*$,

$$W(a) = \ \delta\{\int_{x^*}^{\infty}(1 - G(t|z^*(a),a))dt - (c_p + c_d)G(x^*|z^*(a),a) + x^* + c_d\}.$$

Note that $g(x|z,a) = f(x,a)/(1 - F(z,a))$, $G(x|z,a) = (F(x,a) - F(z,a))/(1 - F(z,a))$, and $z^*(a)$ is determined implicitly by $G(x^*|z,a) = (c_p + c_d)g(x^*|z,a)$ with $z^*(a^*) = 0$.
Using the fact that $F_a(0,a) = 0$, the direct calculation shows the following results. All the derivatives are evaluated at $a = a^*$ and $z^*(a^*) = 0$.

$\partial g(x|z,a)/\partial a = f_a(x,a^*)$, $\partial g(x|z,a)/\partial z = f(x,a^*)\cdot f(0,a^*)$,
$\partial G(x|z,a)/\partial a = F_a(x,a^*)$, $\partial G(x|z,a)/\partial z = -(1 - F(x,a^*))\cdot f(0,a^*)$, and
$\partial z^*(a)/\partial a = -\{f_a(x^*,a^*)F(x^*,a^*) - f(x^*,a^*)F_a(x^*,a^*)\}/f(0,a^*)\cdot f(x^*,a^*)$.

By differentiating $W(a)$ with respect to a and plugging above results, $W'(a^*_+)$ is given as follows;

$$W'(a^*_+) = -\ \delta\{\int_{x^*}^{\infty} F_a(t,a^*)dt + (c_p + c_d)F_a(x^*,a^*)\}$$

$$-\delta\ \frac{\{f_a(x^*,a^*)F(x^*,a^*) - f(x^*,a^*)F_a(x^*,a^*)\}}{f(x^*,a^*)}\{\int_{x^*}^{\infty}(1 - F(t,a^*)dt + (1 - F(x^*,a^*))\}.$$

By Assumption 5, $f_a(x^*,a^*)F(x^*,a^*) - f(x^*,a^*)F_a(x^*,a^*) < 0$, $W'(a^*_+) > W'(a^*_-)$. Kink indeed arises at $a = a^*$. By comparing $V'(a^*)$ with $W'(a^*_+)$ or $W'(a^*_-)$, we can determine whether type 1 or type 2 equilibrium prevails.

***Proposition* 3**
(1) If $V'(a^*) > -W'(a^*_-)$, $a^e < a^*$ and type 1 equilibrium prevails. (2) If $-W'(a^*_+) \leq V'(a^*) \leq -W'(a^*_-)$, $a^e = a^*$ and type 1 equilibrium prevails. (3) If $V'(a^*) < -W'(a^*_+)$, $a^e > a^*$ and type 2 equilibrium prevails.

***Proof*:** Since $C(a) = V(a) + W(a)$ is convex, if $V'(a^*) > -W'(a^*_-)$, for all $a \geq a^*$, $C'(a) > 0$. In other words, by decreasing a, D can reduce the total cost, therefore, $a^e < a^*$ and type 1 equilibrium prevails. If $-W'(a^*_+) \leq V'(a^*) \leq -W'(a^*_-)$, for all $a < a^*$, $C'(a) < 0$, thereby, by increasing a, D can decrease the total cost. For all $a>a^*$, $C'(a)>0$, thereby, by decreasing a, D can reduce the total cost. Hence, $C(a)=V(a)+W(a)$ is minimized at a^*, therefore, $a^e=a^*$ and type 1 equilibrium prevails. If $V'(a^*)<-W'(a^*_-)$, for all $a\leq a^*$, $C'(a)<0$, therefore, $a^e>a^*$ and type 2 equilibrium prevails.

Q.E.D.

VI. The Comparative Statics and Welfare Analysis

In this section, we will give some comparative statics and welfare analysis. First of all, we define the first best level of care denoted by a^f which minimizes $V(a)+\int_0^\infty xf(x,a)dx$. Note that $V(a)$ is an individual cost for the defendant, and $\int_0^\infty xf(x,a)dx$ is a social cost. The first best level of care is defined to be a level which minimizes the sum of these two costs. By Assumptions 1 and 6, a^f is determined by the first order condition:

$$V(a^f)-\int_0^\infty F_a(x,a^f)dx=0. \tag{4}$$

We consider the two types of equilibria separately.

Type 1 equilibrium: $V'(a^*)>-W'(a^*_-)$.

For comparative statics, we makes the following assumption.

***Assumption* 7** For all $a\in(0,\infty)$, $f(x,a)/F_a(x,a)$ is decreasing in x.

For the family of exponential distributions, $f(x,a)=ae^{-ax}$ and $F_a(x,a)=xe^{-ax}$. Hence, $f(x,a)/F_a(x,a)=a/x$, which is decreasing in x for all positive a. So Assumption 7 is satisfied for the family of exponential distribution.

Note that $da/dx|_{F(x,a)=\text{constant}}=f(x,a)/F_a(x,a)$. Therefore, Assumption 7 implies that with a fixed, as x increases, the slope of iso-probability curve in (x,a) plane decreases.

Proposition 4 gives some comparative statics results. In comparative statics, c_p and c_d only appear as a sum, c_p+c_d. Whenever this sum is fixed, the changes in c_p and c_d do not affect the equilibrium level of a and x. Hence, we give a comparative statics result with respect to this sum, c_p+c_d denoted by k. Since proof of Proposition 4 is tedious, it is given in the Appendix.

***Proposition* 4**

1. $\partial a^e/\partial\delta>0$, $\partial x^0(a^e)/\partial\delta<0$.
2. Under Assumption 7, $\partial a^e/\partial\delta>0$, and the sign of $\partial x^0(a^e)/\partial k$ is ambiguous.

Proof: See Appendix.

The intuition behind the Proposition 4 is as follows: The defendant has a trade-off between higher care and a lower settlement offer. In equilibrium, these two countervailing factors are balanced. An increase in δ induces an increase in the expected cost for the defendant in the bargaining stage, which implies that taking care becomes less costly. Hence as δ increases, the defendant takes a higher level of care. Note that $x^0(a^e)$ determined by Equation (1) does not depend on δ. Hence an increase in δ does not have a direct effect on $x^0(a^e)$. However, in an indirect way, an increase in δ increases a^e, which decreases $x^0(a^e)$.

For the effect of k, note that as c_p or c_d increases, litigation becomes more costly than settlement. Hence as k increases, the probability of settlement $F(x^0(a^e),a^e)$ should increase. There are two ways this can happen; either by increasing a or by x. By assumption 7, choosing higher x becomes relatively more costly than taking a higher level of care. Hence in equilibrium, the defendant takes a higher level of care than before. By examining Equation (1), we know that the direct effect of an increase in k on x is positive. However, an increase in the equilibrium level of care has a negative effect on x. Therefore, the total effect of k on $x^0(a^e)$ is ambiguous.

The next proposition gives the comparison between the first best level of care and the equilibrium level of care whose proof is also given in the Appendix.

***Proposition* 5**

Under Assumption 7, there exists $\delta^*\in(0,\infty)$ such that for all $\delta<\delta^*$, $a^e<a^f$, and for all $\delta>\delta^*$, $a^e>a^f$. $a^e=a^f$ when $\delta=\delta^*$.

Proof: See Appendix.

The reason why the defendant takes a higher level of care is that by doing so, he can reduce the cost in the bargaining stage. Therefore, the smaller the discount factor is, the less incentive the defendant has to take a higher level of care. What the Proposition 5 shows is that for smaller discount factors, the defendant takes the smaller level of care in equilibrium than the first best level, and for higher discount factors, he takes a level greater than the first best level.

Proposition 4 combined with Proposition 5 has an interesting policy implication. An increase in the litigation cost does not necessarily induce the defendant to take a level of care closer to the first best level. For low discount factors, it works. But for high discount factors, it worsens matters.

Type 2 equilibrium: $V'(a^*) < -W'(a^*_+)$.

In this case, the conditions which characterize the equilibrium level of a and z are much more complicated. Hence, in order to have some comparative statics results, we have to make somewhat very ad hoc assumptions on the distribution and density function. We don't think the ad hoc assumptions are meaningful, so that in the type 2 equilibrium, we only consider the parameterized example of exponential distribution, to get a sense of possibilities.

For the distribution function $F(x,a) = 1 - e^{-ax}$, and the density function $f(x,a) = ae^{-ax}$, by solving Equation (3), we have $x^*(a,z) =$ $.(ka+1)/a) + z$.

By solving the equation $x^*(a,z) = c/\delta + c_p$, we have $z^*(a) = c/\delta + c_p -$ $.(ka+1))/a$. By a lengthy calculation, we have $W(a) = \delta/a + c$. Let $V(a$ $.a$.[6] Then, a^e is obtained by solving the following equation:

$$\frac{dV(a)}{da} + \frac{dW(a)}{da} = 0, \quad i.e., \quad \frac{1}{a} - \frac{\delta}{a^2} = 0.$$

Hence, we have $a^e = \delta$ and $z^*(a^e) = z^*(\delta) = c/\delta + c_p$ $.(k\delta + 1))/\delta$.

For this case, the first best level of care, a^f is equal to 1. Hence, $a^e = \delta < 1 = a^f$, *i.e.*, the equilibrium level of care is always less than the first best level of care.

Note that $c/\delta + c_p - z^*(a^e$ $.(k\delta + 1))/\delta$ is the proportion of plaintiffs who file nuisance suits. It can be easily checked that $(\partial/\partial k)$

[6]In a does not satisfy the condition that $V_{aa} > 0$. However, since a^{-1} is sufficiently convex, the first order condition is sufficient for the minimization.

$.(k\delta+1))/\delta)=1/(ka+1)>0$, and $(\partial/\partial\delta \quad .(k\delta+1))/\delta)=\{ka-(ka+1)$ $.(ka+1))\}/\delta^2(ka+1)<0$. Note that this proportion is independent of the filing cost, c.

VII. Conclusion

In this paper, we have considered three issues. The first one is whether in the presence of nuisance suit, the court as an incentive device can give suitable incentives to take due care when pre-trial negotiation is possible. The second issue is what incentives the plaintiff with no merit of claim has to file a nuisance suit. Finally, once the a suit is filed, under the circumstances where the defendant cannot distinguish whether the suit is a nuisance one or not, the question is how to resolve the conflict.

For this purpose, we have examined a three-stage game between the defendant and the plaintiff. In this game, the defendant has a trade-off between a level of care and a settlement offer. By taking a high level of care, the defendant reduces the expected cost in the bargaining stage. However, a high level of care is costly. In equilibrium, these two countervailing factors are balanced. In equilibrium, the defendant chooses the level of care such that the marginal cost of increasing the level of care is exactly balanced by the marginal benefit of reducing the expected cost in the bargaining stage. Depending on the relative cost between taking care and the settlement offer in the bargaining stage, we identify two types of equilibria. We have also given some comparative static results and the welfare analysis.

We close by listing possible extension and a further research agenda. For the bargaining stage, although we allow possibility of counteroffer, in equilibrium the plaintiff does not make a counteroffer. However, in many bargaining situations, counteroffer are observed. So we have to explain counteroffer as an equilibrium phenomenon. For this purpose, we may need to consider the model where each party has private information. It will be interesting to repeat the analysis of this three stage model when the defendant also has private information (*e.g.*, about the level of care actually chosen).

In the literature on litigation and the settlement, it is assumed that when two parties go to court, they incur litigation costs. Most of the litigation cost comes from the lawyers' fees. However, the role the lawyer is not examined in our model at all. In a legal dispute, one important reason to hire a lawyer is to let him bargain with the opponents on behalf of the party who hires him. Although the lawyer works for the client who hires him, the interests of the lawyer and the client do not necessarily coincide. An agency problem naturally arises. How to resolve this agency problem in the context of bargaining is an interesting topic for further research on the litigation and settlement.

Appendix

***Proposition* 4**
In type 1 equilibrium,
(1) $\partial a^e/\partial\delta>0$, and $\partial x^0(a^e)/\partial\delta<0$.
(2) Under Assumption 7, $\partial a^e/\partial k>0$, and the sign of $\partial x^0(a^e)/\partial k$ is ambiguous.

***Proof*:** The type 1 equilibrium level of a^e and x are determined by solving the following minimization problem:

$$\min_{a,x} V(a)+\ \delta\{\int_x^\infty (1-F(t,a))dt-(c_p+c_d)F(x,a)+x+c_d\}.$$

By the first order conditions, we have

$$V_a(a^e)-\ \delta\{\int_{x^0(a^e)}^\infty F_a(t,a^e)dt+(c_p+c_d)F_a(x^0(a^e),a^e)\}=0,\quad \text{and} \tag{5}$$

$$\delta\{F(x^0(a^e),a^e)-(c_p+c_d)f(x^0(a^e),a^e)\}=0. \tag{6}$$

Note that the litigation costs c_p and c_d appear in Equations (5) and (6) as a sum only. Hence let $k=c_p+c_d$. Let H be the Hessian matrix of the objective function:

$$H=\begin{bmatrix} V_{aa}(a^e)-\ \delta\{\int_{x^0(a^e)}^\infty F_{aa}(t,a^e)dt+kF_{aa}(x^0(a^e),a^e)\} & F_a(x^0(a^e),a^e)-kf_a(x^0(a^e),a^e) \\ F_a(x^0(a^e),a^e)-kf_a(x^0(a^e),a^e) & f(x^0(a^e),a^e)-kf'(x^0(a^e),a^e) \end{bmatrix}.$$

By differentiating Equations (5) and (6) with respect to δ, we have, in matrix form:

$$H\begin{bmatrix}\dfrac{\partial a^e}{\partial \delta}\\ \dfrac{\partial x^0(a^e)}{\partial \delta}\end{bmatrix}=\begin{bmatrix}\{\int_{x^0(a^e)}^{\infty}F_a(t,a^e)dt+kF_a(x^0(a^e),a^e)\}\\ 0\end{bmatrix}.$$

Solving for $\partial a^e/\partial\delta$ and $\partial x^*(a^e)/\partial\delta$ gives

$$\frac{\partial a^e}{\partial\delta}=\frac{1}{|H|}\{f(x^0(a^e),a^e)-kf'(x^0(a^e),a^e)\}\{\int_{x^0(a^e)}^{\infty}F_a(t,a^e)dt+kF_a(x^0(a^e),a^e)\},\text{ and}$$

$$\frac{\partial x^0(a^e)}{\partial\delta}=\frac{1}{|H|}\{-\{F_a(t,a^e)dt-kf_a(x^0(a^e),a^e)\}\{\int_{x^0(a^e)}^{\infty}F_a(t,a^e)dt+kF_a(x^0(a^e),a^e)\}$$

where $|H|$ is the determinant of matrix H. By the second order condition, $|H|>0$, by Assumption 4, $f(x^0(a^e),a^e)-kf'(x^0(a^e),a^e)>0$ and by Assumption 5, $F_a(x^0(a^e),a^e)-kf_a(x^0(a^e),a^e)>0$ and $\{\int_{x^0(a^e)}^{\infty}F_a(t,a^e)dt+kF_a(x^0(a^e),a^e)\}>0$, $(\partial a^e/\partial\delta)>0$ and $(\partial x^0(a^e)/\partial\delta)<0$.

For the second part, differentiating Equations (5) and (6) with respect to k, yields

$$H\begin{bmatrix}\dfrac{\partial a^e}{ak}\\ \dfrac{\partial x^0(a^e)}{\partial k}\end{bmatrix}=\begin{bmatrix}\delta F_a(x^0(a^e),a^e)\\ f(x^0(a^e),a^e)\end{bmatrix}.$$

Solving for $\partial a^e/\partial k$ and $\partial x^0(a^e)/\partial k$ gives

$$\frac{\partial a^e}{\partial k}=\frac{1}{|H|}\{\delta k\{f(x^0(a^e),a^e\}f_a(x^0(a^e),a^e)-f'(x^0(a^e),a^e)F_a(x^0(a^e),a^e)\}\},\text{ and}$$

$$\frac{\partial x^0(a^e)}{\partial k}=\frac{1}{|H|}\{-\delta F_a(x^0(a^e),a^e)\{F_a(x^0(a^e),a^e)-kf(x^0(a^e),a^e)\}$$
$$+f(x^0(a^e),a^e)[V_{aa}(a^e)-\delta\{\int_{x^0(a^e)}^{\infty}F_{aa}(t,a^e)dt+kF_{aa}(x^0(a^e),a^e)\}]\}.$$

Assumption 7 implies that $f(x^0(a^e),a^e)\cdot f_a(x^0(a^e),a^e)>f'(x^0(a^e),a^e)\cdot F_a(x^0(a^e),a^e)$. Hence $\partial a^e/\partial k>0$. Generally, the sign of $\partial x^0(a^e)/\partial k$ is ambiguous.

Q.E.D.

***Proposition* 5**
Under Assumption 7, there exists $\delta^* \in (0,1)$ such that for all $\delta < \delta^*$, $a^e < a^f$, and for all $\delta > \delta^*$, $a^e > a^f$. $a^e = a$ when $\delta = \delta^*$.

***Proof*:** The first best level of care, a^f is characterized by the following equation:

$$V_a(a^f) - \int_0^\infty F_a(t,a^f)dt = 0. \tag{7}$$

The equilibrium level of care, a^e is determined by Equation (5) where $x^0(a^e)$ satisfies Equation (6): $F(x^0(a^e),a^e) - kf(x^0(a^e),a^e) = 0$. If we rewrite Equation (5), we have

$$V_a(a^e) - \delta\{\int_0^\infty F_a(t,a^e)dt - \int_0^{x^0(a^e)} F_a(t,a^e)dt + kF_a(x^0(a^e),a^e)\} = 0.$$

We will compare $\int_0^x F_a(t,a)dt$ and $kF_a(x,a)$ where (x,a) satisfies Equation (6).

By Assumption 7, for all $(x,a) \in [0,\infty) \times (\infty,0)$, we have

$$f(x,a)f_a(x,a) - f(x,a)F_a(x,a) > 0, \quad \text{or} \quad \frac{f_a(x,a)}{F_a(x,a)} > \frac{f'(x,a)}{f(x,a)}. \tag{8}$$

Note that $f_a/F_a = \partial \quad .F_a/\partial x$ and $f'/f = \partial \quad .f/\partial x$. By integrating both sides of (8) from z to x, we have:

$$\int_z^x \frac{f_a(t,a)}{F_a(t,a)}dt = \ln \frac{F_a(x,a)}{F_a(z,a)} > \int_z^x \frac{f'(t,a)}{f(t,a)}dt = \ln \frac{f(x,a)}{f(z,a)}.$$

Since $\ln(\cdot)$ is an increasing function, we have:

$$\frac{F_a(x,a)}{F_a(z,a)} > \frac{f(x,a)}{f(z,a)}, \quad \text{or} \quad f(z,a)F_a(x,a) > f(x,a)F_a(z,a). \tag{9}$$

Finally, by integrating both sides of (9) with respect to z from 0 to x, we obtain

$$\int_0^x f(z,a)dzF_a(x,a) > f(x,a)\int_0^x F_a(z,a)dz, \quad \text{or} \quad F(x,a)F_a(x,a) > f(x,a)\int_0^x F_a(z,a)dz.$$

Since (x,a) satisfies Equation (6), $kF_a(x,a) > \int_0^x F_a(z,a)dz$.

Let δ^* be defined as a solution to the following equation:

$$V_a(a^f) - \delta^*\{\int_0^\infty F_a(t,a^f)dt - \int_0^{x^0(a^f)} F_a(t,a^f)dt + kF_a(x^0(a^f),a^f)\} = 0.$$

Note that since $kF_a(x^0(a^f),a^f) > \int_0^{x^0(a^f)} F_a(z,a^f)dz$, $\delta^* < 1$. Of course $\delta^* > 0$. If $\delta < \delta^*$,

$$V_a(a^f) - \delta\{\int_0^{\infty} F_a(t,a^f)dt - \int_0^{x^0(a^f)} F_a(t,a^f)dt + kF_a(x^0(a^f),a^f)\}$$

$$> V_a(a^e) - \delta\{\int_0^{\infty} F_a(t,a^f)dt - \int_0^{x^0(a^e)} F_a(t,a^e)dt + kF_a(x^0(a^e),a^e)\}(=0).$$

Hence $a^e < a^f$ whenever $\delta > \delta^*$. Similarly, $a^e > a^f$ whenever $\delta < \delta^*$. When $\delta = \delta^*$, $a^e = a^f$.

Q.E.D.

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