A Bivariate ARFIMA-IGARCH-M Modelling of the Effects of Uncertainty on Inflation and Output Growth

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The main contribution of this paper is to use bivariate ARFIMA-IGARCH-M methods to test four hypotheses about the effects of real and nominal uncertainty on average inflation and output growth in the Korean economy from 1970 to 2002. According to sample types, empirical results show different effects of uncertainty on inflation and growth. Using the producer prices and wholesale prices index, we support all hypotheses considered except Friedman's and reject the Cukierman and Meltzer hypothesis only using consumer prices index.

Keywords: Uncertainty, Inflation, Output growth, Bivariate ARFIMA-IGARCH-M model

JEL Classification: C22, C32, E31

I. Introduction

Friedman (1977) explains a possible positive correlation between inflation and unemployment by arguing that high inflation leads to more uncertainty about future inflation. This uncertainty then

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reduces economic efficiency and temporarily increases unemployment; Increased inflation uncertainty changes optimal contract length and unemployment may be then higher during the transition to this new set of arrangements.[1] Many previous studies have centered on several potential interactions between average inflation or output growth, and uncertainty about inflation or output growth. We consider three other models incorporating these possible interactions. First, Cukierman and Meltzer (1986) model both the policy maker's objective function and the money supply process as random variables. Hence the public has an inference problem when observing higher inflation. They show that inflation uncertainty raises the average inflation rate by increasing the incentive for the

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Second, Deveraux (1989) argues that an exogenous increase in the variability of real shocks lowers the optimal amount of wage indexation, which makes surprise inflation more effective, and thus increasing the incentive to create surprises. Third, Black (1987) argues that the choice of investing in a risky specialized technology will produce an economy with higher average growth and describes a tradeoff between the severity of the business cycle and the average growth rate of output. These papers are all interrelated in that they each propose some link between uncertainty about inflation or growth and inflation or output growth. The four hypotheses are not mutually exclusive. Higher inflation uncertainty could produce both lower average growth (Friedman) and more inflation (Cukierman and Meltzer). Higher real uncertainty could be associated with both higher average growth (Black) and more inflation (Devereux).

In this paper we use integrated GARCH (IGARCH) techniques specifically to estimate a model of the variance of unpredictable innovations. We then use a bivariate ARFIMA-IGARCH-M modelling to simultaneously test the four possible effects of real and nominal uncertainty on inflation and output growth in a single model. The paper proceeds as follows. Sections II and III describe the data analysis and a bivariate ARFIMA-IGARCH-M model, respectively. In

[1]He also points out to the adverse effect that more noise in the price system reduces economic efficiency and raises unemployment during the transitional period as firms adapt to the new environment. In support of this argument, see papers by Foster (1978), Fisher (1981), and Grier and Perry (1998).

Section IV we present the empirical results and concentrate further on our empirical results that according to sample real and nominal uncertainties are associated with average inflation and output growth by studying the size and duration of the effect. Section V provides concluding remarks.

II. Data Analysis

Our data set consists of 388 monthly observations of the producer prices (hereafter PPI), consumer prices (hereafter CPI), wholesale prices (hereafter WPI) and industrial product, covering the period from January 1970 through April 2002. The data sets are from International Financial Statistics.

Table 1 gives the summary statistics for the various inflation types and real output growth. Inflation is the percentage monthly difference of the log of the *PPI*, *CPI*, and *WPI* [$\Pi_t = \log(PPI_t/PPI_{t-1}) \times 100$]. Real output growth ($Y_t$) is the percentage monthly difference in the log of industrial production [$Y_t = \log(IP_t/IP_{t-1}) \times 100$]. We can see from Table 1 that kurtosis is higher than that of a normal distribution, which is 3. The kurtosis and studentized range statistics (which is the range divided by standard deviation) show the characteristic fat-tailed behavior compared with a normal distribution. The Jarque-Bera normality test statistic is far beyond the critical value, which suggests that inflation and output growth are far from a normal distribution.

Figures 1 and 2 give the plots of individual series of *PPI* and industrial production, inflation and output growth, and squared or absolute inflation and output growth.[2] From the figures the individual series movements shows an upward trend but inflation and output growth are rather stable around mean. From the absolute of individual series, we can clearly see the observation of Mandelbrot (1963) and Fama (1965) that large absolute returns are more likely than small absolute returns to be followed by a large absolute return. The market volatility is changing over time, which suggests a suitable model for the data should have a time varying volatility structure as suggested by the ARCH model.

[2]The details for the other two inflation types are available from the author on request but are not reported here for reasons of spaces.

TABLE 1

SUMMARY STATISTICS FOR Π_t AND Y_t

Data	Type	Mean	Skewness	Kurtosis	Q(10)	$Q^2(10)$	JB
Π_t	*PPI*	0.62	5.07	41.39	274.60	28.13	25416.28
	CPI	0.70	1.46	6.07	348.32	317.47	288.96
	WPI	0.62	5.27	44.77	255.85	20.29	29925.32
Π_t^2	*PPI*	2.25	11.86	154.51	28.13	0.27	379219.66
	CPI	1.33	4.16	25.35	317.47	140.23	9173.04
	WPI	2.30	12.13	159.26	20.29	0.16	403196.35
$\lvert\Pi_t\rvert$	*PPI*	0.77	5.78	49.24	232.52	28.13	36634.60
	CPI	0.80	1.97	7.36	454.45	317.47	557.51
	WPI	0.77	6.05	53.60	210.83	20.29	
Y_t		0.94	0.25	10.10	36.64	32.23	3.77
Y_t^2		9.57	8.04	83.81	32.22	10.22	109473.19
$\lvert Y_t\rvert$		2.19	2.99	17.43	21.17	32.23	3931.95

Note: The full Sample is 388 monthly observations from 1970:01 to 2002:04. Π_t denotes the inflation rate and is calculated from the producer price, consumer price, and wholesale price index. Y_t is the growth rate of industrial production. Q(10) and $Q^2(10)$ are the Ljung-Box statistics for tenth-order serial correlation in the residuals. The critical values at the 0.05 significance level is 18.31 for 10 degrees of freedom. All data are from International Financial Statistics.

In the discrete time long-memory fractionally integrated $I(d)$ class of processes, the propagation of shocks to the mean occur at a slow hyperbolic rate of decay when $0<d<1$, as opposed to the extremes of $I(0)$ exponential decay associated with the stationary and invertible ARMA class of processes, or the infinite persistence resulting from an $I(1)$ process. Just as the generalization of the standard ARIMA class of models to the fractionally integrated ARFIMA models have proven empirically important, a corresponding result may hold true when modeling long-term dependence in conditional variances.

In order to motivate the empirical relevance of these ideas both in mean and variance, Figures 3 and 4 plot the lag 1 through 36 sample autocorrelations of inflation and output growth, and their squared values. The autocorrelations do clearly exhibit a pattern of slow decay and persistence in the inflation, but the output growth

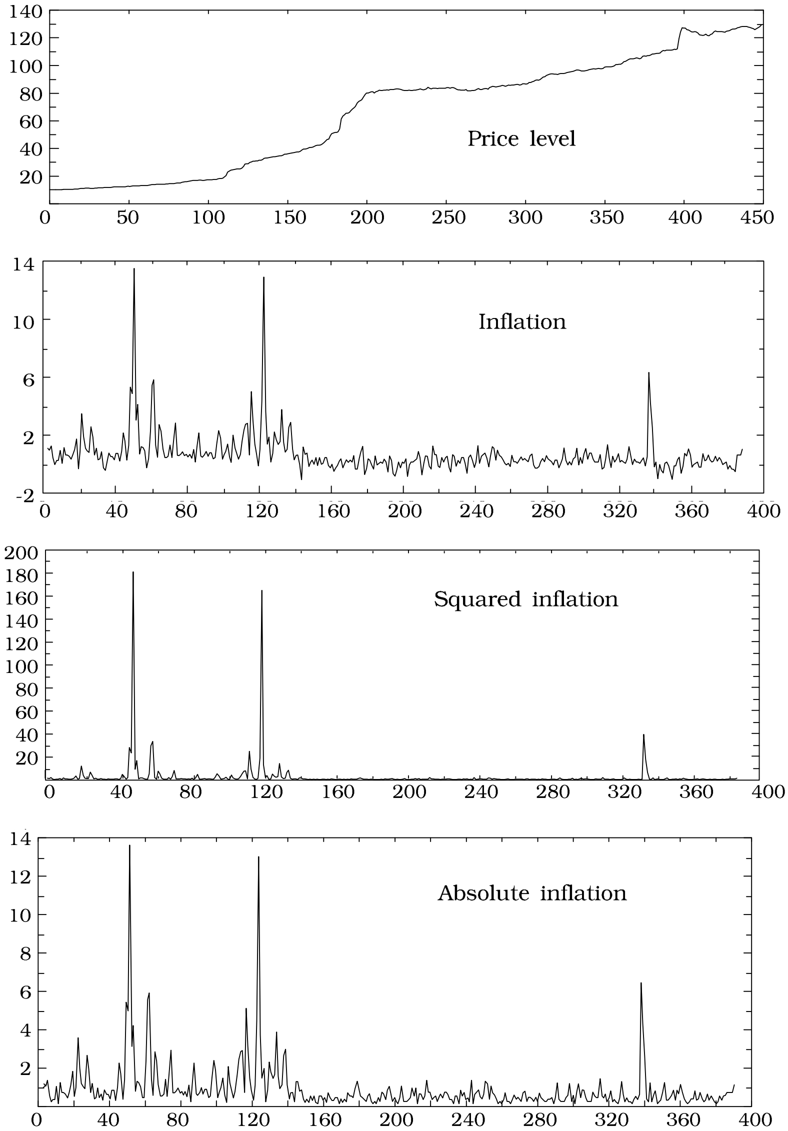

FIGURE 1

PRODUCTION PRICE INDEX, INFLATION, SQUARED INFLATION, AND ABSOLUTE INFLATION: A SAMPLE PERIOD FROM 1970:01-2002:04

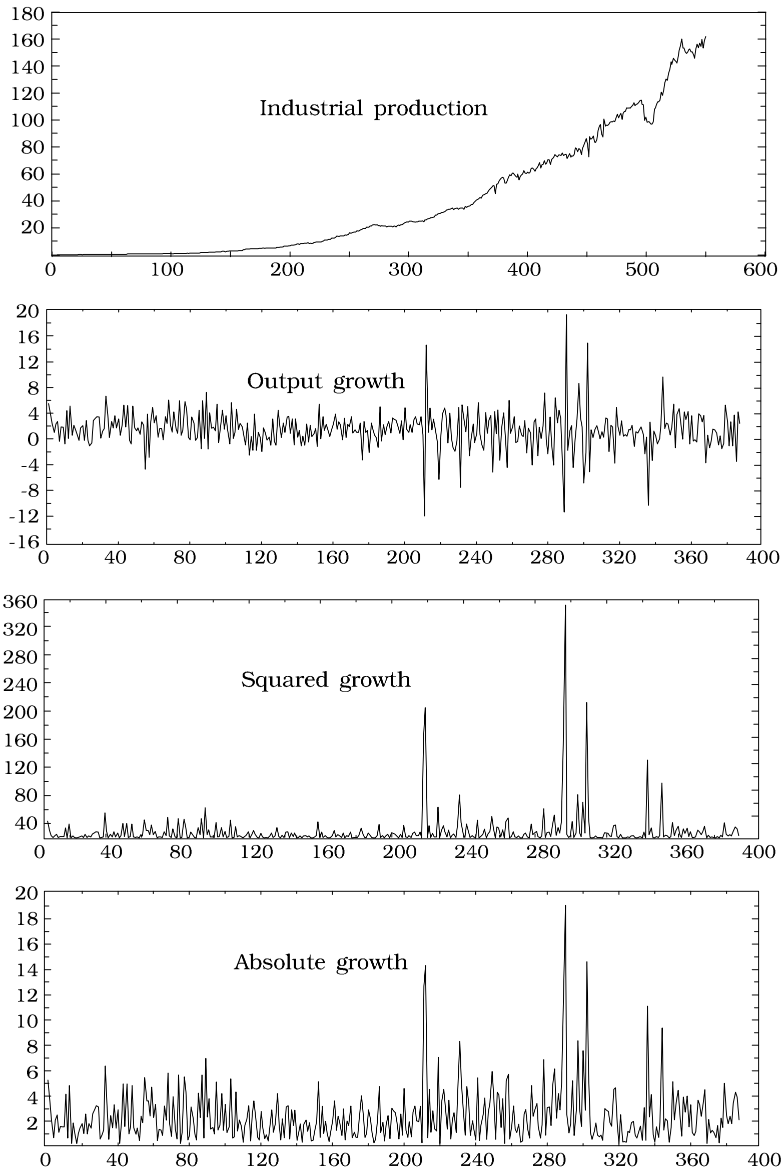

FIGURE 2

INDUSTRIAL PRODUCTION INDEX, GROWTH, SQUARED GROWTH, AND ABSOLUTE GROWTH: A SAMPLE PERIOD FROM 1970:01-2002:04

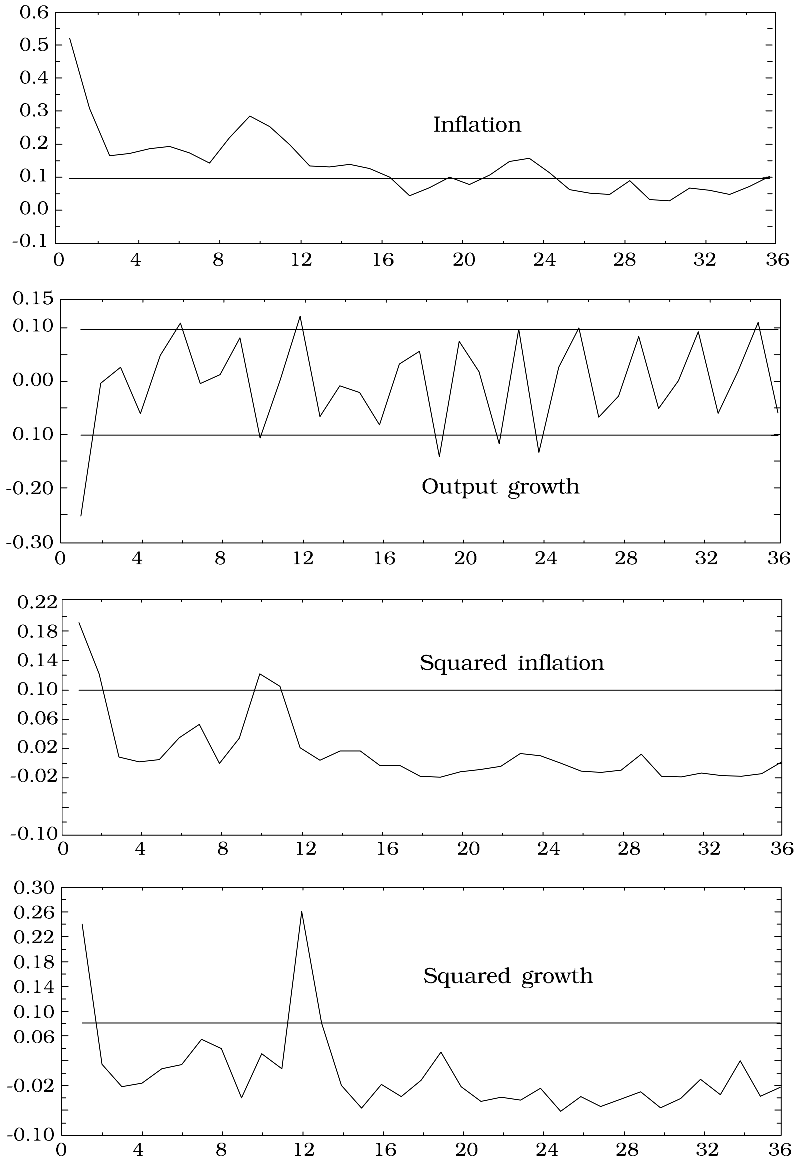

FIGURE 3

AUTUCORRELATIONS OF INFLATION AND GROWTH,
AND THEIR ABSOLUTE VALUES

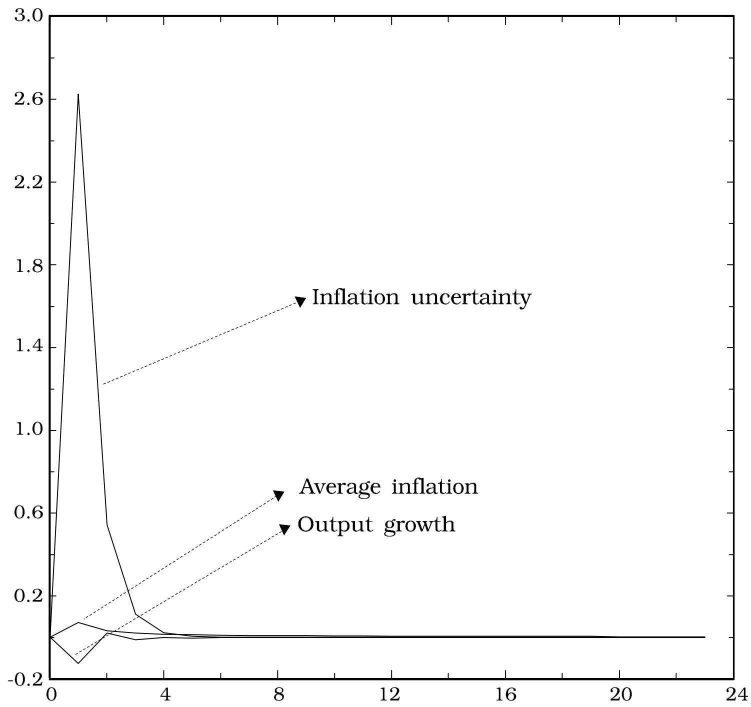

FIGURE 4

THE EFFECT OF A POSITIVE NOMINAL SHOCK ON INFLATION UNCERTAINTY, AVERAGE INFLATION AND OUTPUT GROWTH USING PPI-BASED INFLATION

decays exponentially. The squared correlations for three years appears in the two 95% Bartlett (1946) confidence bands for no serial dependence. Also, Ljung and Box (1978) portmanteau test for the joint significance of lags 1 through 10 equals 274.60 for PPI inflation (36.64 for output growth), which is highly significant when tested in a chi-square distribution with 10 degrees of freedom. Ljung and Box (1978) test of serial correlation assumes homoskedasticity of the error variance and is invalid in the presence of ARCH or other forms of conditional heteroskedasticity. Wooldridge (1991) suggested the regression-based Lagrange multiplier (RB-LM) test, which is robust to conditional heteroskedasticity. RB-LM test statistics of lags 1 through 10 equal 58.25 for PPI inflation and 23.20 for output growth, which are significant when tested in a chi-square distribution with 10 degrees of freedom.[3]

III. Statistical Model of Inflation and Output Growth

Testing any of the above four hypotheses in section I depends on the construction of a specific measure of uncertainty. We use IGARCH techniques to specifically estimate a model of the variance of unpredictable innovations in a variable, rather than simply calculating a variability measure from past outcomes (moving standard deviation) or from conflicting individual forecasts. In this paper IGARCH techniques are very useful for many reasons. Among others IGARCH estimation gives an explicit test of whether the conditional variance of a variable is time-varying. Second, IGARCH allows simultaneous estimation of the conditional variance equations and the mean equations for the variables under consideration.[4] Third, OLS estimation of the inflation and output growth equation is inefficient when inflation and output growth exhibit significant conditional heteroscedasticity.[5] For testing the four possible effects of real and nominal uncertainty calculated from the IGARCH model on inflation and output growth we also consider them in the mean equations and for incorporating long memory properties of inflation process ARFIMA(0,d,1) model is specified in the mean.[6]

Consequently, we estimate bivariate ARFIMA-IGARCH-M models

[3]We would like to thank an anonymous referee for suggesting the serial correlation tests of Diebold (1986) or Wooldridge (1991), which are robust to conditional heteroskedasticity. According to Wooldridge (1991)'s procedure 3.1 on page 16, RB-LM statistics are computed. RB-LM test statistics of lags 1 through 10 equal 23.32 for CPI, 23.46 for WPI inflation.

[4]Pagan (1984) shows that, when working with generated regressors, simultaneous estimation is more efficient than a two-step process.

[5]Engle (1982) shows that the gain in efficiency from using ARCH instead of OLS when there is significant conditional heteroscedasticity can be very large.

[6]An anonymous referee suggests that our ARFIMA-IGARCH-M model would be more convincing if we could provide a comparison of competing models based on the out-of-sample forecast performances. From the model specification perspective, we compare with competing models and our ATFIMA-IGARCH-M model is successful.

for inflation and output growth. The model simultaneously estimates equations for the means of inflation and output growth that include the conditional variance of both series as regressors, along with the time-varying residual covariance matrix. Even though several parameterizations of the general multivariate model are possible, we consider the constant conditional correlation model of Bollerslev (1990) in which the conditional covariance matrix is time-varying but the conditional correlation across equations is assumed to be constant.[7]

Let Π_t represent the calculated inflations from *PPI*, *CPI*, and *WPI* in period t and Y_t represent output growth in period t. We first specify a bivariate model for inflation and real output growth in the mean and test the null hypotheses that each variable has a constant conditional variance.

$$(1-L)^d \Pi_t = \mu_\Pi + \theta_\Pi \varepsilon_{t-1} + \varepsilon_t \tag{1}$$

$$Y_t = \mu_Y + \phi_Y Y_{t-1} + \phi_\Pi \Pi_{t-1} + \upsilon_t \tag{2}$$

Table 2 reports an inflation equation, which includes a first-order moving average term as regressors, along with a long memory term.[8] The residuals are uncorrelated (Q-statistic = 17.18, 7.19, and 17.40 for *PPI*, *CPI*, and *WPI* respectively at 10 lags), but the squared residuals do not show the classic volatility clustering of an ARCH process in *PPI* and *WPI*. However, Jarque-Bera normality test statistic is far beyond the critical value, which suggests that inflation is far from a normal distribution. The output growth equation in Table 2 contains one lag of output growth and one lag of inflation.[9] The growth equation displays correlated residuals and squared residuals. The null hypothesis of a constant residual

[7]The assumption of a constant correlation matrix represents a major reduction in terms of computational complexity and is commonly used in multivariate GARCH models.

[8]Our ARFIMA model is found to be remarkably successful at presenting inflation compared with the ARMA model with long autoregressive lags.

[9]Up to six lags of inflation were considered in the output equation, but only the first lag was statistically significant. Given that the level of inflation is often thought to be related to inflation uncertainty, it is important to include the level of inflation in the output equation to distinguish between the effects of inflation on output and the effects of inflation uncertainty on output.

TABLE 2

BIVARIATE MEAN REGRESSION FOR INFLATION AND OUTPUT GROWTH

	PPI	*CPI*	*WPI*
Inflation equation			
μ_{Π}	0.6873 (0.4130)	0.7798 (0.2052)	0.6844 (0.3982)
d_{Π}	0.2178 (0.0375)	0.2772 (0.0442)	0.2721 (0.0426)
θ_{Π}	0.1934 (0.0456)	0.2470 (0.0560)	0.1504 (0.0551)
m_3	4.75	0.57	5.52
m_4	42.62	4.89	52.73
Q(10)	17.18	7.19	17.40
$Q^2(10)$	17.05	122.35	6.85
Growth equation			
μ_Y	1.2721 (0.1649)	1.2765 (0.2037)	1.2743 (0.1655)
ϕ_Y	-0.2552 (0.0313)	-0.2567 (0.0320)	-0.2548 (0.0313)
ϕ_{Π}	-0.1369 (0.1013)	-0.1405 (0.1853)	-0.1506 (0.1028)
m_3	-0.28	-0.29	-0.28
m_4	8.62	8.78	8.63
Q(10)	19.99	19.68	19.91
$Q^2(10)$	56.67	55.13	56.76
Cross equation			
Q(10)	23.54	9.93	26.50
$Q^2(10)$	73.49	23.90	78.67
Log Likelihood	1555.26	1386.11	1567.85

Note: The standard errors are in parentheses. Q(10) and $Q^2(10)$ are the Ljung-Box statistics for tenth-order serial correlation in the residuals. The critical value at the 0.05 significance level is 18.31 for 10 degrees of freedom. The statistics m_3 and m_4 are the sample skewness and kurtosis of the standardized residuals. Under the null hypothesis of conditional normality, the skewness statistic m_3 is distributed normally with mean zero and variance $6/n$, where n is the sample size, while the kurtosis statistic m_4 is distributed normally with mean 3 and variance $24/n$.

variance is rejected at the 0.01 level for 10 lags.

From the results in Table 2 we consider a model to be integrated in the error variances of both Π_t and Y_t; see important two papers of Engle and Bollerslev (1986) and Bollerslev and Engle (1993). This class of model includes the variance analogue of a unit root in the mean as a special case. Our bivariate ARFIMA-IGARCH-M model for inflation and output growth is:

$$(1-L)^d \Pi_t = \mu_\Pi + \lambda_\Pi \sigma_{\varepsilon_t t} + \delta_\Pi \sigma^2{}_t + \theta_\Pi \varepsilon_{t-1} + \varepsilon_t \tag{3}$$

$$Y_t = \mu_Y + \phi_Y Y_{t-1} + \phi_\Pi \Pi_{t-1} + \lambda_Y \sigma_{\varepsilon_t t} + \delta_Y \sigma^2{}_t + \upsilon_t \tag{4}$$

$$\begin{matrix} \varepsilon_t \\ \upsilon_t \end{matrix} \mid \Omega_{t-1} \sim N\left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} \sigma_{\varepsilon_t t} & \sigma_{\varepsilon \upsilon t} \\ \sigma_{\upsilon \varepsilon t} & \sigma^2_{\upsilon t} \end{bmatrix} \right) \tag{5}$$

$$\sigma_{\varepsilon_t t} = \omega_\Pi + \alpha_{1\Pi} \sigma_{\varepsilon_{t-1} 1} + \beta_{1\Pi} \sigma_{\varepsilon_{t-1} 1} \tag{6}$$

$$\sigma^2{}_t = \omega_Y + \alpha_{1Y} \upsilon_{\upsilon_{t-1} 1} + \beta_{1\Pi} \sigma_{\upsilon_{t-1} 1} \tag{7}$$

$$\sigma_{\upsilon_t t} = \rho_{\varepsilon\upsilon} \ {}_{\varepsilon_t t} \sigma_t , \tag{8}$$

where $\alpha_1 + \beta_1 = 1$ in equations (6) and (7), and we assume that the two error terms, ε_t and υ_t, are jointly conditionally normal with zero means and a conditional variance given by equations (5).

Equation (3) describes the mean inflation rate as a long memory parameter and a first lag of moving average term with the conditional variances of inflation and output growth. Equation (6) gives IGARCH specification of the conditional variance of inflation, which is our time series measure of inflation uncertainty. Equation (4) describes the conditional mean of real output growth as a function of one lag of output growth, one lag of inflation with the conditional variances of inflation and output growth. Equation (7) is the IGARCH equation for the conditional variance of output growth. We estimate the system of equations (3)-(8) using the Berndt *et al.* (1974) numerical optimization algorithm to calculate the maximum likelihood estimates of the parameters.[10]

[10]We would like to thank an anonymous referee for suggesting the possible effects of omitted variables in equations (3) and (4). Bernanke and Blinder (1992) among others show that interest rate spreads are significant

Our statistical model of a bivariate ARFIMA-IGARCH-M incorporates tests of all four theories discussed above in section 1. The coefficient λ_Y on the conditional variance of inflation $\sigma^2{}_t$ in the output equation directly tests Friedman's hypothesis of whether inflation uncertainty has an effect on real output growth. If inflation uncertainty adversely affects real output growth, λ_Y will be negative and significant in equation (4). The Cukierman and Meltzer hypothesis that inflation uncertainty raises average inflation is tested by looking at λ_Π, the coefficient on the conditional variance of inflation in the inflation equation. If λ_Π is a positive and significant, Cukierman and Meltzer hypothesis holds. The coefficient δ_Π for the conditional variance of output in the inflation equation tests Deveraux's hypothesis that increased real uncertainty raises average inflation. Deveraux's hypothesis holds for a positive and significant δ_Π. Finally, the coefficient δ_Y on the conditional variance of output growth in the output growth equation tests Black's hypothesis of whether more risky technology promotes a higher average growth rate. The coefficient δ_Y should be positive and significant to confirm Black's hypothesis.[11]

IV. Empirical Analysis

A. Estimated Results

Table 3 reports estimates of a bivariate ARFIMA-IGARCH-M model shown above. The estimated fractional differencing parameter *d* being equal to 0.120 implying covariance stationarity of the

predictors of real activity. One type of interest rate spread, the difference between private and public securities of the same maturity, is often considered a measure of default risk. However, we could not consider interest rate spreads in equation (4) simply because of trading periods and the condition of same maturity.

[11]The model described above does not include lagged output growth in the inflation equation. We estimated models with such lags, finding that they were not significant and that our results were not sensitive to their exclusion. The model also does not allow the covariance of the errors to affect the conditional means of inflation or output growth. While there are theoretical reasons (described above) for including both conditional variances in each conditional mean equation, there is no such reason for including the covariance, nor would we have any interpretation of the results. See Grier and Perry (2000).

TABLE 3

BIVARIATE ARFIMA-IGARCH-M REGRESSION FOR INFLATION CALCULATED FROM PRODUCER PRICES INDEX AND OUTPUT GROWTH

Inflation equation:

$$(1-L)^{0.120}\Pi_t = \underset{(0.035)}{-0.068} + \underset{(0.256)}{0.409}\,\varepsilon_{t-1} + \underset{(0.042)}{0.144}\,\sigma_{\varepsilon t} + \underset{(0.049)}{0.091}\,\sigma_{vt} + \varepsilon_t \quad (0.033)$$

$$\sigma_{\varepsilon t} = \underset{(0.022)}{0.148} + \underset{(0.041)}{0.793}\,\varepsilon^2_{t-1}$$

$Q(10)=11.714$ $\quad Q^2(10)=2.160$ $\quad$ Skewness $=1.984$ $\quad$ Kurtosis $=12.495$

Growth equation:

$$Y_t = \underset{(0.545)}{0.282} - \underset{(0.072)}{0.274}Y_{t-1} - \underset{(0.162)}{0.188}\Pi_{t-1} - \underset{(0.203)}{0.013}\,\sigma_{\varepsilon t} + \underset{(0.263)}{0.481}\,\sigma_{vt} + v_t$$

$$\sigma_{vt} = \underset{(0.121)}{0.404} + \underset{(0.046)}{0.214}v^2_{t-1}$$

$Q(10)=25.340$ $\quad Q^2(10)=5.871$ $\quad$ Skewness $=-1.157$ $\quad$ Kurtosis $=8.574$

Cross equation:

$$\sigma_{\varepsilon vt} = \underset{(0.065)}{-0.005}\;\; \varepsilon_t\; v_t$$

$Q(10)=10.305$ $\quad Q^2(10)=3.002$

Log likelihood function $=1374.726$

Note: Sample is 388 monthly observations from 1970:01 to 2002:04. The standard errors are in parentheses. $Q(10)$ and $Q^2(10)$ are the Ljung-Box statistics for tenth-order serial correlation in the residuals. The critical value at the 0.05 significance level is 18.31 for 10 degrees of freedom.

inflation process and some mild long-memory features. Interestingly enough, the estimated standard error is small and two-sided confidence intervals for d is correspondingly tight. The IGARCH parameters in the residual variance equation are significant at the 0.01 level. The IGARCH-in-mean variables are both completely significant in inflation equation, showing that in this sample and specification we find effect of either inflation uncertainty or output growth uncertainty on average inflation. Our bivariate ARFIMA-IGARCH-M model strongly provides statistical support for the Cukierman and Meltzer or Deveraux hypotheses.

The estimates of the mean and conditional residual variance of output growth is also reported in Table 3. The lagged output growth coefficients in output growth equation is 0.274, which is similar to the coefficient in the bivariate mean equation (0.255). The lagged level of inflation is negative and significant in both the IGARCH and the bivariate mean models. The IGARCH parameters in the residual variance equation are both significant. However, the coefficient on the lagged residual variance is greater for output growth (0.786) than for inflation (0.207), suggesting that output growth shocks have longer lived effects on output uncertainty than inflation shocks have on inflation uncertainty. The magnitude of α_1, which shows the effect of the last period's shock directly, is found to be considerable. The implication is that volatility is more sensitive to its own lagged values than it is to new surprises in the market place.

The Friedman hypothesis that inflation uncertainty lowers output growth implies a negative and insignificant coefficient on the residual variance of inflation in the output growth equation. The estimated coefficient on $\sigma_{\varepsilon_{\pi t}}$ in the output equation is negative (-0.013) and insignificant. Our results thus do not provide strong empirical confirmation of Friedman's hypothesis. The other three hypotheses including Cukierman and Meltzer's, Deveraux's, and Black's are statistically significant: *i.e.*, there is a clear evidence in Korean economy during the sample period that inflation uncertainty raises average inflation, and increased real uncertainty raises average inflation, and risky output growth is positively correlated with average output growth.

We also calculate Ljung-Box Q-statistics at 10 lags for the levels, squares and cross-equation products of the standardized residuals for the estimated bivariate ARFIMA-IGARCH-M system.[12] The results reported in Table 3 show that the time series models for the conditional means and the IGARCH model for the residual conditional variance-covariance adequately captures the joint distribution of the disturbances. The conditional correlation coefficient is close to zero, suggesting that the residual covariance between

[12]For all our IGARCH estimations, we report Q(10) statistics, which test for serial correlation at ten lags in the standardized inflation residuals, the standardized output growth residuals and the cross-products of those two series. We also report $Q^2(10)$ statistics, which test for tenth-order serial correlation in the squared inflation and output growth residuals. The Q^2-tests are designed to see whether our GARCH model accounts for all the conditional heteroscedasticity in the two series.

TABLE 4

BIVARIATE ARFIMA-IGARCH-M REGRESSION FOR INFLATION CALCULATED FROM CONSUMER PRICES INDEX AND OUTPUT GROWTH: 1970:01-1988:09

Inflation equation:

$$(1-L)^{0.223}\Pi_t = \underset{(0.072)}{1.393} + \underset{(0.346)}{0.254}\,\varepsilon_{t-1} - \underset{(0.093)}{0.024}\,\sigma_{\varepsilon_t^t} - \underset{(0.071)}{0.096}\,\sigma_{v_t^t} + \varepsilon_t \quad (0.044)$$

$$\sigma_{\varepsilon_t^t} = \underset{(0.001)}{0.002} + \underset{(0.013)}{0.050}\,\varepsilon^2_{t-1}$$

$Q(10)=10.147$ $Q^2(10)=4.560$ Skewness$=0.795$ Kurtosis$=5.558$

Growth equation:

$$Y_t = \underset{(0.615)}{0.339} - \underset{(0.070)}{0.269}Y_{t-1} - \underset{(0.164)}{0.410}\,\Pi_{t-1} - \underset{(0.240)}{0.818}\,\sigma_{\varepsilon_t^t} + \underset{(0.329)}{0.819}\,\sigma_{v_t^t} + v_t$$

$$\sigma_{v_t^t} = \underset{(0.138)}{0.314} + \underset{(0.048)}{0.169}v^2_{t-1}$$

$Q(10)=15.523$ $Q^2(10)=4.921$ Skewness$=-0.960$ Kurtosis$=7.559$

Cross equation:

$$\sigma_{\varepsilon v t} = \underset{(0.056)}{0.111} \quad \varepsilon_t^t \ v_t^t$$

$Q(10)=4.612$ $Q^2(10)=3.233$

Log likelihood function$=1324.803$

Note: As for Table 3.

equations is not statistically significant.

To investigate the robustness of our results, we estimate the bivariate ARFIMA-IGARCH-M model discussed above on other inflation type CPI calculated from consumer prices index from 1970:01 through 2002:04. The results are shown in Table 4 for the constant residual correlation model. From the Table 4, we indicate two different facts as in Table 3. The IGARCH parameters in the residual variance equation are significant in both inflation and growth equations. In contrary to Table 3, the coefficient on the lagged residual variance is greater for inflation (0.950) than for output growth (0.831), suggesting that nominal shocks have longer lived effects on inflation uncertainty than real shocks have on output growth uncertainty. Second, our bivariate ARFIMA-IGARCH-M model provides a statistical support for all three hypotheses except Cukierman and Meltzer's hypothesis discussed above.

TABLE 5

BIVARIATE ARFIMA-IGARCH-M REGRESSION FOR INFLATION CALCULATED FROM WHOLESALE PRICES INDEX AND OUTPUT GROWTH: 1988:10-2001:07

Inflation equation:

$$(1-L)^{0.148}\Pi_t = -0.170 + 0.166\,\varepsilon_{t-1} + 0.189\,\sigma_{\varepsilon t} + 0.108\,\sigma_{vt} + \varepsilon_t$$
$$(0.018)\quad(0.143)\quad(0.021)\quad(0.024)\quad(0.029)$$

$$\sigma_{\varepsilon t} = 0.236 + 0.999\,\varepsilon^2_{t-1}$$
$$(0.024)\quad(0.012)$$

$Q(10) = 17.587$ $Q^2(10) = 8.496$ Skewness $= 2.250$ Kurtosis $= 14.670$

Growth equation:

$$Y_t = 0.274 - 0.278Y_{t-1} - 0.187\,\Pi_{t-1} - 0.056\,\sigma_{\varepsilon t} + 0.466\,\sigma_{vt} + v_t$$
$$(0.515)\quad(0.068)\quad(0.130)\quad(0.184)\quad(0.250)$$

$$\sigma_{vt} = 0.385 + 0.219v^2_{t-1}$$
$$(0.012)\quad(0.111)$$

$Q(10) = 25.477$ $Q^2(10) = 5.821$ Skewness $= -1.145$ Kurtosis $= 8.515$

Cross equation:

$$\sigma_{\varepsilon v t} = 0.001 \quad \varepsilon_t \quad v_t$$
$$(0.067)$$

$Q(10) = 16.139$ $Q^2(10) = 1.503$

Log likelihood function $= 1324.803$

Note: As for Table 3.

And we also estimate the bivariate ARFIMA-IGARCH-M model discussed above on other inflation type WPI calculated from wholesale prices index. The results are shown in Table 5 for the constant residual correlation model and similar to the results from PPI-based inflation as in Table 3.

B. Effects of Uncertainty on Inflation and Growth

According to inflation types, empirical results show different effects of uncertainty on inflation and growth. From the sample period of 1970:01-2002:04 our work shows that increases in inflation uncertainty are significantly associated with lower rates of real output growth in the Korean economy. From the *PPI*-based inflation our bivariate ARFIMA-IGARCH-M model provides a statistical support for Cukierman and Meltzer's, Deveraux's, and Black's hypothesis, showing that we find an effect of inflation uncertainty on average inflation and either inflation uncertainty or output growth uncertainty on average inflation and output growth. From the *CPI*-based inflation we statistically supports the Deveraux's, Friedman's and Black's hypothesis, in which inflation uncertainty raises output growth and either nominal or real shocks increase average inflation and output growth. The results from the *WPI*- based inflation is similar to the ones of the *PPI*-based inflation.

To analyze the quantitative importance, consider the effect of an inflation (output) surprise on inflation (output growth) uncertainty, average inflation and output growth in a bivariate ARFIMA-IGARCH-M model presented in section III (equations (3)-(8)). For example an inflation surprise in period t raises the conditional variance of inflation in period $t+1$. The initial increase in inflation uncertainty is therefore given by: $\nabla \sigma_{\varepsilon_{t+1}} = \alpha_1(\varepsilon_t^2)$. The effect of the inflation surprise in period t on inflation uncertainty will persist over time due to the autoregressive term in the conditional variance equation (*i.e.* equation (6)) with coefficient β_1. Similarly the effect of the real shock in period t on growth uncertainty is calculated using equation given by (7).

As for Friedman hypothesis, initially the higher conditional variance lowers output growth (in equation (4)) by λ_Y times the higher variance. For positive shocks, this initial effect is partly offset by the positive coefficient on lagged inflation, λ_Y, in the growth equation (for negative shocks, the initial negative effect is reinforced). However, output growth is also an autoregressive process and the coefficients on lagged output growth create additional real persistence of the effect of inflation uncertainty.[13]

Figure 4 illustrates the effect of a positive nominal shock from the *PPI*-based inflation sample. We assume a one-time inflation surprise of 15.8 percentage points, which is the average absolute value of the inflation residuals, and use the coefficients $\alpha_{1\Pi}$, $\beta_{1\Pi}$, ϕ_Y, ϕ_Π, and λ_Y from the model estimated in Table 3. The nominal shock occurs in period zero. Inflation uncertainty sharply rises and then declines, while we find no serious effect of nominal shock on the average inflation and output growth.

[13]The interpretation of other hypotheses is similar to that of Friedman's. The details for the other three hypotheses are available from the author on request but are not reported here for reasons of space.

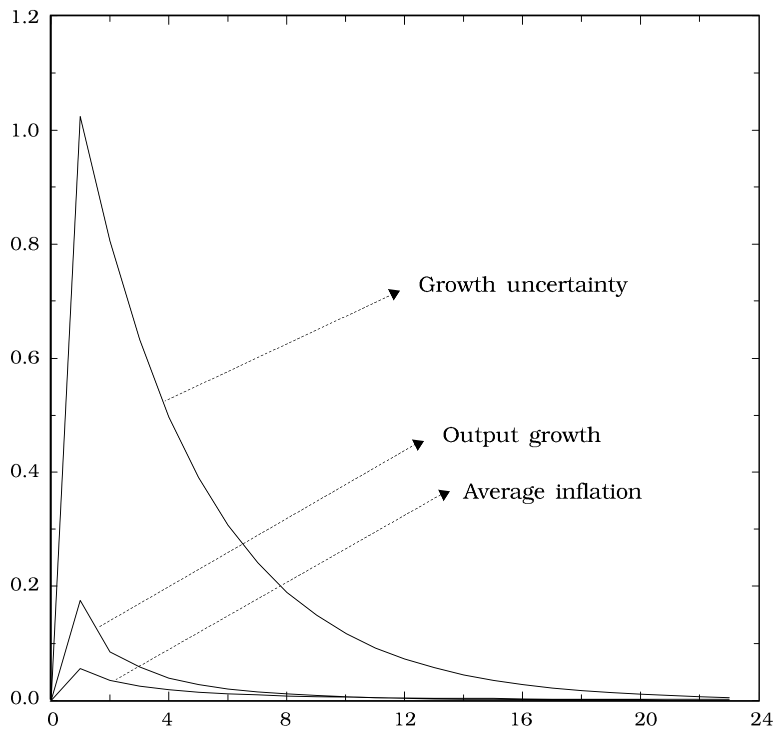

FIGURE 5

THE EFFECT OF A POSITIVE REAL SHOCK ON INFLATION UNCERTAINTY, AVERAGE INFLATION AND OUTPUT GROWTH USING PPI-BASED INFLATION

Figure 5 illustrates the effect of a positive real shock on growth uncertainty, average inflation, and output growth. The real shock occurs in period zero. The output growth uncertainty sharply rises and then declines very slowly, while output growth rises and then falls until 4 months after the shock. It takes almost 12 months for the real effects of the inflation surprise to disappear. Note that there is no effect of the real shock on average inflation.[14]

[14]The details for the other two inflation types are available from the author on request but are not reported here for reasons of spaces.

V. Conclusion

In this paper we consider a bivariate ARFIMA-IGARCH-M model to explain the relationship between uncertainty and average outcomes for inflation and output growth. We first examine that there is significant, persistent conditional heteroscedasticity in both variables and then provide tests of four hypotheses about how uncertainty influences inflation or real growth.

According to inflation types, empirical results show different effects of uncertainty on inflation and growth. From the *PPI*-based inflation our bivariate ARFIMA-IGARCH-M model provides a statistical support for Cukierman and Meltzer's, Deveraux's, and Black's hypothesis, showing that we find an effect of inflation uncertainty on average inflation and either inflation uncertainty or output growth uncertainty on average inflation and output growth. From the *CPI*-based inflation we statistically supports the Deveraux's, Friedman's and Black's hypothesis, in which inflation uncertainty raises output growth and either nominal or real shocks increase average inflation and output growth. The results from the *WPI*-based inflation is similar to the ones of the PPI-based inflation.

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References

Bartlett, M. S. "On the Theoretical Specification of Sampling Properties of Autocorrelated Time Series." *Journal of the Royal Statistical Society 8* (1946): 27-41.

Bernanke, B., and Blinder, A. "The Federal Fund Rates and the Channels of Monetary Transmission." *American Economic Review* 3 (1992): 653-65.

Berndt, E., Hall B., Hall, R., and Hausman, J. "Estimation and Inference in Nonlinear Structural Models." *Annals of Economic and Social Measurement* (No. 3 1974): 653-65.

Black, F. *Business Cycles and Equilibrium*. New York: Basil Blackwell, 1987.

Bollerslev, T. "Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model." *Review of Economics and Statistics* 72 (1990): 498-505.

Bollerslev, T., and Engle, R. F. "Common Persistence in Conditional Variances." *Econometrica* 61 (1993): 167-86.

Cukierman, A., and Meltzer, A. "A Theory of Ambiguity, Credibility, and Inflation under Discretion and Asymmetric Information." *Econometrica* 54 (1986): 1099-128.

Deveraux, M. "A Positive Theory of Inflation and Inflation Variance." *Economic Inquiry* 27 (1989): 105-16.

Diebold, F. X. "Testing for Serial Correlation in the Presence of ARCH." *Proceedings from the American Statistical Association, Business and Economic Statistics Section.* pp. 323-8, 1986.

Engle, R. "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation." *Econometrica* 50 (1982): 987-1008.

Engle, R. F., and Bollerslev, T. "Modeling the Persistence of Conditional Variances." *Econometric Reviews* 5 (1986): 1-50.

Fama, E. F. "The Behavior of Stock Market Prices." *Journal of Business* 38 (1965): 34-105.

Fisher, S. "Relative Shocks, Relative Price Variability, and Inflation." *Brookings Papers Economic Activity 2* (1981): 381-431.

Foster, E. "The Variability of Inflation." *Review of Economics and Statistics 60* (1978): 346-50.

Friedman, M. "Nobel Lecture: Inflation and Unemployment." *Journal of Polical Economy 85* (1977): 451-72.

Grier, K., and Perry, M. J. "On Inflation and Inflation Uncertainty in the G-7 Countries." *Journal of International Money and Finance* 17 (1998): 671-89.

Grier, R., and Perry, J. "The Effects of Real and Nominal Uncertainty on Inflation and Output Growth: Some GARCH-M Evidence." *Journal of Applied Econometrics* 15 (2000): 45-58.

Ljung, G. M., and Box, G. E. P. "On a Measure of Lack of Fit in Time Series Models." *Biometrika* 65 (1978): 297-303.

Mandelbrot, B. B. "The Variation of Certain Speculative Prices." *Journal of Business* 36 (1963): 394-419.

Pagan, A. "Econometric Issues in the Analysis of Regressions with Generated Regressors." *International Economic Review* 25 (1984): 221-47.

Wooldridge, J. M. "On the Application of Robust Regression Based Diagnostics to Models of Conditional Means and Conditional Variances." *Journal of Econometrics* 47 (1991): 5-46.