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Volatility Calibration of the
Commodity Futures Model

(상품 선물옵션 모형의 변동성 추정 및 결과)

2013년 2월

서울대학교 대학원

수리과학부

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Volatility Calibration of the Commodity Futures Model

**A Dissertation
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Master of Science**

**by
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Abstract

In this thesis, we calibrate a volatility function of a commodity future option model with respect to maturity and strike price. Here, the commodity future option model is suggested in 「FinancialModels and Products of Commodity Futures」 by Jun Yeol Kim. First, we assume that the volatility function has only one variable; future maturity, and is cubic equation. Then we calibrate the volatility function of future option based on corn commodity. Secondly, we assume that the volatility function has two variables; future maturity and strike price. Then we calibrate the volatility function of future option based on corn commodity. At last, we do regression of the implied volatilities of the corn future options and compare the volatility approximations to the implied volatilities. Furthermore, we consider the validity of the assumption model and calibration results.

Key words: Commodity Future Model, Commodity Future Option, Black-Scholes Formula, Implied Volatility, Calibration, Regression .

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Chapter 1

Introduction

The Black-Scholes model and its extensions assume that the probability distribution of the underlying asset at any given future time is lognormal. But, traders use volatility smiles to allow for nonlognormality. Furthermore, traders use a volatility term structure. Namely, the implied volatility of an option depends on its maturity. When volatility smiles and volatility term structures are combined, they represent a volatility surface. Thus, implied volatility can be expressed by a function of both strike price and the time to maturity.

In chapter 2, we will consider a theoretical model of a commodity future option suggested in 「Financial Models and Products of Commodity Futures」 by Kim, Jun Yeol.

In chapter 3, we will derive the Black-Scholes formula for the commodity future option.

In chapter 4, we will assume a volatility function of the commodity future option and calibrate them. In addition, we will do regression an implied volatility of future option based on the corn commodity. We will compare implied volatilities to results of approximation volatilities for the corn future option.

In chapter 5, we will consider results of calibration and validity of the assumption of this thesis.

Chapter 2

Commodity Future Option Model

In this chapter, we will propose the generalized cost of carry model which is based on the no-arbitrage assumption.

2.1 Cost of Carry Model

Let (Ω, \mathcal{F}, P) be a probability space which is defined as usual. We are given a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ and W_t is an \mathcal{F}_t -adapted Brownian motion under the martingale measure Q with values in R^d .

Let $F(t, U)$ be a commodity futures price on the underlying S_t at time t with maturity date U . Then $F(t, U)$ is given by $\mathbf{E}_Q[S_U | \mathcal{F}_t]$, so we can model the futures dynamics as

$$dF(t, U) = F(t, U)\sigma(t, U)dW_t$$

with R^d -value function $\sigma(t, U)$

Chapter 3

Black-Scholes Formula for Commodity Future Option

Here we derive a Black-Scholes formula for the call option. (In a future option, call option price and put option price are same.)

By the Risk neutral valuation principle, call option price is

$$C_t = B_t \mathbf{E}\left[\frac{(F(t,U)-K)^+}{B_T} | F_t\right]$$

where $B_t = e^{-\int_s^t r_s ds}$, i.e., bank account.

In addition, let $C(t_0, F_0, T, K)$ be the price at t_0 , of a European call with strike K , and time of maturity T , on a futures maturity U and $F_0 = F(t_0, U)$. Then, under the condition that interest rate r_s is deterministic, call price is

$$C_t = e^{-\int_t^T r_s ds} F(t, U) N(d_1) - e^{-\int_t^T r_s ds} K N(d_2)$$

$$\text{where } d_1 = \frac{\log \frac{F(t,U)}{K} + \frac{1}{2} \int_t^T \sigma^2(s, U) ds}{\sqrt{\int_t^T \sigma^2(s, U) ds}}, \quad d_2 = \frac{\log \frac{F(t,U)}{K} - \frac{1}{2} \int_t^T \sigma^2(s, U) ds}{\sqrt{\int_t^T \sigma^2(s, U) ds}} .$$

Chapter 4

Volatility Calibration of the Commodity Futures Model

In this chapter, we will calibrate a volatility function of the commodity future option and do regression of an implied volatility of future option based on the corn commodity. The key point of our volatility function is that has variables, future maturity and strike price. Furthermore, we will compute approximate values of implied volatilities using our volatility function and compare these calibration results to the implied volatilities.

4.1 Assumption of Volatility Function with respect to Future Maturity

Let U be a maturity of a commodity future and T be a maturity of the commodity future option. In this section, we will assume that a volatility function of the commodity future option is affected by time to future maturity U . Simply, the volatility function is denoted by $\sigma^2(t, U)$.

And let an implied volatility σ_{imp} of the commodity future option be like average of the volatility function from time 0 to time T .

That is,

$$\sigma_{imp}^2 = \frac{1}{T} \int_0^T \sigma^2(t, U) dt$$

For detailed results, we suppose that $\sigma^2(t, U)$ is a polynomial of degree 3. By this assumption, the volatility function is

$$\sigma^2(t, U) = f(U - t) = f(\tau) = -c_0 - 2c_1\tau - 3c_2\tau^2 - 4c_3\tau^3$$

Calculated for the convenience, the coefficients have a minus sign.

Let be future maturity U and option maturity T same. Then,

$$\begin{aligned} \sigma_{imp}^2 &= \frac{1}{T} \int_0^T \sigma^2(t, U) dt = \frac{1}{T} \int_0^T f(T - t) dt = \frac{1}{T} \int_0^T f(\tau)(-d\tau) \\ &= \frac{1}{T} \int_0^T (-c_0 - 2c_1\tau - 3c_2\tau^2 - 4c_3\tau^3)(-d\tau) \\ &= \frac{1}{T} (c_0T + c_1T^2 + c_2T^3 + c_3T^4) = c_0 + c_1T + c_2T^2 + c_3T^3 \end{aligned}$$

4.1.1 Calibration of a Volatility Function for Corn Future Option

In the previous section, we made a volatility function of the commodity future option. Then we will apply to real market data by using the least squares approximation.

These are market data of the corn future option referenced from Bloomberg on 7, Dec, 2012.

Maturity	Implied volatility
0.117808	18.62
0.194521	20.35
0.271233	20.77
0.443836	22.47
0.539726	24.62
0.789041	24.91
1.038356	24.89
1.287671	23.82
1.536986	22.59
2.027397	21.82

[Table4.1: Corn future option data (20120.12.07)]

Due to sufficient data, we can get the optimal coefficients of the volatility function using this data.

c_0	c_1	c_2	c_3
222.19996	11081.779	-905.784	213.6653

[Table4.2: Coefficients of a volatility function]

Thus, volatility function is

$$\sigma^2(t, U) = -222.19996 - 22163.558\tau + 2717.352\tau^2 - 854.6612\tau^3$$

In addition, we can compare the regression of the implied volatilities to actual implied volatilities.

Maturity	Implied volatility	Approximation	Error(%)
0.117808	18.62	19.5672046	-5.087027931
0.194521	20.35	20.39784629	-0.235116901
0.271233	20.77	21.13449055	-1.754889504
0.443836	22.47	22.49423461	-0.107853186
0.539726	24.62	23.0923421	6.204946799
0.789041	24.91	24.18923944	2.893458693
1.038356	24.89	24.68801082	0.811527459
1.287671	23.82	24.62502759	-3.379628842
1.536986	22.59	23.99586677	-6.223403147
2.027397	21.82	20.86686937	4.36815139

[Table4.3: Comparison]

As you can see, an error of approximation for the implied volatility is from -10% to 10% ; it is a small difference. Thus, we have good results of a calibration of the volatility function in the commodity future option.

4.2 Assumption of Volatility Function with respect to Future Maturity and Strike Price

In this section, we will assume that a volatility function of the commodity future option has two variables, future maturity U and strike price K .

First, we suppose that the volatility function of the commodity future option is affected by time to future maturity and strike price. Namely, the volatility function can be denoted by $\sigma^2(t, U, K) = g(U - t, K)$

Secondly, we assume that an implied volatility σ_{imp}^2 of the commodity future option is like average of a volatility function of the commodity future option from time 0 to time T , option maturity.

That is,

$$\sigma_{imp}^2 = \frac{1}{T} \int_0^T \sigma^2(t, U, K) dt$$

For detailed results, we suppose that $\sigma^2(t, U, K)$ is a polynomial of degree 3. Besides, we will consider the volatility smile. The volatility smile defines the relationship between the implied volatility of an option and its strike price.

That is,

$$\begin{aligned} \sigma^2(t, U, K) &= g(U - t, K) = g(\tau, K) = -a(K) - 2b(K)\tau - 3c(K)\tau^2 - 4d(K)\tau^3 \\ &= -(a_0 + a_1K + a_2K^2) - 2(b_0 + b_1K + b_2K^2)\tau \\ &\quad - 3(c_0 + c_1K + c_2K^2)\tau^2 - 4(d_0 + d_1K + d_2K^2)\tau^3 \end{aligned}$$

Calculated for the convenience, the coefficients have a minus sign.

Let be future maturity U and option maturity T same. Then,

$$\begin{aligned}
\sigma_{imp}^2 &= \frac{1}{T} \int_0^T \sigma^2(t, T, K) dt = \frac{1}{T} \int_0^T g(T-t, K) dt = \frac{1}{T} \int_0^T g(\tau, K) d\tau \\
&= \frac{1}{T} \int_0^T \{-a(K) - 2b(K)\tau - 3c(K)\tau^2 - 4d(K)\tau^3\} (-d\tau) \\
&= \frac{1}{T} \int_0^T \{a(K) + 2b(K)\tau + 3c(K)\tau^2 + 4d(K)\tau^3\} d\tau \\
&= \frac{1}{T} \{a(K)T + b(K)T^2 + c(K)T^3 + d(K)T^4\} \\
&= a(K)T + b(K)T^2 + c(K)T^3 + d(K)T^4 \\
&= (a_0 + a_1K + a_2K^2) + (b_0 + b_1K + b_2K^2)T \\
&\quad + (c_0 + c_1K + c_2K^2)T^2 + (d_0 + d_1K + d_2K^2)T^3
\end{aligned}$$

4.2.1 Calibration of a Volatility Function for Corn Future Option

In the previous section, we made a volatility function of the commodity future option with respect to future maturity and strike price. Then we will apply to real market data by using the least squares approximation.

These are market data of the corn future option referenced from Bloomberg on 7, Dec, 2012.

Maturity	Strike price	Implied volatility
0.194520548	0.9	20.92
0.194520548	0.95	20.12
0.194520548	0.975	20.12
0.194520548	1	20.35
0.194520548	1.025	20.75
0.194520548	1.05	21.24
0.194520548	1.1	22.47
0.271232877	0.9	20.44
0.271232877	0.95	20.28
0.271232877	0.975	20.45
0.271232877	1	20.77
0.271232877	1.025	21.17
0.271232877	1.05	21.59
0.271232877	1.1	22.48

[Table4.4: Corn future option data (2012.12.07)]

Maturity	Strike price	Implied volatility
0.443835616	0.8	22.56
0.443835616	0.9	21.94
0.443835616	0.95	22.08
0.443835616	0.975	22.25
0.443835616	1	22.47
0.443835616	1.025	22.72
0.443835616	1.05	22.95
0.443835616	1.1	23.39
0.443835616	1.2	24.19
0.539726027	0.8	24.91
0.539726027	0.9	24.34
0.539726027	0.95	24.39
0.539726027	0.975	24.48
0.539726027	1	24.62
0.539726027	1.025	24.78
0.539726027	1.05	24.94
0.539726027	1.1	25.57
0.539726027	1.2	25.96
0.789041096	0.9	25.06
0.789041096	0.95	24.92
0.789041096	0.975	24.9
0.789041096	1	24.91
0.789041096	1.025	24.94
0.789041096	1.05	24.98

[Table4.4: Corn future option data (2012.12.07)]

Maturity	Strike price	Implied volatility
0.789041096	1.1	25.09
0.789041096	1.2	25.4
1.038356164	0.9	24.99
1.038356164	0.95	24.89
1.038356164	0.975	24.88
1.038356164	1	24.89
1.038356164	1.025	24.92
1.038356164	1.05	24.94
1.038356164	1.1	25.01
1.038356164	1.2	25.18
1.287671233	0.8	23.71
1.287671233	0.9	23.74
1.287671233	0.95	23.78
1.287671233	0.975	23.8
1.287671233	1	23.82
1.287671233	1.025	23.85
1.287671233	1.05	23.87
1.287671233	1.1	23.91
1.287671233	1.2	23.98
1.287671233	1.5	24.01
1.536986301	0.8	22.55
1.536986301	0.9	22.54
1.536986301	0.95	22.56
1.536986301	0.975	22.57

[Table4.4: Corn future option data (2012.12.07)]

Maturity	Strike price	Implied volatility
1.536986301	1	22.59
1.536986301	0.975	22.57
1.536986301	1	22.59
1.536986301	1.025	22.61
1.536986301	1.05	22.63
1.536986301	1.1	22.67
1.536986301	1.2	22.71
2.02739726	0.8	21.78
2.02739726	0.9	21.73
2.02739726	0.95	21.75
2.02739726	0.975	21.78
2.02739726	1	21.82
2.02739726	1.025	21.86
2.02739726	1.05	21.9
2.02739726	1.1	21.98
2.02739726	1.2	22.13

[Table4.4: Corn future option data (2012.12.07)]

Using this data, we can get the optimal coefficients of the volatility function.

a_0	a_1	a_2
1338.411	-2909.97	1795.91
b_0	b_1	b_2
934.2084	1712.54	-1560.44
c_0	c_1	c_2
-2210.51	1627.122	-330.423
d_0	d_1	d_2
777.6723	-911.942	350.554

[Table4.5: Coefficients of volatility function]

Hence, the volatility function is

$$\begin{aligned}
 \sigma^2(t, U, K) &= -(1338.411 - 2909.97K + 1795.91K^2) - (1868.42 + 3425.08K \\
 &\quad - 3120.89K^2)(U - t) + (6631.53 - 4881.37K + 991.28K^2)(U - t)^2 \\
 &\quad - (3110.69 - 3647.77K + 1402.21K^2)(U - t)^3
 \end{aligned}$$

And the following surface is the regression result of the implied volatilities based on corn future option with respect to time to maturity and strike price.

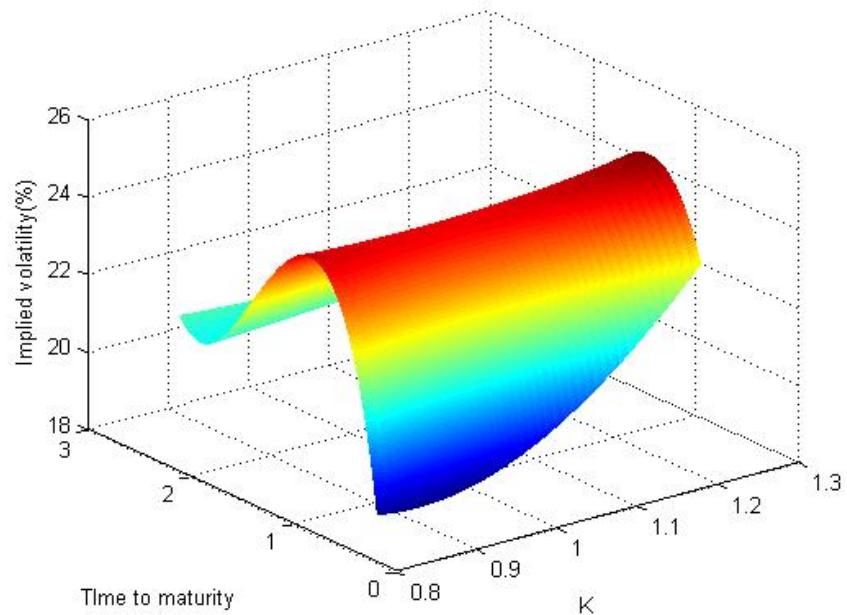


Figure 4.1: Approximation volatility surface

In addition, we can compare the approximate values to the actual implied volatilities.

Maturity	Strike price	Implied volatility	Approximation	Error(%)
0.194520548	0.9	20.92	19.31849749	7.655365726
0.194520548	0.95	20.12	19.60181482	2.575473052
0.194520548	0.975	20.12	19.81222185	1.529712481
0.194520548	1	20.35	20.0666436	1.392414742
0.194520548	1.025	20.75	20.36343038	1.862986143
0.194520548	1.05	21.24	20.70076009	2.538794288
0.194520548	1.1	22.47	21.48918318	4.365005896
0.271232877	0.9	20.44	20.80762929	-1.798577756
0.271232877	0.95	20.28	21.00307199	-3.565443761
0.271232877	0.975	20.45	21.16024851	-3.473097838
0.271232877	1	20.77	21.35596851	-2.821225368
0.271232877	1.025	21.17	21.58918376	-1.980083882
0.271232877	1.05	21.59	21.85869415	-1.24453058
0.271232877	1.1	22.48	22.50120864	-0.094344503
0.443835616	0.8	22.56	23.28957716	-3.233941315
0.443835616	0.9	21.94	23.09125807	-5.247302041
0.443835616	0.95	22.08	23.16514104	-4.914588033
0.443835616	0.975	22.25	23.24514369	-4.472555899
0.443835616	1	22.47	23.3534957	-3.931890061
0.443835616	1.025	22.72	23.48980477	-3.3882252
0.443835616	1.05	22.95	23.65358757	-3.065741056

[Table4.6: Comparison]

Maturity	Strike price	Implied volatility	Approximation	Error(%)
0.443835616	1.1	23.39	24.06123627	-2.869757442
0.443835616	1.2	24.19	25.17693761	-4.079940502
0.539726027	0.8	24.91	24.10296552	3.23980119
0.539726027	0.9	24.34	23.88685388	1.861734247
0.539726027	0.95	24.39	23.92154691	1.920676882
0.539726027	0.975	24.48	23.97458581	2.064600445
0.539726027	1	24.62	24.05123329	2.310181608
0.539726027	1.025	24.78	24.15126456	2.537269714
0.539726027	1.05	24.94	24.27439056	2.668842985
0.539726027	1.1	25.57	24.58847392	3.838584601
0.539726027	1.2	25.96	25.47499694	1.868270636
0.789041096	0.9	25.06	24.84467456	0.859239597
0.789041096	0.95	24.92	24.83355178	0.346902968
0.789041096	0.975	24.9	24.84803644	0.208688977
0.789041096	1	24.91	24.8758612	0.137048584
0.789041096	1.025	24.94	24.91698135	0.092296117
0.789041096	1.05	24.98	24.97133122	0.034702895
0.789041096	1.1	25.09	25.11935571	-0.117001632
0.789041096	1.2	25.4	25.56909339	-0.665721997
1.038356164	0.9	24.99	24.67369792	1.265714625
1.038356164	0.95	24.89	24.66710514	0.895519719
1.038356164	0.975	24.88	24.67185815	0.836583004
1.038356164	1	24.89	24.68197445	0.835779629

[Table4.6: Comparison]

Maturity	Strike price	Implied volatility	Approximation	Error(%)
1.038356164	1.025	24.92	24.69744746	0.89306799
1.038356164	1.05	24.94	24.71826711	0.889065322
1.038356164	1.1	25.01	24.77588897	0.936069684
1.038356164	1.2	25.18	24.95447462	0.895652819
1.287671233	0.8	23.71	23.77298838	-0.265661672
1.287671233	0.9	23.74	23.82391893	-0.353491704
1.287671233	0.95	23.78	23.84732452	-0.283114025
1.287671233	0.975	23.8	23.85851419	-0.245857949
1.287671233	1	23.82	23.86936244	-0.207231058
1.287671233	1.025	23.85	23.87986972	-0.125239911
1.287671233	1.05	23.87	23.89003648	-0.083940026
1.287671233	1.1	23.91	23.90935019	0.002717726
1.287671233	1.2	23.98	23.94390953	0.150502394
1.287671233	1.5	24.01	24.01523296	-0.021794926
1.536986301	0.8	22.55	22.56706742	-0.075687023
1.536986301	0.9	22.54	22.71602666	-0.780952339
1.536986301	0.95	22.56	22.77199727	-0.939704191
1.536986301	0.975	22.57	22.79539837	-0.998663581
1.536986301	1	22.59	22.81575629	-0.999363833
1.536986301	1.025	22.61	22.83307916	-0.98663938
1.536986301	1.05	22.63	22.8473739	-0.960556323
1.536986301	1.1	22.67	22.86690045	-0.868550715
1.536986301	1.2	22.71	22.86978509	-0.703589121

[Table4.6: Comparison]

Maturity	Strike price	Implied volatility	Approximation	Error(%)
2.02739726	0.8	21.78	21.73813069	0.192237425
2.02739726	0.9	21.73	21.69837543	0.145534137
2.02739726	0.95	21.75	21.7122431	0.173594947
2.02739726	0.975	21.78	21.72760588	0.24056072
2.02739726	1	21.82	21.74857352	0.327344083
2.02739726	1.025	21.86	21.77512985	0.388244074
2.02739726	1.05	21.9	21.80725443	0.423495748
2.02739726	1.1	21.98	21.88810607	0.418079761
2.02739726	1.2	22.13	22.11528222	0.066506017

[Table4.6: Comparison]

As you can see, an error of approximation of the implied volatility is approximately from -6% to 8%, which is a small difference. Thus, we have good results of the calibration of the volatility function in the commodity future option.

Chapter 5

Conclusions

We introduce the theoretical models of a commodity future option, cost of carry model and Black formula. We make the volatility function, which is made by assumption that has a general form of polynomial. We notice that there is a small difference between regression results and actual implied volatilities. Furthermore, we notice that our assumptions represent a volatility property; volatility smile. And this assumption results in the appropriate regression for an implied volatility of the commodity future option. Therefore, this volatility function is expected to apply to other market data.

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국문초록

본 논문에서는 상품선물옵션의 내재변동성을 선물만기와 행사가격을 변수로 가지는 변동성 함수로 추정해 본다. 여기서 상품선물모형은 김준열 박사의 논문 「상품 선물 모형 연구 및 결과」에서 제시한 이론을 바탕으로 한다. 첫 번째, 선물만기에 대한 3 차식을 변동성 함수로 가정한다. 그리고 이 변동성 함수로 상품선물옵션의 내재변동성을 추정해 본다. 두 번째, 선물만기와 행사가격을 변수로 하는 변동성 함수를 가정으로 상품선물옵션의 내재변동성을 추정해 본다. 마지막으로 제시된 위의 가정으로 추정한 변동성과 내재변동성과의 오차를 구하고 그 결과에 대한 의미를 알아본다.

핵심 어휘: 상품 선물 모형, 상품 선물 옵션, 블랙숄즈 공식, 내재변동성, 추정, 회귀 .

학번: 2009-20263

감사의 글

친딸처럼 잘해주시고 칭찬을 아끼시지 않았던 최형인 교수님께 감사인사 드립니다. 대학원 다니는 동안 가장 의지가 되었습니다. 교수님 가르침에 누가 되지 않도록 사회에 나가서도 열심히 일하고 영향력 있는 사람으로 성장하겠습니다.

그리고 우리 MTL의 선후배분들께도 감사의 마음 전합니다. 저의 여러 가지 부탁을 잘 들어주신 태형오빠, 철홍오빠, 준열오빠, 정용오빠, 지현언니 고맙습니다. 그리고 연구실 들어와서 즐거운 추억을 만들어준 동기인 영순언니 고마워요. 특히, 마지막 학기까지 정말 많이 도와준 원세오빠 정말 고마워.

취업준비 하느라 많은 시간을 함께 보내진 못했지만 아란, 도영, 시연, 남규, 건우, 시형이도 내 부탁 잘 들어줘서 고맙다. 앞으로 MTL에서 사이 좋게 잘 지내고 열심히 공부하고 성공하길 기도할께. 그리고 복학하고 여로모로 많은 조언과 응원을 아끼지 않았던 09학번 동기들 한솔언니, 경윤언니, 우리언니, 지영언니, 상엽오빠, 병준오빠, 창훈, 병도도 모두 고마워.

나의 대나무숲, 원민아. 지금처럼 서로에게 힘이 되어주는 유일한 친구가 되어주자. 고마워. 사랑한다. 그리고 병선, 상미 졸업과 취업 축하하고 잘되서 나도 기쁘다. 호근이랑 지은이도 아프지 말고 하는 일 모두 잘됐으면 좋겠다.

마지막으로, 막내딸을 변함없이 든든하게 지원해주시고 믿어주신 아빠, 엄마 정말 사랑합합니다. 그리고 우리 큰언니, 작은언니도 고마워. 사랑해.