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Image segmentation using the level set method by the data clustering algorithm

(레벨셋 함수와 데이터 군집화 방법을 이용한 영상 분할에 대한 연구)

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Image segmentation using the level set method by the data clustering algorithm

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Abstract

This paper discusses a variational method for image segmentation by using the level set method with data clustering algorithm. The traditional method of image segmentation has been to separate the given image into two regions-inside and outside. But in the multi-phase segmentation problem, the result is considerably different depending on the number of phases which is given a priori by the user. Therefore, choosing the exact phase number is a critical decision in the multi-phase image segmentation. In this study, we investigated the previous image segmentation models. Then, a multi-phase segmentation method using data clustering algorithms on the image histogram is presented. This study analyzed previous algorithms by giving some changes on the algorithm, and determined the phase number by applying new segmentation method and data clustering methods. Finally, some numerical results were shown for various images.

Key words: basic global thresholding, image segmentation, k-means clustering, level set method, variational method
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Chapter 1

Intoduction

Image segmentation is an art of image analysis. It is the art for detecting objects automatically with various methods. The segmentation is accomplished by the following ideas, separating an image into two regions-inside and outside. Various methods in image segmentation have been widely used in many fields such as image processing, computer vision, and computer graphics.

Many successful methods have been proposed and developed for image segmentation. Out of many relevant works, the primitive approaches were the variational-based and PDE-based method. Almost every model referred to in this paper is based on the method known as deformable model. One of the most popular model is David Mumford and Jayant Shah's Minimum Partition Functional [1]. Based on this functional, Chan and Vese proposed a new active contour model to detect the boundary of objects without edges by approximating the image into a piecewise constant function [2]. Before Chan-vese, There was a method called SNAKE MODEL which is also variational method. On the snake model, boundaries could be detected only if they are defined by a gradient.

For the numerical approximation, Chan and Vese formulated their energy functional in the level set framework. Level set method is a technique for evolving curves and capturing interfaces. It was first proposed in [3]. Level set method has many merits, compared to other methods.

However, Chan-Vese segmentation algorithm has a drawback. Because of a nonlinear parabolic differential equation (PDE) on a large time domain, the algorithm is computationally very expensive. This problem was diminished by Gibou-Fedkiw in [8] by connecting the Chan-vese model to the real-time denoising method and k-means clustering. This revision reduces computational cost needed by the Chan-vese model.

However, general images are not suitable for the previously mentioned segmentation models. The models separate an image into two regions. However, most every images have complex topologies, which means they have various regions-not just inside and outside. Therefore, Chan and Vese extended their idea and generalized their algorithm to multiphase image segmentation [9]. They proposed multiple level set frameworks with numbers of level set functions. Many complex topologies could be described and formulated by their new method.

The fatal drawback of multiphase segmentation is the instability of precisely numbering the distinct regions contained in a given image. This is because there was no reasonable algorithm to obtain the exact number of distinct regions. A user should decide *a priori* number before the implementation. It implies that the segmentation results produced mostly depend on the initial chosen value. Therefore, many methods that has been proposed for multiphase segmentation had tried to improve the unstable problem, finding the exact number of regions before implementing algorithms. The philosophy behind the AGMC algorithm [10] presented in this paper also followed this motivation.

The outline of this paper is as follows: In section 2 we will discuss the relations among the Mumford-shah functional, Chan-vese model and Gibou-Fedkiw model. Section 3 will describe the AGMC algorithm for multiphase segmentation and following segmentation model. In section 4, some results of new research will be provided.

Chapter 2

The Segmentation Using The Level Set Method

2.1 Mumford-Shah Functional

Mumford and Shah [1] proposed a new variational method which is applied to the area of computer vision. If it is possible to assume an image as a function g(x,y) of two variables, we can consider that an image contains a disjoint family of connected sets, each of which has similar intensity. The set is called “a region” when it has pixels having similar level of intensities. The target of the segmentation is to partition an given image into two disjoint, connected regions. From this idea, Mumford and Shah proposed the idea to decompose an image into piecewise smooth approximations by formulating this energy functional

$$E(u,C) = \mu^2 \iint_{\Omega} (u-g)^2 dxdy + \iint_{\Omega - C} ||\nabla u||^2 dxdy + \nu |C|. \tag{2.1.1}$$

where C denotes the boundary of the region, g denotes the function on the image domain Ω, and u is the piecewise smooth approximation to g which has discontinuities on the curve C. The solution image u is obtained by minimizing the energy. The parameters μ and ν are fixed and positive. Each parameter controls the degree of each term’s performance.

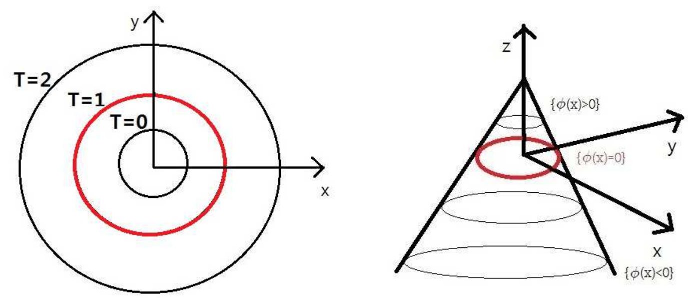


Figure 2.1: Implicit representation and evolution of a curve by using level set function

The first term in (2.1.1) plays a role to measure the deviation between the given image function g and the approximation u. The second term adjusts the smoothness of u. The last term controls the length of the curve C to be as short as possible. It helps us to have smooth segmenting curve. The minimizing process produces optimal u and C as the minimizers or $E(u, C)$. These u and C are the segmentation results. C is sufficiently smooth and u is close to the given image g.

2.2 Level Set Formulation: The Chan-Vese Segmentation Model

Chan and Vese [2] proposed their segmentation model to decompose the image into two regions with piecewise constant approximations. Rather than a piecewise smooth in each region, u is a piecewise constant in each region by approximating u in each region as the mean intensity values. This change can remove the second term from the Mumford-Shah functional (2.1.1). This minimizing process detects a contour C that divides the image into two regions $inside(C)$ and $outside(C)$ and each of them has constant value c_1 and c_2. C is the optimal contour partitioning the given image g. This piecewise constant method of Mumford-Shah is called "minimal partition method."

Then we have a following new energy functional

$$E(C, c_1, c_2) = \lambda_1 \int_{inside(C)} (g - c_1)^2 dxdy + \lambda_2 \int_{outside(C)} (g - c_2)^2 dxdy + \nu |C|. \tag{2.2.1}$$

where $\nu \geq 0$, $\lambda_1 > 0$, $\lambda_2 > 0$ and c_1, c_2 are average intensity values of $inside(C)$ and $outside(C)$ respectively.

The Chan-Vese model makes uses the level set method which was first proposed by Stanley Osher and James Sethian [3] in 1988. The level set method is a technique of tracking interface during the evolution as illustrated in Figure 2.1. The contour we want to capture is embedded as a zero level set in a Lipchitz function which is called a "Level set function." The level set method has many advantages: (1)The ability to handle the topological changes of the level set automatically, (2)The liberation from using complex parameters of space and time, and (3)The possibility to use the fixed grids when it is computed [4]. This model represents the contour C as a zero level set of a level set function ϕ by defining

$$\begin{aligned} C &= \{\mathtt{x} \in \Omega : \phi(\mathtt{x}) = 0\}, \\ in(C) &= \{\mathtt{x} \in \Omega : \phi(\mathtt{x}) > 0\}, \\ out(C) &= \{\mathtt{x} \in \Omega : \phi(\mathtt{x}) < 0\}. \end{aligned} \tag{2.2.2}$$

Their variational active contour model can be described as a level set formulation. With this purpose, they use the Heaviside distribution $H(z)$ and its corresponding Delta distribution $\delta_0(z)$

$$H(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases} \quad \text{and} \quad \delta_0(z) = \frac{d}{dz} H(z). \tag{2.2.3}$$

So the energy functional can be rewritten in the level set framework.

$$\begin{aligned} E(\phi, c_1, c_2) &= \lambda_1 \int_\Omega (g - c_1)^2 H(\phi(\mathtt{x})) dxdy \\ &+ \lambda_2 \int_\Omega (g - c_2)^2 (1 - H(\phi(\mathtt{x}))) dxdy + \nu \int_\Omega |\nabla H(\phi)| \partial\Omega \end{aligned} \tag{2.2.4}$$

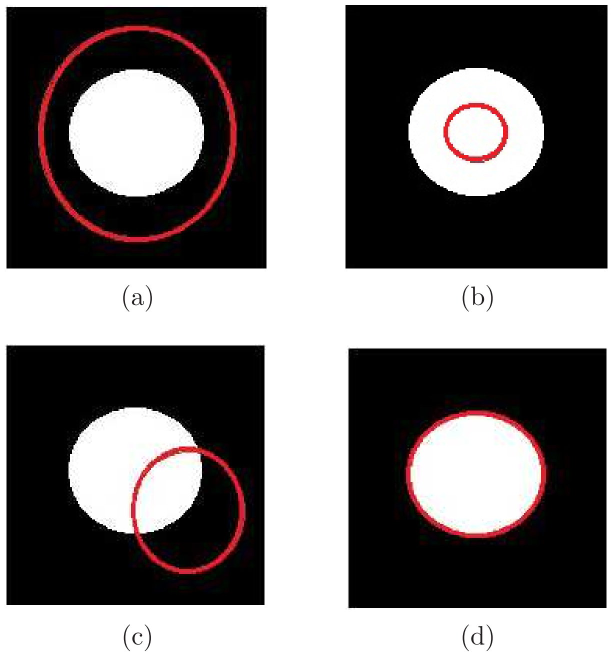


Figure 2.2: The "energy functional" is minimized when the curve C captures the boundary of the object exactly

with c_1 and c_2 being the average intensities of region $inside(C)$ and $outside(C)$ as

$$\begin{aligned} c_1(\phi) &= \frac{\int_\Omega g(\mathtt{x})H(\phi(\mathtt{x}))\,\mathtt{dx}}{\int_\Omega H(\phi(\mathtt{x}))\,\mathtt{dx}} \\ c_2(\phi) &= \frac{\int_\Omega g(\mathtt{x})(1 - H(\phi(\mathtt{x})))\,\mathtt{dx}}{\int_\Omega (1 - H(\phi(\mathtt{x})))\,\mathtt{dx}}, \end{aligned} \tag{2.2.5}$$

which was first suggested in [5]. This can make the minimization problem of one variable ϕ, not three sets of different values c_1,c_1, ϕ.

Now we need the regularized version of H and δ for applying the Euler-Lagrange equation as in [6]. They are

$$H_\varepsilon(z) = \frac{1}{2}\left(1 + \frac{2}{\pi}\arctan\left(\frac{z}{\pi}\right)\right), \qquad \delta_\varepsilon(z) = \frac{1}{\pi}\frac{\varepsilon}{\varepsilon^2 + z^2}. \tag{2.2.6}$$

So we get the regularized energy functional by substituting H and δ with $H_\varepsilon(z)$ and $\delta_\varepsilon(z)$. By applying them, the minimization problem can be treated with the Euler-Lagrange framework. Then we get this associated Euler-Lagrange equation with respect to finding the minimizer of the energy as in [7] which has a specific explanation

$$\phi_t = \delta_\varepsilon(\phi)\left[\mu\ \nabla\cdot\left(\frac{\nabla\phi}{|\nabla\phi|}\right) - \lambda(g-c_1)^2 + \lambda(g-c_2)^2\right] \tag{2.2.7}$$

with artifitial time t , appropriate initian and boundary conditions are

$$\phi(0,\mathtt{x}) = \phi_0(\mathtt{x}) \quad \text{in } \Omega \quad \textit{and} \quad \frac{\partial\phi}{\partial\vec{N}} = 0 \quad \text{on } \partial\Omega, \tag{2.2.8}$$

$\vec{N}$ is the exterior normal vector at the interface. The zero level set of minimizer is the segmentation result which we desired to find.

2.3 Connection to k-means Clustering: Gibou-Fedkiw Model

The main drawback of the Chan-Vese algorithm is the very high computational cost because of the first term $\nabla\cdot\left(\frac{\nabla\phi}{|\nabla\phi|}\right)$ we call the "length scale term." The feature of this term is not to capture noises of images. This term slows down the propagation of the initial curve to the steady state. Frederic Gibou and Ronald Fedkiw [8] tried to overcome this computational cost. They reintroduce the notion of scale using a few time steps of nonlinear diffusion which is like a denoising process instead of introducing the complex term of Chan-vese. This diffusion step allows us to assume that the given image g as a clean image. Then the complex length scale term, which has huge responsibility of high computational cost is not needed any more. The second idea is the connection between Chan-vese algorithm and the k-means clustering. Before implementing the method, k-means clustering splits the image into a number of regions. And we could make initial ϕ as a step function with each cluster centers. In this case, a simple question arouses how the simple setting of ϕ not cause some trouble. Here is the answer. Gibou and Fedkiw realized that the only interest in the segmentation is the steady state of zero level curve, because the steady state is the final result of the segmentation.

The other part of the level set function is relatively not important. For the segmentation, it is enough to capture the evolution of the zero level set which is the union of discontinuities of ϕ. Setting $\delta_\varepsilon(\phi) = 1$ and ignoring the length term deduces the following ODE

$$\frac{\partial \phi}{\partial t}(t, \mathtt{x}) = -\big(g(\mathtt{x}) - c_1(t)\big)^2 + \big(g(\mathtt{x}) - c_2(t)\big)^2. \tag{2.3.1}$$

With these motivations, Gibou and Fedkiw invented the hybrid k-means level set method which is computationally faster than that of the Chan-Vese model. The k-means clustering [11] is an algorithm to split given n data or objects into k clusters with respect to the similarity. In the image processing, the meaning of similarity is the intensity value. This hybrid method corresponds to (2.3.1) which plays a role to measure of distances of each pixel to the average value of each cluster. After this measurement, the mean of the each cluster is updated by new one. When the value of (2.3.1) is positive, ϕ will be increased. If it is negative, ϕ will be decreased. Because our only interest is the steady state of the zero level set, it is sufficient to say that we update ϕ as positive constant value and update it as negative constant value if (2.3.1) is negative. Choosing $\phi = 1$ and $\phi = -1$ makes the computation of centers of each cluster more efficient than that of the Chan-Vese model.

Chapter 3

Adaptive Global Maximum Clustering Algorithm and Its Segmentation Model

3.1 Multiphase Image Segmentation

In the previous chapter, we investigated the history of image segmentation and studied algorithms and following models. The previous algorithms were designed for applying to two-phase images. Those algorithms could be naturally generalized to the multiple image segmentation. Chan and vese [9] extended their model from applying to tow-phase images to the multiphase images by setting multiple level set function. In the multiphase image segmentation, the fundamental assumption was that the pixels with similar intensities are in the same regions. It has been an important but difficult problem to find the exact number of regions in a given image because there has not been a concrete method. So it has been considered as an unstable problem.

Sunhee Kim suggested a new algorithm AGMC [10] which makes it possible to find the appropriate number of distinct regions of multiphase images. Kim revised Gibou and Fedkiw's segmentation model. The continuing chapter will present explanations about the AGMC algorithm and its following segmentation model.

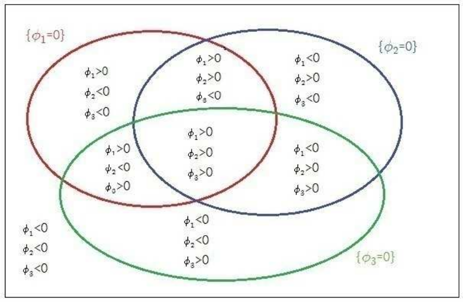


Figure 3.1: 8-phase region by level set function ϕ ($n = 3$).

3.2 Adaptive Global Maximum Clustering Algorithm

Many image segmentation algorithms has been generally based on the properties of intensity value as in [12]. The properties were discontinuity and similarity. In the first approach we found the boundaries of a given image such as edges which have abrupt differences in intensity values. In the second category, the approach was based on partitioning the entire region of the image according to the similarity of the intensity values. In the second approach, a gray level image histogram consists of a light object and a dark background. The goal of this approach is to extract the light object from the background. This approach is accomplished by selecting a threshold T which is the partitioning value. In order to determine the threshold, most of methods analyze the histogram of the image. In the method, every pixels in the image is labeled as an object or background depending on whether the level of each pixel is greater or less than the specific value T, that is, every pixels are classified into two classes. The one is foreground and the other is background [13].

In researches [14] and [15], the authors analyzed the image histograms to determine multi-level thresholds and then to segment the multi-phase images. In their works, pixels were separated into many classes not just foreground-background separation. Sunhee Kim [10] also used the image histogram to find the number of different regions. However, her work was based on the difference method compared to [14] and [15].

The AGMC model assumes that the number of regions is the same as the number of significant maxima of histogram. However there are too many number of local maxima in a single histogram. So we need some reasonable method to detect only significant local maxima. To find only significant local maxima, we first search for subintervals of the histogram which has the subinterval including the maximum of the original histogram. And we fix that special subinterval which is called "a cluster." Next, we move the cluster from the original histogram to have a new reduced histogram and to extract the second cluster. We repeat this process and stop when we get the desired result. So a question arouses to ask what is the desired result and when this process will be stopped. Through the AGMC algorithm, we have two clusters at which the histogram were devided for each step. They have their own maximum, so we have two maxima in total. If the difference between two values of maximm is sufficiently less than the parameter σ, then we stop the iteration process until this condition holds.

$$|\arg\max_{l \in I_1} h(l) - \arg\max_{l \in I_2} h(l)| < \sigma.$$

where $I_1 \bigcup I_2$ is the histogram of each steps. By this way, we can get a cluster including one of the significant local maxima of the original image histogram. Since the histogram is updated at every iteration, the global maximum changes at each steps also. Therefore, the process is called the adaptive global maximum clustering process. Throughout this process, we decompose a gray levels from 1 to 256 into subintervals which we call clusters. The purpose of this process is to split the original histogram as:

$$[1, 256] = (\bigcup_{i=1}^{n} I_i) \bigcup E,$$

each I is the i-th cluster which has the maximum value of each step's adaptive histogram. E is negligible, that is, E is empty or very small set compared to the original histogram.

To accomplish the process, we iteratively implement the k-means clustering with $k = 2$. The k-means clustering is an algorithm to separate data which have similar properties such as intensity values in a digital image data as referred in section 2.3. We splited given data into k groups by minimizing the sum of squares of Euclidean distances between data value and the corresponding cluster centers.

$$\sum_{j=1}^{k}\sum_{x\in I_j}|x-v_j|^2,$$

where $|x-v_j|^2$ is a distance between each data x and the cluster center v_j.We can revise this minimizing problem as the image histogram version described as

$$\sum_{j=1}^{k}\sum_{l\in I_j}h(l)(l-v_j)^2, \tag{3.2.1}$$

where h is an image histogram and $\bigcup_{j=1}^{k} I_j=[1,256]$. And we also compute each cluster center v_j as:

$$v_j=\frac{\sum_{l\in I_j} l\cdot h(l)}{\sum_{l\in I_j} h(l)} \qquad \text{for} \;\; j=1,\ldots,k. \tag{3.2.2}$$

3.3 The Segmentation Model

From this AGMC method, we can find the exact number of distinct regions in any given image. Each cluster is a subinterval and their cluster centers are employed to set an initial level set ϕ which is designed for segmentation model in [10]. By using n distinct regions and following cluster centers, we can define initial ϕ as follows:

$$\phi(x,y)=\begin{cases} 1 & \text{if} \quad g(x,y)\le \frac{v_1+v_2}{2} \\ i & \text{if} \quad \frac{v_{i-1}+v_i}{2}<g(x,y)\le \frac{v_i+v_{i+1}}{2} \\ n & \text{if} \quad \frac{v_{n-1}+v_n}{2}<g(x,y), \end{cases} \tag{3.3.1}$$

where $2\le i\le n-1$.

The energy functional of this situation can be considered as a kind of piecewise constant Mumford-Shah functional in (2.1.1)

$$\inf_{c_i} \sum_{i=1}^{n} \int_{\Omega_i} |c_i - g|^2 dx. \tag{3.3.2}$$

The segmentation model is the modifying version of this functional which is rewritten with the level set function ϕ,

$$\begin{aligned} \min_{c_i,\phi} \Big\{ E(c_i, \phi \mid g) &= \int_{\Omega} \sum_{i=1}^{n} |c_i - u_0|^2 \psi_i(\phi) dx \\ &= \int_{\Omega} \sum_{i=1}^{n} |c_i - u_0|^2 \{H(\phi - i) - H(\phi - (i+1))\} dx \Big\}. \end{aligned} \tag{3.3.3}$$

with

$$\psi_i(\phi) = H(\phi - i) - H(\phi - (i+1)),\ 1 \le i \le n,$$

where $H(\phi)$ equals 1 if $\phi \ge 0$ and equals 0 if $\phi < 0$.

Therefore, we can complete this problem with level set formulation. It is easy to see that consants $c_i(\phi)$ are expressed in terms of fixed ϕ as

$$c_i(\phi) = \frac{\int_{\Omega} g\{H(\phi - i) - H(\phi - (i+1))\} dx}{\int_{\Omega}\{H(\phi - i) - H(\phi - (i+1))\} dx} = average(g) \quad \text{in } \Omega_i.$$

As a result, it is the minimization problem with energy functional (3.3.3) with constants $c_i(\phi)$ and fixed ϕ. By applying the Euler-Lagrange equation, we arrive at the following ordinary differential equation(ODE) in each region Ω_i

$$\phi_t = -|c_{i-1} - g|^2 + |c_i - g|^2 \equiv E_i.$$

which is exactly the same with the two-phase segmentation of Gibou-Fedkiw's model in [8]. Following the idea of [8], we can update ϕ at each iteration:

$$\phi = \begin{cases} \alpha = i & when \quad E_i > 0 \\ \beta = i - 1 & when \quad E_i < 0 \end{cases} \tag{3.3.4}$$

(3.3.5)

where $\alpha > \beta$. The result from this above simple iterative calculation gives the final segmentation. The discontinuities of level set function ϕ are the boundary of the object we want to segment.

Chapter 4

Results of Our Approach

In this chapter, several new experimental results are presented.

4.1 Determining the Initial μ of the k-means Clustering

The result of k-means clustering depends on the initial μ(initial average values of each clusters chosen by a user), which means that different initial values may give different final results. This is a somewhat minor limitation of k-means clustering which makes the algorithm seem unstable. This limitation naturally induces the interest about finding suitable initial value μ for an adequate and exact result of the k-means clustering. Therefore in this chapter, we propose a reasonable method to find appropriate initial value μ.In here the basic global thresholding(BGT) is applied which is referenced in section 3.2. In this section, the BGT algorithm is utilized to find the initial value μ of k-means clustering for each steps of AGMC. The following algorithm was used to this study to obtain T automatically:

1) Select an initial estimated threshold T.

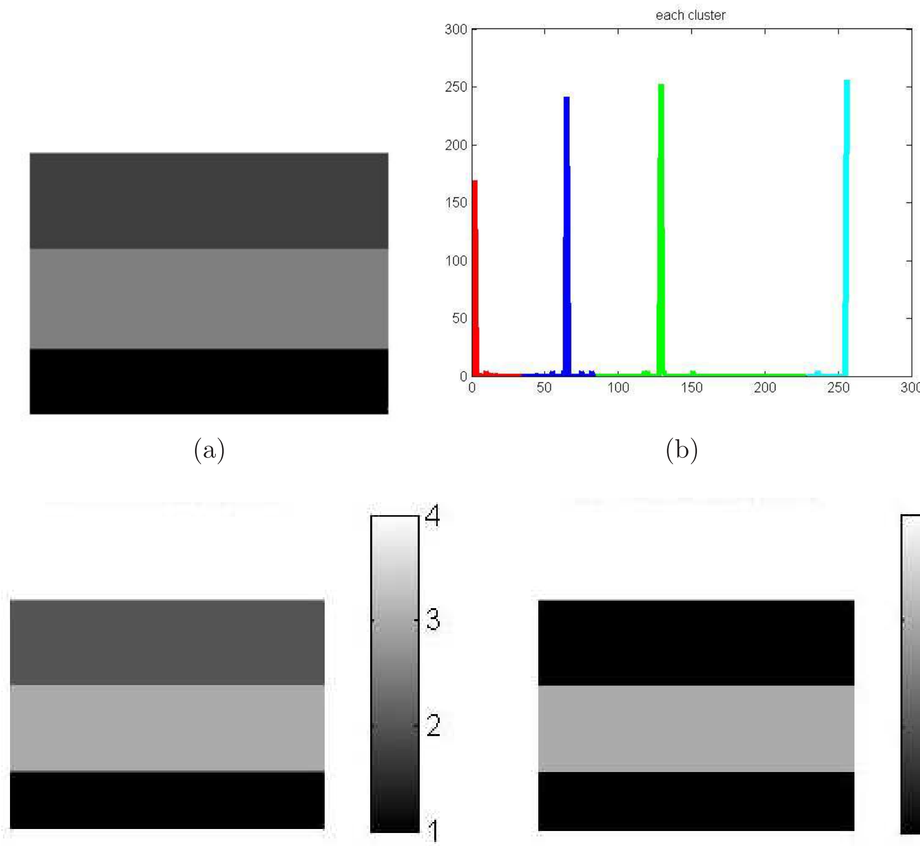
2) Segment the signal using T, which will yield two groups,
I_1, consists of all points with values $< T$ and I_2, consists of points with values $>= T$.

3) Compute an average distance between points of I_1 and T, and points of I_2 and T.

4) Compute a new threshold T=(M1+M2)/2

5) Repeat steps from 2 to 4 until difference between T and T_0(which is value of T before updating) is small enough.

In [16], the initial threshold T can be chosen at the bottom of the sharp valley of the histogram which is supposed to have deep valley between two peaks. But many real image have complex topologies, which means that their histogram would have many deep and sharp valleys. So it is difficult to find the exact bottom of the valley. In order to overcome this problem, Otsu [16] designed a calculation algorithm to obtain a proper threshold T. The value T is calculated by the function "graythresh" which is built in MATLAB toolbox. MATLAB toolbox is a collection of MATLAB functions. This function follows Otsu's method [9].



(a) (b)

(c) (d)

Figure 4.1: (a) Original image. (b) Clusters corresponding to our segmentation result. (c) Segmentation result from the AGMC method. (d) Segmentation result from our approach. It is almost same as the result (c) from the AGMC method.

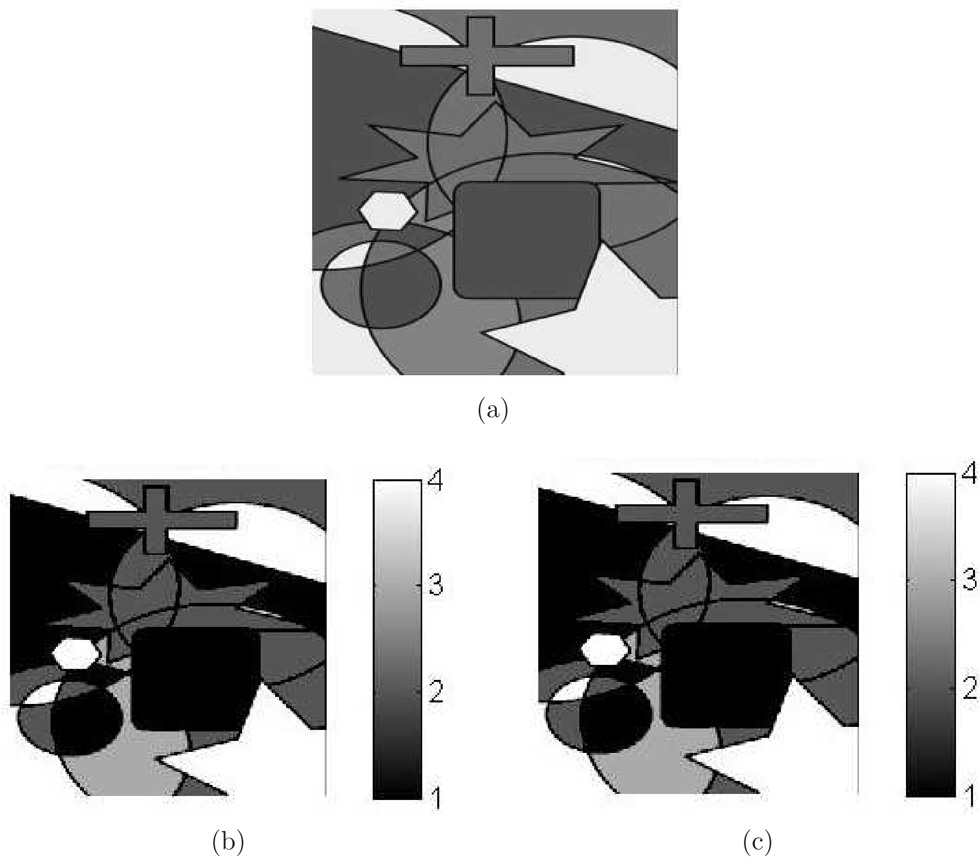
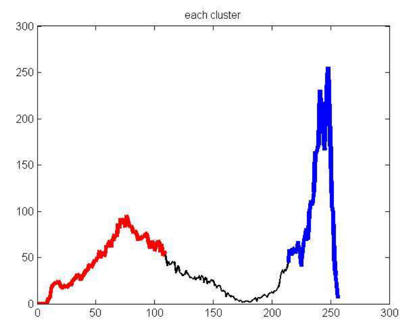
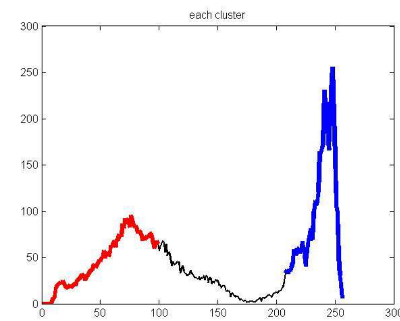


Figure 4.2: (a) Original image. (b) Segmentation result from the AGMC method. (c) Segmentation result from our approach. The results are similar.

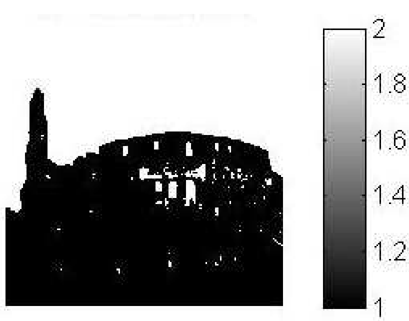
(a)



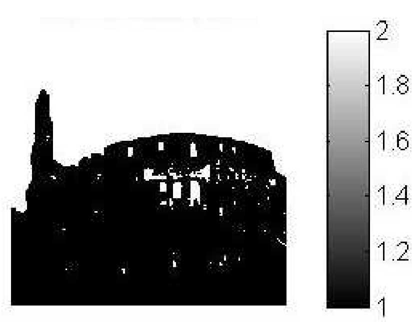
(b)



(c)



(d)



(e)

Figure 4.3: (a) Original image. (b) Clusters corresponding to AGMC segmentation result. (c) Clusters corresponding to our segmentation result. (d) Segmentation result from the AGMC method. (e) Segmentation result from our approach.

4.2 Revising the Algorithm Using Another Clustering Method Instead of the k-means Clustering

In this section, a new approach for obtaining the exact number of distinct regions is suggested. This is accomplished by using bgt algorithm instead of k-means clustering. As shown in the previous section, bgt is a very simple and fast algorithm of data clustering. So it is also a common algorithms applied to image processing(in particular, image segmentation operation). By using bgt algorithm, we can also separate a histogram into two classes just like k-means clustering do likewise. The fact that we analyze the image histogram to determine the threshold T as in AGMC algorithm implies that we can apply bgt instead of k-means clustering. By implementing this new method, we can get the following numerical results.

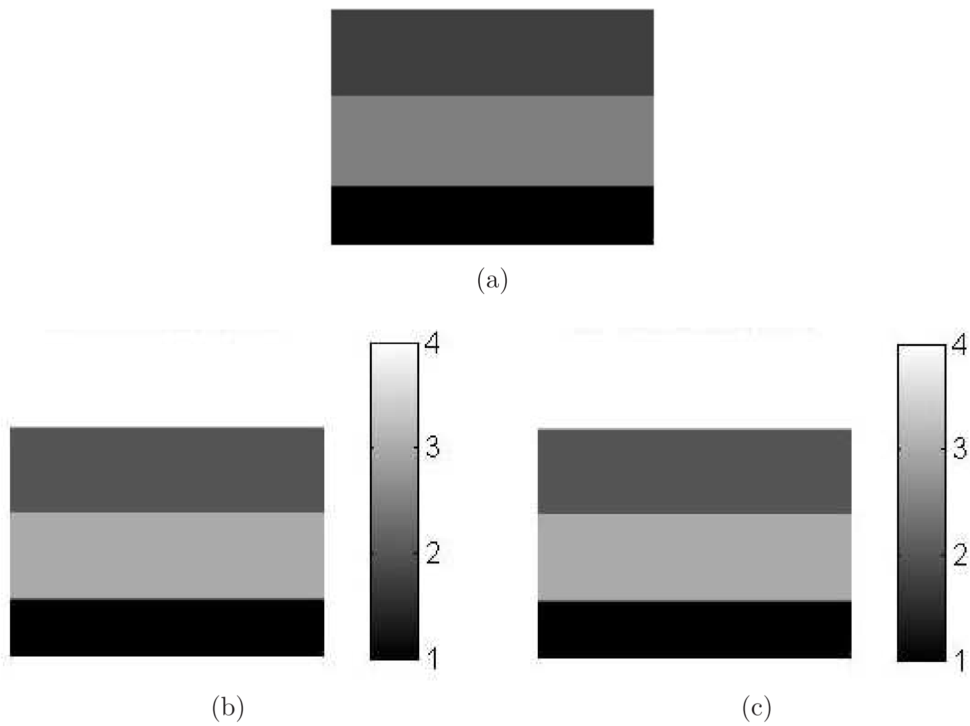
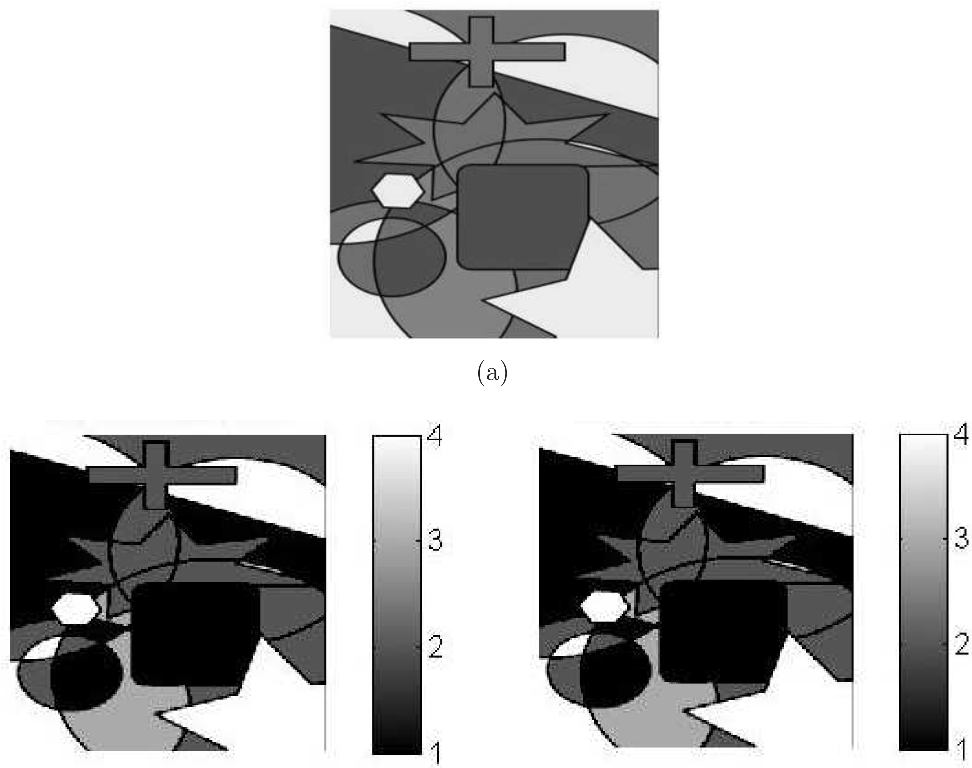


Figure 4.4: (a) Original image. (b) Segmentation result from the AGMC method. (c) Segmentation result from our second approach. It is almost same as the result (b) from the AGMC method.



(a)

(b) (c)

Figure 4.5: (a) Original image. (b) Segmentation result from the AGMC method. (c) Segmentation result from our second approach. The reasonable results are similar.

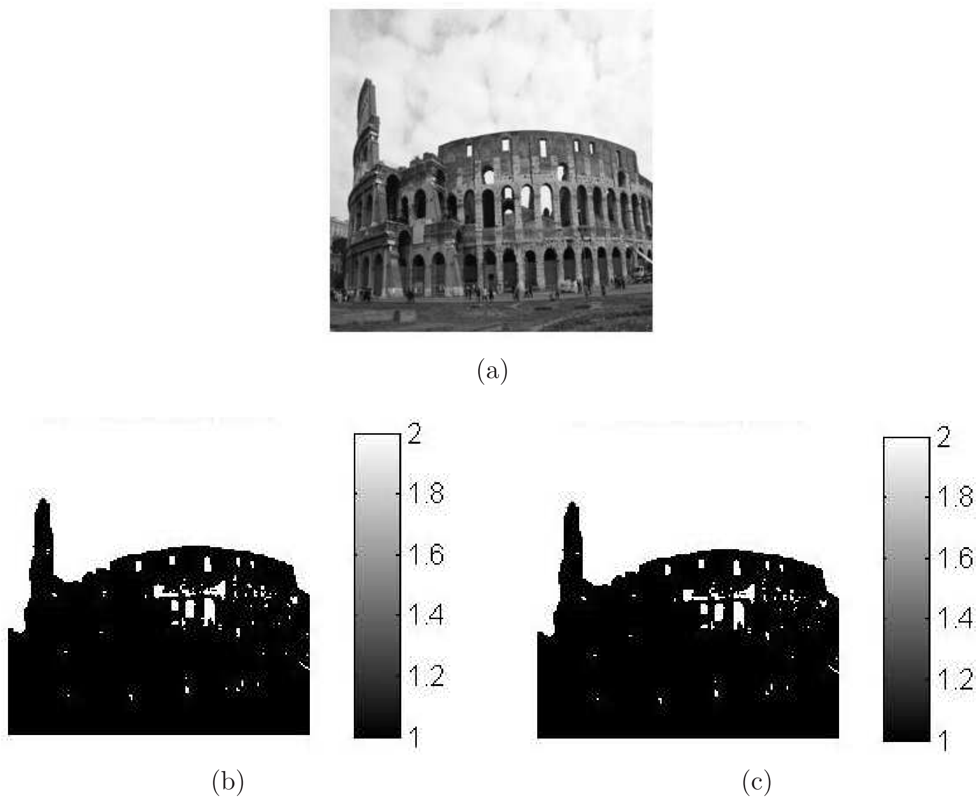
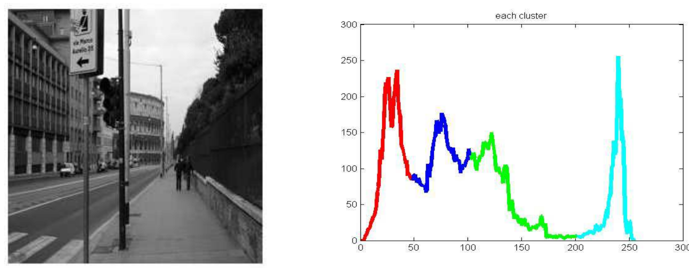


Figure 4.6: (a) Original image. (b) Segmentation result from the AGMC method. (c) Segmentation result from our second approach. The results are similar.

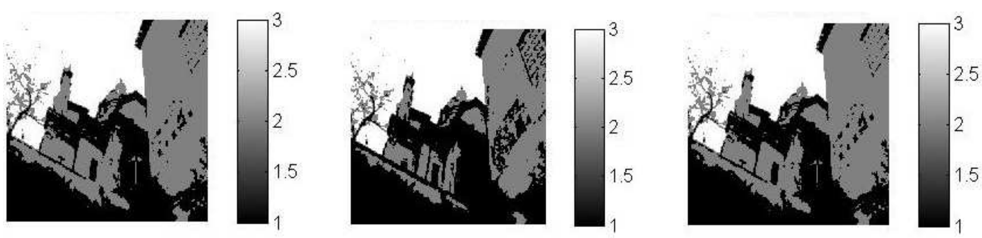


(a) (b)

(c) (d) (e)

Figure 4.7: (a) Original image. (b) Clusters corresponding to the segmentation result from our second approach. (c) Segmentation result from the AGMC method. (d) Segmentation result from our first approach. (e) Segmentation result from our second approach. This approach gives a better result compared to (c) and (d). They could not segment telegraph pole which is located at a distance.

(a)



(b) (c) (d)

Figure 4.8: (a) Original image. (b) Segmentation result from the AGMC method. (c) Segmentation result following our first approach. (d) Segmentation result following our second approach. (c) gives a better result, particularly on a textured region compared to (b) and (d).

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국문초록

본 논문에서는 레벨셋 함수와 데이터 군집화 방법을 이용한 영상 분할을 위한 변분법을 다룬다. 하나의 영상을 분할하는 고전적인 방법은 영상을 내부와 내부의 두 영역으로 나누는 것이다. 그러나 다상 영상을 분할하는 과정에서는 주어진 영상이 몇 개의 상으로 이루어졌는지의 초기 정보에 따라 분할 결과가 상당히 달라지게 된다. 그래서 주어진 이미지가 몇 개의 상으로 이루어졌는지 결정하는 것은 영상 분할에서 굉장히 중요한 문제이다. 먼저 기존의 영상 분할 알고리즘들을 소개하고, 이미지 히스토그램에 데이터 군집화 방법을 사용하여 영상을 이루는 상이 몇 개인지를 알아내는 기존의 연구를 소개한다. 이 논문에서는 기존의 연구에서 사용된 데이터 군집화 방법을 변화시켜 비교 분석하고, 새로운 데이터 군집화 방법을 사용하여 다상 영상을 분할하여 본다. 또한 다양한 이미지에 알고리즘을 적용한 결과를 제시한다.

주요어휘: 전역적 문턱치 처리, 영상 분할, k-평균 군집화, 레벨셋 방법, 변분법
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