Long-Run Comparative Advantage and Transitional Dynamics after Free Trade in an Endogenous Growth Model

Noh-Sun Kwark*

This paper examines the determinants of long-run comparative advantage and analyzes transitional dynamics after free trade in an endogenous growth model. It shows that a country with favorable conditions for capital accumulation has a long-run comparative advantage in capital-intensive goods. Transitional dynamics shows that the growth rate of a more patient country tends to be higher while a less patient country experiences a slowdown in its output growth rate during the transition path. In this study, it is shown that an endogenous growth model with international trade can produce the three empirically observed facts in the rapidly growing East Asian countries: high savings rates, high growth rates, and a significant change in the trade pattern.

Keywords: Long-run comparative advantage, Endogenous growth, Transitional dynamics, Capital accumulation

JEL Classification: F43, F40, O10

*Associate Professor, Department of Economics, Sogang University, 1 Shinsoo-dong, Mapo-gu, Seoul 121-742, Korea, (Tel) +82-2-705-8770, (Fax) +82-2-704-8599, (Email) kwark@sogang.ac.kr. I am grateful to Marianne Baxter, Mark Bils, Yongsung Chang, João Gomez, Insoo Kang, Sergio Rebelo, Changyong Rhee, Alan Stockman, Farshid Vahid, and two anonymous referees for helpful comments and suggestions. All remaining errors are of course mine.

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I. Introduction

The rapid growth of some Asian countries in recent decades has been one of the most puzzling issues in the growth literature (see, *e.g.*, Lucas (1993), Pack and Page (1994), Young (1994, 1995)). One direction of research explaining the rapid economic growth in these Asian countries is to emphasize the role of international trade (see International Monetary Fund (1993) for discussions). However, a significant change in the trade pattern of those countries has not been examined very closely. The East Asian countries which accomplished rapid growth in recent decades have displayed changing trade patterns: from exporting agricultural goods to exporting manufactures.[1]

Table 1 shows the percentage shares of manufactures in merchandise exports for four rapidly growing Asian economies from 1960 to 1990. The real GDP of all four countries grew at a rate of about six percent during this period. Korea shows a significant change in the share of manufactured goods in its total merchandise exports during this period, rising from 14 percent to 94 percent. The other three countries show a similar trend over the period.[2] In particular, there is a significant increase in the share of capital-intensive goods, *i.e.*, machinery and transport equipment in merchandise exports. Although an increase in the portion of manufactures in trade is a general trend for almost all countries, these figures represent a significant change. Thus, it seems that there is a close relationship between economic growth and changes in the pattern of trade. Another feature of the rapidly growing Asian economies is high savings rates. The high savings rates in these countries have been emphasized as an important factor to which the rapid growth might be attributed. Table 1 also provides savings rates and investment-output ratios of the rapidly growing East Asian countries. The savings rates and investment-output ratios of the NIES are significantly higher than the averages of the OECD countries or the world economy.[3]

[1] Ventura (1997) explains the high performance of the East Asian countries focusing on changing trade patterns.

[2] Traditionally, the share of textiles and clothings in the exports of Hong Kong has been large (45 percent in 1960 and 39 percent in 1990). Excluding textiles and clothings, Hong Kong also shows the same trend in the structure of its exports.

TABLE 1
PERCENTAGE SHARE OF MANUFACTURES IN MERCHANDISE EXPORTS AND CAPITAL ACCUMULATION

Country	Share of Manufactures[1)]		Saving[2)]	Investment[2)]
	1960	1990	1975-90	1975-90
Korea	14 (0)	94 (37)	30.6	31.3
Taiwan	41 (4)[3)]	93 (36)	32.9	25.6
Hong Kong	80 (4)	96 (23)	38.6	29.5
Singapore	26 (7)	73 (48)	36.8	36.7
OECD	59 (29)	81 (42)	21.7	22.1
World[4)]	58 (25)[3)]	75 (36)	23.3	23.7

Notes: 1) The percentage shares of machinery and transport equipment in merchandise exports are provided in parentheses.
2) In percent of GDP or GNP.
3) 1965.
4) A weighted average.

Source: *World Development Report*, 1982, 1992.
World Economic Outlook, 1992.
IFS data set.

This study attempts to construct an endogenous growth model which can explain the empirically observed changing trade patterns in the high-performing East Asian countries. I will focus on three facts in those Asian countries: high growth rates, high savings rates, and a significant change in the trade pattern. This study argues that the East Asian countries with rapid growth have had high capital accumulation (through low time preference, for example) relative to others, and this is the main reason for the high performance and the changing trade pattern from exporting agricultural goods (land-intensive or labor-intensive goods) to exporting manufactures (or capital-intensive goods).

The traditional neoclassical trade model suggests that the pattern of trade depends on the relative amount of endowment. This, in turn, determines an autarky relative price of traded goods and comparative advantage, which is the main content of the Heckscher -Ohlin model.[4] However, this analysis is limited to static trade

[3] Japan has also shown a high savings rate over the high growth period (an average of 33.9 percent from 1960 to 1990).

[4] MacDonald and Markusen (1985) and Markusen (1983) show some cases

models. The implication of the neoclassical trade model, when capital accumulation and dynamic optimization are included, is very different from that of static trade models, as examined by Baxter (1992). Baxter (1992) shows with a two-good, two-factor, and two-country model that the neoclassical model can explain many phenomena (*e.g.* increasing volume of trade, two-way trade in goods with similar factor content, welfare benefit of trade liberalization, *etc.*) that are inexplicable within the traditional Heckscher-Ohlin-Samuelson model. This paper differs from her model in the sense that this paper can analyze the effect of international trade on economic growth while in Baxter (1992)'s neoclassical model, the international trade only has a level effect on output. Thus, it may be useful to discuss the rapid growth in the East Asian countries in an endogenous growth context. Another important implication in the transitional dynamics incorporating endogenous growth is that this model can show a reversal in the trade pattern without exogenous shocks such as government tax policies. As capital accumulates, a country can be switched from a land (or labor)-abundant country to a capital-abundant country, which may be sped up by international trade.

The importance of capital accumulation in determining long-run trade patterns has been emphasized by many studies such as Oniki and Uzawa (1965), Findlay (1970), Clarida and Findlay (1991), and Fisher (1995). This study differs from these previous studies in the following sense. First, this study considers a general equilibrium endogenous growth model based on dynamic optimization so that the relationship between growth and patterns of trade can be explicitly examined.[5] The closed economy steady state and the free trade steady state are compared in terms of output growth, incorporating changes in the patterns of trade from the beginning of free trade to the free trade steady state. Second, the transitional dynamics is extensively investigated, analytically as well as numerically. The free

in which the traditional Heckscher-Ohlin model fails in the sense that the pattern of trade is not determined by the relative factor endowments. However, in most cases the pattern of trade is determined by autarky relative prices, except when there are increasing returns to scale.

[5] The endogenous growth model of the study is an extension of Rebelo (1991) by incorporating international trade into the model. Thus, the net value added can be found in the aspects of international trade and the transitional dynamics after free trade described in the following sections.

trade steady state is analytically shown to be reached in a finite time rather than approached asymptotically in an infinite time. Applying a solution method of differential equations, the evolution of all variables of interest is described numerically with some parameterization.

The economy considered in this paper has three production sectors (two consumption goods and one investment good) with a constant returns to scale technology as in Rebelo (1991). This paper shows that parameters related to capital accumulation play a crucial role in the determination of the long-run comparative advantage. A country where conditions favor capital accumulation will develop a long-run comparative advantage in the capital-intensive good regardless of initial comparative advantage. The new steady state, after free trade, is described by specialization of at least one country. This study will show that the growth rate of the country with favorable conditions for capital accumulation tends to be higher over the transition path after free trade, while the country with lower capital accumulation experiences lower output growth. Therefore, high growth rates and changing trade patterns associated with high savings rates observed in the East Asian countries can be explained by high capital accumulation in the endogenous growth model.

The remainder of this paper is organized as follows. Section II describes an endogenous growth model and derives the steady state balanced growth path in a closed economy. Transitional dynamics after free trade in a small open economy and a two-country economy is examined in Section III. Section IV concludes.

II. An Endogenous Growth Model

A. Economic Environment

There is a representative consumer (with no population growth for convenience) who chooses her consumption of c_{1t} and c_{2t}, and savings in the form of an increase in assets, $\dot{a}_t (=da/dt)$, given the initial asset holdings, a_t, to maximize her lifetime discounted utility by solving the following problem:

$$\max_{\{c_{1t},\, c_{2t},\, \dot{a}_t\}} \int_0^{\infty} e^{-\rho t} u(c_{1t}, c_{2t})dt, \quad \rho>0, \tag{1}$$

$$u(c_{1t}, c_{2t}) = \frac{(c_{1t}{}^{\beta} c_{2t}{}^{1-\beta})^{1-\sigma} - 1}{1-\sigma}, \quad \sigma > 0, \ 0 < \beta < 1,$$

subject to

$$\dot{a}_t \leq r_t a_t - c_{1t} - p_{2t} c_{2t}, \quad c_{1t}, c_{2t} \geq 0 \text{ for all } t,$$

where $u(c_{1t}, c_{2t})$ is a momentary utility function, ρ is a discount factor, r_t is an interest rate on assets, and p_{2t} is a relative price of consumption good 2 in terms of consumption good 1. Consumption good 1 is chosen as a numeraire ($p_1 = 1$). Consumer's assets consist of claims on two production factors, capital and a non-reproducible factor.

In this economy, there are three production sectors: two consumption goods, X_1, X_2 and one investment good, I, with the following constant returns to scale production functions:

$$X_{1t} = F_1(\phi_{1t} Z_t, \psi_t T) = A_1(\phi_{1t} Z_t)^{\alpha_1} (\psi_t T)^{1-\alpha_1}, \quad (2)$$

$$X_{2t} = F_2(\phi_{2t} Z_t, (1-\psi_t) T) = A_2(\phi_{2t} Z_t)^{\alpha_2} ((1-\psi_t) T)^{1-\alpha_2}, \quad (3)$$

$$I_t = F_3((1-\phi_{1t}-\phi_{2t}) Z_t) = B(1-\phi_{1t}-\phi_{2t}) Z_t. \quad (4)$$

Consumption goods are produced with two factors: capital, Z, and a non-reproducible factor, T. In this model, capital is a composite including all kinds of factors to be accumulated (*e.g.*, physical and human capital). The non-reproducible factor represents all factors that are not depreciated and cannot be reproduced, such as land or raw labor (from now on, I will simply call this fixed production factor as land for convenience). Consumption good 1 uses ϕ_{1t} fraction of total capital at time t, and ψ_t fraction of land. In producing consumption good 2, ϕ_{2t} fraction of capital and $(1-\psi_t)$ fraction of land are used. The investment good (capital) is produced with the remaining fraction $(1-\phi_{1t}-\phi_{2t})$ of capital in a linear technology. A_1, A_2, and B are constants representing technology parameters. Without loss of generality, assume that consumption good 1 is produced with a relatively more capital-intensive technology and consumption good 2 is produced with a land-intensive technology, that is, α_1 is greater than α_2. The investment good is produced with capital only. This investment production function is similar to

Lucas (1988)'s human capital accumulation equation. Capital depreciates at a rate δ; thus the accumulation of capital is defined as

$$\dot{Z}_t = I_t - \delta Z_t. \tag{5}$$

A representative firm maximizes its current profits by hiring capital and land and allocating these factors among three production sectors as in the following:

$$\max_{\{\phi_{1t},\, \phi_{2t},\, \psi_t\}} \quad X_{1t} + p_{2t} X_{2t} + p_{zt} I_t - \delta p_{zt} Z_t - R_{zt} p_{zt} Z_t - R_{Tt} p_{Tt} T, \tag{6}$$

where p_{zt} and p_{Tt} are relative prices of capital and land in terms of consumption good 1, and R_{zt} and R_{Tt} are rental rates on capital and land, respectively. The competitive equilibrium in this closed economy is defined as follows.

A competitive equilibrium is defined by a set of allocation rules, $c_{1t}=C_1(Z_t,T)$, $c_{2t}=C_2(Z_t,T)$, $\dot{a}_t=A(Z_t,T)$, $\dot{Z}_t=Z(Z_t,T)$, $\phi_{1t}=\Phi_1(Z_t, T)$, $\phi_{2t}=\Phi_2(Z_t,T)$, and $\psi_t=\Psi(Z_t,T)$, and pricing functions, $p_{2t}=P_2(Z_t,T)$, $p_{zt}=P_z(Z_t,T)$, $p_{Tt}=P_T(Z_t,T)$, $r_t=r(Z_t,T)$, $R_{zt}=R_z(Z_t,T)$, and $R_{Tt}=R_T(Z_t,T)$ such that

i) consumers solve problem (1),

ii) firms solve problem (6), and

iii) the goods market and assets market clear each period, implying that

$$X_{1t} = c_{1t}, \quad X_{2t} = c_{2t}, \quad a_t = p_{zt} Z_t + p_{Tt} T.$$

The following relations are obtained from the first order conditions of the competitive equilibrium:

$$\xi_{11} \gamma_{c_{1t}} + \xi_{12} \gamma_{c_{2t}} = \gamma_{p_{2t}} + \xi_{21} \gamma_{c_{1t}} + \xi_{22} \gamma_{c_{2t}} = r_t - \rho, \tag{7}$$

$$\begin{aligned} R_{zt} p_{zt} &= F_1^1(\phi_{1t} Z_t, \psi_t T) - \delta p_{zt} \\ &= p_{2t} F_2^1(\phi_{2t} Z_t, (1-\psi_t) T) - \delta p_{zt} \\ &= p_{zt}[F_3^1((1-\phi_{1t}-\phi_{2t}) Z_t) - \delta], \end{aligned} \tag{8}$$

$$R_{Tt} p_{Tt} = F_1^2(\phi_{1t} Z_t, \psi_t T) = p_{2t} F_2^2(\phi_{2t} Z_t, (1-\psi_t) T), \tag{9}$$

$$r_t = \gamma_{p_{zt}} + R_{zt} = \gamma_{p_{Tt}} + R_{Tt}, \tag{10}$$

$$\lim_{t \to \infty} p_{zt} Z_t u_1(c_{1t}, c_{2t}) e^{-\rho t} = 0, \tag{11}$$

where ξ_{ij} denotes the elasticity of marginal utility of consumption of good i with respect to good $j(\partial u_i / \partial c_j)$, F_i^1 and F_i^2 are the marginal products of capital and land, respectively, for production sector i, γ is the growth rate of the subscript variable, and u_1 is the partial derivative of the momentary utility function with respect to the first argument (c_1). Equation (7) provides the conditions that an optimal path of consumption and wealth accumulation must satisfy. Resource allocation conditions (8) require that the value of the marginal product of capital should be equalized across the three production sectors, and also be equal to rental rates of capital. Conditions (9) imply that the value of marginal product of the fixed factor should be equal between the two consumption sectors and also be equal to rental rates of the fixed factor. The arbitrage condition implies equality of yield across assets as in equation (10). Equation (11) is the transversality condition.

B. Steady State in a Closed Economy

In the steady state, growth rates of fraction variables, ϕ_{1t}, ϕ_{2t}, and ψ_t, are zero.[6] The steady state equilibrium of a closed economy requires that the production of each good be equal to the consumption of the representative consumer. Then, in the steady state, $\gamma_{X_1} = \gamma_{c_1} = \alpha_1 \gamma_z$ and $\gamma_{X_2} = \gamma_{c_2} = \alpha_2 \gamma_z$. National income is defined as $Y = X_1 + p_2 X_2 + p_z I$. Then, the growth rate of national income is given by

$$\gamma_y = \alpha_1 \gamma_z = \frac{\alpha_1 (B - \delta - \rho)}{1 - (1 - \sigma)(\beta \alpha_1 + (1 - \beta) \alpha_2)}. \tag{12}$$

[6] In general, there is no guarantee that the steady state exists because there is no reason that the efficiency conditions from the consumer side, such as equation (7), should hold on the producer side, as pointed out by Rebelo (1991, pp. 516-7). However, this system, with the specification of functions described above (a Cobb-Douglas production function and a CRRA utility function), guarantees the existence of the steady state path.

If this growth rate is lower than $-\alpha_1\delta$ (which is the growth rate with no production of capital, *i.e.*, $\phi_1+\phi_2=1$), then the economy grows at a rate $-\alpha_1\delta$. In the following analysis, this corner solution will be ruled out. Equation (11) demonstrates a common property of endogenous growth models: namely, the economy grows persistently without exogenous shocks. This economy has no transitional dynamics; it always grows at rate γ_y. The source of economic growth comes from the accumulation of capital. As the investment good is produced (that is, capital is accumulated), the production of consumption goods increases and national income grows. In a closed economy, the growth rate depends on tastes and technology parameters describing the condition for capital accumulation. The lower the time preference (the smaller is ρ), the larger the elasticity of intertemporal substitution (the smaller is σ), and the more biased the tastes are to the capital-intensive consumption good (the bigger is β), the higher the growth rate.[7] The higher the capital share in the production of the relatively capital-intensive consumption good (the bigger is α_1), and the more advanced the technology of investment good production (the bigger is B), the faster the economy grows. The growth rate does not depend on the amount of the fixed factor. The amount of the fixed factor determines the level of output, but not the growth rate. With this benchmark model, a free trade equilibrium is examined in Section III. Along the balanced growth path, the interest rate is constant (given by $B-\delta+(\alpha_1-1)\gamma_z$). The relative price of capital is decreasing at a rate of $(\alpha_1-1)\gamma_z$.

The effect of taxation on goods can be easily examined as in Rebelo (1991). Only a tax (or subsidy) on capital (or investment good production) affects the steady state growth rate of output. With a balanced government budget and no effect of government spending on the marginal utility of consumption or the marginal cost of production, it is easy to show that only the tax rate on the investment good affects the growth rate. Suppose that the tax rate on the investment good is τ_z, but the tax is refunded for the depreciation δZ_t at the same rate. Then, only the rental rate of

[7] This statement is valid only when the economy grows at a positive rate ($B-\delta-\rho>0$). When the growth rate is negative (but higher than $-\alpha_1\delta$), the lower time preference, the smaller intertemporal substitution, and the more biased tastes to the land (fixed factor)-intensive consumption good lead to a higher growth rate.

capital in equation (8) needs to be modified: $R_z = (1-\tau_z)(B-\delta)$. Then, the growth rate of national income is changed to the following:

$$\gamma_y = \frac{\alpha_1[(1-\tau_z)(B-\delta)-\rho]}{1-(1-\sigma)[\beta\alpha_1+(1-\beta)\alpha_2]}. \tag{13}$$

Thus, a higher (lower) tax rate on the investment good (or capital income tax) reduces (increases) the incentive to accumulate capital, which results in a lower (higher) growth rate. This result supports justification of the government subsidy policy to capital-intensive industries (here, corresponds to the investment production sector). This might be an explanation for the high performance of some Asian countries (for a related study, see Easterly and Rebelo (1993), which examines the role of government s public investment on growth).

III. Transitional Dynamics after Free Trade

A. A Small Open Economy Model

The model considers a world market where free trade of consumption goods is possible at the world relative price of consumption goods. Assume that only consumption goods are traded. Capital is not traded.[8] A small open economy model is first analyzed and a two-country model will be examined later. A small open economy is defined as a country that takes the path of the world relative price of consumption goods as given, and its supply and demand do not affect the world price. Thus, the growth rate of the relative price of consumption goods is given exogenously as a constant γ_{p_2} and the goods market clearing conditions now change to a trade balance equation, $X_{1t}+p_{2t}X_{2t}=c_{1t}+p_{2t}c_{2t}$. Without loss of generality assume that the small economy is showing a higher growth rate of the relative price in autarky than that of the world relative price ($\gamma_{p_2^A} > \gamma_{p_2^W}$). This implies that the small open economy has a favorable

[8] Capital in this paper includes human capital and physical capital. Thus, it is reasonable to assume that capital is immobile across countries at least partially. Barro *et al.* (1995) show that a partial immobility of capital (human capital) results in a slow convergence over transitional dynamics.

condition for capital accumulation relative to the world economy so that it grows faster than the world economy before the free trade.[9, 10] This small economy accumulates capital faster than the world economy over time.

With higher capital accumulation than the world economy, the relative cost of producing good X_1 in the small economy is decreasing faster than in the world economy since the endowment ratio becomes more favorable to capital-intensive good X_1. Therefore, this economy tends to produce more of consumption good X_1 over time and eventually specializes in consumption good X_1. That is, this economy has a long-run comparative advantage in producing consumption good X_1 compared to the world market. This result is summarized in the following proposition:

Proposition 1

Assume that consumption good X_1 is produced with a relatively more capital-intensive technology $(\alpha_1 > \alpha_2)$ and that the growth rate of the relative price of consumption goods in the small economy is higher in autarky than that of the world economy $(\gamma_{p_2^A} > \gamma_{p_2^W})$. If this economy is small, so that it cannot affect the world relative price, then after free trade of the two consumption goods, this small open economy produces an increasing amount of consumption good X_1, and eventually specializes in the production of consumption good X_1 and investment good I within a finite time.

Proof: Appendix.

The convergence of this economy to the steady state in a finite time is different from the conventional argument of asymptotic convergence to the steady state in an infinite time. The slope of the

[9] This case may violate the small economy assumption of no effect of the supply and demand of the small economy on the world market. However, if the size of the economy is assumed to be very small, this case can still be consistent with the small economy assumption.

[10] The price of consumption good 2 is endogenous. Thus, it may be more accurate to state that if the small open economy accumulate faster than the world economy (due to some factor such as patience, *etc.*, then the relative price of consumption good 2 in terms of consumption good 1 in the small open economy increases faster than that of the world economy since the consumption good 2 is less capital-intensive than the consumption good 1.

production possibility frontier at the specialization (in X_1) point in the plane of the two consumption goods becomes flatter at a faster speed than the growth rate of the world relative price as the small open economy accumulates capital faster than the world economy. Eventually, this economy reaches the specialization point in a finite time. After then, this economy produces only consumption good X_1 and investment good I in a new steady state described by the following proposition.

Proposition 2

When a small country specializes in X_1, in the sense that it produces only consumption good X_1 and the investment good I, the new steady state growth rate of output (γ_y^s) is lower than that of autarky if and only if the inverse of the elasticity of intertemporal substitution (σ) is greater than one. However, if a small economy specializes in consumption good X_2 (land-intensive good) and the investment good I, then the new steady state growth rate of output is always higher than that of autarky.

Proof: Appendix.

The above result is also summarized in Table 2 for the two cases of specialization. This result is interesting because it may be different from the conventional view in two ways. First, it has been thought that specialization might have a positive effect on the growth rate of output. Second, the effect of trade on the growth rate of a small open economy is not symmetric between the two directions of specialization: specialization in capital-intensive goods and specialization in land-intensive goods. Recent studies such as Grossman and Helpman (1991) and Matsuyama (1992) have shown that trade may reduce the growth rate of a small open economy. However, the second result has not been shown in the literature.

The above result may be explained intuitively as follows. In the new steady state where the small open economy is specialized in producing X_1 and I, the law of motion for the price of capital is dictated by the world economy. The relative price of capital in the world economy is decreasing slower than that of the small economy in autarky. This implies that the interest rate ($B-\delta+\gamma_{p_*}$) is higher in the small economy after it reaches specialization than that of

TABLE 2
SPECIALIZATION IN A SMALL OPEN ECONOMY

	Case 1	Case 2
Before Trade (Autarky)		
Growth Rate	$\gamma_y^A > \gamma_y^w$	$\gamma_y^A < \gamma_y^w$
Growth Rate of p_2	$\gamma_{p_2^A} > \gamma_{p_2^w}$	$\gamma_{p_2^A} < \gamma_{p_2^w}$
Static Comparative Advantage		
i) $p_{2t} > p_{2t}^w$	X_1	X_1
ii) $p_{2t} < p_{2t}^w$	X_2	X_2
Long-Run Comparative Advantage	X_1	X_2
After Specialization		
Growth Rate	$\gamma_y^s \gtreqless \gamma_y^A$ *as* $\sigma \lesseqgtr 1$	$\gamma_y^s > \gamma_y^A$

autarky. With an increase in the interest rate, there are two effects on the allocation of resources between the consumption good production sectors and the investment good production sector: a wealth effect and a (intertemporal) substitution effect. The wealth effect of an increase in the interest rate increases resources producing the consumption goods and the substitution effect increases resources producing the investment good. Thus, the small economy may increase consumption good production relative to investment good production ($\phi_1^s > \phi_1^A + \phi_2^A$) or may increase investment good production relative to consumption good production ($\phi_1^s < \phi_1^A + \phi_2^A$). Whether this small open economy accumulates capital faster or slower depends on its behavior in response to an increase in the interest rate, that is, the elasticity of intertemporal substitution $(1/\sigma)$. With a high intertemporal substitution $(\sigma < 1)$, the small open economy accumulates faster than in autarky, so that the growth rate is higher after specialization. However, for an economy with low capital accumulation in autarky, the interest rate after specialization in X_2 is lower than that in autarky, which makes resource allocation in producing the investment good greater through the wealth effect and the substitution effect. Therefore, for a country with low capital accumulation, the net effect of trade on the growth rate is always positive.

The next question is what happens to key variables of interest during the transition path. Figure 1 shows an example for the small open economy model. The free trade equilibrium over the

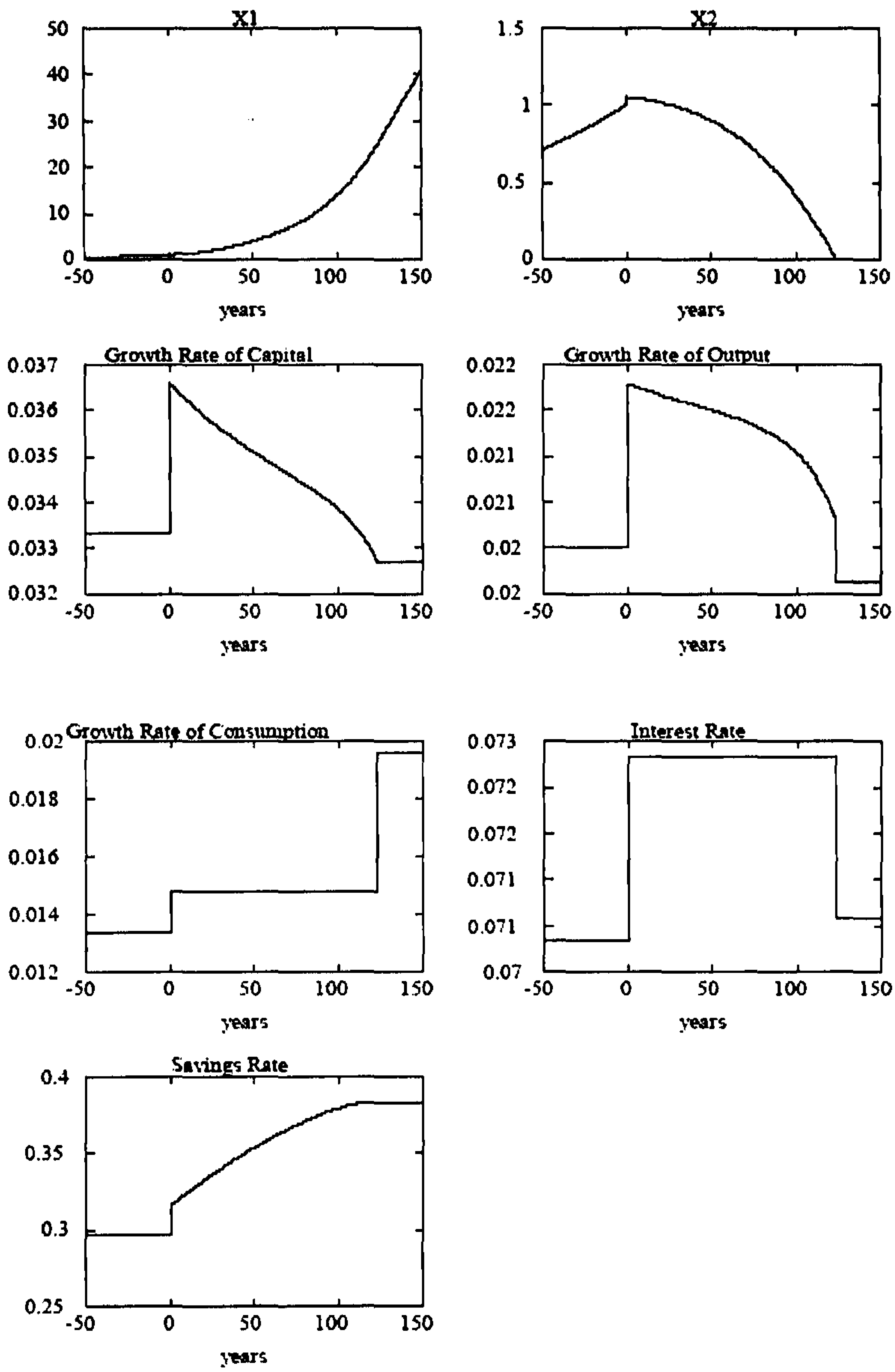

FIGURE 1
TRANSITIONAL DYNAMICS IN A SMALL OPEN ECONOMY

$=0.037$, $B=0.104$, $\delta=0.02$, $\sigma=2$, $\beta=0.5$, and $A_1=A_2=1$. This parameterization leads to an autarky steady state growth rate equal to 0.02, a steady state interest rate of 0.07, a savings rate of 0.30, and a c_1/y ratio of 0.35 in autarky. The growth rate of the world relative price is set constant such that it corresponds to a growth rate of world output of 0.017. The capital-land ratio is exogenously given such that the autarky relative price of the small economy is matched with the world relative price in period zero when international trade starts. Under this condition, static trade models predict that no trade occurs. However, this model has an additional production sector, the investment good sector. The shadow price of the investment good in this country rises during the transition path, and thus capital accumulation increases. The only way to accommodate the increase in demand for capital in the investment good production sector in period zero is for the consumption good X_1 (the capital-intensive good) to decrease and X_2 (the land-intensive good) to increase given the fixed world relative price. This is a direct result of the capital-land ratio in production sector X_1 being higher than that of production sector X_2. This is a version of the Rybczynski (1955) theorem. Thus, the production of consumption good X_1 decreases at the initial point of free trade, even though this small economy eventually specializes in producing good X_1 and the investment good. The production of consumption good X_2 jumps up at period zero.

In Figure 1, X_1 and X_2 are normalized to one at the time of starting free trade. After the initial jump, the inputs shift continuously from the production of X_2 to the production of X_1 during the transition path until the economy reaches the new steady state. It is easy to see a possibility of reversal of the trade pattern. In period zero the trade pattern depends on the relative price of consumption goods compared to that of the world economy. For example, suppose that the small open economy has a relatively large endowment of land which gives it an initial comparative

[11] This method is similar to the Time-Elimination method suggested by Mulligan and Sala-i-Martin (1991, 1993) in the sense that differential equations are solved backward. However, unlike the conventional transitional dynamics where the steady state is approached asymptotically in an infinite time, this economy reaches the new steady state in a finite time. Thus, the differential equation solution method can be applied directly to this model, starting from the steady state backward. The program is available upon request.

advantage in producing the land-intensive consumption good X_2. However, with higher capital accumulation the small economy will eventually have a comparative advantage in producing the capital-intensive good X_1. Thus, this model may explain the significant change in the trade pattern of the East Asian countries.

The growth rate of capital is higher at the starting point of free trade. The growth rate of output is also higher over the transition path, but the new steady state growth rates of output and capital are lower than those of autarky since σ is assumed to be two (that is, not a high intertemporal substitution). However, the growth rate of composite consumption defined by $c_1^{\beta}\, c_2^{1-\beta}$ is higher over the transition and in the new steady state, which means international trade is always welfare-improving. The savings rate (defined by $1 - (c_1 + p_2^w c_2)/y$) rises markedly in the new steady state as well as during the transition reflecting the incentive of high capital accumulation. The interest rate is also higher during the transition path but it follows the path of the world economy after specialization. Note that the switch to specialization takes a long (in this example, about 125 years) but finite time. Sensitivity analysis shows that a smaller difference in growth rates between the small economy and the world economy before trade results in a longer transition path.

B. A Two-Country Model

In a two-country world economy, the world relative price (p_2^w) is endogenously determined. The maximization problem of the foreign country's representative agent is as follows:

$$\max_{\{c_{1t}^*,\, c_{2t}^*,\, \dot{a}_t^*\}} \int_0^{\infty} e^{-\rho^* t} \frac{(c_{1t}^{*\beta^*} c_{2t}^{*1-\beta^*})^{1-\sigma^*}}{1-\sigma^*}\, dt,$$

subject to

$$\dot{a}_t^* \leq r_t^* a_t^* - c_{1t}^* - p_{2t}^w c_{2t}^*, \qquad c_{1t}^*,\ c_{2t}^* \geq 0 \text{ for all } t,$$

$$X_{1t}^* = A_1^*(\phi_{1t}^* Z_t^*)^{\alpha_1^*}(\psi_t^* T^*)^{1-\alpha_1^*},$$

$$X_{2t}^* = A_2^*(\phi_{2t}^* Z_t^*)^{\alpha_2^*}((1-\psi_t^*)T^*)^{1-\alpha_2^*},$$

$$I_t^* = B^*(1 - \phi_{1t}^* - \phi_{2t}^*)Z_t^*,$$

$$\dot{Z}_t^* = I_t^* - \delta^* Z_t^*,$$

$$X_{1t}^* + p_{2t}^w X_{2t}^* = c_{1t}^* + p_{2t}^w c_{2t}^*,$$

$$a_t^* = p_{zt}^* Z_t^* + p_{Tt}^* T^*,$$

$$\lim_{t \to \infty} Z_t^* u_1(c_{1t}^*, c_{2t}^*) e^{-\rho^* t} = 0,$$

where asterisks denote foreign country variables and parameters. If the tastes and technology for capital accumulation are the same across the two countries so that the growth rates of the relative price of consumption goods are the same, then international trade determines a comparative advantage for each country depending on the factor endowment ratios (the capital-land ratios) of the two countries at the time of trading. With a one-time change in the relative price of consumption goods the two countries follow the same balanced growth path as in autarky.

Consider a world economy composed of two countries with different autarky growth rates (or different capital accumulation rates) arising from differences in preferences or capital income tax rates. It is easy to show that the long-run trade pattern is determined by capital accumulation summarized by the growth rate of the relative price of the two consumption goods as follows:

$$\gamma_{p_2} = \frac{(\alpha_1 - \alpha_2)((1-\tau_z)(B-\delta) - \rho)}{1-(1-\sigma)(\beta\alpha_1 + (1-\beta)\alpha_2)},$$

$$\gamma_{p_2^*} = \frac{(\alpha_1^* - \alpha_2^*)((1-\tau_z^*)(B^* - \delta^*) - \rho^*)}{1-(1-\sigma^*)(\beta^*\alpha_1^* + (1-\beta^*)\alpha_2^*)}.$$

Thus, the determination of long-run comparative advantage is closely related to taxation on the investment good. A subsidy on the investment good sector may cause the country to have a long-run comparative advantage in the production of the capital -intensive good even though it has a long-run comparative advantage in the production of the land-intensive consumption good without the subsidy. The following proposition summarizes this argument.

Proposition 3

Assume that $\alpha_1 > \alpha_2$, $\alpha_1 = \alpha_1^*$, and $\alpha_2 = \alpha_2^*$. Under free trade of the two consumption goods, the long-run comparative advantage is determined by the relative magnitude of γ_{p_z} and $\gamma_{p_z^*}$ in autarky of the two countries. If the home country's growth rate of the relative price of the two consumption goods in autarky is higher than the foreign country's ($\gamma_{p_z} > \gamma_{p_z^*}$), then the home country produces relatively more and more of consumption good X_1 and relatively less and less of consumption good X_2 over the transition path.

Proof: Appendix.

An intuitive explanation may be provided as follows. For simplicity, suppose that the home country has a lower time preference (other differences can be analyzed in a similar fashion). Then the home country tends to accumulate capital faster than the foreign country. This higher capital accumulation leads to a higher growth rate of the relative price of the two consumption goods in autarky, which, in turn, implies a long-run comparative advantage in producing the capital-intensive good X_1. The possibility of reversal of the trade pattern can be discussed in the same way as in the small open economy model. In summary, long-run comparative advantage is determined by the growth rates of the autarky relative prices of the two consumption goods, since it contains information on all parameters describing capital accumulation.

The new steady state in the two-country open economy model is different from that of the small open economy model in the sense that the law of motion of the world market equilibrium is endogenously determined and that specialization in both countries is unlikely to happen. The new steady state is described in the following proposition.

Proposition 4

Given the assumptions of Proposition 3, the new steady state is reached in a finite time by specialization of one country and incomplete specialization of the other country. The law of motion of the world economy after specialization follows the specialized country's autarky growth path. If the home country first reaches specialization (*e.g.* in X_1 due to the long-run comparative advantage in X_1) and the foreign country incompletely specializes, then the growth rates of the two countries in the new steady state are given by the home country's autarky growth rate. If the foreign country first reaches specialization in X_2 and the home country incompletely specializes, the growth rates are given by the foreign country's autarky growth rate.

Proof: Appendix.

Table 3 summarizes the above proposition. After free trade, the countries start to produce more of the good in which they have a long-run comparative advantage. For example, a more patient country starts to produce more of consumption good X_1 and the other country produces more of X_2. The country that will completely specialize in one consumption good and the investment good depends on the convergence speed and the initial difference in endowment ratios across countries. It is worth noting that complete specialization by both countries happens only if both countries reach their specialization points simultaneously. It is interesting that after a country reaches a specialization point, the two countries follow the law of motion given by the specialized country's autarky balanced growth path. If a slowly growing country specializes first (Case 1), the growth rate of the more rapidly growing country is lower. In contrast, if a country with more rapid growth specializes first (Case 2), then the slowly growing country grows faster than in autarky. If both countries reach specialization simultaneously (Case 3), the growth rates of the two countries are higher than in autarky as long as they have a high elasticity of intertemporal substitution (*i.e.*, $\sigma < 1$ and $\sigma^* < 1$). In the new steady state after free trade, the growth rate of output is the same across both countries.

A numerical example of the transitional dynamics in the two -country open economy model is shown in Figure 2. In addition to the same parameterization as in the small open economy, the foreign country is assumed to be less patient than the home country ($\rho^* = 0.04 > \rho$). The new steady state is assumed to be specialization by the foreign country and incomplete specialization by the home country. As in the example for the small open economy, the initial endowment ratios are specified such that the

TABLE 3
SPECIALIZATION IN A TWO-COUNTRY WORLD ECONOMY

	Home Country	Foreign Country
Assumption (Autarky)		
Technology Parameters	$\alpha_1 > \alpha_2,$	$\frac{\alpha_1}{\alpha_2} = \frac{\alpha_1^*}{\alpha_2^*}$
Growth Rates of Output	$\gamma_y^A = \frac{\alpha_1 (B-\delta-\rho)}{1-(1-\sigma)[\alpha_1 \beta + \alpha_2(1-\beta)]}$	$> \gamma_{y^*}^A = \frac{\alpha_1^*(B^*-\delta^*-\rho^*)}{1-(1-\sigma^*)[\alpha_1^* \beta^* + \alpha_2^*(1-\beta^*)]}$
Growth Rates of p_2	$\gamma_{p_2}^A = \frac{(\alpha_1-\alpha_2)(B-\delta-\rho)}{1-(1-\sigma)[\alpha_1 \beta + \alpha_2(1-\beta)]}$	$> \gamma_{p_2^*}^A = \frac{(\alpha_1^*-\alpha_2^*)(B^*-\delta^*-\rho^*)}{1-(1-\sigma^*)[\alpha_1^* \beta^* + \alpha_2^*(1-\beta^*)]}$
Static Comparative Advantage		
i) $p_{2t} > p_{2t}^*$	X_1	X_2^*
ii) $p_{2t} < p_{2t}^*$	X_2	X_1^*
Long-Run Comparative Advantage	X_1	X_2^*
New Steady State		
Case 1	Incomplete Specialization	Complete Specialization
Production	X_1 X_2 I	X_2^* I^*
Growth Rates of Output	$\gamma_y^s = \gamma_{y^*}^s = \gamma_{y^*}^A =$	$\frac{\alpha_1^* (B^*-\delta^*-\rho^*)}{1-(1-\delta^*)[\alpha_1^* \beta^* + \alpha_2^*(1-\beta^*)]}$
Comparison with Autarky	$\gamma_y^s < \gamma_y^A$	$\gamma_{y^*}^s = \gamma_{y^*}^A$
Case 2	Complete Specialization	Incomplete Specialization
Production	X_1 I	X_1^* X_2^* I^*
Growth Rates of Output	$\gamma_y^s = \gamma_{y^*}^s = \gamma_y^A =$	$\frac{\alpha_1 (B-\delta-\rho)}{1-(1-\delta)[\alpha_1 \beta + \alpha_2(1-\beta)]}$
Comparison with Autarky	$\gamma_y^s = \gamma_y^A$	$\gamma_{y^*}^s > \gamma_{y^*}^A$
Case 3	Complete Specialization	Complete Specialization
Production	X_1 I	X_2^* I^*
Growth Rates of Output	$\gamma_y^s = \gamma_{y^*}^s = \frac{\alpha_1[1-\alpha_2^*(1-\beta^*)(1-\sigma^*)](B-\delta-\rho)+\alpha_1\alpha_2^*(1-\beta)(1-\sigma)(B^*-\delta^*-\rho^*)}{1-\alpha_1\beta(1-\sigma)-\alpha_2^*(1-\beta^*)(1-\sigma^*)+\alpha_1\alpha_2^*(1-\sigma)(1-\sigma^*)(\beta-\beta^*)}$	
Comparison with Autarky	ambiguous	ambiguous
if $\sigma < 1$, $\sigma^* < 1$	$\gamma_y^s > \gamma_y^A$	$\gamma_{y^*}^s > \gamma_{y^*}^A$

autarky relative prices between the two consumption goods are equal across both countries. Figure 2 shows that the patient country (the home country) produces relatively more and more of consumption good X_1 and less and less of consumption good X_2,

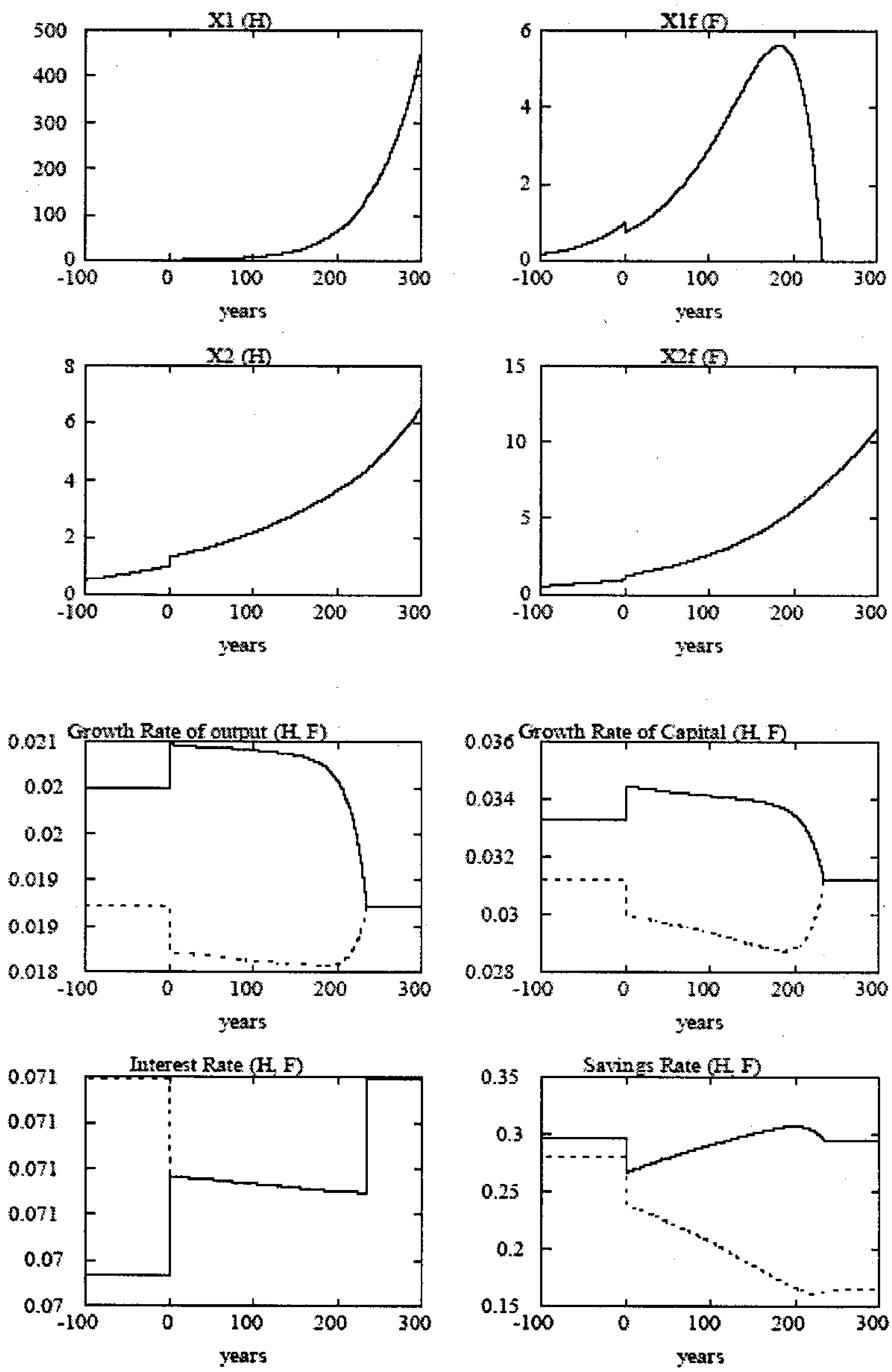

FIGURE 2

TRANSITIONAL DYNAMICS IN A TWO-COUNTRY WORLD ECONOMY

(HOME: ——, FOREIGN: - - -)

while the impatient country (the foreign country) eventually specializes in producing the land-intensive good X_2. The initial jumps of X_1^* and X_2^* in the foreign country do not seem to satisfy the argument of the Rybczynski theorem. One reason is that the relative price changes if there is a shift in resource allocation in the two countries (p_2^w jumps up in this example). This violates the condition of the fixed relative price needed for the Rybczynski theorem to hold. The source of these dynamics is the difference in capital accumulation. After free trade starts, the home country (the patient country) accumulates more capital, while the foreign country (the impatient country) accumulates less capital. These two different capital accumulation behaviors result in different growth rates of output across countries. The growth rate of the home country jumps up, while the growth rate of the foreign country falls at the start of free trade. This difference is also shown in the growth rates of capital across the two countries. After free trade, the interest rate is equalized across the two countries. The reason that factor-price equalization still holds in the specialized economy is that the specialization (defined above) is not complete in the sense that the specialized country still produces a consumption good and the investment good. In the new steady state, the growth rates of output, capital accumulation, and composite consumption of the two countries are equal to those of the specialized (here the foreign) country in autarky. The savings rate of the home country is higher in the transition path while that of the foreign country is lower.

IV. Summary and Concluding Remarks

The close relation between trade and growth has been emphasized in the recent trade literature. However, the transitional dynamics of moving from autarky to the new free trade steady state has not been fully investigated. In this paper a free trade equilibrium was analyzed in an endogenously growing economy with an investment production sector *à la* Rebelo (1991). In a closed economy the steady state growth rate depends on preference and technology parameters describing capital accumulation behavior. The economy with a lower time preference, a larger elasticity of intertemporal substitution, and more biased tastes toward capital-intensive consumption goods grows faster. Once a country opens its domestic market, the country starts to reallocate production factors depending on capital accumulation behavior, which determines the long-run comparative advantage. Along the transition path, a country with higher capital accumulation in autarky increases its use of inputs for producing a capital-intensive consumption good in which it has a long-run comparative advantage.

Transitional dynamics shows that the growth rate of output of the country with high capital accumulation tends to be higher over the transition path than that in autarky (that is, amplified through international trade). On the other hand, the country with low capital accumulation experiences a lower output growth rate over the transition path than that in autarky. Therefore, the endogenous growth model described above can explain many real economic phenomena such as the high performance of the East Asian countries with changes in patterns of trade through high capital accumulation (or savings rates).

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Appendix

Proof of Proposition 1

The whole system of the economy over the transition path is as follows.

$$\gamma_{x_1} = \alpha_1(\gamma_{\phi_1} + \gamma_z) + (1-\alpha_1)\gamma_\psi \tag{A1}$$

$$\gamma_{x_2} = \alpha_2(\gamma_{\phi_2} + \gamma_z) + (1-\alpha_2)\left(-\frac{\psi}{1-\psi}\right)\gamma_\psi \tag{A2}$$

$$\gamma_z = B(1-\phi_1-\phi_2) - \delta \tag{A3}$$

$$\begin{aligned}(\alpha_1-1)(\gamma_{\phi_1}+\gamma_z)+(1-\alpha_1)\gamma_\psi \\ = \gamma_{p_2^{w}} + (\alpha_2-1)(\gamma_{\phi_2}+\gamma_z) + (1-\alpha_2)\left(-\frac{\psi}{1-\psi}\right)\gamma_\psi \\ = \gamma_{p_z}\end{aligned} \tag{A4}$$

$$\alpha_1(\gamma_{\phi_1}+\gamma_z) - \alpha_1\gamma_\psi = \gamma_{p_2^{w}} + \alpha_2(\gamma_{\phi_2}+\gamma_z) - \alpha_2\left(-\frac{\psi}{1-\psi}\right)\gamma_\psi \tag{A5}$$

$$-\rho-(1-\beta+\beta\sigma)\gamma_{c_1}+(1-\beta)(1-\sigma)\gamma_{c_2}=-r \tag{A6}$$

$$-\rho+\beta(1-\sigma)\gamma_{c_1}-(\beta+\sigma-\beta\sigma)\gamma_{c_2}=\gamma_{p_2^{w}}-r \tag{A7}$$

$$r=R_z+\gamma_{p_z}=B-\delta+\gamma_{p_z} \tag{A8}$$

$$\frac{\alpha_1\phi_2}{\alpha_1\phi_2+\alpha_2\phi_1}(\gamma_{\phi_2}-\gamma_{\phi_1})+\gamma_{x_1}=\gamma_{c_1} \tag{A9}$$

All variables indicated by γ are growth rates (*e.g.* $\gamma_{x_1}=\dot{X}_{1t}/X_{1t}$), and in general, all variables except parameters depend on time t; for notational convenience, the time subscript t is dropped. With the assumption of a small open economy, $\gamma_{p_2^{w}}$ is set to a fixed constant. From (A4) and (A5)

$$\gamma_{\phi_1}-\gamma_{\phi_2}=\frac{\psi}{1-\psi}\gamma_{\psi}. \tag{A10}$$

Substituting this into (A4) and (A5) gives

$$\gamma_{\phi_1}=-\gamma_z+\gamma_{\psi}+\frac{1}{\alpha_1-\alpha_2}\gamma_{p_2^{w}},$$

$$\gamma_{\phi_2}=-\gamma_z-\frac{\psi}{1-\psi}\gamma_{\psi}+\frac{1}{\alpha_1-\alpha_2}\gamma_{p_2^{w}}.$$

Plugging these relations into (A1) and (A2),

$$\gamma_{x_1}=\gamma_{\psi}+\frac{\alpha_1}{\alpha_1-\alpha_2}\gamma_{p_2^{w}},$$

$$\gamma_{x_2}=-\frac{\psi}{1-\psi}\gamma_{\psi}+\frac{\alpha_2}{\alpha_1-\alpha_2}\gamma_{p_2^{w}}.$$

From (A6) and (A7), $\gamma_{c_1}=\gamma_{c_2}+\gamma_{p_2^{w}}$ and from (A4), $\gamma_{p_z}=(\alpha_1-1)/(\alpha_1-\alpha_2)\gamma_{p_2^{w}}$; substituting these into (A7) and combining (A7) and (A8) give

$$\gamma_{c_1}=\frac{1}{\sigma}\{(B-\delta-\rho)+[\beta(1-\sigma)+\sigma-\frac{1-\alpha_2}{\alpha_1-\alpha_2}]\gamma_{p_2^{w}}\}.$$

Then, from (A9) γ_ψ is given by

$$\gamma_\psi = \frac{\frac{1}{\sigma}\{(B-\delta-\rho)-\frac{1}{\alpha_1-\alpha_2}[1-(1-\sigma)(\beta\alpha_1+(1-\beta)\alpha_2)]\gamma_{p_2^{W}}\}}{1-\frac{\alpha_1\phi_2}{(1-\psi)(\alpha_1\phi_2+\alpha_2\phi_1)}},$$

$$= C \cdot (\gamma_{p_2^A}-\gamma_{p_2^W})$$

where C is a positive constant since from the efficiency conditions (9),

$$\frac{\phi_1}{\phi_2} = \frac{\alpha_1\psi(1-\alpha_2)}{\alpha_2(1-\psi)(1-\alpha_1)} > \frac{\alpha_1}{\alpha_2}(\frac{\psi}{1-\psi}).$$

This relation implies $\gamma_\psi > 0$ over the transition path with the assumption of $\gamma_{p_2^A} > \gamma_{p_2^W}$ in autarky. Then, it follows that $\gamma_{X_1} > 0$ and $\gamma_{X_2} < 0$. Therefore, this economy produces more and more X_1 and less and less X_2 over the transition path. Eventually this economy specializes in producing good X_1. Also, from the efficiency condition (9),

$$\psi = \frac{(1-\alpha_1)\alpha_2\phi_1}{(1-\alpha_2)\alpha_1\phi_2+(1-\alpha_1)\alpha_2\phi_1},$$

$$\lim_{\psi\to 1}\gamma_\psi = \lim_{\phi_2\to 0}\gamma_\psi = C' \cdot \frac{1-\alpha_2}{\alpha_1-\alpha_2}(\gamma_{p_2^A}-\gamma_{p_2^W}) > 0,$$

where C' is a positive constant. This implies that specialization is reached in a finite time.

Proof of Proposition 2

The new steady state growth rate of output after specialization can be derived as follows. First, if the small open economy specializes in X_1 due to its long-run comparative advantage in X_1 (that is, $\gamma_{p_2^A} > \gamma_{p_2^W}$ in autarky), then the new steady state growth rate of output after specialization is given by

$$\gamma_y^s = \frac{\alpha_1\{(B-\delta-\rho)-(1-\beta)(1-\sigma)\gamma_{p_2^W}\}}{1-\alpha_1(1-\sigma)}.$$

This growth rate is higher than γ_y^A in autarky given by equation (11) under the assumption that $\alpha_1 > \alpha_2$ if and only if $\sigma < 1$. Second, if the small open economy specializes in X_2 due to its long-run comparative advantage in X_2 ($\gamma_{p_2^A} < \gamma_{p_2^w}$ in autarky), then the new steady state growth rate of output is derived as

$$\gamma_y^S = \frac{\alpha_2\{(B-\delta-\rho)+[1-\alpha_2(1-\beta)(1-\sigma)]\gamma_{p_2^w}}{1-\alpha_2(1-\sigma)},$$

which is always higher than the autarky growth rate γ_y^A given by equation (11).

Proof of Proposition 3

The whole system of transition paths for the home country is described by the equations (A1) - (A9). The transition path of the foreign country is given as follows.
Foreign country:

$$\gamma_{x_1^*} = \alpha_1^*(\gamma_{\phi_1^*} + \gamma_{z^*}) + (1-\alpha_1^*)\gamma_{\psi^*}$$

$$\gamma_{x_2^*} = \alpha_2^*(\gamma_{\phi_2^*} + \gamma_{z^*}) + (1-\alpha_2^*)\left(-\frac{\psi^*}{1-\psi^*}\right)\gamma_{\psi^*}$$

$$\gamma_{z^*} = B^*(1-\phi_1^*-\phi_2^*) - \delta^*$$

$$\begin{aligned}(\alpha_1^*-1)(\gamma_{\phi_1^*} + \gamma_{z^*}) &+ (1-\alpha_1^*)\gamma_{\psi^*} \\ &= \gamma_{p_2^w} + (\alpha_2^*-1)(\gamma_{\phi_2^*} + \gamma_{z^*}) + (1-\alpha_2^*)\left(-\frac{\psi^*}{1-\psi^*}\right)\gamma_{\psi^*} \\ &= \gamma_{p_z^*}\end{aligned}$$

$$\begin{aligned}\alpha_1^*(\gamma_{\phi_1^*} &+ \gamma_{z^*}) - \alpha_1^*\gamma_{\psi^*} \\ &= \gamma_{p_2^w} + \alpha_2^*(\gamma_{\phi_2^*} + \gamma_{z^*}) - \alpha_2^*\left(-\frac{\psi^*}{1-\psi^*}\right)\gamma_{\psi^*}\end{aligned}$$

$$-\rho^* - (1-\beta^*+\beta^*\sigma^*)\gamma_{c_1^*} + (1-\beta^*)(1-\sigma^*)\gamma_{c_2^*} = -r_c^*$$

$$-\rho^* + \beta^*(1-\sigma^*)\gamma_{c_1^*} - (\beta^*+\sigma^*-\beta^*\sigma^*)\gamma_{c_2^*} = \gamma_{p_2^w} - r_c^*$$

$$r_c^* = R_z^* + \gamma_{p_z^*} = B^* - \delta^* + \gamma_{p_z^*}$$

$$\frac{\alpha_1^* \phi_2^*}{\alpha_1^* \phi_2^* + \alpha_2^* \phi_1^*} (\gamma_{\phi_2^*} - \gamma_{\phi_1^*}) + \gamma_{x_1^*} = \gamma_{c_1^*}$$

For notational convenience, it is assumed that the difference in autarky growth rates of the two countries comes from a difference in time preference rates ($\rho \neq \rho^*$). This simplification can be easily extended to the general case where any parameter describing capital accumulation behavior can be different across the two countries. The world market equilibrium condition gives

$$p_2^w = \left(\frac{1-\beta}{\beta}\right) \frac{X_1 + X_1^*}{X_2 + X_2^*}.$$

Then, the growth rate of the relative price of the consumption good X_2 is given by

$$\gamma_{p_2^w} = \theta_1 \gamma_{x_1} + \theta_1^* \gamma_{x_1^*} - \theta_2 \gamma_{x_2} - \theta_2^* \gamma_{x_2^*},$$

where p_2^w is the world relative price, $\gamma_{p_2^w}$ is the growth rate of world relative price and θ_1, θ_1^*, θ_2, and θ_2^* are the production shares of each country in world production of goods 1 and 2 ($\theta_1 + \theta_1^* = 1$, $\theta_2 + \theta_2^* = 1$).

As in the proof of Proposition 1, the following relations are derived.

$$\gamma_{\phi_1} - \gamma_{\phi_2} = \frac{1}{1-\phi} \gamma_\phi, \quad \gamma_{\phi_1^*} - \gamma_{\phi_2^*} = \frac{1}{1-\phi^*} \gamma_{\phi^*}$$

$$\gamma_{x_1} = \gamma_\phi + \frac{\alpha_1}{\alpha_1 - \alpha_2} \gamma_{p_2^w}, \quad \gamma_{x_1^*} = \gamma_{\phi^*} + \frac{\alpha_1}{\alpha_1 - \alpha_2} \gamma_{p_2^w}$$

$$\gamma_{x_2} = -\frac{\phi}{1-\phi} \gamma_\phi + \frac{\alpha_2}{\alpha_1 - \alpha_2} \gamma_{p_2^w}, \quad \gamma_{x_2^*} = -\frac{\phi^*}{1-\phi^*} \gamma_{\phi^*} + \frac{\alpha_2}{\alpha_1 - \alpha_2} \gamma_{p_2^w}$$

$$\frac{\gamma_\phi}{\gamma_{\phi^*}} = \frac{\theta_1^* + \theta_2^* \left(\frac{\phi^*}{1-\phi^*}\right)}{\theta_1 + \theta_2 \left(\frac{\phi}{1-\phi}\right)}$$

$$\gamma_{p_2} = \frac{\alpha_1 - 1}{\alpha_1 - \alpha_2}\, \gamma_{p_2^{w}} = \gamma_{p_2^*}$$

$$\gamma_{p_2^{w}} = \frac{(\alpha_1-\alpha_2)(B-\delta-\rho) - \sigma(\alpha_1-\alpha_2)\left[1 - \frac{\alpha_1\phi_2}{(1-\psi)(\alpha_1\phi_2+\alpha_2\phi_1)}\right]\gamma_\psi}{1-(1-\sigma)(\alpha_1\beta+\alpha_2(1-\beta))}$$

$$= \frac{(\alpha_1-\alpha_2)(B-\delta-\rho^*) - \sigma(\alpha_1-\alpha_2)\left[1 - \frac{\alpha_1\phi_2^*}{(1-\psi^*)(\alpha_1\phi_2^*+\alpha_2\phi_1^*)}\right]\gamma_{\psi^*}}{1-(1-\sigma)(\alpha_1\beta+\alpha_2(1-\beta))}$$

$$\gamma_\psi = \frac{\frac{1}{\sigma}(\rho^*-\rho)}{\left[1-\frac{\alpha_1\phi_2}{(1-\psi)(\alpha_1\phi_2+\alpha_2\phi_1)}\right] + \frac{\theta_1+\theta_2\left(\frac{\psi}{1-\psi}\right)}{\theta_1^*+\theta_2^*\left(\frac{\psi^*}{1-\psi^*}\right)}\left[1-\frac{\alpha_1\phi_2^*}{(1-\psi^*)(\alpha_1\phi_2^*+\alpha_2\phi_1^*)}\right]}$$

$$= C'' \cdot (\gamma_{p_2} - \gamma_{p_2^*}),$$

where C'' is a positive constant. As in the proof of Proposition 1, from these relations, it follows that $\gamma_\psi > 0$, $\gamma_{\psi^*} < 0$, $\gamma_{p_2^{w}} > 0$, and $\gamma_{\phi_1} > \gamma_{\phi_2}$, $\gamma_{\phi_1^*} < \gamma_{\phi_2^*}$. Therefore, the home country uses a higher proportion of factors in producing good X_1 relative to good X_2 over the transition path.

Proof of Proposition 4

The proposition is proved in two steps. Assume that the new steady state described by a complete specialization of the home country in the consumption good X_1 and the investment good I and an incomplete specialization of the foreign country in the two consumption goods and the investment good (Case 2 in Table 3). First, it will be shown that the steady state is reached in a finite time, and second, it will be proved that there is a steady state (balanced growth path) with the specialization described above.

It can be proved in a similar way to the proof of Proposition 1 that the steady state is reached in a finite time. From Proposition 3, we get the expression for the growth rate of the fraction of land in producing consumption good 2. As the economy approaches the steady state, the growth rate converges to a constant, which implies that specialization is reached in a finite time.

$$\lim_{\psi \to 1} \gamma_{\psi} = \lim_{\phi_2 \to 0} \gamma_{\psi} =$$

$$\frac{\frac{1}{\sigma}(\rho^* - \rho)(1-\alpha_2)}{\alpha_1 - \alpha_2 + (1-\alpha_2)\theta_1 + \frac{A_2 Z^{\alpha_2} T^{1-\alpha_2}}{\theta_1^* + (\frac{\psi^*}{1-\psi^*})} (\frac{\alpha_2(1-\alpha_1)\phi_1}{\alpha_1(1-\alpha_2)})^{\alpha_2} [1 - \frac{\alpha_1 \phi_2^*}{(1-\psi^*)(\alpha_1 \phi_2^* + \alpha_2 \phi_1^*)}]} > 0.$$

The next step of the proof is to show that there exists a steady state balanced growth path after specialization. Assuming the home country produces consumption good 1 and the investment good and the foreign country produces consumption goods 1, 2, and investment good, a steady state balanced growth path can be derived as follows. The balanced growth path is described by the equations (A1)-(A9) for the home country except that the growth rates of fraction variables are zero and the consumption good 2 is not produced any more and the similar equations for the foreign country. The world market equilibrium condition requires

$$\gamma_{p_2^w} = \theta_1 \gamma_{x_1} + \theta_1^* \gamma_{x_1^*} - \gamma_{x_2^*}.$$

Then, we get the growth rate of the new steady state balanced growth path as

$$\gamma_y^s = \gamma_{y^*}^s = \gamma_y^A = \frac{\alpha_1(B - \delta - \rho)}{1-(1-\delta)[\alpha_1 \beta + \alpha_2(1-\beta)]}.$$

The balanced growth paths in Case 1 and Case 3 in Table 3 can also be derived in the similar way.

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