Abstract

The Economics of Child Labor Revisited : A Bargaining Approach

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Gupta (2000) considered child worker's wage determination process as a Nash bargaining game between the employer and the guardian of the child worker. In our paper, we first solve Gupta's bargaing problem with minimum wage by using Nash solution and Kalai Smorodinsky solution. We compare these two solutions to examine the effect of minimum wage policy: which solution type is favorable to the guardian depending on the amount of the minimum wage. Then, we develop Gupta's model by considering the guardian's altruistic concern toward the child worker. In this new model, with regard to the child labor supply, we find consistency with Basu and Van (1998)'s luxury axiom. We also show that the subsidy provided to the poor guardian may induce Pareto improvement.

Keywords: Child labor; Nash bargaining game; Kalai Smorodinsky solution; Altruism; Minimum wage policy; Subsidy policy.

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Contents

List of Figures

List of Tables

1 Introduction

Child labor market is an unusual market in a sense that the decision of labor supply is not determined by child workers, but usually by the parents of the child. Most research on child labor pays attention to this feature and cares about what they have assumed on household behavior. On the other hand, since firms consider child labor as a factor of production, assumptions on child labor production play an important role.

Basu and Van (1998) have captured these two features and established a model analyzing child labor. Luxury axiom, which means that child labor occurs only when the household income from non-child labor is lower than a certain level, is an assumption on household behavior. Substitution axiom, which means that adult labor and child labor are substitutes from the firm's point of view, is an assumption on labor market.

Since these features had priority in analyzing child labor, most research on child labor has followed traditional market equilibrium concepts. Basu and Van (1998) and Baland and Robinson (2000), the most influential papers in this area, assume perfectly competitive labor market, which implies that a child worker is paid as much as what he has produced. Many papers analyzing child labor use perfectly competitive labor market assumptions since it makes the analysis simple and follows the tradition.

There has been an attempt to modify this perfectly competitive market assumption. Grossmann and Michaelis (2007) have used monopolistic competition model. Bhalotra and Heady (2003) have suggested failures of the markets for labor and land in order to explain their empirical findings.

In labor economics literature, Nash bargaining game is commonly used in wage determination. Nash (1950) proposed theoretical discussion of two person bargaining problem. McDonald and Solow (1981) provided a formal bargaining model of wage determination between the employer and the labor union. Starting from Diamond (1981) and Mortensen (1982a, 1982b), analysis that uses a matching function for determining meetings and bargaining for determining wages became popular.[1]

Based on this literature, Gupta (2000) suggested to use a simple Nash bargaining game in determining the child workers' wage. In his model, the employer and the guardian of the child have a certain amount of bargaining power, so that they jointly determine the wage of the child. Moreover, he analyzed his model with a minimum wage and with an adult labor market respectively. Although Gupta's framework is well designed, there are few succeeding works analyzing child labor using a bargaining approach.

This paper modifies two features of Gupta's model. First, Gupta (2000) uses Nash solution that maximizes the product of each player's payoff in order to solve Nash bargaining problem. Kalai and Smorodinsky (1975) provided another solution type that solves Nash bargaining problem. Both Nash and Kalai Smorodinsky solutions are most popular solution types of Nash bargaining problem. In the first part of this paper, we will solve Gupta's bargaining problem by using Kalai Smorodinsky solution and compare this result with Nash solution. In particular, we will pay attention to the bargaining problem with a minimum wage and compare these solutions with respect to the minimum wage level.

The second feature is that nobody cares about the child's welfare in Gupta's model. This point has been much criticized since the parents just treat their

[1] See Osborne and Rubinstein (1990) and Rogerson, Shimer, and Wright (2005).

child as a tool for earning household income.[2] Most research analyzing child labor considers parents' altruistic concern about their child to some extent. In the second part of this paper, we will provide a new bargaining model considering parents' altruism toward their child. Then, we will also solve this bargaining model by using Nash solution and Kalai Smorodinsky solution and compare these two solutions.

In section 2, we will introduce Gupta (2000)'s bargaining game. Then, we will solve the bargaining problem by using two representative solution concepts, Nash solution and Kalai Smorodinsky solution, and compare two solutions, in section 3 and 4. In section 5, we will provide a new bargaining model, and then we will also solve the new bargaining problem by using two solution types and compare these two solutions in section 6.

2 Model

2.1 Bargaining problem without minimum wage

In this section, we will see a Nash bargaining problem proposed by Gupta (2000). There are two players (the employer and the guardian of the child worker) in the bargaining game. Let w be the wage of the child worker. Part of the wage, t, is used for the provision of meal to the child. Assume $0 \leq t \leq w$.[3] The rest of the wage payment $w-t$ is transferred to the guardian. We assume that the amount of meal consumption determines child labor output under the production function $f(t)$, which is increasing and concave, that is, $f' > 0$, $f'' < 0$.

[2] See Basu (1999) and Krueger and Donohue (2005).

[3] Gupta(2000) notated t as λw, where λ is the fraction of the wage given to the child worker and $1 \geq \lambda \geq 0$.

Given w and t, the employer's profit, E, is given by $E = f(t) - w$.[4] The income of the guardian, G, is given by $G = w - t$. If the employer and the guardian do not reach an agreement, they earn zero payoff, that is, the disagreement point is $(E^N, G^N) = (0, 0)$.

The employer and the guardian need to reach an agreement on w and t, through bargaining. Their feasible set, $\mathcal{F}$, is the set of (E, G) that can be realized by a pair (w, t) such that $E = f(t) - w$ and $G = w - t$;

$$\mathcal{F} = \{(E, G) \,|\, E = f(t) - w,\, G = w - t,\, E \geq 0,\, G \geq 0\}.$$

Individual rationality requires that $E \geq 0$, $G \geq 0$, which is $f(t) \geq w \geq t \geq 0$. Then, we also define the bargaining set, $\mathcal{B}$, the efficient subset of $\mathcal{F}$, as the set of pareto optimal elements of the bargaining set $\mathcal{B}$;

$$\mathcal{B} = \left\{(E, G) \in \mathcal{F} \,\middle|\, \nexists (E', G') \in \mathcal{F} \text{ s.t. } (E', G') \neq (E, G),\, E' \geq E \text{ and } G' \geq G \right\}.$$

We can obtain $\mathcal{B}$ by maximizing the joint payoff $E + G = f(t) - t$, which drives t to be equal to t^* satisfying $f'(t^*) = 1$.[5] Then, the bargaining set is the set of pairs (E, G) satisfying $E + G = f(t^*) - t^*$, that is,

$$\mathcal{B} = \{(E, G) \in \mathcal{F} \,|\, E + G = f(t^*) - t^*,\, E \geq 0,\, G \geq 0\}.$$

Therefore, the bargaining set is the line which slope is -1 and the bargaining game is symmetric. This is illustrated in Figure 2.1.

[4] We assume output price is given by 1.
[5] Assume that $f(t^*) - t^* \geq 0$.

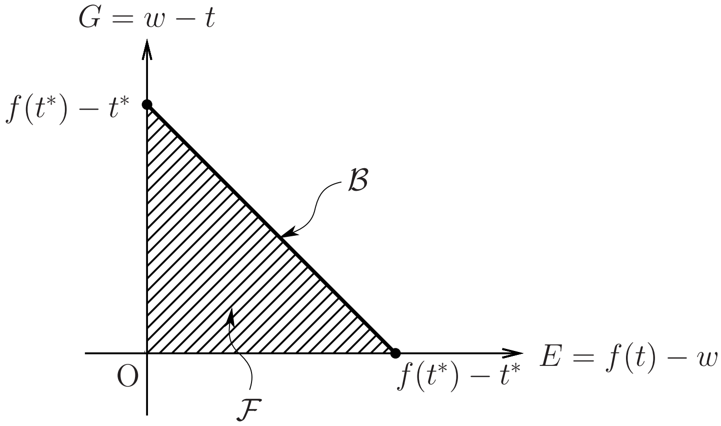

Figure 2.1: The bargaining set without minimum wage

2.2 Bargaining problem with minimum wage

Gupta(2000) introduced two variations of this model. The first variation is the imposition of the minimum wage and the other variation is the introduction of the adult labor market. In this paper, we will only look at the first case.

Let the minimum wage be $\bar{w}$, that is, $w \geq \bar{w}$. Then, we should redefine the feasible set, $\mathcal{F}(\bar{w})$, as follows,

$$\mathcal{F}(\bar{w}) = \{(E, G) \,|\, E = f(t) - w,\, G = w - t,\, 0 \leq t \leq w \leq f(t),\, w \geq \bar{w}\}\,.$$

When w is less than or equal to t^*, $E + G = f(t) - t$ is less than or equal to $f(t^*) - t^*$ because $t^* \geq w \geq t$ and $f'(t^*) = 1$. Therefore, the imposition of $\bar{w} \leq t^*$ does not affect the bargaining set, i.e., $\mathcal{B}(\bar{w}) = \mathcal{B}$.[6]

When we impose $\bar{w} > t^*$, for any $\bar{w} > t \geq t^*$ and sufficiently small $\epsilon > 0$, $(f(t) - \bar{w} + \epsilon,\, \bar{w} - \epsilon - t)$ is no longer in the feasible set, so the feasible set

[6]For a pair (E, G), which is realized by $w < \bar{w}$ and $0 \leq t \leq w$, there may or may not exists $w' \geq \bar{w}$ and $0 \leq t' \leq w'$ realizing (E, G). If there does not exist such (w', t'), the bargaining set shrinks. However, even in this case, if $\bar{w} \leq t^*$, the frontier remains same and the deleted area does not play a critical role.

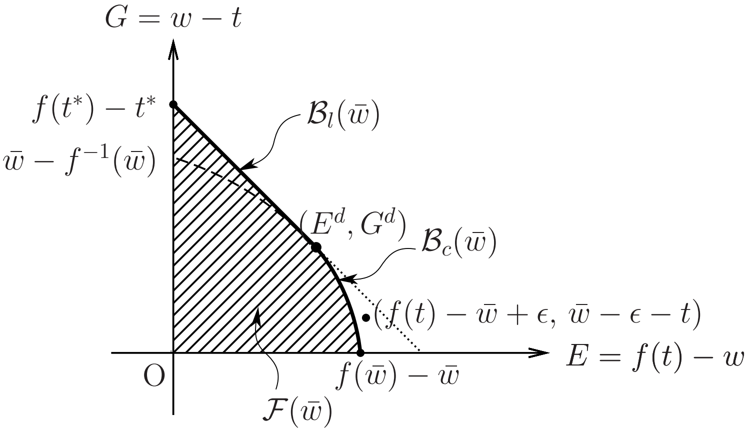

Figure 2.2: The bargaining set with minimum wage

reduces.[7] $\mathcal{B}(\bar{w})$, the bargaining set, is composed of two parts - a line part, $\mathcal{B}_l(\bar{w})$, and a curve part, $\mathcal{B}_c(\bar{w})$, i.e.,

$$\mathcal{B}(\bar{w}) = \mathcal{B}_l(\bar{w}) \cup \mathcal{B}_c(\bar{w}).$$

$\mathcal{B}_l(\bar{w})$ is the set of pairs (E, G) of which the sum is equal to $f(t^*) - t^*$, i.e.,

$$\mathcal{B}_l(\bar{w}) = \{(E, G) \in \mathcal{F} \mid E + G = f(t^*) - t^*,\ f(t) \geq w \geq \bar{w}\}.$$

$\mathcal{B}_c(\bar{w})$ is the set of pairs (E, G) that is realized by $\bar{w}$ and $t \geq t^*$, i.e.,

$$\mathcal{B}_c(\bar{w}) = \{(E, G) \in \mathcal{F} \mid w = \bar{w} \text{ and } w \geq t \geq t^*\}.$$

In the line part, t is fixed to t^* and w varies on $[\bar{w},\ f(t)]$. On the other

[7] If there exists (w', t') realizing $(f(t) - \bar{w} + \epsilon, \bar{w} - \epsilon - t)$, $f(t') - t' = f(t) - t$ should be satisfied. From this, t' is equal to t or $t^* > t'$. If $t' = t$, w' is equal to $\bar{w} - \epsilon$, which is less than the minimum wage, $\bar{w}$. If $t^* > t'$, $f(t') - w' = f(t) - \bar{w} + \epsilon$, so that $w' = \bar{w} + f(t') - f(t) - \epsilon$, which is less than the minimum wage, $\bar{w}$, since $t \geq t^* > t'$. Therefore, $(f(t) - \bar{w} + \epsilon,\ \bar{w} - \epsilon - t)$ is not in the bargaining set anymore.

hand, in the curve part, w is fixed to $\bar{w}$ and t varies on $[t^*,\, \bar{w}]$. We consider (E^d, G^d), which is realized by $\bar{w}$ and t^*, i.e.,

$$(E^d, G^d) = (f(t^*) - \bar{w}, \bar{w} - t^*).$$

We will call this point 'the division point', since this point divides $\mathcal{B}_l(\bar{w})$ and $\mathcal{B}_c(\bar{w})$. Moreover, (E^d, G^d) is the only element that belongs to the intersection of $\mathcal{B}_l(\bar{w})$ and $\mathcal{B}_c(\bar{w})$. This is illustrated in Figure 2.2.

3 Nash Solution

Nash(1950) proposed a bargaining problem and the solution concept of this problem. Nash solution maximizes the product of two individuals' payoffs and this solution is the unique solution satisfying two axioms - independence of irrelevant alternatives (IIA) and scale independence. Independence of irrelevant alternatives axiom characterizes that the reduction of the feasible set does not change the solution of the bargaining problem if the solution still remains in the reduced feasible set.

In this section, we will look at the Nash solution of the bargaining problem suggested in the previous section. The results of this section is mainly referred from Gupta(2000).

3.1 Nash solution of the bargaining problem without minimum wage

As mentioned in the previous section, the bargaining problem without minimum wage is symmetric and this induces equal payoffs for the employer and the guardian in Nash bargaining solution. This is illustrated in Figure 3.1.

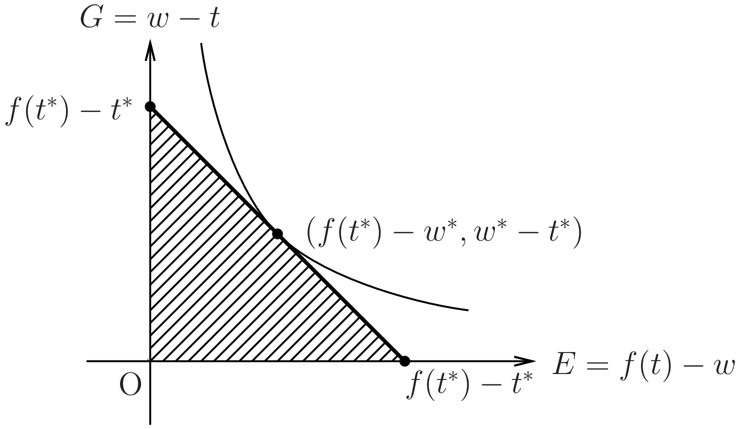

Figure 3.1: Nash solution in the bargaining game without minimum wage

Therefore, $E^* = G^* = \frac{1}{2}\{f(t^*) - t^*\}$ and from this we can derive that

$$\frac{w^*}{t^*} = \frac{1}{2}\left\{\frac{f(t^*)}{t^*} + 1\right\}. \tag{3.1}$$

This result is consistent with McDonald and Solow(1981)'s condition of wage determination in the bargaining model in the sense that the effective cost ($\frac{w^*}{t^*}$) is equal to the mean of the average productivity ($\frac{f(t^*)}{t^*}$) and the marginal productivity ($1 = f'(t^*)$).

3.2 Nash solution of the bargaining problem with minimum wage imposition

3.2.1 When $0 \leq \bar{w} \leq w^*$

If we introduce minimum wage constraint, $w \geq \bar{w}$, the feasible set reduces by eliminating the subset of the feasible set that can only be obtained by agreeing with the wage below $\bar{w}$. Reduction of the feasible set reminds us of the IIA axiom. When the minimum wage is less than or equal to w^*, the original

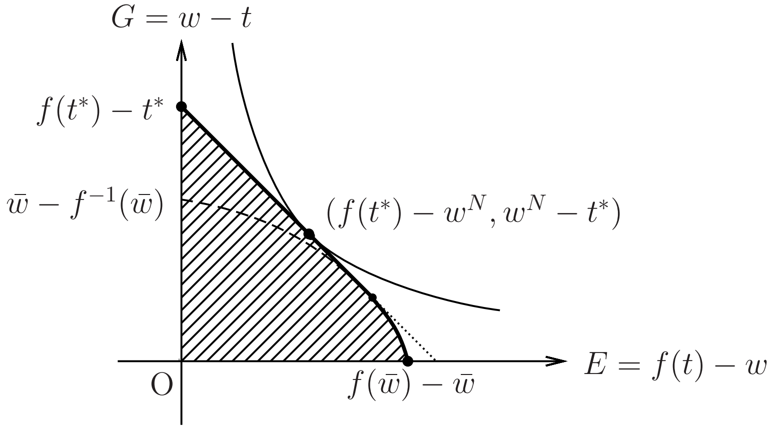

Figure 3.2: Nash solution in the bargaining game with minimum wage $\bar{w} \leq w^*$

game's Nash solution (E^*, G^*) is still in the feasible set so that the modified game's Nash solution (E^N, G^N) is equal to (E^*, G^*) and the equilibrium wage w^N is equal to w^*. Figure 3.2 shows this case. This result implies that the minimum wage policy does not influence the Nash solution at all.

3.2.2 When $w^* < \bar{w}$

When the minimum wage is above w^*, IIA axiom does not play any role since (E^*, G^*) is no longer in the feasible set. In order to obtain the Nash solution in this case, E and G will solve

$$(w^N,\, t^N) = \arg \max_{\substack{w \geq \bar{w} \\ w \geq t \geq 0}} (f(t) - w)(w - t).$$

Note that Nash solution is on the bargaining set. Thus, it is on either $\mathcal{B}_l(\bar{w})$ or $\mathcal{B}_c(\bar{w})$. When $\bar{w} > w^*$, the point on $\mathcal{B}_l(\bar{w})$ that maximizes $E \cdot G$ is the division point (E^d, G^d), the intersection of $\mathcal{B}_l(\bar{w})$ and $\mathcal{B}_c(\bar{w})$. This implies that Nash solution is on $\mathcal{B}_c(\bar{w})$. We can see this from Figure 3.3. Since w is fixed

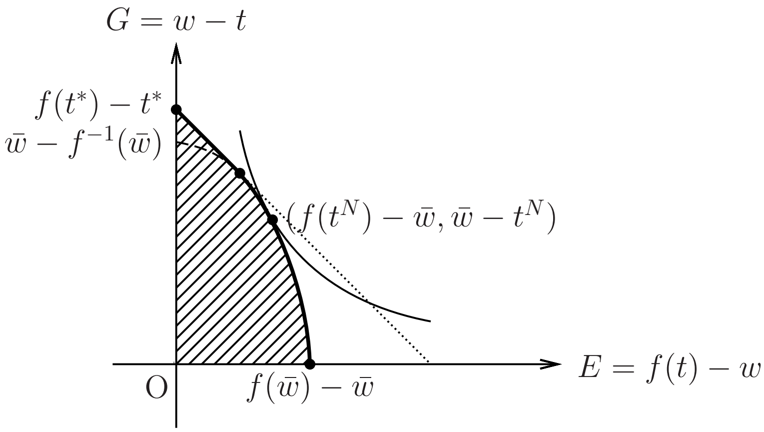

Figure 3.3: Nash solution in the bargaining game with minimum wage $\bar{w} > w^*$

to $\bar{w}$ and t varies on $\mathcal{B}_c(\bar{w})$, the optimal w^N is $\bar{w}$. Therefore, the maximization problem turns to

$$\max_{\bar{w} \geq t \geq 0} (f(t) - \bar{w})(\bar{w} - t).$$

Then, the first order condition on t is

$$\frac{\bar{w}}{t} = \frac{f'(t) + \frac{f(t)}{t}}{f'(t) + 1}. \tag{3.2}$$

This equation implies that the effective cost ($\frac{\bar{w}}{t}$) is equal to $\frac{MP+AP}{MP+1}$. In order to solve t in terms of $\bar{w}$, let us define a function $g(t)$ as follows;

$$g(t) = \frac{f(t) + t \cdot f'(t)}{f'(t) + 1}.$$

Then, by differentiating g, we can derive that g is an increasing function as long as $f(t) \geq t$, which is true from the assumption $f(t) \geq w \geq t$. Thus, the inverse function of g exists and we can get the optimal t^N as follows;

$$t^N = g^{-1}(\bar{w}). \tag{3.3}$$

Since $g(t^*)$ is equal to w^*, the optimal t^N is greater than t^*. Moreover, because g is increasing, the optimal t^N increases as $\bar{w}$ increases. The joint income of the employer and the guardian $E+G$ is $f(t^N) - t^N$ and it declines as t^N increases since $f'(t^N) < 1$ from $t^N > t^*$. Employer's payoff is $E^N = f(t^N) - \bar{w}$ and this can be written as $E^N = f(t^N) - g(t^N)$. By differentiating E^N once with respect to t^N and using $f'(t^N) < 1$, we can derive that the employer's payoff decreases as t^N increases.[8] These results imply that as the wage bound $\bar{w}$ rises the joint income and the employer's payoff become smaller.

Guardian's payoff is $G^N = \bar{w} - t^N = g(t^N) - t^N$ and increases only when $g'(t^N) > 1$, which is equivalent to $f'^2 + (t^N - f) \cdot f'' > 1$. The inequality is satisfied around t^* since $f'(t^*) = 1$ and $(t-f) \cdot f'' > 0$. However, the inequality may not be satisfied as t^N increases, and this induces the guardian's payoff to decrease. The following proposition summarize preceding results.

Proposition 1. *When the minimum wage $\bar{w}$ is imposed on the child labor bargaining model and the solution type of the bargaining game is Nash bargaining solution,*

1. *if $0 \leq \bar{w} \leq w^*$, then $w^N = w^*$ and $t^N = t^*$, and thus everything remains the same as $\bar{w}$ varies.*

2. *if $w^* < \bar{w}$, then $w^N = \bar{w}$ and $t^N = g^{-1}(\bar{w})$. As $\bar{w}$ increases, wage (w) and meal provided to the child worker (t) increase and the employer's payoff (E) and the sum of two indiviiduals' payoffs $(E+G)$ decrease. Guardian's payoff increases near w^*, however it may decrease as $\bar{w}$ becomes larger.*

[8] By differentiating E^N by t^N, $\frac{dE^N}{dt} = f' - \frac{2f'^2+2f'+(t-f)f''}{(f'+1)^2} = \frac{f'^3-f'-(t-f)f''}{(f'+1)^2} < 0$.

where $g(t) = \frac{f(t)+t\cdot f'(t)}{f'(t)+1}$.

4 Kalai Smorodinsky Solution

Although Nash solution is a representative solution of the bargaining problem, it is not a unique solution. As McDonald and Solow (1981) have pointed out, IIA axiom has been much criticized. Many researchers believed that the deleted alternatives may be concerned with determining the bargaining power of a player, since the notion of "bargaining power" may include what one can do alternatively.

As an alternative of IIA axiom, Kalai and Smorodinsky (1975) introduced monotonicity axiom, which is based on this philosophy. The monotonicity axiom says that as the maximum of the player's possible utility increases, the player should be paid more. Kalai Smorodinsky solution is the unique solution that satisfies the monotonicity axiom, pareto optimality, symmetry, and the invariance with respect to affine transformations. In their solution, the bargaining power of each player is determined by the maximum of the player's possible utility. In practice, when the employer's maximum payoff is $\hat{E}$ and that of the guardian is $\hat{G}$, we draw a line connecting the disagreement point $(0,0)$ and $(\hat{E},\hat{G})$. Hereafter we will call this line 'the ratio line'. Kalai Smorodinsky solution is the intersection of this line and the bargaining set.

4.1 Kalai Smorodinsky solution of the bargaining problem without minimum wage

We will solve Gupta's bargaining problem by using Kalai and Smorodinsky solution concept and compare this with the Nash solution. Now, let us de-

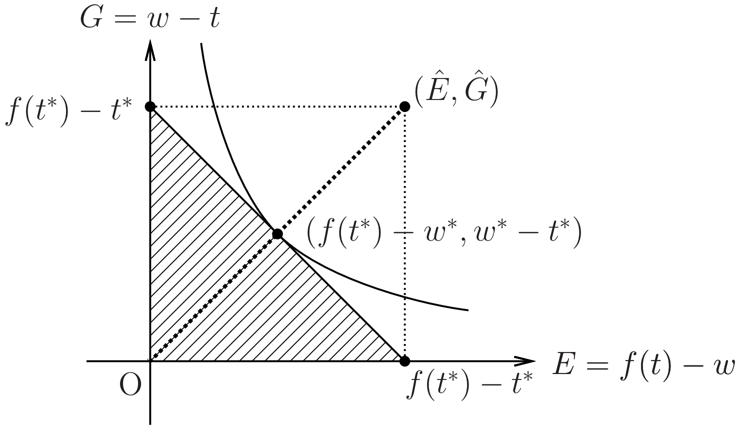

Figure 4.1: Nash solution and Kalai Smorodinsky solution in the bargaining game without minimum wage.

rive the Kalai Smorodinsky solution of the bargaining problem in section 2. Since the bargaining game is symmetric, Kalai Smorodinsky solution gives the employer and the guardian same payoffs and is equal to the Nash solution. Therefore, (E^*, G^*) is Kalai Smorodinsky solution and $w^*$is the equilibrium wage. This is illustrated in Figure 4.1.

4.2 Kalai Smorodinsky solution of the bargaining problem with minimum wage

When the minimum wage is imposed, in order to get Kalai Smorodinsky solution, the range of $\bar{w}$ should be partitioned into three parts. When $\bar{w}$ is small enough, the imposition of minimum wage would not affect the feasible set, so that Kalai Smorodinsky solution remains the same. When $\bar{w}$ is in the intermediate part, Kalai Smorodinsky solution lies on $\mathcal{B}_l(\bar{w})$, the line part of the bargaining set. When $\bar{w}$ is large enough, Kalai Smorodinsky solution lies on $\mathcal{B}_c(\bar{w})$, the curve part of the bargaing set.

4.2.1 When $0 \leq \bar{w} \leq t^*$

When w is less than or equal to t^*, we know that the imposition of $\bar{w}$ less than or equal to t^* does not affect the feasible set. Indeed, w^{KS}, the wage of Kalai Smorodinsky solution with minimum wage $\bar{w} \leq t^*$ is equal to w^*, the wage of the bargaining game's solution without minimum wage. t^{KS}, the meal provision of Kalai Smorodinsky solution with minimum wage $\bar{w} \leq t^*$ is equal to t^*, the meal provision of the bargaining game's solution without minimum wage. Since the wage and the meal provision do not vary in this range, the payoff of the employer and the guardian and the sum of both individual's payoffs are also fixed.

4.2.2 When $t^* < \bar{w} \leq \hat{w}$[9]

When we impose $\bar{w} > t^*$, the feasible set reduces and Kalai Smorodinsky solution is not equal to (E^*, G^*) anymore. The closed form of Kalai Smorodinsky solution would vary according to the location of the division point with respect to the ratio line. If the division point is below the ratio line, Kalai Smorodinsky solution is on $\mathcal{B}_l(\bar{w})$, the line part of the bargaining set. On the other hand, if the division point is above the ratio line, Kalai Smorodinsky solution is on $\mathcal{B}_c(\bar{w})$, the curve part of the bargaining set. Therefore, finding the cutoff level of $\bar{w}$, which makes the division point to be on the ratio line, is essential in order to derive the closed form of Kalai Smorodinsky solution. Let the minimum wage $\bar{w}$ be $\hat{w}$. Then $\hat{w}$ satisfies the following equation

$$\frac{\hat{w} - t^*}{f(t^*) - \hat{w}} = \frac{\hat{G}}{\hat{E}} = \frac{f(t^*) - t^*}{f(\hat{w}) - \hat{w}}. \tag{4.1}$$

[9] $\hat{w}$ is defined on 4.1.

For notational convenience, let $h(x) = f(x) - x$. Then, h is decreasing on $x > t^*$ and the preceding equation can be transformed to

$$\frac{h(t^*)}{f(t^*) - \hat{w}} - \frac{h(t^*)}{h(\hat{w})} = 1. \tag{4.2}$$

When $\hat{w} = t^*$, the left hand side of the equation is equal to 0. When $\hat{w}$ is close to $f(t^*)$, the left hand side of this equation goes to infinity due to the term $\frac{h(t^*)}{f(t^*)-\hat{w}}$. By differentiating the left hand side of (4.2) by $\bar{w}$, we get $h(t^*) \cdot \left(\frac{1}{(f(t^*)-\hat{w})^2} + \frac{f'(\hat{w})-1}{h(\hat{w})^2}\right)$. This is positive on $\hat{w} \in (t^*, f(t^*))$. Thus, the left hand side of the equation is continuous and increasing with respect to $\hat{w}$ on $(t^*, f(t^*))$, and there exists the unique $\hat{w}$ satisfying (4.2) on $(t^*, f(t^*))$ by the intermediate value theorem.

By using the fact that the right hand side of (4.1) is greater than 1, since t^* maximizes $h(t)$, and $\frac{w^*-t^*}{f(t^*)-w^*} = 1$, we get $\frac{\hat{w}-t^*}{f(t^*)-\hat{w}} > \frac{w^*-t^*}{f(t^*)-w^*}$. From this we get

$$\hat{w} > w^*. \tag{4.3}$$

Now we return to Kalai Smorodinsky solution. When $t^* < \bar{w} \leq \hat{w}$, as we can see in Figure 4.2, Kalai Smorodinsky solution would be on $\mathcal{B}_l(\bar{w})$. This implies that the meal provision t^{KS} of Kalai Smorodinsky solution should be equal to t^*. From the fact that Kalai Smorodinsky solution is on the ratio line, we can derive

$$\frac{w^{KS} - t^*}{f(t^*) - w^{KS}} = \frac{\hat{G}}{\hat{E}} = \frac{h(t^*)}{h(\bar{w})}, \tag{4.4}$$

and (4.4) can be transformed to

$$w^{KS} = t^* + \frac{h(t^*)^2}{h(\bar{w}) + h(t^*)}. \tag{4.5}$$

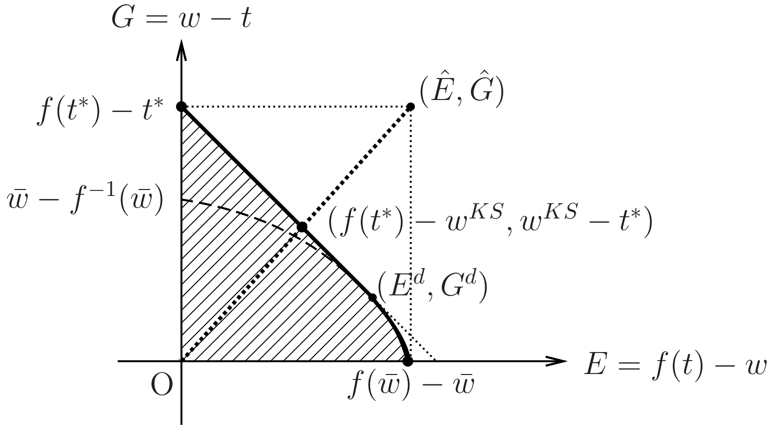

Figure 4.2: Kalai Smorodinsky solution in the bargaining game with minimum wage $\hat{w} \geq \bar{w} > t^*$

As $\bar{w}$ increases in this range, the transfer t^{KS} is fixed and the wage w^{KS} increases from (4.5) and $h' < 0$. Therefore, $E + G = f(t^{KS}) - t^{KS}$ is fixed, $E = f(t^{KS}) - w^{KS}$ decreases and $G = w^{KS} - t^{KS}$ increases.

4.2.3 When $\hat{w} < \bar{w} \leq f(t^*)$

When $\hat{w} < \bar{w} \leq f(t^*)$, as we can see in Figure 4.3, Kalai Smorodinsky solution would be on $\mathcal{B}_c(\bar{w})$. This implies that the wage w^{KS} of Kalai Smorodinsky solution should be equal to $\bar{w}$. From the fact that Kalai Smorodinsky solution is on the ratio line, we can derive that

$$\frac{\bar{w} - t^{KS}}{f(t^{KS}) - \bar{w}} = \frac{\hat{G}}{\hat{E}} = \frac{h(t^*)}{h(\bar{w})}. \tag{4.6}$$

For convenience, let us define $k_{\bar{w}}(t) = \frac{\bar{w}-t}{f(t)-\bar{w}}$, which is decreasing with respect to t on $(f^{-1}(\bar{w}), \bar{w})$. Then $k_{\bar{w}}^{-1}$ exists on $(f^{-1}(\bar{w}), \bar{w})$ and by using the

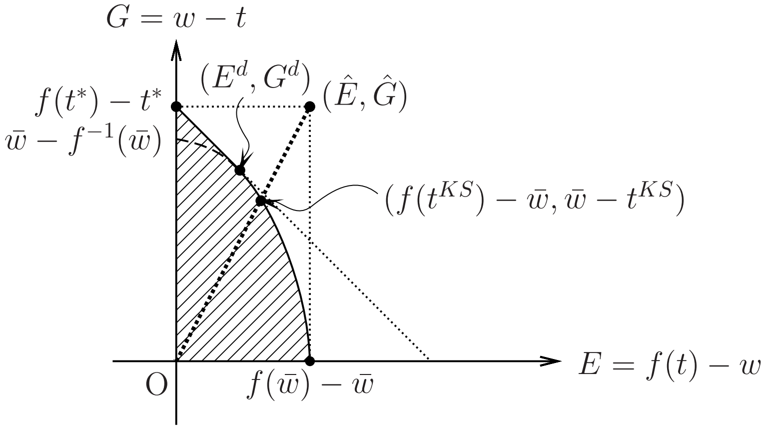

Figure 4.3: Kalai Smorodinsky solution in the bargaining game with minimum wage $f(t^*) \geq \bar{w} > \hat{w}$

fact that $\lim_{t \to f^{-1}(\bar{w})} k_{\bar{w}}(t) = \infty$ and $k_{\bar{w}}(\bar{w}) = 0$, (4.6) can be transformed to

$$t^{KS} = k_{\bar{w}}^{-1}\left(\frac{h(t^*)}{h(\bar{w})}\right). \tag{4.7}$$

As $\bar{w}$ increases in this range, the wage w^{KS} increases since it is equal to $\bar{w}$. From (4.7), we know that $k_{\bar{w}}(t^{KS}) = \frac{h(t^*)}{h(\bar{w})}$ and by differentiating this with respect to $\bar{w}$ we get

$$\frac{\partial k_{\bar{w}}}{\partial t} \cdot t' + \frac{\partial k_{\bar{w}}}{\partial \bar{w}} = -\frac{h(t^*)}{h(\bar{w})^2} \cdot h'(\bar{w}). \tag{4.8}$$

We can simply derive that $\frac{\partial k_{\bar{w}}}{\partial t} < 0$ and with some manipulation we also can get $-\frac{\partial k_{\bar{w}}}{\partial \bar{w}} - \frac{h(t^*)}{h(\bar{w})^2} \cdot h'(\bar{w}) < 0$. From this, we can get $t' > 0$.[10] Thus, in this case, the meal provided to the child worker increases. Therefore, $E + G = f(t^{KS}) - t^{KS}$ decreases and $E = f(t^{KS}) - w^{KS}$ decreases as $\bar{w}$ increases.

[10] From the definition of $k_{\bar{w}}$ and h , $-\frac{\partial k_{\bar{w}}}{\partial \bar{w}} - \frac{h(t^*)}{h(\bar{w})^2} \cdot h'(\bar{w}) = -\frac{h(t)}{(f(t)-\bar{w})^2} + \frac{h(t^*)}{h(\bar{w})^2} - \frac{h(t^*) \cdot f'(\bar{w})}{h(\bar{w})^2}$ and by using (4.6), the equation becomes $\frac{-(h(\bar{w})+h(t^*))^2 + h(t) \cdot h(t^*)}{h(t) \cdot h(\bar{w})^2} - \frac{h(t^*) \cdot f'(\bar{w})}{h(\bar{w})^2}$. It is negative because $-h(t^*)^2 + h(t) \cdot h(t^*) \leq 0$ from $t \geq t^*$.

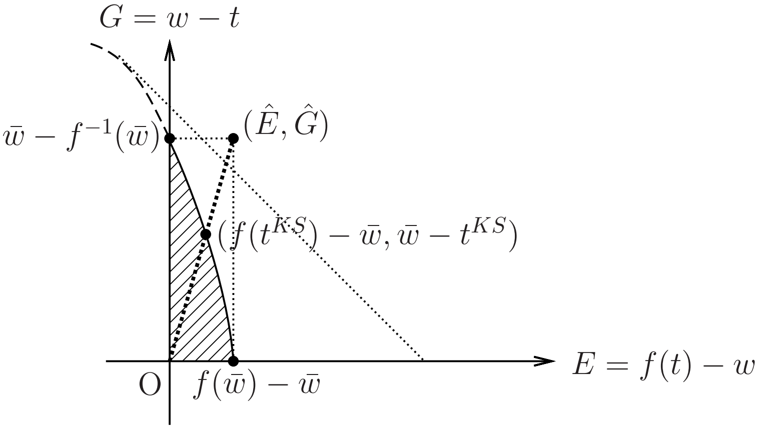

Figure 4.4: Kalai Smorodinsky solution in the bargaining game with minimum wage $f(t^*) \geq \bar{w} > \hat{w}$

However, the movement of $G = w^{KS} - t^{KS}$ with respect to increasing $\bar{w}$ is ambiguous; G increases if and only if t' from (4.8) is smaller than 1.

4.2.4 When $f(t^*) < \bar{w}$

When $f(t^*) < \bar{w}$, as we can see in figure 4.4, the line part of the bargaining set does not exist anymore. Then the wage w^{KS} of Kalai Smorodinsky solution should be equal to $\bar{w}$ since Kalai Smorodinsky solution would be on $\mathcal{B}_c(\bar{w})$. Moreover, the maximum payoff that the guardian obtains would decrease to $\bar{w} - f^{-1}(\bar{w})$, which is equal to $h(f^{-1}(\bar{w}))$.[11] Then, the equation of the ratio line induces

$$\frac{\bar{w} - t^{KS}}{f(t^{KS}) - \bar{w}} = \frac{\hat{G}}{\hat{E}} = \frac{h(f^{-1}(\bar{w}))}{h(\bar{w})}. \tag{4.9}$$

Using similar steps of the preceding case, we can derive that

$$t^{KS} = k_{\bar{w}}^{-1}\left(\frac{h(f^{-1}(\bar{w}))}{h(\bar{w})}\right) \quad \text{and} \quad t' > 0. \tag{4.10}$$

[11] By the individual rationality of the employer, $f(t) - \bar{w} \geq 0$, so that $t \geq f^{-1}(\bar{w}) > t^*$. Therefore, the maximum payoff of the guardian is $\bar{w} - f^{-1}(\bar{w})$.

Therefore, as $\bar{w}$ increases, $E+G$ and E also decrease in this case, and G is still ambiguous and increases if and only if $t' < 1$.

The following proposition summarizes the results of this section.

Proposition 2. *When the minimum wage $\bar{w}$ is imposed on the child labor bargaining model and the solution type of the bargaining game is Kalai Smorodinsky solution,*

1. *if $0 \le \bar{w} \le t^*$, $w^{KS} = w^*$ and $t^{KS} = t^*$, so that everything remains same as $\bar{w}$ varies.*

2. *if $t^* < \bar{w} \le \hat{w}$, $w^{KS} = t^* + \frac{h(t^*)^2}{h(\bar{w})+h(t^*)}$ and $t^{KS} = t^*$.*
   As $\bar{w}$ increases, only the wage of the child worker increases and the meal provision is fixed. The sum of two individual's payoffs is fixed, the employer's payoff decreases and the guardian's payoff increases.

3. *if $\hat{w} < \bar{w} \le f(t^*)$, $w^{KS} = \bar{w}$ and $t^{KS} = k_{\bar{w}}^{-1}\left(\frac{h(t^*)}{h(\bar{w})}\right)$.*
   As $\bar{w}$ increases, the wage of the child worker and the meal provision increase. Employer's payoff and the sum of two individual's payoffs decrease. Guardian's payoff is ambiguous and increases if and only if $\frac{\partial t^{KS}}{\partial \bar{w}} < 1$.

4. *if $f(t^*) < \bar{w}$, $w^{KS} = \bar{w}$ and $t^{KS} = k_{\bar{w}}^{-1}\left(\frac{h(f^{-1}(\bar{w}))}{h(\bar{w})}\right)$.*
   As $\bar{w}$ increases, the wage of the child worker and the meal provision increase. Employer's payoff and the sum of two individual's payoffs decrease. Guardian's payoff is ambiguous and increases if and only if $\frac{\partial t^{KS}}{\partial \bar{w}} < 1$.

where $h(x) = f(x) - x$, $\hat{w}$ satisfying $\frac{h(t^*)}{\hat{w}-t^*} - \frac{h(t^*)}{h(\hat{w})} = 1$, $k_{\bar{w}}(t) = \frac{h(t)}{\bar{w}-t}$.

4.3 Comparison with Nash solution

We have solved the bargaining problem by using Nash solution and Kalai Smorodinsky solution. In this section, we will compare these two solutions. As we can see in the preceding sections, the results may differ by the range of $\bar{w}$. The following proposition summarizes the results and proof is provided in appendix.

Proposition 3. *When the minimum wage $\bar{w}$ is imposed on the child labor bargaining model,*

1. *When $0 \leq \bar{w} \leq t^*$,*

   the wages, the meal provisions, the employer's profit, the guardian's payoff, and the sums of each player's payoff of both solutions are equal to w^*, t^*, $\frac{1}{2}(f(t^*) - t^*)$, $\frac{1}{2}(f(t^*) - t^*)$, and $f(t^*) - t^*$ respectively.

2. *When $t^* \leq \bar{w} \leq w^*$,*

   the wage, w, and the guardian's payoff, G, of Kalai Smorodinsky solution are greater than those of Nash solution,

   and the employer's profit, E, of Kalai Smorodinsky solution is less than that of Nash solution,

   and the meal provision, t, and the sum of each player's payoff, $E + G$, of Kalai Smorodinsky solution are equal to those of Nash solution.

3. *When $w^* < \bar{w} < \hat{w}$,*

   the wage, w, and the guardian's payoff, G, and the sum of each player's payoff, $E + G$, of Kalai Smorodinsky solution are greater than those of Nash solution,

   and the employer's profit, E, and the meal provision, t, of Kalai Smorodinsky solution are less than that of Nash solution,

4. *When $\hat{w} \leq \bar{w}$,*

   the wages, w, of both solutions are equal to $\bar{w}$,

   however, the other variables are ambiguous to compare.

The first part of the proposition shows that the imposition of small minimum wage does not affect each solution and both solutions induce same payoff to the guardian and the employer.

When $t^* \leq \bar{w} \leq w^*$, which is the condition of the second part of the proposition, Nash solution induces both players to get same payoff, however, Kalai Smorodinsky solution induces the guardian to get more payoff than the employer. Since the meal provisions, t, of both solutions are same, the sums of each player's payoff are same. This implies that the main difference between two solutions in this range is how the employer and the guardian divide the revenue of child labor, but how much they enforce child to produce. Another implication is that the minimum wage policy affects only Kalai Smorodinsky solution in this range.

When $w^* < \bar{w} < \hat{w}$, which is the condition of the third part of the proposition, the guardian's payoff and the sum of two players' payoff of Kalai Smorodinsky solution are greater than those of Nash solution, on the other hand, the employer's payoff of Nash solution is greater than that of Kalai Smorodinsky solution. This implies that solution type not only affect how two players divide the revenue of child labor, but also how much they enforce child to produce.

When $\hat{w} \leq \bar{w}$, which is the condition of the fourth part of the proposition, wages, w, of both solutions are equal to $\bar{w}$. Which solution's meal provision, t, is greater than the other's varies according to the form of f. Therefore, it is also ambiguous that which solution gives more payoff to the employer and the guardian respectively.

Based on these results we know that Kalai Smorodinsky solution gives more or equal payoff to the guardian than Nash solution gives when the government imposes $\bar{w} < \hat{w}$. This implies that Kalai Smorodinsky solution is more favorable to the guardian than Nash solution in this range. However, when $\hat{w} \leq \bar{w}$, it is ambiguous that which solution is more favorable to the guardian.

5 Bargaining model with an altruistic guardian

Gupta (2000) has assumed that the guardian does not care about the child's welfare. This is contrary to recent studies, which consider the guardian's altruism.[12] Basu and Van (1998) and Baland and Robinson (2000), the most influential theoretic papers in child labor analysis, are founded on the parent's altruistic concern about the child's welfare. In this section, we will adapt the guardian's altruism towards the child to Gupta (2000)'s bargaining model.

The first step of modifying the model is reconsidering the role of the meal provision t. In Gupta (2000)'s model, the variable t plays two roles - one is the guardian's transfer towards the child worker and the other is the factor of production. In the new model, we will separate these two roles and use two variables corresponding to each role. This feature is important to analyze the child's welfare. The two roles that we have mentioned work oppositely to the child's welfare in a sense that the increase of the guardian's transfer towards the child is positive and the increase of labor supply as a factor of production

[12] Basu (1999) has describe that the child in Gupta (2000)'s model is valued in the same way as the goose that lays the golden eggs.

is negative to the child worker. Thus, when we consider the child's welfare, it is impossible to treat the two roles as one variable.

The second step is to introduce the guardian's income. Basu and Van (1998) assume luxury axiom which implies that the guardian sends his child out to work if and only if the income from non-child labor sources drops low. This can be viewed as a selective altruism towards the child. In Gupta (2000)'s model, we do not have the opportunity to consider non-child labor income and even if we introduce the guardian's income in the model, the effect would just be shifting up to the disagreement point and the feasible set. However, in the new model, since we have separated the guardian's transfer from the factor of production in the first step, the introduction of the guardian's income may influence the disagreement point and the shape of the feasible set. In the next section, we will introduce the new model considering these two features.

5.1 Bargaining model

We consider a bargaining problem of the employer and the guardian as in section 2. We introduce the child labor level $l \in [0, L]$, and production function $f(l)$ with $f' > 0$ and $f'' < 0$, and cost function for child worker $c(l)$ with $c' > 0$. We are assuming that $f'(0) - c'(0) > 0$ and $f'' - c'' < 0$, so that there exists $l_0 \in (0, L]$ maximizing $f(l) - c(l)$.[13] Moreover, we are assuming that $f(l_0) - c(l_0) < 0$.[14] This assumption implies that child labor will never occur if the players of bargaining game are the employer and child worker.[15] However,

[13] From $f'(0) - c'(0) > 0$ and $f'' - c'' < 0$, l_0 satisfies either $f'(l_0) = c'(l_0)$ or $l_0 = L$ and $f'(L) - c'(L) > 0$.

[14] Here, we assume that $c(0) > 0$, which is the fixed cost of child labor.

[15] When the wage is w, the employer's profit is $f(l) - w$ and child's payoff is $w - c(l)$ and the disagreement point is $(E^N, G^N) = (0, 0)$. They will not cooperate since the sum of each player's payoff at disagreement point is greater that of cooperative situation.

our model considers a bargaining game of the employer and the guardian and this feature drives child worker to work.

In section 2, the employer and the guardian make a contract on w and t, and in this model, the employer and the guardian make a contract on w and l. When the wage is w and the labor level is l, the employer's profit is $E = f(l) - w$ and the guardian receives all the wage of the child worker. Then, the guardian decides how much transfer he would give to his child and let us call this amount b. We also assume that guardians already have their own income and we call it A and assume that $A \geq 0$. Then, b should be constrained to $0 \leq b \leq A + w$. In contrast to the model in section 2, the guardian cares about the utility of his child and assumes that the child's utility is decided by b and l. In this paper, we simply assume that the child's utility is separable, so that $u(b, l) = v(b) - c(l)$.[16] Here we assume that $v(0) = 0$, $v' > 0$, $v'' < 0$, and there exists b_0 such that $v'(b_0) = 1$ and it satisfies $v(b_0) - b_0 > 0$. By summing up all the preceding features, the guardian's payoff is $G = A + w - b + v(b) - c(l)$.

The payoff of the employer at the disagreement point, E^N, is always fixed to 0. However, the payoff of the guardian at the disagreement point, G^N, varies with respect to A. Moreover, whether $A \geq b_0$ or $A < b_0$ would affect the representation of the disagreement point.

$A \geq b_0$ means that the guardian has enough money to give his child the optimal transfer b_0. At the disagreement point, the guardian will choose the optimal b^*, which maximizes his payoff subject to $w = 0$ and $l = 0$. Since $0 \leq b \leq A$ and $A \geq b_0$, the optimal b^* is equal to b_0. Therefore, when $A \geq b_0$, the disagreement point is $(E^N, G^N) = (0,\ A - b_0 + v(b_0))$.

$A < b_0$ means that the guardian does not have enough money to give his

[16]More precisely, u is the guardian's altruistic utility toward child.

child the optimal transfer b_0. In this case, from $0 \leq b \leq A < b_0$, the optimal b^*, for deciding the disagreement point, is equal to A. Then, the disagreement point is $(E^N, G^N) = (0,\, v(A))$.

5.2 Child labor supply

In this bargaining model, child labor is supplied if and only if the sum of the payoffs of the employer and the guardian in an agreement point is greater than that in the disagreement point, i.e.,

$$E + G > E^N + G^N.$$

We will consider two cases as in the preceding section.

5.2.1 When $b_0 \leq A$

When $A \geq b_0$, the following inequality implies that guardian's payoff at the disagreement point is greater than guardian's payoff at any agreement point. child labor does not occur at all in this case.

$$A - b_0 + v(b_0) > A - b + v(b) + f(l_0) - c(l_0) \geq A + w - b + v(b) - c(l).$$

This result implies that child labor does not occur at all when the guardian has enough money to give his child the optimal transfer b_0. This is consistent with Basu and Van (1998)'s Luxury axiom. Moreover, it is also similar to Baland and Robinson (2000). They demonstrated that child labor level is efficiently high when the capital market is perfect and parents leave positive bequests to their child. However the child labor level may be inefficiently high when the family is so poor that parents cannot leave bequests to their child.

5.2.2 When $0 \leq A < b_0$

When $0 \leq A < b_0$, since $(E^N, G^N) = (0, v(A))$, child labor is supplied when

$$E + G > v(A).$$

Therefore, we need to check whether the maximized $E + G$ is greater than $v(A)$ or not. The maximization problem of $E + G$ is

$$\begin{array}{cc} \max & (f(l) - w) + (A + w - b + v(b) - c(l)), \\ \text{subject to} & 0 \leq l \leq L, \quad 0 \leq w \leq f(l), \quad 0 \leq b \leq A + w. \end{array}$$

In order to maximize $f(l) - c(l)$, l should be equal to l_0. Since w is canceled out, w does not directly affect to the maximization process, but influence indirectly by forming the constraint of b, so that $w = f(l_0)$ is optimal. Therefore, the maximization problem turns to $\max_{0 \leq b \leq A + f(l_0)} (-b + v(b)) + (A + f(l_0) - c(l_0))$. In order to obtain optimal b, we should consider two cases. For notational convenience, let us define $g = f(l_0) - b_0 + v(b_0) - c(l_0)$.

Case A. $f(l_0) + A \geq b_0$

In this case, the optimal b^* is b_0. Then, child labor is supplied only when

$$A + g > v(A),$$

which is equivalent to

$$\tilde{v}^{-1}(g) > A \geq b_0 - f(l_0),$$

where $\tilde{v}(x) = v(x) - x$, which is increasing on $A \in [0, b_0]$.

Case B. $b_0 > f(l_0) + A$

In this case, the optimal b^* is $f(l_0) + A$. Then, child labor is supplied only when

$$v(f(l_0) + A) - c(l_0) > v(A),$$

which is equivalent to

$$\min\left\{b_0 - f(l_0),\ \bar{v}_{f(l_0)}^{-1}(c(l_0))\right\} > A,$$

where $\bar{v}_k(x) = v(k + x) - v(x)$, which is decreasing.

The following proposition summarizes the preceding results.

Proposition 4. *When the employer and the guardian play the child labor bargaining model with an altruistic guardian,*

1. *When $f(l_0) \geq b_0$, child labor is supplied if and only if $\tilde{v}^{-1}(g) > A \geq 0$.*
2. *When $b_0 > f(l_0)$,*
   (a) *if $c(l_0) \geq v(b_0) - v(b_0 - f(l_0))$, child labor is supplied if and only if $\bar{v}_{f(l_0)}^{-1}(c(l_0)) > A \geq 0$.*
   (b) *if $c(l_0) < v(b_0) - v(b_0 - f(l_0))$, child labor is supplied if and only if $\tilde{v}^{-1}(g) > A \geq 0$.*

Proof. When $f(l_0) \geq b_0$, for all $A \geq 0$, $f(l_0) + A \geq b_0$. Therefore, only case 1 happens in this extent. We can derive that $\tilde{v}^{-1}(g) \geq \bar{v}_{f(l_0)}^{-1}(c(l_0)) > 0 \geq b_0 - f(l_0)$ and this implies that when $\tilde{v}^{-1}(g) \geq A \geq 0$ child labor occurs and when $A > \tilde{v}^{-1}(g)$ child labor never occurs.

When $b_0 > f(l_0)$, if $c(l_0) \geq v(b_0) - v(b_0 - f(l_0))$, we can show that $b_0 - f(l_0) \geq \tilde{v}^{-1}(g) \geq \bar{v}^{-1}_{f(l_0)}(c(l_0))$. From this, we know that child labor never occurs in case 1 since $A > b_0 - f(l_0)$ implies $A > \tilde{v}^{-1}(g)$. Child labor can occur only in case 2. From this, we obtain that child labor is supplied if and only if $\bar{v}^{-1}_{f(l_0)}(c(l_0)) > A \geq 0$.

If $c(l_0) < v(b_0) - v(b_0 - f(l_0))$, we can derive $\tilde{v}^{-1}(g) \geq \bar{v}^{-1}_{f(l_0)}(c(l_0)) > b_0 - f(l_0)$. When $\tilde{v}^{-1}(g) > A \geq b_0 - f(l_0)$, it belongs to case 1 and child labor occurs. When $\bar{v}^{-1}_{f(l_0)}(c(l_0)) > A \geq 0$, it belongs to case 2 and child labor occurs. From this, we can obtain that child labor is supplied if and only if $\tilde{v}^{-1}(g) > A \geq 0$. □

This proposition leads us to the luxury axiom in the sense that there exists a subsistence level such that child labor is supplied if and only if guardian's income is less than that level. In Basu and Van (1998), the luxury axiom operates as an axiom imposed on parental preference and the subsistence level is given exogenously. On the other hand, in this model, we get the luxury axiom as a result of analysis. Moreover, if we know function f, v, and c, we can get the subsistence level endogenously.

There are several empirical findings that support the luxury axiom. Bhalotra (2007) has shown that poverty forces child, especially a boy, to work from household survey for Pakistan. Edmonds and Schady (forthcoming) have randomly selected poor women with children in Ecuador for a cash transfer. They found that poor families with children in school at the time of the award use the extra income to postpone the child's entry into the labor force.

5.3 The feasible set and bargaining set of the bargaining problem

In this section, we will look into the bargaining game defined on the previous section more precisely. The employer and the guardian need to reach an agreement on w and l, through bargaining and the guardian chooses optimal b maximizing his utility. Individual rationality requires that $E \geq 0$ and $G \geq 0$. Therefore, the bargaining set, $\mathcal{F}(A)$, for the employer and the guardian with income A is

$$\mathcal{F}(A) = \left\{ (E, G) \;\middle|\; \substack{E=f(l)-w,\; G=A+w-b+v(b)-c(l), \\ G \geq v(A),\, 0 \leq l \leq L,\, 0 \leq w \leq f(l),\, 0 \leq b \leq A+w} \right\}$$

We also define the bargaining set, $\mathcal{B}(A)$ as in section 2. From $E + G = A + f(l) - c(l) - b + v(b)$, l maximizes $f(l) - c(l)$ and b maximizes $v(b) - b$, on $\mathcal{B}(A)$, so that l is always equal to l_0, and b is equal to $\min\{b_0, A + w\}$.[17] Therefore, $\mathcal{B}(A)$ is

$$\mathcal{B}(A) = \left\{ (E, G) \in \mathcal{F}(A) \;\middle|\; \substack{E=f(l_0)-w,\; G=A+w-b+v(b)-c(l_0), \\ G \geq v(A),\, b=\min\{b_0, A+w\},\, 0 \leq w \leq f(l_0)} \right\}$$

Due to $b = \min\{b_0, A + w\}$, $\mathcal{B}(A)$ is divided into two parts - $\mathcal{B}_l(A)$, of which b is equal to b_0 and $b_0 - A \leq w \leq f(l_0)$, and $\mathcal{B}_c(A)$, of which b is equal to $A + w$ and $0 \leq w \leq b_0 - A$. the shape of $\mathcal{B}(A)$ varies according to the range of A. The sets $\mathcal{B}_l(A)$ and $\mathcal{B}_c(A)$ are defined as follows,

$$\mathcal{B}_l(A) = \left\{ (E, G) \in \mathcal{F}(A) \;\middle|\; \substack{E=f(l_0)-w,\; G=A+w+g-c(l_0), \\ G \geq v(A),\, b_0-A \leq w \leq f(l_0)} \right\}$$

$$\mathcal{B}_c(A) = \left\{ (E, G) \in \mathcal{F}(A) \;\middle|\; \substack{E=f(l_0)-w,\; G=v(A+w)-c(l_0), \\ G \geq v(A),\, 0 \leq w \leq b_0-A} \right\}$$

[17] When $A + w \geq b_0$, the optimal b is b_0, and when $b_0 \geq A + w$, the optimal b is $A + w$.

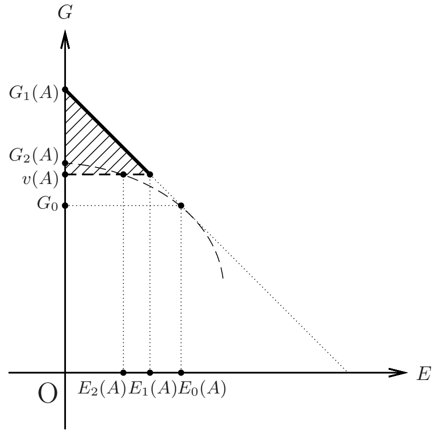

Figure 5.1: The bargaining set when $\tilde{v}^{-1}(g) > A > v^{-1}(G_0)$

We can easily see that $\mathcal{B}_l(A)$ is a segment and we call this a line part of the bargaining set and $\mathcal{B}_c(A)$ is a curve and we call this a curve part of the bargaining set.

For notational convenience, let us define $G_0 = v(b_0) - c(l_0)$, $G_1(A) = A + g$, and $G_2(A) = v(A + f(l_0)) - c(l_0)$, where $g = f(l_0) - b_0 + v(b_0) - c(l_0)$. Also define $E_0(A) = A + f(l_0) - b_0$, $E_1(A) = -v(A) + A + g$, $E_2(A) = A + f(l_0) - v^{-1}(v(A) + c(l_0))$. Then, these values play an important role in analyzing the bargaining set of the game and we can see this from figures 5.1, 5.2, 5.3. For example, we can see that (E_0, G_0) is realized by $w = b_0 - A$, $l = l_0$, and $b = b_0$, so that (E_0, G_0) would be the division point of the bargaining set.[18] According to the range A belongs to, the shape of $\mathcal{B}(A)$ would vary. Now we will consider three ranges where A can belong to.

Case 1. When $\tilde{v}^{-1}(g) > A > v^{-1}(G_0)$

In this case, $v(A) > v(b_0) - c(l_0) \geq v(A + w) - c(l_0)$ for all $0 \leq w \leq b_0 - A$,

[18] Due to the constraint $G \geq v(A)$, there would be some cases that (E_0, G_0) is not in the frontier of the bargaining set. Except for those cases, (E_0, G_0) is the division point of the bargaining set's frontier and the only element of $\mathcal{B}_l(A) \cap \mathcal{B}_c(A)$.

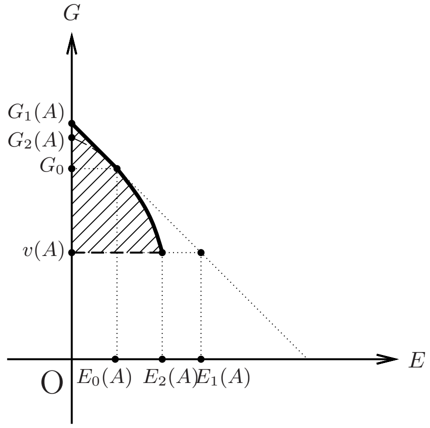

Figure 5.2: The bargaining set when $v^{-1}(G_0) \geq A \geq b_0 - f(l_0)$

and this implies that the bargaining set is only composed of the line part, i.e., $\mathcal{B}(A) = \mathcal{B}_l(A)$ and $\mathcal{B}_c(A) = \emptyset$. This is illustrated in Figure 5.1.

Case 2. When $v^{-1}(G_0) \geq A \geq b_0 - f(l_0)$

In this case, $v^{-1}(G_0) \geq A$ implies that $\mathcal{B}_c(A) \neq \emptyset$ and $A \geq b_0 - f(l_0)$ implies that $\mathcal{B}_l(A) \neq \emptyset$. This means that the bargaining set is composed of both the line part, $\mathcal{B}_l(A)$, and the curve part, $\mathcal{B}_c(A)$. This is illustrated in Figure 5.2.

Case 3. When $b_0 - f(l_0) > A \geq 0$

In this case, $b_0 > A + f(l_0) \geq A + w$ for all $0 \leq w \leq f(l_0)$, and this implies that the bargaining set is only composed of the curve part, i.e., $\mathcal{B}(A) = \mathcal{B}_c(A)$ and $\mathcal{B}_l(A) = \emptyset$. This is illustrated in Figure 5.3.

According to f, c, and v, the realized feasible set can vary. When $f(l_0) \geq b_0$, the condition of the first part of proposition 4, is satisfied, only case 1 and

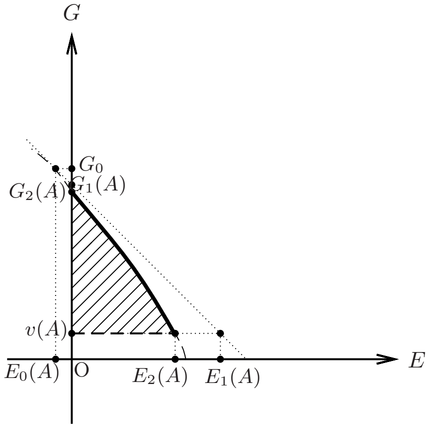

Figure 5.3: The bargaining set when $b_0 - f(l_0) > A \geq 0$

case 2 may happen since $0 \geq b_0 - f(l_0)$ in this case. When $f(l_0) < b_0$ and $c(l_0) \geq v(b_0) - v(b_0 - f(l_0))$, the condition of 2-(a) part of proposition 4, are satisfied, only case 3 can happen since $b_0 - f(l_0) \geq \tilde{v}^{-1}(g)$ in this case. When $f(l_0) < b_0$ and $c(l_0) < v(b_0) - v(b_0 - f(l_0))$, the condition of 2-(b) part of proposition 4, is satisfied, all cases may arise.

6 The solutions of the bargaining model with an altruistic guardian

In this section, we will solve the bargaining problem with an altruistic guardian by using Nash solution and Kalai Smorodinsky solution. No matter which solution we use, the bargaining solution is on the bargaining set. Therefore, the child labor level of every bargaining solution is equal to l_0. This means that the child labor level is the same for all households that supply child labor.[19]

[19]In reality, the child labor level seems to decrease as household's income increases. This is true when we consider child worker's domestic work. However, in this paper, we are focusing on the market work that the employer and the guardian bargain.

6.1 Nash solution

In this section, we will solve the bargaining problem with an altruistic guardian by using Nash solution. The range of A is partitioned into two parts. Let $\tilde{A}$ satisfy $\frac{v(\tilde{A})+G_1(\tilde{A})}{2} = G_0$. Whether the guardian's income, A, is greater than $\tilde{A}$ or not makes a difference in solving the problem.

6.1.1 When $\tilde{A} \leq A < \tilde{v}^{-1}(g)$

In this case, $\left(\frac{G_1(A)+v(A)}{2}, \frac{E_1(A)}{2}\right)$ belongs to the bargaining set, $\mathcal{B}(A)$. Then, we consider a set, $\mathcal{B}'(A)$, which is a convex hull of $(0, G_1(A))$, $(0, v(A))$, $(E_1(A), v(A))$. Then, $\mathcal{B}'(A)$ is the symmetric bargaining set and the Nash solution of $\mathcal{B}'(A)$ is $\left(\frac{G_1(A)+v(A)}{2}, \frac{E_1(A)}{2}\right)$. Since $\mathcal{B}(A) \subset \mathcal{B}'(A)$ and $\left(\frac{G_1(A)+v(A)}{2}, \frac{E_1(A)}{2}\right) \in \mathcal{B}(A)$, from the IIA axiom, Nash bargaining solution of $\mathcal{B}(A)$ is

$$\left(G^N, E^N\right) = \left(\frac{G_1(A)+v(A)}{2}, \frac{E_1(A)}{2}\right),$$

and w^N is

$$w^N = f(l_0) - \frac{E_1(A)}{2}.$$

From this and $v'(A) > 1$ in this range, we can derive that the guardian's payoff, G^N, and the wage, w^N, increases and the employer's payoff, E^N, decreases as A increases. The sum of each individuals' payoff, $E^N + G^N$, is equal to $G_1(A)$, which is increasing with respect to A.

6.1.2 When $0 \leq A < \tilde{A}$

In this case $\left(\frac{G_1(A)+v(A)}{2}, \frac{E_1(A)}{2}\right)$ is not in the bargaining set $\mathcal{B}(A)$ anymore, and Nash bargaining solution is not on the line part of the bargaining set,

$\mathcal{B}_l(A)$, but on the curve part of the bargaining set, $\mathcal{B}_c(A)$. Then, w of Nash solution solves

$$w^N = \arg \max_{0 \le w \le b_0 - A} \left(f(l_0) - w\right) \cdot \left(v(A+w) - c(l_0)\right).$$

The first order condition of w induces that

$$c(l_0) = v(A + w^N) - (f(l_0) - w^N) \cdot v'(A + w^N).$$

Let us define $g_A(w) = v(A+w) - (f(l_0) - w) \cdot v'(A+w)$. By differentiating $g_A(w)$ by A and w, respectively, we can get

$$\tfrac{\partial g_A(w)}{\partial A} = v'(A + w) - (f(l_0) - w) \cdot v''(A + w),$$

$$\tfrac{\partial g_A(w)}{\partial w} = 2v'(A + w) - (f(l_0) - w) \cdot v''(A + w),$$

which implies that $g_A(w)$ is an increasing function with respect to A and w. Therefore, we can get $w^N = g_A^{-1}(c(l_0))$ and E and G of Nash bargaining solution are $E^N = f(l_0) - w^N$ and $G^N = v(A + w^N) - c(l_0)$.

From $g_A(w^N) = c(l_0)$ and by differentiating this with respect to A, we can derive that

$$\frac{\partial w^N}{\partial A} = -\frac{\frac{\partial g_A(w^N)}{\partial A}}{\frac{\partial g_A(w^N)}{\partial w^N}}.$$

Thus w^N decreases as A increases and $\frac{\partial w^N}{\partial A} > -1$. Since $\frac{\partial E^N}{\partial A} = -\frac{\partial w^N}{\partial A}$ and $\frac{\partial G^N}{\partial A} = v'(A + w^N) \cdot \left(1 + \frac{\partial w^N}{\partial A}\right)$, E^N and G^N increases as A increases and $E + G$ also increases.

The following proposition summarizes the result of this section.

Proposition 5. *When the solution type of the bargaining problem with an* *altruistic guardian is Nash solution,*

1. *When* $0 \leq A < \tilde{A}$,
   $w^N = g_A^{-1}(c(l_0))$, $G^N = v(A + w^N) - c(l_0)$ *and* $E^N = f(l_0) - w^N$.
   As the guardian's income A increases, the wage, w^N, decreases, and the guardian's payoff, G^N, the employer's payoff, E^N, and the sum of each individual's payoff, $E^N + G^N$, increase. This implies that the subsidy policy in this range may induce Pareto improvement.

2. *When* $\tilde{A} \leq A < \tilde{v}^{-1}(g)$,
   $w^N = f(l_0) - \frac{E_1(A)}{2}$, $G^N = \frac{G_1(A)+v(A)}{2}$ *and* $E^N = \frac{E_1(A)}{2}$.
   As the guardian's income A increases, the wage, w^N, the guardian's payoff, G^N, the sum of each individual's payoff, $E^N + G^N$, increase and the employer's payoff, E^N, decreases.

where $g_A(w) = v(A + w) - (f(l_0) - w) \cdot v'(A + w)$.

Proposition 5 suggests a fresh viewpoint about child labor analysis. The proposition shows that child labor wage of Nash solution decreases until the guardian's income reaches some level. Then it increases as the guardian's income exceeds that level. In the first part of proposition 5, under some level of guardian's income, the guardian would get smaller additional possible payoff, and the employer would get more additional possible payoff on the bargaining game, and this implies that the bargaining power of the guardian is lessened and that of the employer rises.[20] This derives the child labor wage of Nash bargaining solution to decrease as the guardian's income increases. On the other hand, above that level, both the guardian and the employer would

[20] More precisely, $G_2(A) - v(A) = v(A + f(l_0)) - c(l_0) - v(A)$ decreases and $E_2(A) - 0$ increases as A increases.

get smaller additional possible payoffs. However, since the employer should compensate on increased $v(A)$, the guardian's payoff at the disagreement point, the child labor wage of Nash bargaining solution increases.[21]

Let us consider the government's policy that gives subsidy to the poor guardian as in Edmonds and Schady (forthcoming). When $0 \leq A < \tilde{A}$, from the first part of the proposition 5, the government's subsidy to the guardian may induce Pareto improvement. Since the guardian's payoff is equal to the child's welfare in this case, the definition of Pareto improvement is not constrained to the improvement of the employer and the guardian, but includes the improvement of the child worker. However, when A is above $\tilde{A}$, the subsidy policy is no longer Pareto improving policy. Only the guardian and the child worker get better off and the employer becomes worse off by the subsidy policy in this case.

6.2 Kalai Smorodinsky solution

In this section, we will solve the bargaining problem with an altruistic guardian by using Kalai Smorodinsky solution. The range of A is partitioned into four parts. When $\tilde{v}^{-1}(g) > A \geq v^{-1}(G_0)$, the bargaining set is only composed of the line part. When $v^{-1}(G_0) > A > b_0 - f(l_0)$, the bargaining set is composed of the line part and the curve part. Let $\hat{A}$ satisfy $\frac{G_0 - v(\hat{A})}{E_0(\hat{A})} = \frac{E_1(\hat{A})}{E_2(\hat{A})}$. Then, when $A = \hat{A}$, the division point is on the ratio line. When $A \geq \hat{A}$, the solution is on the line part. On the other hand, when $A < \hat{A}$, the solution is on the curve part. When $b_0 - f(l_0) \geq A$, the bargaining set is only composed of the curve part.

[21]Actually, there exists a range of A such that both the guardian and the employer get smaller additional possible payoff but the child labor wage increases.

6.2.1 When $v^{-1}(G_0) \leq A < \tilde{v}^{-1}(g)$

When $v^{-1}(G_0) \leq A < \tilde{v}^{-1}(g)$, the bargaining problem is symmetric. Therefore, the employer and the guardian get same additional payoffs, i.e.,

$$G^{KS} - v(A) = E^{KS} = \frac{1}{2} \cdot E_1(A).$$

Thus, from $G_1(A) - v(A) = E_1(A)$, Kalai Smorodinsky solution is

$$\left(G^{KS}, E^{KS}\right) = \left(\frac{G_1(A) + v(A)}{2}, \frac{E_1(A)}{2}\right),$$

and w^{KS} is

$$w^{KS} = f(l_0) - \frac{E_1(A)}{2}.$$

As in 6.1.1, G^{KS}, w^{KS}, and $E^{KS} + G^{KS}$ increases and E^{KS} decreases as A increases.

6.2.2 When $\hat{A} \leq A < v^{-1}(G_0)$

When $\hat{A} \leq A < v^{-1}(G_0)$, (E^{KS}, G^{KS}) is the intersection of $\mathcal{B}_l(A)$ and the ratio line. Hence, E^{KS} and G^{KS} satisfies following two equations.

1. $E^{KS} + G^{KS} = G_1(A)$, since (E^{KS}, G^{KS}) is on $\mathcal{B}_l(A)$.

2. $\frac{G^{KS} - v(A)}{E^{KS}} = \frac{G_1(A) - v(A)}{E_2(A)}$, since (E^{KS}, G^{KS}) is on the ratio line.

From $G_1(A) - v(A) = E_1(A)$, we can derive that

$$E^{KS} = \frac{E_1(A) \cdot E_2(A)}{E_1(A) + E_2(A)} = \frac{1}{\frac{1}{E_1(A)} + \frac{1}{E_2(A)}},$$
$$G^{KS} = v(A) + \frac{E_1(A)^2}{E_1(A) + E_2(A)},$$
$$w^{KS} = f(l_0) - E^{KS}.$$

Since $E^{KS} + G^{KS} = G_1(A)$, the sum of each individuals' payoff increases as A increases. With some manipulation, we can show that both $E_1(A)$ and $E_2(A)$ are decreasing with respect to A. From this and $E^{KS} = \frac{1}{\frac{1}{E_1(A)} + \frac{1}{E_2(A)}}$, we can show that E^{KS} decreases as A increases. Thus, from $w^{KS} = f(l_0) - E^{KS}$, w^{KS} increases as A increases. Moreover, since $E^{KS} + G^{KS}$ is increasing and E^{KS} is decreasing with respect to A, G^{KS} increases as A increases.

6.2.3 When $b_0 - f(l_0) < A < \hat{A}$

In this case, (E^{KS}, G^{KS}) is on $\mathcal{B}_c(A)$. Hence, E^{KS} and G^{KS} satisfies following equations.

1. $E^{KS} = f(l_0) - w^{KS}$, $G^{KS} = v(A + w^{KS}) - c(l_0)$, since (E^{KS}, G^{KS}) is on $F_c(\mathcal{B}(A))$.

2. $\frac{G^{KS} - v(A)}{E^{KS}} = \frac{G_1(A) - v(A)}{E_2(A)}$, since (E^{KS}, G^{KS}) is on the ratio line.

From these equations, we can derive that

$$k_A(w^{KS}) = c(l_0), \tag{6.1}$$

where $k_A(w) = v(A+w) - v(A) - R_1(A) \cdot (f(l_0) - w)$ and $R_1(A) = \frac{E_1(A)}{E_2(A)}$. $R_1(A)$ is relative bargaining power of the guardian with respect to the employer in this range of A.

Since $k_A(0) < c(l_0) < k_A(f(l_0))$ and k_A is increasing with respect to w, there exists $w^{KS} \in (0,\ f(l_0))$ such that $w^{KS} = k_A^{-1}(c(l_0))$.

In order to see the implication of the subsidy policy as in 6.1.2, we should derive $\frac{\partial w^{KS}}{\partial A}$. E^{KS} increases as A increases if and only if $\frac{\partial w^{KS}}{\partial A} < 0$. On the other hand, G^{KS} increases as A increases if and only if $\frac{\partial w^{KS}}{\partial A} > -1$. By differentiating equation 6.1 with respect to A, we get

$$\left(v'(A+w)+R_1(A)\right)\cdot\frac{\partial w^{KS}}{\partial A}=v'(A)+\left(R_1(A)\right)'(f(l_0)-w).$$

Thus, the subsidy policy may induce Pareto improvement if and only if

$$-1<\frac{v'(A)+\left(R_1(A)\right)'(f(l_0)-w)}{v'(A+w)+R_1(A)}<0.$$

6.2.4 When $0\leq A\leq b_0-f(l_0)$

In this case, (E^{KS}, G^{KS}) is on $\mathcal{B}_c(A)$. Hence, E^{KS} and G^{KS} satisfies following equations.

1. $E^{KS}=f(l_0)-w^{KS}$, $G^{KS}=v(A+w^{KS})-c(l_0)$, since (E^{KS}, G^{KS}) is on $F_c(\mathcal{B}(A))$.
2. $\frac{G^{KS}-v(A)}{E^{KS}}=\frac{G_2(A)-v(A)}{E_2(A)}$, since (E^{KS}, G^{KS}) is on the ratio line.

From these equations, we can derive that

$$l_A(w^{KS})=c(l_0) \tag{6.2}$$

where $l_A(w)=v(A+w)-v(A)-R_2(A)\cdot(f(l_0)-w)$ and $R_2(A)=\frac{G_2(A)-v(A)}{E_2(A)}$. $R_2(A)$ is relative bargaining power of the guardian with respect to the employer in this range of A.

Since $l_A(0)<c(l_0)<l_A(f(l_0))$ and l_A is increasing, there exists $w^{KS}\in(0,\ f(l_0))$ such that $w^{KS}=l_A^{-1}(c(l_0))$.

As in the preceding section, by differentiating equation 6.2 with respect to A, we get

$$\left(v'(A+w)+R_2(A)\right)\cdot\frac{\partial w^{KS}}{\partial A}=v'(A)+\left(R_2(A)\right)'(f(l_0)-w).$$

Thus, the subsidy policy may induce Pareto improvement if and only if

$$-1 < \frac{v'(A) + (R_2(A))' (f(l_0) - w)}{v'(A + w) + R_2(A)} < 0.$$

The following proposition summarizes the result of this section.

Proposition 6. *When the solution type of the bargaining problem with an altruistic guardian is Kalai Smorodinsky solution,*

1. *When* $0 \leq A \leq b_0 - f(l_0)$,

   $w^{KS} = l_A^{-1}(c(l_0))$, $G^{KS} = v(A + w^{KS}) - c(l_0)$ *and* $E^{KS} = f(l_0) - w^{KS}$,

   where $l_A(w) = v(A + w) - v(A) - R_2(A) \cdot (f(l_0) - w)$ *and* $R_2(A) = \frac{G_2(A) - v(A)}{E_2(A)}$.

   The subsidy policy may induce Pareto improvement if and only if $-1 < \frac{v'(A) + (R_1(A))' (f(l_0) - w)}{v'(A+w) + R_1(A)} < 0$.

2. *When* $b_0 - f(l_0) < A < \hat{A}$, $w^{KS} = k_A^{-1}(c(l_0))$,

   $G^{KS} = v(A + w^{KS}) - c(l_0)$, *and* $E^{KS} = f(l_0) - w^{KS}$,

   where $k_A(w) = v(A+w) - v(A) - R_1(A) \cdot (f(l_0) - w)$ *and* $R_1(A) = \frac{E_1(A)}{E_2(A)}$.

   The subsidy policy may induce Pareto improvement if and only if $-1 < \frac{v'(A) + (R_2(A))' (f(l_0) - w)}{v'(A+w) + R_2(A)} < 0$.

3. *When* $\hat{A} \leq A < v^{-1}(G_0)$,

   $G^{KS} = v(A) + \frac{E_1(A)^2}{E_1(A) + E_2(A)}$, $E^{KS} = \frac{E_1(A) \cdot E_2(A)}{E_1(A) + E_2(A)}$, *and* $w^{KS} = f(l_0) - E^{KS}$.

4. *When* $v^{-1}(G_0) \leq A < \tilde{v}^{-1}(g)$,

   $w^{KS} = f(l_0) - \frac{E_1(A)}{2}$, $G^{KS} = \frac{G_1(A) + v(A)}{2}$ *and* $E^{KS} = \frac{E_1(A)}{2}$.

5. *In case of* 3 *and* 4, *as the guardian's income* A *increases, the wage,* w^N,

the guardian's payoff, G^N, the sum of each individual's payoff, $E^N + G^N$, increase and the employer's payoff, E^N, decreases.

6.3 Comparison of Nash solution and Kalai Smorodinsky solution

In order to compare Nash solution and Kalai Smorodinsky solution, it is essential to determine which one of $\tilde{A}$ and $\hat{A}$ is greater than the other. Since $\frac{G_0 - v(\tilde{A})}{E_0(\tilde{A})} = 1$ and $\frac{E_1(\tilde{A})}{E_2(\tilde{A})} > 1$, $\frac{E_1(\tilde{A})}{E_2(\tilde{A})} > \frac{G_0 - v(\tilde{A})}{E_0(\tilde{A})}$ and this implies that

$$\tilde{A} > \hat{A}.$$

When $0 \leq A \leq \hat{A}$, both Nash and Kalai Smorodinsky solutions are on $\mathcal{B}_c(A)$, the curve part of the bargaining set. Even though we have obtained the closed form of Kalai Smorodinsky solution in section 6.2, it is not easy to characterize the solution and compare with Nash solution. As in the last part of proposition 3, both solutions are on the curve part of the bargaining set and it is ambiguous to compare the two solutions.

When $\hat{A} \leq A \leq \tilde{A}$, Nash solution is on $\mathcal{B}_c(A)$ and Kalai Smorodinsky solution is on $\mathcal{B}_l(A)$. Thus,

$$\begin{aligned} E^{KS} &\leq f(l_0) - b_0 + A \leq E^N, \\ G^N &\leq v(b_0) - c(l_0) \leq G^{KS}, \end{aligned}$$

and

$$w^N \leq b_0 - A \leq w^{KS}.$$

In each inequality, only one of the equalities can be satisfied. Hence, we can obtain that $E^{KS} < E^N$, $G^N < G^{KS}$, and $w^N < w^{KS}$.

When $\tilde{A} < A < v^{-1}(G_0)$, both Nash and Kalai Smorodinsky solutions are on $\mathcal{B}_l(A)$, the line part of the bargaining set. From $E_1(A) > E_2(A)$, we can show that

$$E^{KS} = \frac{E_1(A) \cdot E_2(A)}{E_1(A) + E_2(A)} < \frac{1}{2} \cdot E_1(A) = E^N. \tag{6.3}$$

Since both solutions are on $\mathcal{B}_l(A)$, $E^{KS} + G^{KS} = E^N + G^N$. Thus, from this and 6.3, $G^{KS} > G^N$. From $E = f(l_0) - w$ and 6.3, $w^{KS} > w^N$.

When $v^{-1}(G_0) \leq A < \tilde{v}^{-1}(g)$, the bargaining problem is symmetric. Thus, the employer and the guardian get same payoffs at both solutions.

The following proposition summarizes the result of this section.

Proposition 7. *In the bargaining model with an altruistic guardian,*

1. *When $0 \leq A < \hat{A}$,*

   both solutions are on the curve part of the bargaining set.

   It is ambiguous to compare two solutions.

2. *When $\hat{A} \leq A \leq \tilde{A}$,*

   Nash solution is on the curve part of the bargaining set and Kalai Smorodinsky solution is on the line part of the bargaining set.

   Thus, $E^N > E^{KS}$, $G^N < G^{KS}$, $w^N < w^{KS}$, and $E^N + G^N < E^{KS} + G^{KS}$.

3. *When $\tilde{A} < \bar{w} < v^{-1}(G_0)$,*

   both solutions are on the line part of the bargaining set.

   $G^N < G^{KS}$, $E^N > E^{KS}$, $w^N < w^{KS}$, and $E^N + G^N = E^{KS} + G^{KS}$.

4. *When $v^{-1}(G_0) < A < \tilde{v}^{-1}(g)$,*

   both solutions are the mid point of the line part of the bargaining set.

Thus, $E^N = E^{KS}$, $G^N = G^{KS}$, *and* $w^N = w^{KS}$.

7 Conclusion

In this paper, we have revisited the analysis of child labor by using a bargaining approach. First, we looked Gupta (2000)'s model and solved the bargaining game with and without the minimum wage by using Nash and Kalai Smorodinsky solutions. When the minimum wage is below a certain level, the wage of the child worker, the guardian's payoff, and the sum of the employer and the guardian's payoffs of Kalai Smorodinsky solution are greater than or equal to those of Nash solution. On the other hand, the meal provision and the employer's profit of Kalai Smorodinsky solution are less than or equal to those of Nash solution. Thus, Kalai Smorodinsky solution is more favorable to the guardian in comparison with Nash solution. Moreover, the range of the minimum wage where the policy is effective is broader at Kalai Smorodinsky solution than at Nash solution. The minimum wage policy induces the reduction of the employer's profit. However, it does not always imply the increase of the guardian's payoff. The policy may induce the decrease of both players' payoffs.

The second part of this paper has modified Gupta (2000)'s model and introduced the guardian's income, bequest, and child altruism. This model suggests a consistent result with Basu and Van (1998)'s luxury axiom about child labor supply in that there exists a subsistence level of the guardian's income, below which the child labor is supplied. Then, we have solved the bargaining solution by using two solution concepts. The comparative statics of Nash solution on A shows that child labor wage decreases until the guardian's income reaches some level, then it increases as the guardian's income exceeds that level. Thus, when A is below some level, as A increases, both individuals' payoffs increase. This implies that the subsidy policy toward a poor guardian induces Pareto improvement. However, at Kalai Smorodinsky solution, the policy may not induce Pareto improvement. We have found the condition that the subsidy policy induces both players' payoffs to increase under sufficiently small A. When A is small, it is not easy to compare Nash solution and Kalai Smorodinsky solution. However, when A is above some level and still so small that child is working, the guardian's payoff of Kalai Smorodinsky solution is greater than or equal to that of Nash solution. Still in this case, Kalai Smorodinsky solution is more favorable to the guardian than Nash solution.

8 Appendix

Proof of Proposition 3 When $0 \leq \bar{w} \leq t^*$, the wages and the meal provisions of both solutions are equal to w^* and t^* respectively. Thus, E, G, and $E + G$ of both solutions are exactly same and they are equal to E^*, G^*, $E^* + G^*$.

When $\bar{w}$ is greater than t^* and less than or equal to w^*, both Nash bargaining solution and Kalai Smorodinsky solution are on $\mathcal{B}_l(\bar{w})$, which implies that the meal provisions of both solutions are equal to t^*. However, the wage of Kalai Smorodinsky solution is greater than that of Nash bargaining solution, i.e., $w^{KS} \geq w^N = w^*$. This can be easily shown by using $w^* = t^* + \frac{1}{2}h(t^*)$, (4.5), and $h(t^*) \geq h(\bar{w})$. From $w^{KS} > w^N$ and $t^{KS} = t^N$, it can be derived that $E^{KS} < E^N$, $G^{KS} > G^N$, and $E^{KS} + G^{KS} = E^N + G^N$.

When $\bar{w}$ is greater than w^* and less than $\hat{w}$, Nash bargaining solution is on

$\mathcal{B}_c(\bar{w})$ and Kalai Smorodinsky solution is on $\mathcal{B}_l(\bar{w})$. Thus, $w^N = \bar{w}$ and $t^{KS} = t^*$. The condition $\bar{w} < \hat{w}$ implies that $\frac{\bar{w}-t^*}{f(t^*)-\bar{w}} < \frac{h(t^*)}{h(\bar{w})}$. By using this and (4.5), it can be shown that the wage of Kalai Smorodinsky solution is greater than $\bar{w}$, i.e., $w^{KS} > w$. We already have shown that t^N is greater than t^*, which is equal to the transfer of Kalai Smorodinsky solution, on section 4.2. From $w^{KS} > w^N$ and $t^{KS} < t^N$, we can easily show that $E^{KS} = f(t^{KS}) - w^{KS}$ is less than $E^N = f(t^N) - w^N$, $G^{KS} = w^{KS} - t^{KS}$ is greater than $G^N = w^N - t^N$, and $E^{KS} + G^{KS} = f(t^{KS}) - t^{KS}$ is greater than $E^N + G^N = f(t^N) - t^N$.

When $\bar{w}$ is greater than $\hat{w}$, both Nash solution and Kalai Smorodinsky solution are on $\mathcal{B}_c(\bar{w})$, which implies that the wages of both solutions are equal to $\bar{w}$. Thus, it remains to compare t^N and t^{KS}. However, unlike the previous cases, it is ambiguous which one is greater. It is obvious that t^N is greater than t^{KS} near $\hat{w}$, since $t^N > 1$ and $t^{KS} = 1$ at $\hat{w}$. However, we cannot predict which one is greater.

We will provide two examples which induce opposite results. First, let us consider $f_1(t) = \frac{1}{10} \cdot \ln(10t - 9) + 2$, of which t^* is equal to 1 and $f(t^*) = 2$. Table 1 provides the approximated meal provision level of Nash bargaining solution and Kalai Smorodinsky solution. As we can see in the table, when $\bar{w}$ is near $\hat{w}$, t^N is greater than t^{KS}, however, when as $\bar{w}$ increases, t^{KS} becomes greater than t^N.

We also consider another production function $f_2(t) = 20 \cdot \ln\left(\frac{t+19}{20}\right) + 2$, which has smaller curvature than $f_1(t)$, so that the reduced area of the feasible set produced by $f_2(t)$ is less than that by $f_1(t)$. t^* of $f_2(t)$ is also equal to 1 and $f(t^*) = 2$. Table 2 provides the approximated meal provision levels of both solutions. We do not exactly know which one is greater, since they are approximated value. However, we can see that, when $\bar{w}$ is less than $f(t^*)$,

t^N and t^{KS} are close. On the other hand, when $\bar{w} > f(t^*)$, t^N is apparently greater than t^{KS}, which is an opposite result to $f_1(t)$.

$\bar{w}$	$\hat{w} = 1.641$	1.7	1.8	1.9	1.95	2.1	2.2
t^N	1.049	1.075	1.134	1.217	1.273	1.544	1.899
t^{KS}	1	1.041	1.128	1.233	1.294	1.553	1.900

Table 1: The meal provision levels when $f_1(t) = \frac{1}{10} \cdot \ln(10t - 9) + 2$

$\bar{w}$	$\hat{w} = 1.502$	1.6	1.8	1.9	1.95	2.1	2.5
t^N	1.002	1.099	1.297	1.397	1.447	1.597	2.000
t^{KS}	1	1.098	1.297	1.397	1.447	1.479	1.881

Table 2: The meal provision levels when $f_2(t) = 20 \cdot \ln\left(\frac{t+19}{20}\right) + 2$

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