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Regulation of Natural Monopolies, Self-Selection, and Optimal Taxation

자연독점 규제, 자기선택, 그리고 최적조세

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고 지 현

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Abstract

Regulation of Natural Monopolies, Self-Selection, and Optimal Taxation

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It is well-known that the increasing returns-to-scale (IRS) property accounts for the presence of natural monopolies, which usually become public enterprises or are subject to regulations. This paper argues that public enterprises provide private goods not only for the IRS property, but also for relaxing the incentive problem of the tax system: they help relax the self-selection constraint of the optimal income tax problem through nonlinear pricing. The intuition is that when some private goods with IRS properties (e.g., public transportation service) relative to other goods are more valuable to low-ability individuals than the high-ability counterparts in terms of the marginal rate of substitution (MRS), the high-ability individuals are discouraged to mimic low-ability ones. Our results provide theoretical underpinning for the low price of publicly provided private goods for low-income individuals, breaking the p=MC rule for efficient redistribution. The optimal nonlinear pricing allows low marginal tax rates for both types, leading to greater work incentives.

Key Words: public provision of private goods; self-selection; optimal taxation
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I would like to inform you that this paper is worked with my thesis advisor Chul-In Lee.

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I Introduction

Some private goods are publicly provided. Textbook microeconomic theory shows that the increasing returns-to-scale (henceforth, IRS) property gives rise to natural monopolies, and the regulation of natural monopolies is an important subject in applied economic analysis. Examples of the goods or industries with IRS properties include the utility, communications, and transportation which are highly regulated in most countries. In many instances, regulations of them take the form of public enterprises providing those private goods/services with IRS properties, i.e., public provision of private goods on the basis of the rationale that the competitive market principle of price equaling marginal cost (the $p = MC$ rule) does not allow operation of firms due to negative profits.[1]

This paper argues that public enterprises provide private goods not only because of the IRS property, but also because they help relax the self-selection constraint of the optimal income tax problem. The intuition is that when some private goods (e.g., public transportation service) relative to other goods are more valuable to low-ability individuals than high-ability ones in terms of the marginal rate of substitution (henceforth, MRS), an appropriate pricing scheme can induce high-ability individuals to be discouraged to mimic low-ability ones, allowing for a room for Pareto improvement. In this case, our results provide a theoretical explanation for why public enterprises provide private goods at a low price for low-income individuals. In fact, the violation of the $p = MC$ rule implies that the production efficiency theorem by Diamond and Mirrlees (1971) does not

[1] An alternative to this marginal cost pricing is two-tier pricing system, charging some users a higher price while maintaining the price equaling marginal cost for other users. With this pricing system, profits on the high-price demanders compensate the losses incurred on the low-priced sales.

hold. While this result appears surprising, the presence of IRS does not necessarily guarantee the production efficiency theorem by Diamond and Mirrlees (1971) to hold in our setting. In this particular environment, utilizing the information about consumption of the goods produced by public enterprises can offer the opportunities for more efficient redistribution.[2]

There is a vast literature on optimal income taxation where efficient redistribution is the key issue in the presence of heterogeneity in earnings ability (see e.g., Mirrlees, 1971; Stiglitz, 1982). To our knowledge, however, there is little research addressing the interaction between optimal income tax structure and pricing of publicly provided private goods. We build upon the standard optimal income taxation models by Stiglitz (1982) and Boadway and Keen (1993), and then demonstrate the optimal tax and regulation policy combinations. In particular, we show the structure of nonlinear pricing of publicly provided private goods, and then how it interacts with optimal income taxation. At the least, this exercise broadens the horizon of optimal taxation and can shed light on the importance of policy mixes.

The paper is organized as follows. Section II presents our models and optimization results. Section III describes the optimal pricing structure of the goods with IRS properties and its implications. Section IV concludes with some discussions.

[2] In a two-sector model with constant-returns-to-scale technologies in both sectors, Naito (1999) shows that, in addition to a non-linear income tax system, subsidizing the wages for the workers in public enterprises that employ a labor-intensive technology can enhance social welfare. This violates the production efficiency theorem but by inducing a factor price structure that is favorable to low-skilled labor the social welfare can improve. In our paper, however, we use the information about a particular type of consumption that is favorable to low-income individuals for enhancing efficient redistribution.

II The Model

Drawing on Stiglitz (1982) and Boadway and Keen (1993), we build a model with government in which optimal taxation and optimal regulation of public enterprises are both considered. In a model with income taxes, Stiglitz (1982) analyzes the set of Pareto efficient tax structure and formulates the canonical problem with self-selection. Later, Boadway and Keen (1993) consider a model with income taxes and public goods, and find that public good provision with optimal nonlinear taxes can deviate from the Samuelson Rule when two types of households are allowed to value a public good differently, using the self-selection approach. While sharing a similar spirit with the previous studies, we focus on optimal provision of private goods with the IRS property in the presence of another essential government instrument, income taxes. In this study, we explore optimal taxation with pricing of a publicly provided 'private' good, not a pure public good, which has not been studied yet to our knowledge.

a Environment

Consider an economy with two types of households with different ability: low ability and high ability. Production is linear in labor supplies and individuals are paid according to their ability level: w_1 for the low ability and w_2 for the high ability. Type-i person with $i = 1, 2$ provides an amount of labor L_i, and consumes X_i of a private good and Q_i of another private good with the IRS property provided by a government. The pre-tax income for type-i person is

$$Y_i = w_i L_i. \tag{1}$$

There are N_i households of type i. Persons of type-i maximize their utility, given by strictly concave utility function $U^i(X_i, L_i, Q_i)$. As in Boadway and Keen (1993), we allow the two types to value Q_i differently. Different tastes may arise from either different utility function or different abilities under identical utility. In the latter case, income differences will lead to different tastes. Including Q with Q_i's being different across individuals is a new feature, compared to the traditional optimal income taxation literature. In addition to that, the prices of Q_i for each individuals are determined by a price function for privately provided good, denoted by $p(Y_i)$. The social planner determines the structure of $p(Y_i)$ in accordance with individuals' income levels. The budget constraint of household i in the presence of the income tax is given by

$$X_i + p(Y_i)Q_i \leq Y_i - T(Y_i), \tag{2}$$

where $T(Y_i)$ is a nonlinear tax function, $p(Y_i)$ is a price of Q_i which implies the price relies on the type of a household and X_i is numeraire. Natural monopolies appear in the production of Q, exhibiting IRS property and decreasing average costs over a broad range of output levels we consider. Marginal cost pricing of Q is, therefore, efficient but involves an operating loss which needs to be subsidized by the government.

The planner who chooses the function of nonlinear taxes is not able to identify individuals by their ability, i.e., asymmetric information. That is, w_i and L_i are not observable but only information regarding Y_i is revealed. Hence, writing the utility function in terms of observable elements–Q_i, X_i and Y_i–is useful in self-selection problem where asymmetric information between households and the government exists. Following Stiglitz (1982), we posit type-i individual's utility function as:

$$U^i(X_i, L_i, Q_i) = U^i(X_i, Y_i/w_i, Q_i) \equiv V^i(X_i, Y_i, Q_i), \tag{3}$$

where $V_x^i = U_x^i, V_y^i = U_L^i/w_i$ and $V_Q^i = U_Q^i$.

b Individual Problem

We first characterize the individual problem. The individual chooses $\{X_i, Y_i, Q_i\}$ to maximize his or her utility V^i subject to his or her budget constraint. The Lagrangian expression for individual i can be written

$$\mathcal{L} \ = V^i(X_i, Y_i, Q_i) + \lambda_i\{Y_i - T(Y_i) - X_i - p(Y_i)Q_i\}, \tag{4}$$

where λ_i is a Lagrange multiplier for the individual i's budget constraint. We obtain the first-order conditions of X_i, Q_i and Y_i :

$$V_X^i = \lambda_i, \tag{5-1}$$

$$V_Y^i = -\lambda_i(1 - T' - p'Q_i), \tag{5-2}$$

$$V_Q^i = \lambda_i p(Y_i), \tag{5-3}$$

where $T' = \frac{\partial T(Y_i)}{\partial Y_i}$ and $p' = \frac{\partial p(Y_i)}{\partial Y i}$.

For later reference, we combine first-order conditions properly and have the following marginal rate of substitutions:

$$\frac{V_Y^i}{V_X^i} = -(1 - T' - p'Q_i), \tag{6}$$

$$\frac{V_Q^i}{V_X^i} = p(Y_i), \tag{7}$$

$$\frac{V_Y^i}{V_Q^i} = -\frac{(1 - T' - p'Q_i)}{p(Y_i)}. \tag{8}$$

c Constraints

Before writing the planner's problem, we conceptualize the problem and constraints. The social planner chooses the tax function $T(Y_i)$, price function $p(Y_i)$ for Q and the total level of publicly provided private goods, Q which becomes $N_1 Q_1 + N_2 Q_2$ by definition. The planner maximizes the objective function—the Paretian social welfare function—following the work of Boadway and Keen (1993). Precisely, we consider the problem of maximizing V^1 subject to a given level of V^2 denoted by $\overline{V^2}$. There are three constraints the planner faces: one is a revenue constraint and the other two are self-selection constraints. The revenue constraint reflects the government budget constraint:

$$\begin{aligned} &\underbrace{AC(Q)Q}_{\text{total production cost}} - \{\underbrace{p(Y_1)N_1Q_1 + p(Y_2)N_2Q_2}_{\text{revenue of natural monopoly}}\} \\ &\quad \leq \underbrace{N_1\{Y_1 - X_1 - p(Y_1)Q_1\}}_{\text{tax revenue from type-1 individuals}} + \underbrace{N_2\{Y_2 - X_2 - p(Y_2)Q_2\}}_{\text{tax revenue from type-2 individuals}}, \end{aligned} \tag{9}$$

where $AC(Q)$ is the average cost of producing Q. Equation (9) implies that the operating loss of the natural monopoly, i.e., total cost − revenue, should be subsidized by the tax revenue for continuation of production. Note that average cost of producing Q is expected to decrease because of the IRS property, or economies of scale in production of Q. This revenue constraint, (9) can be simplified to:

$$C(Q) \leq N_1(Y_1 - X_1) + N_2(Y_2 - X_2), \tag{9$'$}$$

where $C(Q)$ is total cost of producing Q. Self-selection constraints that make the households be at least well-off by consuming the consumption bundle meant for them are needed since persons are not identified by ability. Since there are two types, we need two self-selection constraints,

$$V^1(X_1, Q_1, Y_1) \geq V^1(X_2, Q_2, Y_2), \tag{10}$$
$$V^2(X_2, Q_2, Y_2) \geq V^2(X_1, Q_1, Y_1). \tag{11}$$

Most of the studies in this literature, including Boadway and Keen (1993), however, assume that the first self-selection constraint is always satisfied since it is hard for a low-ability person to mimic a high-ability person. We will study with focus the 'normal' case where only the second self-selection constraint is binding.

d Optimization Problem

The social planner maximizes V^1 subject to a given level of V^2 denoted as $\overline{V^2}$ under the two constraints explained above. Putting the elements together, the Lagrangian expression for the planner problem can be written as:

$$\Omega(X_i, Y_i, Q_i, \mu, \xi, \gamma) = V^1(X_1, Y_1, Q_1) + \mu[V^2(X_2, Q_2, Y_2) - \overline{V^2}] + \xi[V^2(X_2, Y_2, Q_2) - V^2(X_1, Y_1, Q_1)] + \gamma[N_1(Y_1 - X_1) + N_2(Y_2 - X_2) - C(Q)], \quad (12)$$

where μ, ξ and γ are Lagrange multipliers. Now, optimization of the problem yields the first-order conditions for X_i, Y_i and Q_i:

$$V_X^1 - \xi \widehat{V}_X^2 - \gamma N_1 = 0, \quad (13\text{-}1)$$

$$V_Y^1 - \xi \widehat{V}_Y^2 + \gamma N_1 = 0, \quad (13\text{-}2)$$

$$V_Q^1 - \xi \widehat{V}_Q^2 - \gamma N_1 MC(Q) = 0,^{2} \quad (13\text{-}3)$$

$$\mu V_X^2 + \xi V_X^2 - \gamma N_2 = 0, \quad (13\text{-}4)$$

$$\mu V_Y^2 + \xi V_Y^2 + \gamma N_2 = 0, \quad (13\text{-}5)$$

$$\mu V_Q^2 + \xi V_Q^2 - \gamma N_2 MC(Q) = 0, \quad (13\text{-}6)$$

where $MC(Q) = \frac{\partial C(Q)}{\partial Q}$ and $\widehat{V}^2 = V^2(X_1, Y_1, Q_1)$ representing the utility of the high-ability person when mimicking the low-ability one.

[2] $\frac{\partial C(Q)}{\partial Q_i} = \frac{\partial C(Q)}{\partial Q} \frac{\partial Q}{\partial Q_i}$ and $\frac{\partial Q}{\partial Q_i} = N_i$ since $Q = N_1 Q_1 + N_2 Q_2$.

We rearrange these first-order conditions for later reference as follows:

$$\begin{aligned}
\frac{V_Y^1}{V_X^1} &= -\frac{-\xi\widehat{V}_Y^2+\gamma N_1}{\xi\widehat{V}_X^2+\gamma N_1}, &(14)\\
\frac{V_Q^1}{V_X^1} &= \frac{\xi\widehat{V}_Q^2+\gamma N_1 MC(Q)}{\xi\widehat{V}_X^2+\gamma N_1}, &(15)\\
\frac{V_Y^1}{V_Q^1} &= -\frac{-\xi\widehat{V}_Y^2+\gamma N_1}{\xi\widehat{V}_Q^2+\gamma N_1 MC(Q)}, &(16)\\
\frac{V_Y^2}{V_X^2} &= -1, &(17)\\
\frac{V_Q^2}{V_X^2} &= MC(Q), &(18)\\
\frac{V_Y^2}{V_Q^2} &= -\frac{1}{MC(Q)}. &(19)
\end{aligned}$$

Analyzing these conditions yields the conventional optimal income taxation results of Stiglitz (1982) and when combined with the optimal pricing conditions for Q_i, they yield some new results. To be concrete, equations (17) and (6) yield $T'+p'Q_i=0$ for the high-ability individuals, which does not necessarily implies that $T'=0$ and $p'=0$. We will discuss this issue in a later subsection. From (14) and (6), one can deduce that $T'+p'Q_i>0$ for the low-ability individuals, which is greater than that of the high-ability ones. To see this, following Stiglitz (1982), define

$$\begin{aligned}
\alpha^i &= -\frac{\partial V^i/\partial Y_1}{\partial V^i/\partial X_1}, &(20)\\
\nu &= \frac{\xi\partial V^2/\partial X_1}{\gamma N_1}. &(21)
\end{aligned}$$

Then, (14) can be written as:

$$-\frac{V_Y^1}{V_X^1} = \alpha^1 \tag{22}$$

$$= \frac{1 - \xi(\partial V^2/\partial Y_1)/\gamma N_1}{1 + \xi(\partial V^2/\partial X_1)/\gamma N_1} \tag{23}$$

$$= \frac{1 + \nu\alpha^2}{1 + \nu} \tag{24}$$

$$= \alpha^2 + \frac{1 - \alpha^2}{1 + \nu}. \tag{25}$$

By invoking the "well-known" single crossing property, $\alpha^1 > \alpha^2$, we can obtain:

$$\alpha^2 < \alpha^1 < 1. \tag{26}$$

From this, we can prove the positive marginal tax rate on low-ability individuals as follows:

$$\alpha^1 = 1 - T' - p'Q_1 \text{ (from (6))} \tag{27}$$

$$< 1, \tag{28}$$

which implies $T' + p'Q_1 > 0$.

As (16) is redundant with (14) and (15), we can leave out (16). In the same way, (19) can be omitted for convenience. Therefore, using (15) and (18), we can discuss the optimal pricing of Q_i below.

III Optimal Pricing

Now, we can discuss optimal pricing of the publicly provided private good, Q by combining (15) and (7), and (18) and (7), respectively. First, combining (18) and (7), we obtain:

$$MC(Q) = p(Y_2), \tag{29}$$

which is a well-known standard result for efficiency of $p = MC$. Charging the marginal cost to high-ability individuals is optimal. Next, combining (15) and (7) in a similar way yields

$$\frac{\xi \widehat{V}_Q^2 + \gamma N_1 MC(Q)}{\xi \widehat{V}_X^2 + \gamma N_1} = p(Y_1), \tag{30}$$

which looks complex but contains a key implication of the paper.

a Two interesting cases

To interpret (30), we first need to analyze (15), the MRS_{QX} of type 1 by considering two cases:

$$(a)\ \frac{\widehat{V}_Q^2}{\widehat{V}_X^2} = \frac{V_Q^1}{V_X^1},\ or\ \widehat{MRS}_{QX}^2 = MRS_{QX}^1,$$ [3]

[3] $\widehat{MRS}_{QX}^2 = MRS_{QX}^1$ means the consumption goods X_i and Q_i are weakly separable from L in the utility of the households. This happens when both a mimicker and a low-ability person have the same valuation about X_i and Q_i regardless of L. Given that some publicly provided private goods are not irrelevant to labor supply or intrinsically more valuable to low-income individuals, the preference restriction of $\widehat{MRS}_{QX}^2 = MRS_{QX}^1$ is unrealistic in our context.

$$(b)\ \frac{\widehat{V}_Q^2}{\widehat{V}_X^2} < \frac{V_Q^1}{V_X^1},\ or\ \widehat{MRS}_{QX}^2 < MRS_{QX}^1,$$

where the latter case, (b) is more interesting for our study because some goods are more preferred by low-income individuals in a relative sense.[4]

The case of equal MRS. First, we start with the case (a) and have

$$\begin{aligned} \frac{V_Q^1}{V_X^1} &= \frac{\xi \widehat{V}_Q^2 + \gamma N_1 MC(Q)}{\xi \widehat{V}_X^2 + \gamma N_1} \quad \text{(from (15))} \\ &= \frac{\widehat{V}_Q^2}{\widehat{V}_X^2} \quad \text{(case } a\text{)}, \end{aligned} \tag{31}$$

where the first line of (31) is equal to (15), and the second line is simply the case (a). Therefore, we obtain $\widehat{V}_Q^2/\widehat{V}_X^2 = \{\xi \widehat{V}_Q^2 + \gamma N_1 MC(Q)\}/\{\xi \widehat{V}_X^2 + \gamma N_1\}$, which is simplified to $\widehat{V}_Q^2/\widehat{V}_X^2 = MC(Q)$, or $\widehat{V}_Q^2/\widehat{V}_X^2 = V_Q^1/V_X^1 = MC(Q)$. With this and an optimizing process equating MRS_{QX}^i to the price ratio, we obtain a conventional result, $MC(Q) = p(Y_1)$ and $MC(Q) = p(Y_2)$ from (20). It implies that the $p = MC$ rule still holds under the case (a) of $\widehat{MRS}_{QX}^2 = MRS_{QX}^1$.

The realistic case. Next, we consider the case (b) which is more realistic for our case. Through a similar analysis, we obtain

[4] This is type of relative comparison typically arises in discussions of international trade issues in a Heckman-Ohlin model.

$$
\begin{aligned}
\frac{V_Q^1}{V_X^1} &= \frac{\xi \widehat{V}_Q^2 + \gamma N_1 MC(Q)}{\xi \widehat{V}_X^2 + \gamma N_1} \quad \text{(from (15))} \\
&> \frac{\widehat{V}_Q^2}{\widehat{V}_X^2} \quad \text{(case } b\text{)},
\end{aligned} \tag{32}
$$

where the first line of (32) is equal to (15), and the second line is simply the case (b). Therefore, we obtain $\widehat{V}_Q^2/\widehat{V}_X^2 < \{\xi \widehat{V}_Q^2 + \gamma N_1 MC(Q)\}/\{\xi \widehat{V}_X^2 + \gamma N_1\}$, which is simplified to $\widehat{V}_Q^2/\widehat{V}_X^2 < MC(Q)$.[4] Substituting $\widehat{V}_Q^2 < \widehat{V}_X^2 MC(Q)$ into $\widehat{V}_Q^2$ in $\{\xi \widehat{V}_Q^2 + \gamma N_1 MC(Q)\}/\{\xi \widehat{V}_X^2 + \gamma N_1\}$ yields:

$$
\begin{aligned}
\frac{\xi \widehat{V}_Q^2 + \gamma N_1 MC(Q)}{\xi \widehat{V}_X^2 + \gamma N_1} &< \frac{\xi \widehat{V}_X^2 MC(Q) + \gamma N_1 MC(Q)}{\xi \widehat{V}_X^2 + \gamma N_1} \\
&= MC(Q).
\end{aligned} \tag{33}
$$

Given

$$
\frac{V_Q^1}{V_X^1} = \frac{\xi \widehat{V}_Q^2 + \gamma N_1 MC(Q)}{\xi \widehat{V}_X^2 + \gamma N_1}, \tag{34}
$$

we obtain the result:

$$
\frac{V_Q^1}{V_X^1} < MC(Q). \tag{35}
$$

[4] $\frac{\widehat{V}_Q^2}{\widehat{V}_X^2} < \frac{\lambda \widehat{V}_Q^2 + \gamma N_1 MC(Q)}{\lambda \widehat{V}_X^2 + \gamma N_1}$ becomes $\widehat{V}_Q^2(\lambda \widehat{V}_X^2 + \gamma N_1) < \widehat{V}_X^2\{\lambda \widehat{V}_Q^2 + \gamma N_1 MC(Q)\}$ by cross-multiplication, given both $\widehat{V}_X^2$ and $\lambda \widehat{V}_X^2 + \gamma N_1$ are positive. It reduces to $\widehat{V}_Q^2 < \widehat{V}_X^2 MC(Q)$.

The last result, $\frac{V_Q^1}{V_X^1} < MC(Q)$, arises when the marginal evaluation to the mimicker is less than that to the low-ability person: $\widehat{MRS}_{QX}^2 < MRS_{QX}^1$. As optimizing households equate MRS_{QX} to the price ratio, we obtain:

$$
\begin{aligned}
p(Y_1) &< MC(Q), &\quad (36)\\
p(Y_2) &= MC(Q) \quad \text{(from (20))}, &\quad (37)
\end{aligned}
$$

which means $p(Y_1) < p(Y_2)$. The deviation from the standard result, $p = MC$ and $MRS_{QX}^i = MRS_{QX}^j$ is for "efficient redistribution" in the presence of a private good with the IRS property and $\widehat{MRS}_{QX}^2 < MRS_{QX}^1$. Note that the private good with a IRS property here is comparatively more useful for low-income individuals, and this property has been utilized when setting the optimal pricing for $Q_1 < Q_2$. Figure 1 describes the equilibrium for case (b). The diagram (a) depicts aggregate demand curve of the low-ability persons, (b) depicts that of the high-ability ones and (c) contains marginal cost and average cost curve for Q. In equilibrium, the price for the high-ability equals to $MC(Q)$ and that of the low-ability is lower than $MC(Q)$.

We summarize the discussion above as follows:

Proposition 1 *In the presence of optimal nonlinear income taxation, the rule for optimal provision of publicly provided private goods with the IRS property involves the pricing* $p(Y_1) <,=,> MC(Q)$ *as the marginal evaluation of the goods to the mimicker is less than, equal to, or greater than that to the low-ability person (i.e.,* $\widehat{MRS}_{QX}^2 <,=,> MRS_{QX}^1$*).*

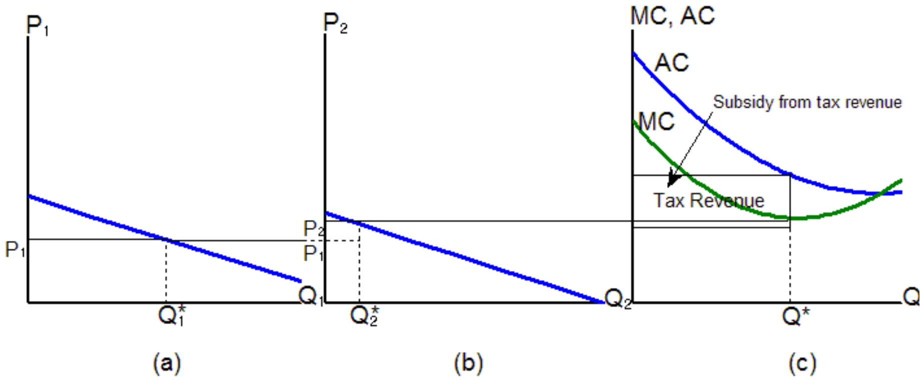

Figure 1: Note that $Q^* \equiv Q_1^* N_1 + Q_2^* N_1$ and p_1 is lower than p_2. The planner's solution is where $p_2 = MC(Q^*)$ holds. Tax revenue partially covers total production cost.

Interpretation. An intuitive interpretation applies to the optimal taxation and pricing result, explained above. To be concrete, consider the case in which the low-ability person values the publicly provided private good more than the mimicker, so $\widehat{MRS}^2_{QX} < MRS^1_{QX}$. Here, Proposition 1 tells that at the planner's optimum, $p(Y_1) < MC(Q)$ holds so the good should be "over-consumed" by the low-ability person in contrast to the traditional rule. For this to do so, it should be possible to show that a Pareto improvement is possible starting at a traditional optimal income tax equilibrium with the $p = MC(Q)$ rule satisfied. To see this is possible indeed, consider the following thought experiment. We start at $p(Y_1) = p(Y_2) = MC(Q)$, or $MRS^1_{QX} = MRS^2_{QX} = MC(Q)$, where an efficient public provision of private good has been believed to occur in accordance with the literature. Next, imagine increasing Q_i with $i = 1, 2$ incrementally and adjusting income tax structure such that each person pays as much as his or her

MRS_{QX} for the additional Q_i. There will be no change in the welfare of either person 1 or 2 and government budget will not have changed. However, since the additional Q costs a mimicking person MRS^1_{QX} which is greater than the mimicker's valuation, $\widehat{MRS}^2_{QX}$, the mimicker will be worse off. That is, the self-selection constraint is relaxed (i.e., mimicking is less attractive) and a change in the optimal tax structure and pricing function of Q can be undertaken which will make both persons better off. That is, $T' + p'Q_i$ for person 1 can be lowered with the same total tax revenue being collected. Person 1 is made better off, without inducing person 2 to mimic. At the same time, the higher preference for Q by low-ability individuals allows for $p(Y_1) < p(Y_2)$ for efficient redistribution.

b Unobservability of consumption

The previous section is based on the presumption that the government is able to observe individuals' consumption including Q. In some instances, however, consumption of Q may be unobservable due to lack of information or difficulties of collecting information about consumption. It is interesting to examine whether unobservability of Q makes a difference to our earlier results.

Proposition 2 *In the absence of information about consumption of Q, the rule for optimal provision of publicly provided private goods with the IRS property involves the uniform pricing, $p(Y_1) = p(Y_2) = MC(Q)$. See the proof at the appendix.*

The intuition behind Proposition 2 is that since Q does not play the role of relaxing the self-selection constraint, the efficiency gain from non-uniform pricing no longer exists, which justifies the uniform pricing.

c Complementarity between Q and L

Often public provision of private goods is justified when those goods help labor supply, i.e., complementarity between Q and L. This subsection will discuss how the complementarity affects our results about optimal income tax and pricing of Q, $T' + p'Q_i$. Complementarity of Q to labor – marginal willingness to pay in terms of MRS_{QX} rises with labor – implies that differentiating MRS^i_{QX} from (5-1) and (5-3) with respect to labor yields a positive sign:

$$\begin{aligned} \frac{\partial MRS^i_{QX}}{\partial L_i} &= p'_i w_i \qquad \text{(differentiating } MRS^i_{QX} \text{ w.r.t labor)} \qquad (38) \\ &> 0, \qquad \text{(complementarity between } Q \text{ and } L\text{)} \end{aligned}$$

where $p'_i = \partial p(Y_i)/\partial Y_i$, and w_i is non-negative. From this, one can deduce that $p_i\prime$ should be positive when Q is complementary to labor. Given $T' + p'Q_i = 0$ for the high-ability person due to (17) combined with (6), the marginal income tax rate of the high-ability person is negative in this case. Similarly, since $p_i\prime$ is also positive for low-ability individuals, T' can go down further leading to a greater supply of labor, i.e., less distortion in work. Therefore, we can find that the complementarity reinforces our earlier results with greater work incentives. This can be seen as an application of Corlette and Hague (1953): since Q is complementary to labor supply, favoring Q through offering a lower price or a better tax treatment is justified. As we specified the preference in the previous section, if the preference difference arises from different earnings ability and hence different labor supply, the complementarity of Q to labor occurs which leads to greater incentives to work to both low and high ability individuals.

On the other hand, if Q is independent of labor, we obtain:

$$\begin{aligned} \frac{\partial MRS^i_{QX}}{\partial L_i} &= p'_i w_i \\ &= 0, \end{aligned} \tag{39}$$

where $p'_i = \partial p(Y_i)/\partial Y_i$, and w_i is non-negative. From this, one can deduce that $p_i\prime$ should be zero which leads to that the marginal income tax rate of the high-ability individuals is zero, as in the literature. In this case, other things being constant, T' goes up, compared to the complementarity case, leading to less work. In a similar way, the marginal income tax rate of low-ability individuals remains positive which discourages their labor supply more compared to the case above.

Proposition 3 *With a complementarity between Q and L, $p_i\prime$ is positive, T' is negative for high-ability individuals, and T' gets smaller for low-ability individuals, boosting work incentives for all individuals.*

d Discussion

Using Propositions 1 and 2, we can offer a more specific explanation about the public provision of a private good, given the good is complementary to labor and more preferred by the low-ability individuals. Proposition 1 shows that an optimal pricing of Q is "discriminatory" with $p(Y_1) <$

$MC(Q) = p(Y_2)$ when the low-ability individuals' valuation of Q is higher than the high-ability ones' valuation. Furthermore, this difference in preferences generates an Pareto improvement—represented by a lowered value of $T' + p'Q_i$ for the low-ability ones—at the traditional optimal income tax equilibrium with the $p(Y_1) = p(Y_2) = MC(Q)$ rule satisfied. This implies the possibility of reduced marginal income tax rate of the low-ability individuals, when p' is fixed. From Proposition 2, we can deduct the rule for determining $p_i\prime$ and it relies on the complementarity of Q to labor, which, for example, makes $p_i\prime$ positive as Q is complementary to labor. Taking advantage of these results, we can conclude that given the characteristics of Q—complementary to labor and more valuable to the low-ability individuals, providing Q at a lower price to the low-ability persons makes them better off with a reduced marginal income tax rate, which eventually leads to greater labor supply. Even with these benefits that the low-ability individuals newly enjoy—the lower price and marginal income tax rate, the high-ability individuals are not willing to mimic since Q is less valuable to them. This is another interpretation of relaxation of the self-selection constraint. Resulting welfare improvement occurs when there is a difference in preferences for Q and a government provides that good.

In fact, some countries implement subsidy programs on the consumption of electricity, public transportation, etc., based on the income of households. Promising is empirical investigation of whether those publicly-provided private goods exhibit an IRS feature, and whether consumption of those goods really favor low-income individuals, which is the topic of the next section. Finally, it should also be noted that implementation of a non-linear pricing policy requires observability of consumption of Q. Otherwise, the usual uniform $p = MC$ rule for all individuals is still the social-welfare maximizing result.

IV Summary and Conclusions

We have studied the policy mix of optimal income taxation and pricing of publicly provided private goods with the IRS property. Our main results have established that public enterprises provide private goods not only for the IRS property, but also for relaxing the incentive problem of the tax system: they help relax the self-selection constraint of the optimal income tax problem through nonlinear pricing.

The intuition behind the results is that when some private goods with IRS properties (e.g., public transportation) relative to other goods are more valuable to low-ability individuals than high-ability ones in terms of the marginal rate of substitution (MRS), high-ability individuals are discouraged to mimic low-ability ones. In this particular case, our results provided theoretical underpinning for the low price of publicly provided goods for low-income individuals. Owing to the nonlinear pricing, the optimal income tax rate for low-income individuals can go down, allowing for welfare improvement. We believe that the highlighted policy mix between optimal income tax structure and pricing of publicly provided private goods can offer new insight into optimal tax and expenditure policy combinations.

Appendix

Proof of Proposition 2 In the previous section, we have assumed a government can obtain information of each individuals' consumption, which is not available in reality. As the government cannot observe consumption of individuals, the only information the government uses to discern type is the pre-tax income, Y_i. That is, a mimicker does not have to consume as much as X_1 and Q_1 any more but adjust his or her labor hours to obtain the pre-tax income, Y_1. In this case, the mimicker is able to choose amount of X and Q according to his or her preference under the given budget constraint

$$Y_1 - T(Y_1) = \widetilde{X_2}(Y_1) - P(Y_1)\widetilde{Q_2}(Y_1), \tag{44}$$

where $\widetilde{X_2}$ and $\widetilde{Q_2}$ represent choices of X and Q of the high-ability individuals, when mimicking the low-ability ones with pre-tax income, income tax and price of Q given as $Y_1, T(Y_1)$ and $P(Y_1)$. Note that $\widetilde{X_2}$ and $\widetilde{Q_2}$ should be the optimal function of Y_1 as the mimicker's choices of X and Q now depend on the income of low-ability individuals. We can rewrite the mimicker's optimality problem with the modified budget constraint as follows:

$$\mathcal{L}' = V^2(\widetilde{X_2}(Y_1), \widetilde{Q_2}(Y_1), Y_1) + \lambda_2'\{Y_1 - T(Y_1) - \widetilde{X_2}(Y_1) - P(Y_1)\widetilde{Q_2}(Y_1)\}, \tag{45}$$

where $\lambda_2\prime$ is the Lagrangian multiplier in the case of unobservability of Q. We can derive first order conditions for $\widetilde{X_2}$ and $\widetilde{Q_2}$ which simply become

$$\widetilde{V_X^2} - \lambda_2' = 0, \tag{46}$$
$$\widetilde{V_Q^2} - P(Y_1)\lambda_2' = 0, \tag{47}$$

where $\widetilde{V_X^2} = \frac{\partial V^2}{\partial \widetilde{X_2}}$ and $\widetilde{V_Q^2} = \frac{\partial V_Q^2}{\partial \widetilde{Q_2}}$. With this newly defined optimal problem of mimicker, the social planner has to change its self-selection constraint (10) into

$$V^2(X_2, Q_2, Y_2) \geq V^2(\widetilde{X_2}(Y_1), \widetilde{Q_2}(Y_1), Y_1), \tag{48}$$

and the Lagrangian expression for the social planner should be

$$\begin{aligned}
\Omega^{'}(X_i, Y_i, Q_i, \mu, \xi, \gamma) \quad = \quad & V^1(X_1, Y_1, Q_1) + \mu[V^2(X_2, Q_2, Y_2) - \overline{V^2}] \\
& + \xi[V^2(X_2, Y_2, Q_2) - V^2(\widetilde{X_2}(Y_1), Y_1, \widetilde{Q_2}(Y_1))] \\
& + \gamma[N_1(Y_1 - X_1) + N_2(Y_2 - X_2) - C(Q)], \quad (49)
\end{aligned}$$

where $\mu\prime, \xi\prime$ and $\gamma\prime$ are Lagrangian multipliers. With this, one can derive another first order conditions on X_i, Y_i and Q_i

$$V_X^1 - \gamma N_1 = 0, \tag{50-1}$$

$$V_Y^1 - \xi \frac{\partial V^2(\widetilde{X_2}(Y_1), Y_1, \widetilde{Q_2}(Y_1))}{\partial Y_1} + \gamma N_1 = 0, \tag{50-2}$$

$$V_Q^1 - \gamma N_1 MC(Q) = 0, \tag{50-3}$$

$$\mu V_X^2 + \xi V_X^2 - \gamma N_2 = 0, \tag{50-4}$$

$$\mu V_Y^2 + \xi V_Y^2 + \gamma N_2 = 0, \tag{50-5}$$

$$\mu V_Q^2 + \xi V_Q^2 - \gamma N_2 MC(Q) = 0, \tag{50-6}$$

where $MC(Q) = \frac{\partial C(Q)}{\partial Q}$. As there is no change in optimal conditions for the high-ability individuals, we will now restrict attention to equation (50-1),(50-2) and (50-3). Rearranging the first-order conditions of low-ability ones, we obtain

$$\frac{V_Y^1}{V_X^1} = -\frac{-\xi\frac{dV^2(\widetilde{X_2}(Y_1),Y_1,\widetilde{Q_2}(Y_1))}{dY_1}+\gamma N_1}{\gamma' N_1}, \tag{51}$$

$$\frac{V_Q^1}{V_X^1} = MC(Q), \tag{52}$$

$$\frac{V_Y^1}{V_Q^1} = -\frac{-\xi\frac{dV^2(\widetilde{X_2}(Y_1),Y_1,\widetilde{Q_2}(Y_1))}{dY_1}+\gamma N_1}{\gamma' N_1 MC(Q)}. \tag{53}$$

Analyzing these conditions yields different results of optimal pricing of Q and income taxation. To be concrete, equation (52) and (7) yield $p(Y_1) = MC(Q)$, which implies the uniform price for good Q regardless of difference in preference of each types. The intuition is that since Q does not play the role of relaxing the self-selection constraint, the efficiency gain from non-uniform pricing no longer exists, which justifies this uniform pricing. The optimal taxation for low-ability individuals is derived from equation (51) and (6) ,which can be written as follows:

$$\begin{aligned}
\frac{V_Y^1}{V_X^1} &= -\frac{-\xi\frac{dV^2(\widetilde{X_2}(Y_1),Y_1,\widetilde{Q_2}(Y_1))}{dY_1}+\gamma N_1}{\gamma N_1} && \text{(from equation (50))} \\
&= -(1-\frac{\xi}{\gamma N_1}\frac{\partial V^2(\widetilde{X_2}(Y_1),Y_1,\widetilde{Q_2}(Y_1))}{\partial Y_1}) \\
&= -(1-T'-p'Q_1) && \text{(from equation (6))}, \quad (54)
\end{aligned}$$

where the first line of (54) is equal to equation (51), the second line simply a rearrangement of equation (51) and the last line equation (6). From (51) one can deduce $T' + p'Q_1 = \frac{\xi}{\gamma N_1}\frac{\partial V^2(\widetilde{X_2}(Y_1),Y_1,\widetilde{Q_2}(Y_1))}{\partial Y_1}$ where

$\frac{\xi}{\gamma N_1} > 0$ but the sign of $\frac{\partial V^2(\widetilde{X_2}(Y_1), Y_1, \widetilde{Q_2}(Y_1))}{\partial Y_1}$ is unclear. For analytic purpose, we need to specify $\frac{dV^2(\widetilde{X_2}(Y_1), Y_1, \widetilde{Q_2}(Y_1))}{dY_1}$ and we obtain

$$\frac{dV^2(\widetilde{X_2}(Y_1), Y_1, \widetilde{Q_2}(Y_1))}{dY_1} = \frac{\partial V^2}{\partial \widetilde{X_2}} \frac{d\widetilde{X_2}}{dY_1} + \frac{\partial V^2}{\partial \widetilde{Q_2}} \frac{d\widetilde{Q_2}}{dY_1} + \frac{\partial V^2}{\partial Y_1}. \tag{55}$$

Using equation (46) and (47), and definition of Lagragian multiplier, we can rewrite and rearrange equation (55) as follows:

$$\begin{aligned} \frac{\partial V^2(\widetilde{X_2}(Y_1), Y_1, \widetilde{Q_2}(Y_1))}{\partial Y_1} &= \lambda_2\prime \frac{\partial \widetilde{X_2}}{\partial Y_1} + P(Y_1)\lambda_2\prime \frac{\partial \widetilde{Q_2}}{\partial Y_1} + \lambda_2\prime \\ &= \lambda_2\prime [\frac{\partial \widetilde{X_2}}{\partial Y_1} + P(Y_1)\frac{\partial \widetilde{Q_2}}{\partial Y_1} + 1] \end{aligned} \tag{56}$$

where $\lambda_2\prime$ and $\frac{\partial \widetilde{X_2}}{\partial Y_1}$ are expected to be positive while $\frac{\partial \widetilde{Q_2}}{\partial Y_1}$ is possibly negative since publicly provided public good Q is usually favorable for low-income individuals and has negative income elasticity. Following the definition of income elasticity, we can rewrite the last line of equation (56) as

$$\frac{\partial V^2(\widetilde{X_2}(Y_1), Y_1, \widetilde{Q_2}(Y_1))}{\partial Y_1} = \lambda_2\prime [\frac{\partial \widetilde{X_2}}{\partial Y_1} + P(Y_1)\frac{\partial \widetilde{Q_2}}{\partial Y_1} + 1] \tag{57}$$

$$= \lambda_2\prime [\epsilon_{X,Y} \frac{Y_1}{\widetilde{X_2}} + \epsilon_{Q,Y} P(Y_1) \frac{Y_1}{\widetilde{Q_2}} + 1] \tag{58}$$

where $\epsilon_{X,Y}$ and $\epsilon_{Q,Y}$ are income elasticities of demand for Q and X respectively. Without loss of generality, we can assume that consumption of X and Q rises with increase in income, equivolently X and Q are normal goods. This leads to $\epsilon_{X,Y} > 0$ and $\epsilon_{Q,Y} > 0$. In this case, as equation (58)leads to $\frac{\partial V^2(\widetilde{X_2}(Y_1),Y_1,\widetilde{Q_2}(Y_1))}{\partial Y_1} > 0$ and eventually $\frac{\xi}{\gamma N_1}\frac{\partial V^2(\widetilde{X_2}(Y_1),Y_1,\widetilde{Q_2}(Y_1))}{\partial Y_1} = T' + p'Q_1 > 0$ holds, we can conclude that the marginal tax rate combined with the pricing condition has to be positive for the low-ability individuals, i.e., the same result of the previous observable case.

References

[1] Atkinson, A., J. Gomulka, and N. Stern, "Spedning on alcohol: evidence from the family expenditure survey 1970-1983," Economic Journal, 100, 1990, 808-827.

[2] Blomquist, S and Christiansen, V, (1998), "Topping up or opting out? The optimal design of public provision schemes", *International Economic Review*, Vol. 39, No 2, 399-411.

[3] Boadway, R. and Keen, M., (1993), "Public goods, self-selection and optimal income taxation", *International Economic Review*, Vol. 34, No. 3, 463-478.

[4] Corlette, W. J and D. C. Hague, (1953), "Complementarity and the excess burden of taxation", *Review of Economic Studies*, 21:21-30.

[5] Diamond, P. and J. A. Mirrlees, (1971), "Production efficiency", *American Economic Review*, 61(1), 8 - 27.

[6] Gordon, Roger H. and Wojciech Kopczuk, (2008), "The choice of the personal income tax base", NBER working paper.

[7] Mirrlees, J. A., (1971), "An exploration in the theory of optimal income taxation", *Review of Economic Studies*, 38, 175-208.

[8] Naito, H., (1999), "Re-examination of uniform commodity taxes under a non-linear income tax system and its implication for production efficiency", *Journal of Public Economics,* 71, 165–188.

[9] Pirttilä, J. and Tenhunen, S., (2007), "Pawns and queens revisited: public provision of private goods when individuals make mistakes", *International Tax Public Finance* 15, 599-619.

[10] Preston, A. E, (1993), "Efficiency, quality and social externalities in the provision of day care: comparisons of nonprofit and for-profit firms", *The Journal of Productivity Analysis*, 4, 165-182.

[11] Ringstad, V. and LØyland, K., (1998), "Cost-optimal scale of child care institutions", *Journal of Productivity Analysis*, 10, 305-311.

[12] Snyder, C. and W. Nicholson (2008), Microeconomic theory: Basic principles and extension (10th ed.), Belmont, CA : Thomson Business and Economics.

[13] Stiglitz, J. E., (1982), "Self-selection and Pareto efficient taxation", *Journal of Public Economics* 17, 213-240.

국문초록

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규모수익체증은 자연독점의 존재를 설명하고 이 때 생산은 주로 공기업을 통해서나 정부의 규제를 하에 이루어진다는 것은 잘 알려진 사실이다. 이 논문은 공기업이 사적 재화를 공급하는 것은 규모수익증가의 성격뿐만 아니라 조세제도에서 incentive problem을 완화하는 측면도 있기 때문이라는 것을 보여준다. 즉, 공기업이 비선형 가격책정을 통해서 사적재화를 공급하는 것이 최적조세에서의 자기선택제약을 완화해주는 역할을 한다는 것이다. 그 이면의 직관은 대중교통서비스와 같이 규모수익증가의 성격을 가진 사적재화가 한계대체율의 측면에서 상대적으로 고소득층에 비해 저소득층에게 더 가치 있는 경우에는 고소득층이 저소득층을 따라하는 행위가 억제된다는 것이다. 우리가 얻은 결과는 공적으로 제공되는 사적재화에 대해서 효율적 분배 법칙인 p=MC rule을 깨고 저소득층에게 더 낮은 가격에 재화를 공급하는 것에 대한 이론적 타당성을 제공해준다. 더불어, 최적 비선형 가격 책정은 저소득층과 고소득층 모두의 한계세율을 낮춰주어 근로 장려 효과를 가져 온다.

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