경제학석사 학위논문

Steady-State Solutions of Optimal Tax Mixes for the Pension

균제상태에서의 연금과 최적조세

2013년 8월

서울대학교 대학원

경제학부 경제학전공

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Abstract

Steady-State Solutions of Optimal Tax Mixes for the Pension

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This paper studies the optimal tax policy in welfare states where the nominal pension level is higher than the optimum level, but the government has difficulties reforming the pension because of political voices which advocate expansionary fiscal policies. This research provides proper tax policy directions for the many countries experiencing this problem, especially developed countries in Europe. I built a recursive optimization problem based on a modified version of two-period OLG model to which I added the pension system and lifetime uncertainty. By using an analytical approach and calibration, it is found out that a social planner can improve welfare by increasing the consumption tax, decreasing the labor income tax, and retaining the capital income tax in the situation of an excessively high level of pension. Besides uncovering the optimal taxation, this research also gained a new insight that a proper tax mix can develop welfare when expenditure-side rigidities are present.

Key Words : pension; consumption tax; optimal policy mix; OLG model

Student Number : 2011-23161

I would like to inform you that this paper is worked with my thesis advisor Chul-In Lee.

1. Introduction

In many welfare states, the main issue related with a pension system is that young generations are more and more burdened so to support their old generation.; they are imposed a high level of distortionary taxes to finance the pension. Such distortionary taxes thus decreases young generations' utility and then, reduce their work motivation. Factors that contribute to this socioeconomic problem are 1) `Pay as You Go(PYAGO)' system and 2) demographic trend, falling birth rate and aging populations. Most of welfare states operate PYAGO pension systems where the current workers should financially support the current pensioners. As the elder lives longer and longer, i.e. aging populations and the birth rates decrease, the relative number of the young to the old becomes smaller. Due to the coexistence of these two factors, young generations are more and more burdened.

One very simple solution to this issue is just reducing the level of pension to alleviate the young's financial load, but it is hardly achieved because of political movements such as citizens' strong desire to accomplishing a high level of social welfare and governments' solid orientation to expansionary fiscal policy. Moreover, in the demographic trend, a falling birth rate and aging populations, the elderly become more relatively compared to the young and they give a strong voice to liberal pension system even though the optimal pension level should be diminished because young workers who can financially support elderly people become less. These make it more difficult to reform pension system.

In this paper, I analyze how tax structures should be in this situation where the pension system deviates above the optimal level but it is hard to reduce its nominal value because of political reasons. Prior to answer this question, it needs to be analyzed what tax systems welfare states have in reality. The Organization for Economic Cooperation and Development (OECD) data show that the countries with generous pension systems tend to have high consumption taxes. (See Figure1)

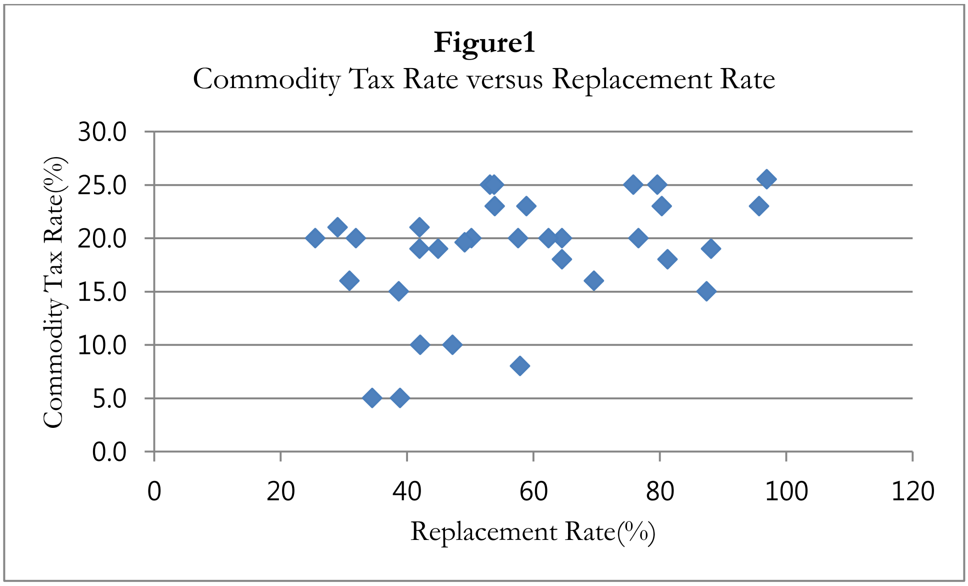

Figure1
Commodity Tax Rate versus Replacement Rate

The replacement rate here is gross replacement rate of OECD countries.[1]

This paper examines whether or not such empirical tax trends are optimal and why the patterns are optimal if it is true. By doing so, it is automatically answered how tax structures should be in welfare countries where people want benevolent pension system and then, the government cannot adjust the nominal value of pension to be optimal.

The main economic tool to find optimal taxes in this paper is two periods Overlapping Generations Model(OLG), i.e. people live two periods, youth and old age. At one point, young and old generations thus live simultaneously. The young generation works and bears labor income and commodity taxes to fund pension for

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[1] There are three kinds of pension schemes, public, mandatory private, and voluntary private pension scheme. I regard public and mandatory private pension scheme in the real economy as public pension in this model and then, the gross replacement rate is yielded by benefit provided from public and mandatory private pension. It is reasonable because government directly or indirectly affect the two pension schemes to secure agent's future consumption and such pension schemes are under government's control.

the old generations. At the same time, the old one lives in their savings and pension without working and pay capital income and commodity taxes to fund pension for themselves. Such distortionary taxes to finance pension detract economic efficiency, but the pension system secure people's income when they retire. The social planner wants to maximize their social welfare function which consists of not only current young and old generations but also unborn future generations. To find the optimal level of pension, they have to consider that different levels of pension bring about disparate effects. If pension level is quite low, it creates a little financial burden to young generation, but does not secure old one's income. On the contrary, if the pension level is quite high, it surely secures old generation's income, but it gives a big financial load to young one. As I said above, the main question in this paper is how government should construct tax structures to maximize social welfare when the pension level is set high by political processes. They can escalate social welfare by reforming tax mixes, because taxes can adjust real value of pension to be the optimum indirectly. In the OLG model here, the high pension level is exogenously given and the optimal fiscal policies, i.e. taxes, are endogenously determined to achieve social welfare maximizing.

Using analytical tool, I proved that the steady-state optimal tax mixes exist. To investigate how tax structures should be, I used qualitative analysis and so, found out that commodity taxes should increase, labor incomes taxes should decrease, and capital incomes taxes should not be changed in the steady state to maximize social welfare when the pension level is higher than the optimal one. The intuition behind raising commodity taxes is that even if it is impossible to reduce nominal value of pension, the real value can be indirectly lessened by increasing commodity taxes because such taxes raise price level, so lower purchasing power of pension. Using commodity taxes, the government let pension be close to the optimal level. The implication behind lowering labor income taxes is to motivate young generations to work. To financially support old generation, young generation already pays commodity and labor income taxes. When the pension level goes up, the young group should pay high commodity taxes, i.e. goods are expensive and their work incentive shrinks. To motivate the young to work and then increase the total output, the labor income taxes go down. The other reason for decreasing labor income taxes is to alleviate moral hazard. The young generation does not have a strong desire to saving for the future and thus, does not work hard because the high level of pension secures their future income. To reduce such laziness, the labor income taxes increase. The unchanged capital income taxes at the optimum in the steady state is consistent with Chamley(1986)'s result: to raise government revenue, labor income taxes can be used, but capital income taxes should not be exploited in the long run. Financing through capital income taxes reduces motivation to save, hinders capital accumulation, and so damages production efficiency. For these reasons, the capital income taxes should be fixed whatever the pension level is. Both young and old generations pay commodity taxes and only the former group pays labor income taxes. Combining the two results of increasing commodity taxes and decreasing labor incomes taxes, we have a simple but very important implication: as pension level becomes higher and higher above the optimal one, current pensioners receiving such increased pension should bear more and more financial burden themselves at the optimum.

This paper is arranged as follows. Section 2 explains the OLG model and assumptions used to analyze optimal tax schemes in detail. Section 3 proves the existence of optimal tax mixtures in the steady state based on the section 2's model. Section 4 shows very important results for main questions here using qualitative method with some specifications. Section 5 summarizes the key implications found out in this paper.

ABBREVIATIONS

PYAGO: Pay as You Go pension system
OECD: Organization for Economic Cooperation and Development
OLG: Overlapping Generations Model
CRS: Constant Returns to Scale
FOC: First Order Condition
RC: Resource Constraint
GBC: Government Budget Constraint
IBC: Individual Budget Constraint

2. The Model

A. Environment

The key economic tool is OLG model. Based on this useful model, I set some assumptions to make the model simple but still describing the real economy. The assumptions are as follows.

People live two periods which are youth and old age and then, young and old generations live contemporaneously. People work only when they are young and they pay labor income and commodity taxes to finance pension for the old generation. When people become old, they lose their jobs and thus, they live in their savings and pension. In this period, they pay capital income and commodity taxes to fund pension for themselves.

There are identical people in terms of both earning ability and preference for consumption and leisure, i.e. they have the same wage and utility function. This homogeneity assumption deprives pension of redistributive function within generation. In the qualitative analysis, we will see how this supposition affects the optimal labor income taxes.

Those who die right after the first period can consume their private savings for the second period consumption before dying. They cannot bequeath their savings to the next generation as they do not have children. This assumption limits the agents' behavior into consuming. However, these two behaviors do not make difference because people can use their money for their utility therefore their saving motivations are not changed.

The agents have no borrowing constraint, because it is not perfectly sure to live the second period and so, they cannot borrow money in the first period on security of pension in the next period. This assumption is realistic. In the real economy, financial market does not lend money on security of only pension to people who do not have any property and children who pay their parent's debt.

In this model, pension is exogenously set above the optimal level to describe the situation that pension level is determined by political process, not economic process and established higher than the optimal level to satisfy the public's desire to benevolent welfare systems.

Government can only raise their revenue through age-independent consumption, labor income and capital income taxes and there is no depreciation on capital.

B. Individual's Decisions.

(1) $Max\ U(C_t, Z_{t+1}, l_t) = \theta \cdot (U(C_t, l_t)^1 + \beta \cdot U(Z_{t+1}, 1)^2) + (1-\theta) \cdot U\left(\frac{(1-\tau_{w,t})\, w_t l_t}{(1+\tau_{c,t})}, l_t\right)^3$

$s.t.$

(2) $$\left(1+\tau_{c,t}\right)C_t + a_t = \left(1-\tau_{w,t}\right) w_t l_t$$

(3) $$\left(1+\tau_{c,t+1}\right)Z_{t+1} = \left(1+\left(1-\tau_{r,t+1}\right)r_{t+1}\right) a_t + m_{t+1}$$

(4) $$\left(1+\tau_{c,t}\right)C_t + \frac{(1+\tau_{c,t+1})Z_{t+1}}{(1+(1-\tau_{r,t+1})r_{t+1})} = \left(1-\tau_{w,t}\right) w_t l_t + \frac{m_{t+1}}{(1+(1-\tau_{r,t+1})r_{t+1})}$$

A representative agent's total expected utility function is given as (1). Here, θ is a survival probability to live the second period. So the former part in (1) is the utility that people face when they live until the second period with probability θ. The utility is additively separable with the first and second period utility. The latter part is the utility that people face when they die right after the first period with probability $(1-\theta)$. They consume their whole savings in the first period before dying and thus, the first period consumption here is different with the one when agent lives until the second period. Both periods utility functions are strictly concave and twice differentiable. $C, Z,$ and l respectively indicate current(1st period) consumption, future(2nd period) consumption, and leisure. The subscript t is the time when variables are realized. β is time preference for the second period utility. In the youth age, people work and allocate their earnings into current consumption and savings, a, for the future consumption – their saving is invested as the next period's capital stock or is borrowed by government issuing one period risk free bond.. They pay labor incomes and commodity taxes at that time. Individuals, thus, face the first period budget constraint (2). In the old age, if people survive, they live in their savings and pension. They pay capital incomes and commodity taxes at that time. Agents, thus, face the second period budget constraint (3), but their action in the second period is predetermined because they just consume their savings already allocated in the first period. The representative individual chooses $C, Z,$ and l to maximize their expected utility in their youth age under the life-time budget constraint (4) which combines the first and second period budget constraints. In fact, we need no borrowing constraint in addition to (4) because we have an assumption that people cannot borrow due to their life uncertainty in above environment part. However, if this constraint is non-binding, i.e. people save not borrow to maximize their utility, we can ignore this constraint. The numerical analysis in section 4 supports that the constraint is non-binding, so we ignore it from now on.

Individual optimization gives two Euler equations as follows.

(5) $$\theta \cdot {U^1}_{l,t} + (1-\theta) \cdot \frac{dU^3}{dl} + \theta \cdot {U^1}_{c,t} \frac{(1-\tau_{w,t})\,w_t}{(1+\tau_{c,t})} = 0, \quad \frac{dU^3}{dl} = {U^3}_{c,t} \frac{(1-\tau_{w,t})\,w_t}{(1+\tau_{c,t})} + {U^3}_{l,t}$$

$$\left(\theta \cdot {U^1}_{c,t} + (1-\theta){U^3}_{c,t}\right) \frac{(1-\tau_{w,t})\,w_t}{(1+\tau_{c,t})} = -\left(\theta \cdot {U^1}_{l,t} + (1-\theta) \cdot {U^3}_{l,t}\right)$$

(6) $$\theta\beta {U^2}_{z,t+1} - \frac{\theta \cdot {U^1}_{c,t}}{(1+\tau_{c,t})} \cdot \frac{(1+\tau_{c,t+1})}{(1+(1-\tau_{r,t+1})r_{t+1})} = 0$$

The first one (5) is intra-temporal choice between current consumption and leisure. The second one (6) is inter-temporal choice between current and future consumption.

To address that our two periods model is consistent with the real economy, assuming the first period is equivalent to working period (25-60 years, at this age people work and save for their future) and the second one is equivalent to non-working period (60-95 years, at this age people receive pension and live in their savings and pension without working). In working period, people's behavior is almost the same - they work and save - and then, we consider this period as the first period in our model. In non-working period, people's activity is also almost the same - they live in their savings and pension - and thus, we regard this period as the second period in our model. However, people die at different times during non-working period in general. In other words, some people die long before the expected life of the economy and others die long after it. This allows pension to function as an insurance to deal with such a stochastic life expectancy. The main weak point in this model is that we assume people die at the same period of time i.e. those who survive after the first period die altogether at the end of the second period. This implies that anyone who survives until 60 years cannot die at different times during non-working period, so they die at the same period of time after living until 95 years. This setting does not permit a stochastic shock upon their life expectancy and so, it restricts the pension's role. However, the pension still secures income when people retire (the second period in this model and 60-95 years in the real economy). In other words, even under above strong setting, pension still plays its fundamental role as income guarantee during the non-working period and then, the optimal tax mixtures for the pension in this setting are not different with the setting where pension functions as insurance.

C. Firm's Decisions.

$$(7) \qquad F(K_t, L_t{}^d) \rightarrow F(k_t, l_t{}^d)$$

$$(8) \qquad Max\ \pi_t = \left[F\left(k_t, l_t{}^d\right) - w_t l_t{}^d - \gamma_t k_t\right],\ \gamma_t = r_t$$

$$(9) \qquad w_t = F_l\left(k_t, l_t{}^d\right),\ \gamma_t = F_k\left(k_t, l_t{}^d\right)$$

Following the neo-classical growth model, we presume that the production function (7) is constant returns to scale(CRS) function and a single product and factors(labor and capital) markets are competitive. The production function can be changed into per capita version. Here, k and l^d respectively refer per capita capital and per capita labor demand. Firm's profit function is given as (8). w and γ each indicate wage and rental rate. The rental rate is equivalent to interest rate r. Firm' optimization behavior creates two first order conditions(FOCs) (9). Marginal product of labor and marginal product of capital are equal to wage and rental rate respectively at the optimum.

D. Labor and Capital Market Conditions

$$(10) \qquad {l_t}^d = l_t$$

$$(11) \qquad (1+n)k_{t+1} = a_t - b_t$$

$$(12) \qquad (1+n)k_{t+1} = \left(1-\tau_{w,t}\right) w_t l_t - \left(1+\tau_{c,t}\right) C_t - b_t$$

In the labor market, the labor supply from individual's decision and the labor demand from firm's decision should be equated at the equilibrium (10). In the capital market, the next period's per capita capital is equal to private saving minus government's debt (11). This means the net capital accumulation in the next period is the rest of private savings after borrowing money to government. Here, multiplying $(1+n)$ is for reflecting population growth between two generations and the net savings of one agent is divided into $(1+n)$ agents in the next period. We can replace a with individual's first period budget constraint (2) and so, (12) is appeared.

E. Resource Constraint

$$(13) \qquad F(k_t, l_t) + k_t = C_t + \frac{\theta}{1+n} Z_t + (1+n)k_{t+1}$$

The resource constraint tells that the left hand side (LHS) is the total available products and the right hand side (RHS) is the total expenditures. In detail, the LHS is the sum of the output and the capital at this period and the RHS is the sum of young generation's consumption, old one's consumption, and next period's capital.

F. Government Budget Constraint

$$(14)\qquad b_t + \left(\tau_{w,t}\ w_t\, l_t + \tau_{c,t}\left(\theta \cdot C_t + (1-\theta)\cdot \frac{(1-\tau_{w,t})\, w_t\, l_t}{(1+\tau_{c,t})}\right)\right) + \frac{1}{1+n}\left(\theta \cdot \tau_{r,t} r_t a_{t-1} + \theta \cdot \tau_{c,t} Z_t\right) = \frac{1+r_t}{1+n} b_{t-1} + \frac{\theta}{1+n} m_t$$

The government budget constraint is divided into two parts, revenues and expenditures. They raise money through debt – one period risk free bond – and distortionary taxes – we set distortionary taxes as linear or flat taxes not non-linear taxes. The first bracket in the left hand side is the taxes from young generation and the second bracket is the taxes from old one. The government uses their money for repaying their debt and giving pension. Here, pension is all of government purchases.

G. Equilibrium

Given price w, γ and fiscal policies $\tau_c, \tau_w, \tau_r, m$, a representative individual chooses $C, Z,$ and l to maximize their utility and a firm chooses k and l^d to maximize their profit. If fiscal policies $\tau_c, \tau_w, \tau_r, m$ satisfy government budget constraint given such C, Z, l, k, and l^d and such quantity values satisfy capital and labor market conditions, and resource constraint, price systems w, γ, fiscal policies $\tau_c, \tau_w, \tau_r, m$, and quantity values C, Z, l, k, and l^d are competitive equilibrium. For a pair of fiscal policies, there is a competitive equilibrium in general. Among a lot of competitive equilibrium, government wants to find optimum which maximizes their social welfare function which consists of not only current young and old generations but also unborn future generations. We call such an optimal allocation as Ramsey allocation. However, the main question in this paper is how government should construct tax structures to maximize social welfare when the pension level is set high by political processes. So, using this model, I will solve the social planner's problem that high pension level is exogenously given and the optimal fiscal taxes are endogenously determined to maximize social welfare.

In the next section, I analytically prove the existence of the Ramsey allocation in the steady-state when high pension level is exogenously given. For this case, section 4 shows that how optimal tax mixtures should be in the long-run, using qualitative model with some specifications.

3. Analytical Discussion

A. The existence of Steady-State Optimal Tax Mixes.

To find the steady state optimal tax systems for pension in this dynamic problem, we use a dynamic program so called bellman equation. To construct the equation, we need to trim individual's decisions and RC, capital and labor market equilibrium conditions as follows.

(a). Rearranging Individual's Decisions.

(1) $Max\ U(C_t, Z_{t+1}, l_t) = \theta \cdot (U(C_t, l_t)^1 + \beta \cdot U(Z_{t+1}, 1)^2) + (1-\theta) \cdot U\left(\frac{(1-\tau_{w,t})\, w_t l_t}{(1+\tau_{c,t})}, l_t\right)^3$

$s.t.$

(4) $\left(1+\tau_{c,t}\right)C_t + \frac{\left(1+\tau_{c,t+1}\right)Z_{t+1}}{\left(1+\left(1-\tau_{r,t+1}\right)r_{t+1}\right)} = \left(1-\tau_{w,t}\right) w_t l_t + \frac{m_{t+1}}{\left(1+\left(1-\tau_{r,t+1}\right)r_{t+1}\right)}$

(15) $\left(1+\left(1-\tau_{r,t+1}\right)r_{t+1}\right) \cdot \left(1+\tau_{c,t}\right)C_t + \left(1+\tau_{c,t+1}\right)Z_{t+1} = \left(1-\tau_{w,t}\right)\left(1+\left(1-\tau_{r,t+1}\right)r_{t+1}\right) w_t l_t + m_{t+1}$

(16) $P_t C_t + \left(1+\tau_{c,t+1}\right)Z_{t+1} = \omega_t l_t + m_{t+1},\ P_t = \left(1+\left(1-\tau_{r,t+1}\right)r_{t+1}\right) \cdot \left(1+\tau_{c,t}\right),\ \omega_t = \left(1-\tau_{w,t}\right)\left(1+\left(1-\tau_{r,t+1}\right)r_{t+1}\right)$

(17) $X_t = X_t\left(P_t, \tau_{c,t+1}, \omega_t, m_{t+1}; \theta, \beta\right), for\ X = C, Z, l$

(18) $V_t = V_t(k_t, P_t, \tau_{c,t+1}, \omega_t, m_{t+1}; \theta, \beta)$

By multiplying $\left(1+\left(1-\tau_{r,t+1}\right)r_{t+1}\right)$ to both sides of (4), we can obtain

(15). Replacing the coefficients of C_t and $w_t l_t$ in (15) with P_t and ω_t respectively gives (16). Through individual's optimization process, the optimizing choices are represented by price systems and fiscal policies as (17). The maximized utility or indirect utility is also denoted by such exogenous parameters and capital stock which they face.

(b). RC, Capital and Labor market equilibrium conditions combining

$$(19) \qquad k_t + F(k_t, l_t) + \tau_{c,t} C_t + b_t - \frac{\theta}{1+n} Z_t - \left(1 - \tau_{w,t}\right) w_t l_t = 0$$

By combining three conditions, RC, Capital and Labor market equilibrium conditions, they are summarized as one simple equation and then it reduces the number of conditions which have to be considered.

(c). Social Welfare Function

$$(20) \qquad W_t\left(k_t, \tau_{r,t}, \tau_{c,t}, \tau_{w,t-1}, m_t\right) = Max \sum_{j=t}^{\infty} \left(\frac{1}{1+\rho}\right)^{j-t} \cdot V_j$$

The social welfare function at time t consists of indirect utilities of not only current young and old generations but also future unburned generations. ρ is the social discount rate or social rate of time preference. The value function at time t is represented by the state variables at the time. $k_t, \tau_{r,t}, \tau_{c,t}, \tau_{w,t-1}, m_t$.

(d). Dynamic Programming (Bellman Equation)

$$(21) \qquad W_t\left(k_t, \tau_{r,t}, \tau_{c,t}, \tau_{w,t-1}, m_t\right) = Max \Big[V_t + \mu_t\{k_t + F(k_t, l_t) - C_t - \frac{\theta}{1+n} Z_t - (1+n) k_{t+1}\} + \frac{1}{1+\rho} W_{t+1}\left(k_{t+1}, \tau_{r,t+1}, \tau_{c,t+1}, \tau_{w,t}, m_{t+1}\right)\Big]$$

To find the steady state Ramsey allocation which maximizes the social welfare, we firstly have to build a dynamic social planner's problem as (21). We should consider IBC, RC, GBC, Capital and Labor market equilibrium conditions in maximizing the social welfare function, but Walars's law allows to eliminate GBC, and IBC is already included in the indirect utility. We, thus, consider only the summarized equation (19) to optimize the social welfare function. The bellman eq. (21) is appeared where μ_t is the lagrange multiplier for the combining eq. (19). One very interesting property is that using either the combining eq. or RC yields the same result and so, for convenience we use the latter condition in the dynamic programming.

(e). Finding Steady State Solutions.

The FOCs. of (21) with respect to control variables $\tau_{r,t+1}, \tau_{c,t+1}, \tau_{w,t}$ are as follows.

(22-1) $\partial\tau_{r,t+1}: -\frac{\partial V_t}{\partial\tau_{r,t+1}} = \mu_t\left[\left(w_t l_{t,P} - C_{t,P}\right)P_{\tau_{r,t+1}} + \left(w_t l_{t,\omega} - C_{t,\omega}\right)\omega_{\tau_{r,t+1}}\right] + \frac{1}{1+\rho}W_2(t+1)$

(22-2) $\partial\tau_{c,t+1}: -\frac{\partial V_t}{\partial\tau_{c,t+1}} = \mu_t\left[\left(w_t l_{t,\tau_{c,t+1}} - C_{t,\tau_{c,t+1}}\right) + \left(w_t l_{t,P} - C_{t,P}\right)P_{\tau_{c,t+1}} + \left(w_t l_{t,\omega} - C_{t,\omega}\right)\omega_{\tau_{c,t+1}}\right] + \frac{1}{1+\rho}W_3(t+1)$

(22-3) $\partial\tau_{w,t}: -\frac{\partial V_t}{\partial\tau_{w,t}} = \mu_t\left[\left(w_t l_{t,P} - C_{t,P}\right)P_{\tau_{w,t}} + \left(w_t l_{t,\omega} - C_{t,\omega}\right)\omega_{\tau_{w,t}}\right] + \frac{1}{1+\rho}W_4(t+1)$

(22-4) $\mu_t = W_1(t+1)/\left((1+n)(1+\rho)\right)$

The lagrange multiplier of RC is given by (25). The intuitive meaning of the eq. is that the social marginal utility of unit capital at this period should be equivalent to the present value of social marginal utility of $\frac{1}{1+n}$ capital in the next period. The left hand side is cost and the right hand side is benefit when we invest unit capital at this period for the next period.

The state valuation function series W given by differentiating the bellman eq. with respect to $\tau_{r,t}, \tau_{c,t}, \tau_{w,t-1}, k_t$ are as follows.

(23-1) $\partial\tau_{r,t}: W_2(t) = -\frac{\theta}{1+n}\mu_t\,[Z_{t,P}P_{\tau_{r,t}} + Z_{t,\omega}\omega_{\tau_{r,t}}]$

(23-2) $\partial\tau_{c,t}: W_3(t) =$
$-\frac{\theta}{1+n}\mu_t\,[Z_{t,\tau_{c,t}} + Z_{t,P}P_{\tau_{c,t}} + Z_{t,\omega}\omega_{\tau_{c,t}}] + \mu_t\,\big[\big(w_t l_{t,P} - C_{t,P}\big)P_{\tau_{c,t}} +$
$\big(w_t l_{t,\omega} - C_{t,\omega}\big)\omega_{\tau_{c,t}}\big]$

(23-3) $\partial\tau_{w,t-1}: W_4(t) = -\frac{\theta}{1+n}\mu_t\,[Z_{t,P}P_{\tau_{w,t-1}} + Z_{t,\omega}\omega_{\tau_{w,t-1}}]$

(23-4) $\partial k_t: [W_1(t) - \mu_t(1+r)] = \frac{\partial V_t}{\partial k_t} + \mu_t\,\Big[\big(w_t l_{t,P} - C_{t,P}\big)P_{k_t} +$
$\big(w_t l_{t,\omega} - C_{t,\omega}\big)\omega_{k_t} - \frac{\theta}{(1+n)(1+\rho)}(Z_{t,P}P_{k_t} + Z_{t,\omega}\omega_{k_t})\Big]$

Using Roy's identities, we can express the partial derivatives of indirect utilities with respect to $\tau_{r,t+1}, \tau_{c,t+1}, \tau_{w,t}, k_t$ as follows:

(24-1) $\frac{\partial V_t}{\partial\tau_{r,t+1}} = -\lambda_t\big[C_t P_{\tau_{r,t+1}} - l_t\omega_{\tau_{r,t+1}}\big]$

(24-2) $\frac{\partial V_t}{\partial\tau_{c,t+1}} = -\lambda_t\big[C_t P_{\tau_{c,t+1}} + Z_t - l_t\omega_{\tau_{c,t+1}}\big]$

(24-3) $\frac{\partial V_t}{\partial\tau_{w,t}} = -\lambda_t\big[C_t P_{\tau_{w,t}} - l_t\omega_{\tau_{w,t}}\big]$

(24-4) $\frac{\partial V_t}{\partial k_t} = -\lambda_t\big[C_t P_{k_t} - l_t\omega_{k_t}\big]$

By substituting the state valuation function series W in the difference equations, (23) and Roy's identities' result (24) into the steady-state version of the FOCs, in the steady state we obtain the following homogeneous-equation system:

(25-1) $A_1 P_{\tau_r} - A_3\omega_{\tau_r} = 0$

(25-2) $A_1 P_{\tau_c} + A_2 - A_3\omega_{\tau_c} + \frac{\mu}{1+\rho}\big[(wl_P - C_P)P_{\tau_c} + (wl_\omega - C_\omega)\omega_{\tau_c}\big] =$
0

(25-3) $A_1 P_{\tau_w} - A_3 \omega_{\tau_w} = 0$

(25-4) $[W_1(t) - \mu_t(1+r)] = \mu[(1+n)(1+\rho) - (1+r)] = A_1 P_k - A_3 \omega_k$

Here, we ignore the time subscript because we focus on steady-state. The only solution to this homogeneous-equation system is as follows:

(26-1) $A_1 = \lambda C - \mu\big(wl_P - C_P + \big(K - (1+\tau_c)\big) \cdot \theta Z_P\big) = 0$

(26-2) $A_2 = \lambda Z - \mu\big(wl_{\tau_c} - C_{\tau_c} + \big(K - (1+\tau_c)\big) \cdot \theta Z_{\tau_c}\big) = 0$

(26-3) $A_3 = \lambda l - \mu\big(C_\omega - wl_\omega - \big(K - (1+\tau_c)\big) \cdot \theta Z_\omega\big) = 0$

(26-4) $\frac{\mu}{1+\rho}\big[(wl_P - C_P)P_{\tau_c} + (wl_\omega - C_\omega)\omega_{\tau_c}\big] = 0$

(26-5) $(1+n)(1+\rho) = (1+r)$

where, $K = (1+\tau_c) - \frac{1}{(1+n)(1+\rho)}$. (26-5) is often referred to as the modified golden rule which determines the optimal capital stock per capita. This result indicates that the capital stock is set at its first-best level. Using the partial derivatives of the steady-state version of the individual lifetime budget constraint, (16), we can rewrite the A_i as follows:

(27-1) $A_1 = \lambda C - \mu\big(C\theta + C_P(P\theta - 1) + Z_P K\theta + l_P(w - \theta\omega)\big) = 0$

(27-2) $A_2 = \lambda Z - \mu\left(Z\theta + C_{\tau_c}(P\theta - 1) + Z_{\tau_c} K\theta + l_{\tau_c}(w - \theta\omega)\right) = 0$

(27-3) $A_3 = \lambda l - \mu\big(l\theta - C_\omega(P\theta - 1) - Z_\omega K\theta - l_\omega(w - \theta\omega)\big) = 0$

Note that the terms C_P, Z_P and l_P denote the direct marginal effect of a change in the rate of P, not the total marginal effect. Using the properties of the Slutsky equations. eq (27) can be simplified in matrix form:

(28) $$\begin{bmatrix} C_P & Z_P & l_P \\ C_{\tau_c} & Z_{\tau_c} & l_{\tau_c} \\ -C_\omega & -Z_\omega & -l_\omega \end{bmatrix} \begin{bmatrix} P\theta - 1 \\ K\theta \\ w - \omega\theta \end{bmatrix} = \begin{bmatrix} S_{C_P} & S_{Z_P} & S_{l_P} \\ S_{C_{\tau_c}} & S_{Z_{\tau_c}} & S_{l_{\tau_c}} \\ -S_{C_\omega} & -S_{Z_\omega} & -S_{l_\omega} \end{bmatrix} \begin{bmatrix} P\theta - 1 \\ K\theta \\ w - \omega\theta \end{bmatrix} =$$

$$\left(\frac{\lambda}{\mu}-\theta\right)\begin{bmatrix} C \\ Z \\ l \end{bmatrix}$$

The coefficient matrix of (28) is a matrix of substitution terms which is negative semi-definite and symmetric. Then, the slutsky terms shows the following properties. $S_{Z_P} = S_{C_{\tau_c}}$ $S_{l_P} = -S_{C_\omega}$ $S_{l_{\tau_c}} = -S_{Z_\omega}$ and the only two terms $S_{Z_P}, S_{C_{\tau_c}}$ are positive and the all other terms are negative. The first principal sub-matrix of the coefficient matrix of (28) is negative because $S_{C_P} < 0$. The second principal sub-matrix is positive to make the coefficient matrix negative semi-definite. We sure the third principal sub-matrix non-positive, but we are not sure that it is negative. With the assumption that the determinant of the coefficient matrix is not zero, we sure the third principal sub-matrix is negative and the coefficient matrix is negative definite, so non-singular, i.e. invertible.

Proposition 1. With the assumption that the determinant of the coefficient matrix is not zero, the optimal steady-state fiscal policies can be determined.

The solutions are

$$(29) \qquad P\theta - 1 = \frac{D_1}{D}, K\theta = \frac{D_2}{D}, w - \omega\theta = \frac{D_3}{D}$$

where, D denotes the determinant of the coefficient matrix of (28) and D_1 denotes the determinant of the coefficient matrix with the first column replaced by the matrix on the right-hand side of (28), and similarly for D_2 and D_3. Substituting τ_r, τ_c and τ_w into P, K and ω in eq. (29) and rearranging such eq. create the optimal steady-state fiscal policies $\tau_r{}^*, \tau_c{}^*$ and $\tau_w{}^*$ represented by D, D_1, D_2, D_3 and θ.

4. Quantitative Analysis

A. Data

In the previous section, the existence of steady-state optimal tax mixtures for pension is proved, but we do not know the relationships between pension and consumption, labor income and capital income tax rates from the analysis. However, the empirical finding from OECD sample countries data(2011) tells that as pension level(here, replacement rate) rises, the commodity tax rate also increases. The simulation results of a qualitative model with few specifications in this section are consistent with the empirical finding and explaining why it is optimal. It, thus, is automatically answered how tax structures should be when high pension level is exogenously given.

B. A Numerical Model

$$(30)\qquad U(C,l)=\frac{C^{1-\gamma}}{1-\gamma}\left[1+M\left(1-\frac{1}{\gamma}\right)\frac{l^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right]^{\gamma},with\,\gamma>0,M>0,\eta>0$$

$$(31)\qquad F(k,l)=A\,k^{\alpha}l^{1-\alpha},with\,\alpha>0,A>0$$

For quantitative analysis, we need specifications about utility and production function forms. The preferences are depicted by Kimball and Shpiro's (2008) form of the King-Plosser-Rebelo (KPR) utility function which is widely used in macro economic literatures (30): where γ is the relative risk aversion parameter, M is the work aversion parameter, and η is the labor supply elasticity. Consumption and labor are not separable in this preference and so, the properties of this model are very general. The production is described by Cobb-Douglas production function (31): where A is total factor productivity, α is capital share in production.

Calibration parameters' values are set as follows (see Table 1). As the benchmark relative risk aversion and elasticity of effort supply, $\gamma=2$ and $\eta=0.6$ are set. They are the values widely used in many empirical literatures. We normalize total factor productivity A as 1. Survival probability and population growth rate –

respectively, $\theta = 0.94$(94%)[2] and $n = 0.006$(0.6%) - arise from OECD countries' data. The time preference β is 0.4. The widely used time preference is between 0.96 and 0.99 for one year, but we have to consider the one period in this model as 35years and we accept $\beta = 0.4$ which is 0.975 to the power of 35 as a time preference. Finally, exogenously given pension level $m = 4.966$, social discount rate $\rho = 0.009$, and work aversion parameter $M = 0.65$ are set to make simulated value - commodity taxes, pension replacement ratio and real interest rate - be consistent with benchmark values 18.5%, 57.0%, and 1.5% which come from OECD data.[3]

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2 The survival probability between 15-60years is 0.92(92%) from OECD countries' data. In this model, the first period matches the working period, 25-60years and the survival probability for this working period can be regarded as 0.94(94%). A very important assumption related with θ is that even though people die at different time during either working period or non-working period in real economy, in this model they die at the end of each period simultaneously. Hence, 100% of agent lives the first period and 94% of agent who survive lives the second period and die at the end of the second period.

3 Commodity tax rate is average of all countries' tax rates. Pension replacement ratio is gross pension replacement rate which include both public pension and mandatory private pension. The real interest rate is yielded by long term interest rates of 10 years treasury bond minus inflation rate. Except for population growth rate, all data is collected in 2011 and only the population growth rate is gathered in 2010.

TABLE 1. **The Parameters**

Parameter	Definition	Calibrated Value
γ	The relative risk aversion	2.000
η	The effort supply elasticity	0.600
M	The work aversion parameter	0.650
A	Total Factor Productivity	1.000
α	Capital Share in Production	0.425
m	Given pension level	4.966
θ	Survival probability	0.940
n	Population growth rate	0.006
ρ	Social discount rate	0.009
β	Time Preference	0.400

C. Simulation Results

Indeterminacy. In finding the Ramsey allocation of this problem, if the all fiscal policies $\tau_{r,}\tau_c, \tau_w, b$ and m are free variables, there is indeterminacy issue, i.e. no unique solution. However, if one of them is predetermined when finding optimal allocation, a unique solution is appeared.

Dividing both sides of (16) by $(1+\tau_{c,t+1})$ yields following eq:

(32) $$\frac{P_t}{(1+\tau_{c,t+1})}C_t + Z_{t+1} = \frac{\omega_t}{(1+\tau_{c,t+1})}l_t + \frac{m_{t+1}}{(1+\tau_{c,t+1})}$$

(33) $$P_t{}'C_t + Z_{t+1} = \omega_t{}'l_t + m_{t+1}{}'$$

where, the coefficient of C_t and l_t and $\frac{m_{t+1}}{(1+\tau_{c,t+1})}$ are replaced by $P_t{}'$, $\omega_t{}'$ and $m_{t+1}{}'$. The unique transformed fiscal polices $P_t{}'$, $\omega_t{}'$ and $m_{t+1}{}'$ which achieve social welfare maximization can be realized from several combinations $\tau_{r,}\tau_c, \tau_w$ and m. The reason why there are various combinations is that there are three constraints for $P_t{}'$, $\omega_t{}'$ and $m_{t+1}{}'$, but four control variables $\tau_{r,}\tau_c, \tau_w$ and m exist. The GBC restrictS one variable's freedom among such variables and so, a unique Ramsey allocation is appeared. However, if there exists b as another control variable, b is perfectly free variable and then, indeterminacy issue occurs.

For this reason, by setting commodity tax rate as a benchmark rate 18.5% from OECD countries' sample data, we can find the optimal pension level of our numerical model. By setting pension exogenously given in the calibration model higher than the optimal level found when commodity tax rate is 18.5%, it can be analyzed how the steady state optimal tax mixtures respond to the change of pension level.

The Base Case Numerical Results. Under the set of calibrated parameters value, we obtain the steady-state Ramsey allocations which maximize the social welfare (see Table 2). The optimal equilibrium on key variables is as follows: $\{\tau_c = 0.185, \tau_w = -0.179, \tau_r = -0.171, a = 0.215\}$. The positive saving supports the idea that no borrowing constraint is non-binding and then we can ignore it – this result is used for analytical discussion in section 2 and 3. Using many different initial values in numerical model, the numerical result is a stable optimal point. This, thus, supports the proposition 1 that optimal tax mixtures can be determined.

TABLE 2. **The Base Case Results**

Simulation Result (Steady-State)	τ_c	τ_w	τ_r	***C***	***Z***
	0.185	-0.179	-0.171	8.480	4.375
	l	***a***	***RR***	***k***	***r***
	1.282	0.215	0.570	427.6	0.015

$$\text{Replacement Ratio(RR)} = \text{pension benefit} / \text{income} = m / (w * l)$$

Labor income taxes here are unrealistically lower than ones in the real economy. The homogeneity in terms of productivity accounts for such low taxes. A crucial role of pension is to redistribute from high earning ability people to low ones. The redistribution, here, means that almost all people receive similar level of pension but their contributions to the social welfare system are quite different depending on their wages. To redistribute, labor income taxes should be highly positive and then, it is possible to collect more money from high ability people and less money from low ability people. At the same flat labor income taxes, it is true that those who have higher wage pay more taxes. This model, however, ignore the heterogeneity and so, pension does not have the redistribution function. Hence, it is not natural that labor income taxes in this numerical model are different from the ones in welfare states of real economy.

Capital income tax rates are positive in almost all welfare countries. However, the tax rate is strangely negative in this numerical model. This phenomenon also can be originated from our assumptions to make model simple, but I will deal with it in the end of this section.

Comparative Statics Analyses. I present comparative statics analysis which is how optimal Ramsey allocations including tax rates respond to the changes in pension benefit and parameter values.

Optimal Ramsey allocations. In this part, we increase all parameters and pension benefit by 5% except for survival probability θ, and increase the survival probability by 0.03 to make it less than unity. Considering both 'incentive' and 'income guarantee' effects of pension, social planner chooses optimal tax mixtures which maximize social welfare to respond the changes in parameter values. The comparative statics results are shown in Table 3.

TABLE 3. **Comparative Statics at the Base Case Equilibrium** (5% increase)

Variable	Base Case	$\uparrow$m	$\uparrow$n	$\uparrow\boldsymbol{\theta}$(0.97)	$\uparrow\boldsymbol{\gamma}$	$\uparrow$M
$\boldsymbol{\tau_c}$	0.185	0.304	0.256	0.221	0.169	0.228
$\boldsymbol{\tau_w}$	-0.179	-0.296	-0.246	-0.216	-0.161	-0.222
$\boldsymbol{\tau_r}$	-0.171	-0.171	-0.238	-0.128	-0.229	-0.171
$\boldsymbol{C}$	8.480	8.480	8.175	8.445	8.514	8.182
$\boldsymbol{Z}$	4.375	4.375	4.213	4.345	4.474	4.222
$\boldsymbol{l}$	1.282	1.282	1.288	1.289	1.295	1.237
$\boldsymbol{k}$	427.6	427.6	400.9	429.7	431.9	412.6
$\boldsymbol{a}$	0.215	0.236	0.317	0.332	0.260	0.215
$\boldsymbol{RR}$	0.570	0.627	0.585	0.567	0.564	0.591
$\boldsymbol{r}$	0.015	0.015	0.015	0.015	0.015	0.0157

(a) In the third column of table 3, we see that commodity tax rates goes up, labor income tax rates goes down, and capital income tax rates and other variables are not changed as pension is set above the optimal level. Pension has an unique optimal level in real term, but it is exogenously determined above the optimum point. In this situation where government cannot control pension benefit to be close to the optimal point, adjusting commodity tax rates plays the role to control the real value of pension indirectly because they can change prices and so modify purchasing power of pension. Increasing commodity tax rate returns the exogenously highly determined pension to the optimal level. Because of the higher commodity tax rates, young generations lose willingness to work. The higher pension demotivates them to work more and then, save more for their future consumption. In this context, to give young generations work motivation, the labor income taxes are diminished and therefore, their labor supply is not changed at the new optimum. The newly optimized labor income tax rate functions to perfectly neutralize the young worker's moral hazard from the higher commodity taxes and so, not only labor supply but also other variables including capital stock are not changed in the new optimal equilibrium. The total output in the new optimum is still the same because labor supply and capital stock are unchanged. The fixed capital income taxes at the optimum in the long run is congruous with Chamley(1986)'s result: to finance government budget, labor income taxes can be exploited but capital income taxes should not be used in the steady state. Raising revenue through capital income taxes demotivate to save, prevent capital accumulation, and so harm production efficiency.

For these reasons, the capital income taxes should not be changed whatever the pension level is.

(b) In the steady-state, as n rises, capital stock per capita decreases, because one unit investment for the next period capital stock is allocated into more people who live in the next period. As a result, total output per capita in the economy shrinks and current and future consumption also diminish. This result is compatible with the Solow growth model that depreciation rate on capital and population growth rate lower the capital stock per capita.

(c) As the relative risk aversion parameter γ increases, agents become more risk-averse and thus strongly want consumption smoothing. They hope to escalate the purchasing power of pension in preparation for losing their jobs in the second period. Setting lower commodity tax rate and higher labor income tax rate satisfy agent's needs and so the social welfare goes up.

(d) Next, we will see the effects of work aversion parameter M. Commodity tax rate goes up and labor income tax rate goes down to respond the increase in M. We can find a convincing explanation why the new equilibrium is so from the role of M. M measures how individual avoids to work and so, a higher M means that agent hates to work. In a high M case, agent take relatively a lot of leisure and a little of current and future consumption. By decreasing labor income taxes, government can stimulate agent's work motivation and let them consume more. These new tax systems raise social welfare in response to the increase in M.

(e) As θ goes up, the outstanding effect is that capital income tax rate rises. In the Euler equation (6) between current and future consumptions, we can delete from both sides and thus, it means that the survival probability has no impact on the inter-temporal choice. This is clear. Assuming that our total income fixed and we face the decision to allocate our income into current and future consumptions. Regardless of survival probability, individual always save the same amount of money with the saving that people do to maximize their utility when survival probability is 100%. By doing so, no matter how the survival probability is, they consume the same amount in the second period if they survive and they consume the fixed total income in the first period if they do not survive. This leads them to achieving maximal social welfare. Hence, the influence of θ on agent's behavior can be found from the other Euler equation (5) between current consumption and leisure. θ in the left hand side functions as a weight value between two different marginal utility of consumptions. If θ increases, the LHS - expected marginal utility of current consumption - goes up because $U^1{}_{c,t}$ is higher than $U^3{}_{c,t}$ - consuming all income before dying diminishes the marginal utility of consumption, $U^3{}_{c,t}$. In this case, the right hand side also goes up as leisure declines and so it equates the two sides. The intuition beyond the positive relationship between θ and labor supply is that the higher θ makes current consumption more attractive and so, it induces individual to work harder. As a result, when θ is 1, the labor supply, private saving, and capital accumulation arrive at the maximum amount. However, if θ goes down, the labor supply also diminishes and the capital accumulates less than the optimum level. So, the government stimulates workers to supply more labor and save more money by reducing capital income tax rates. This is an explanation for the negative capital income tax. Combining two Euler eq. (5) and (6) creates following eq.:

$$(34)\quad \left(\theta\beta U^2{}_{z,t+1}\cdot\left(1+\tau_{c,t}\right)\cdot\frac{\left(1+\left(1-\tau_{r,t+1}\right)r_{t+1}\right)}{\left(1+\tau_{c,t+1}\right)}+\right.$$
$$\left.(1-\theta)U^3{}_{c,t}\right)\frac{\left(1-\tau_{w,t}\right)w_t}{\left(1+\tau_{c,t}\right)}$$
$$=-\left(\theta\cdot U^1{}_{l,t}+(1-\theta)\cdot U^3{}_{l,t}\right)$$

We can check that as τ_r decreases, left hand side rises and so labor supply should increase to equate right hand side with left hand side.

As θ goes up, the government's desire to facilitate the willingness to work of agent becomes weaker and then increases the capital income tax rates. Hence, when θ is close to 1, the capital income tax is close to 0. (see table 4).

TABLE 4. **Comparative Statics at the Base Case Equilibrium**

(survival probability, given m)

Variable	Base Case	$\uparrow\boldsymbol{\theta}$(0.98)	$\uparrow\boldsymbol{\theta}$(0.99)	$\uparrow\boldsymbol{\theta}$(0.995)	$\uparrow\boldsymbol{\theta}$(1)
$\boldsymbol{\tau_c}$	0.185	0.234	0.248	0.255	0.262
$\boldsymbol{\tau_w}$	-0.179	-0.230	-0.246	-0.254	-0.262
$\boldsymbol{\tau_r}$	-0.171	-0.096	-0.053	-0.028	-1.355×10^{-14}
$\boldsymbol{C}$	8.480	8.437	8.431	8.430	8.428
$\boldsymbol{Z}$	4.375	4.336	4.327	4.322	4.318
$\boldsymbol{l}$	1.282	1.291	1.294	1.295	1.297
$\boldsymbol{k}$	427.6	430.5	431.4	431.9	432.3
$\boldsymbol{a}$	0.215	0.377	0.425	0.450	0.477
$\boldsymbol{RR}$	0.570	0.566	0.565	0.565	0.564
$\boldsymbol{r}$	0.015	0.015	0.015	0.015	0.015

(f) The another very interesting implication stemmed from θ is that as θ increases, commodity tax rate decreases. This means that the current pensioner's financial burden for their pension should rise. The economic intuition beyond this result is as follows: as elderly people become more – we can regard θ as the size of the elderly or the level of population aging, because it indicates the proportion of people who live non-working period i.e. old age to total population – the young worker's financial load to support the current pensioners is to be more severe and reducing nominal pension level can raise social welfare. However, it is impossible due to political reason like strong voice of elderly people to liberal pension system. In such case, increasing commodity tax can indirectly reduce pension's real value and transfer the financial burden from the young to the old. The following table 5 shows that as θ rises, pension level should decrease at the optimum. This result backs up the idea that when it is impossible to reduce pension level, increasing commodity tax rate raise social welfare above.

TABLE 5. **Comparative Statics at the Base Case Equilibrium**

(survival probability, given $\boldsymbol{\tau_c}$)

Variable	Base Case	$\uparrow\boldsymbol{\theta}$(0.98)	$\uparrow\boldsymbol{\theta}$(0.99)	$\uparrow\boldsymbol{\theta}$(0.995)	$\uparrow\boldsymbol{\theta}$(1)
$\boldsymbol{m}$	4.966	4.770	4.717	4.690	4.662
$\boldsymbol{\tau_w}$	-0.179	-0.181	-0.183	-0.184	-0.185
$\boldsymbol{\tau_r}$	-0.171	-0.096	-0.053	-0.028	-8.420×10^{-15}
$\boldsymbol{C}$	8.480	8.437	8.431	8.430	8.428
$\boldsymbol{Z}$	4.375	4.336	4.327	4.322	4.318
$\boldsymbol{l}$	1.282	1.291	1.294	1.295	1.297
$\boldsymbol{k}$	427.6	430.5	431.4	431.9	432.3
$\boldsymbol{a}$	0.215	0.362	0.404	0.425	0.449
$\boldsymbol{RR}$	0.570	0.544	0.537	0.533	0.529
$\boldsymbol{r}$	0.015	0.015	0.015	0.015	0.015

Because of indeterminacy issue, we should fix one of policy variables to give pension variable freedom, i.e. make it endogenously determined. Here, we fix commodity tax rate.

5. Conclusion

In section 3, the main result is that with the assumption that the determinant of the coefficient matrix is not zero, the optimal steady-state fiscal policies can be determined. In section 4, there are various results, but the key implications are related with tax structures when pension is exogenously set above the optimal level – this is the motivation of this paper. In such case, increasing commodity tax, decreasing labor income tax, and not altering capital income tax achieve social welfare maximization. This result explains why the positive relationship between commodity taxes and pension benefit (gross replacement rate in this paper) in the real economy exist.

The contribution of this paper is that it teaches us how tax structures should be in many welfare states where government or strong political parties seek expansionary fiscal policy or the elderly gives a strong voice to liberal pension system and then, reforming pension is hardly achieved.

By expanding our model in this paper, we can make it more realistic and obtain result consistent with the real economy. First, if we introduce heterogeneity in terms of productivity i.e. earning ability, pension here can have redistribution function within generation. It makes the unrealistically low labor income tax be close to the tax rate in reality. Second, placing stochastic shock on total factor productivity allows us to see how transfer or redistribution from the young generation who face the shock to the old one is changed. For example, by imposing a positive shock which yields more output, we can expect that the transfer from the young who become rich as a result of such a good shock to the old rises.

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국문초록

균제상태에서의 연금과 최적조세

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본 논문은 연금이 최적 수준 이상이나 정부가 팽창적 재정정책을 옹호하는 정칙적 압력에 부딪혀 연금수준을 개혁할 수 없을 때의 최적 조세 정책을 연구한다. 이 연구는 이러한 문제를 겪고 있는 유럽의 복지선진국들에게 바람직한 조세제도의 방향에 대해 제시할 것이다. 본 연구를 위해 2기간 중첩세대모형에 연금과 생애불확실성을 도입한 모형을 설정하여 이를 해석 및 수치적 방법으로 풀었으며, 그 결과 소비세를 높이고 노동 소득세를 낮추고 자본 소득세를 그대로 유지할 때, 지나치게 과도한 연금수준이 낳는 경제적 비효율성이 개선됨이 밝혀졌다. 본 논문은 최적 조세의 방향뿐 아니라, 정부 재정 지출에 경직성이 존재할 때 수입측면인 조세제도를 조절함으로써 사회후생을 개선할 수 있다는 통찰을 제시하는데 의의를 지닌다.

주요단어 : 연금; 소비세; 최적 정책 조합; 중첩세대 모형

학　　번 : 2011-23161