Endogenous Growth and the Real Interest Rate: Evaluating Korea's Low Interest Rate Regime

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This paper attempts to evaluate Korea's low interest rate regime of the post-2000 period by estimating the long-run equilibrium real rates using dynamic macroeconomic models. A unified endogenous growth model incorporating both R&D and human capital is constructed to ascertain the relative importance of various determinants of the real rate. The estimates of the long-run equilibrium rates for the unified model, the Solow model, and the Schumpeterian model are presented. The results show that (1) in all cases, the gap between the actual rate and the equilibrium rate seems to have been widening, and (2) estimates for the long-run equilibrium rates indicate that there has been no significant regime shift that would have justified the low interest rates of the post-2000 period, although the unified model shows that there has been a downward shift in the real rate in the 1990s mainly due to changes in human capital productivity.

Keywords: Real interest rate, Natural rate, Endogenous growth model

JEL Classification: E43, O16, O41

I. Introduction

Until around 2000, the real interest rate in Korea has been quite stable, fluctuating most of the time between 5 and 10%. Afterwards,

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however, the real interest rate fell significantly, approaching zero by the end of 2004. A near-zero real rate had never been observed since the early 1980s, a time when Korea was experiencing political turmoil. For this reason, it is often said that the Korean economy has set up a low interest rate regime, although there has been serious debate on whether or not the interest rate has been too low.

In order to determine if the low interest rate regime is justified by the fundamentals of the economy, this paper attempts to ascertain the long-run equilibrium real interest rates for various dynamic macroeconomic models, and compare the equilibrium rates with the actual rate. Analytical solutions for various models are obtained, and their estimates are computed. Unlike previous efforts such as Laubach and Williams (2003) and Bernhardsen (2005) that try to estimate the equilibrium real rate by smoothing the actual rates with time series techniques, this paper utilizes more fundamental information than just the time series of actual interest rates or growth rates.

Dynamic macroeconomic theories, such as the Ramsey model, generally offer explanations for the determinants of the real rate that rely on rather vague concepts such as time preference rates and exogenously given growth rates. These variables are supposed to be stable over time, preventing us from analyzing regime shifts in the real rate. Therefore, I turn to endogenous growth theories of the changes in growth rates, which would explain shifts in the long-run real rates. For example, Lucas (1988) type human capital-based models imply that the real rate should follow the productivity of human capital formation, while the Schumpeterian models or R&D-based models such as Aghion and Howitt (1998) show that the real rate is ultimately determined by R&D intensity or innovative capacity of the economy.

To see the relative importance of those factors and to consider more factors that would affect the real rate, I develop a unified framework that addresses both R&D and human capital. In this model, returns to human capital play an important role as they surely affect the return to physical capital or the real rate by an arbitrage process. The model also takes into account the technological distance to the world frontier, which certainly affects growth rates in developing countries.

This paper is structured as follows. Section II investigates the determinants of the real interest rate from the view point of various growth theories, and presents a unified endogenous model. Section III shows long-run equilibrium levels of the real rate derived from each theory and compares them with the actual real rate. Section IV concludes.

II. Growth Theories and the Determinants of the Real Interest Rate

The real interest rate has traditionally been regarded as relatively stable around a constant value. However, numerous empirical studies show that the real rate has not been stationary. Walsh (1987) and Rose (1988) failed to reject the hypothesis that real rates are not stationary. Garcia and Perron (1996) point out that there have been regime shifts in the trends in U.S. real rates. Many other studies concede implicitly or explicitly that the natural rate or equilibrium real rate can vary over time in response to shifts in preferences and technology.[1]

According to growth theories, long-run real rates can shift due to changes in the fundamentals in the economy. The determinants of the real interest rate can be analyzed through macroeconomic models. Here, we will consider various types of growth models, including the exogenous growth model of Solow, the Ramsey model, the human capital-based model of Lucas (1988), the R&D-based model of Aghion and Howitt (1992) with an extension for developing countries proposed by Ha (2004b), and a unified model that incorporates both R&D and human capital.

A. The Solow Model

The Solow model is the simplest framework for analyzing the determinants of the real rate. Consider the following differential equation that describes the motion of capital per worker k.

$$\dot{k}=sf(k)-(n+\delta)k \tag{1}$$

where s is the savings (and investment) rate, $f(k)$ is the normalized

[1] Some economists like Bomfim (1997) define the natural rate as a short-term real rate, while Laubach and Williams (2003) see it as a medium-run real rate. However, it is more common to define it as a long-run equilibrium real interest rate.

aggregate production function, n is the population growth rate, and δ is the depreciation rate of physical capital.

Differentiating Equation (1) with respect to k in the steady state where $\dot{k}=0$, one can easily obtain the real interest rate r as follows.

$$r \equiv f'(k) - \delta = \frac{n+\delta}{s} - \delta = \frac{n}{s} + \left(\frac{1-s}{s}\right)\delta \tag{2}$$

The Solow model shows that the real interest rate is a decreasing function of s, and an increasing function of n and δ. Although this model is simple, the determinant of the key variable s is still to be explained.

B. The Ramsey Model

The Ramsey model endogenizes the savings rate s of the Solow model using a dynamic optimization framework. In this framework, we consider a utility maximization problem of a dynasty where members enter and exit constantly. I will specifically consider a continuous time overlapping generations model as follows.

$$\begin{aligned} &\max \int_0^{\infty} \ln c \cdot L \cdot e^{-\rho t} dt \\ &s.t. \\ &\dot{a} = ra + w - c - na \\ &\dot{L} = nL \end{aligned} \tag{3}$$

where c is consumption per worker, L is the size of the population, ρ is the time preference rate, a is the amount of physical assets per worker, r is the real interest rate, and w is the wage rate. A log utility function is assumed for simplicity. The current-value Hamiltonian for this problem is:

$$H = \ln c \cdot L + \mu_a [ra + w - c - na] \tag{4}$$

where μ_a is the shadow price of a.

Now, the head of the dynasty chooses c so that the utility of the dynasty is maximized. The first order condition for this problem is:

$$\frac{dH}{dc}=\frac{L}{c}-\mu_a=0 \Leftrightarrow \mu_a=\frac{L}{c} \tag{5}$$

Using this result, the corresponding Euler equation can be written as:

$$\rho=\frac{\dot{\mu}_a}{\mu_a}+\frac{dH/da}{\mu_a}=n-\frac{\dot{c}}{c}+r-n \tag{6}$$

From Equation (6) one can easily obtain the following equation.[2]

$$r=\frac{\dot{c}}{c}+\rho \tag{7}$$

Equation (7) tells us that the real interest rate is determined by the growth rate of consumption per worker, $\dot{c}/c$ and the rate of time preference ρ. Here, people allow consumption to grow (a deviation from consumption smoothing) only when the return to savings r exceeds the required rate of compensation ρ.

Also, on a balanced growth path where the growth rate of consumption per worker is a constant γ, Equation (8) becomes:

$$r=\gamma+\rho \tag{8}$$

In Equation (8), the major determinant of r would be ρ, which, however, the Ramsey model does not analyze further. In order to analyze other fundamentals behind the real rate, one has to adopt an endogenous framework, to which we now turn.

C. Lucas-Type Human Capital-Based Growth Model

Lucas (1988) proposed an endogenous growth model in which the engine of growth is human capital accumulation and its speed is determined by the fraction of time devoted to education. The most crucial part of such models is the dynamics of human capital accumulation, which can be written as follows.

[2] The transversality condition for this problem $\lim \mu_{a,t}\, a_t\, e^{\rho t}=0$, which is that the growth rate of a is smaller than $r-n$ is assumed to be satisfied.

$$\dot{h} = \theta(1-\upsilon)h - nh \tag{9}$$

where h is human capital per person, θ is the productivity of human capital formation, and υ is the share of time devoted to work instead of education, so that $(1-\upsilon)$ is interpreted as the investment rate for education. The population growth rate n is the depreciation rate of human capital as human capital h is expressed in *per capita* terms. In fact, it would be more appropriate to use the birth rate as the depreciation rate of human capital since human capital is not the same as physical capital in that the human capital of a dead person just disappears as human capital is, by definition, the skills and knowledge embodied in living people. However, in Lucas-type models, human capital is interpreted as something that includes both disembodied knowledge - or technology - and embodied knowledge, justifying the use of n as the depreciation rate.

Applying the Lucas model to a continuous time overlapping generations model yields the following optimization problem.

$$\begin{aligned} &\max \int_0^{\infty} \ln c \cdot L \cdot e^{-\rho t} dt \\ &s.t. \\ &\dot{a} = ra + w\upsilon h - c - na \\ &\dot{h} = \theta(1-\upsilon)h - nh \\ &\dot{L} = nL \end{aligned} \tag{10}$$

The corresponding current-value Hamiltonian is:

$$H = \ln c \cdot L + \mu_a\{ra + w\upsilon h - c - na\} + \mu_h\{\theta(1-\upsilon)h - nh\} \tag{11}$$

where μ_a is the shadow value of a and μ_h is the shadow value of h.

Now, the head of the dynasty chooses υ as well as c so that the utility of the dynasty is maximized. The first order condition with respect to c is:

$$\frac{dH}{dc} = \frac{L}{c} - \mu_a = 0 \Leftrightarrow \mu_a = \frac{L}{c} \tag{12}$$

Combining this result and the first order condition with respect to v:

$$\frac{dH}{dv} = \mu_a wh - \mu_h \theta h = 0 \Leftrightarrow \mu_h = \mu_a \frac{w}{\theta} = \frac{wL}{\theta c} \tag{13}$$

Using this result, the corresponding Euler equations are:

$$\rho = \frac{\dot{\mu}_a}{\mu_a} + \frac{dH/da}{\mu_a} = n - \frac{\dot{c}}{c} + r - n = r - \frac{\dot{c}}{c} \tag{14}$$

$$\rho = \frac{\dot{\mu}_h}{\mu_h} + \frac{dH/dh}{\mu_h} = \frac{\dot{w}}{w} + n - \frac{\dot{c}}{c} + \frac{\mu_a}{\mu_h} wv + \theta(1-v) - n = \frac{\dot{w}}{w} - \frac{\dot{c}}{c} + \theta \tag{15}$$

Equation (14) is the same as Equation (7) of the Ramsey model, but this Lucas-type model has another sector which is represented by Equation (15). Combining Equations (14) and (15), we have:

$$r = \frac{\dot{c}}{c} + \rho = \frac{\dot{w}}{w} + \theta \tag{16}$$

Equation (16) shows that the return on physical assets r must equal the return on human capital $\dot{w}/w + \theta$, all of which must be equal to the target rate of return ρ plus adjustments in consumption growth. This reflects an arbitrage trade between physical and human capital, or the forces that equalize the return on two different assets; the dynasty must be indifferent between investments in human and physical capital at the margin.[3]

Now, what happens to the real rate when the economy is on a balanced growth path, where c grows at the long-run growth rate of the economy and w is a constant? Note that the long-run growth rate in this economy is equal to the steady-state growth rate of h, and the

[3] The transversality condition for this problem $\lim \mu_{a,t} a_t e^{-\rho t} = \lim \mu_{h,t} h_t e^{-\rho t} = 0$, which means that the condition of the growth rate of a lower than $r-n$ is assumed to be satisfied.

steady state value of w is constant, since the income from one unit of human capital wh grows with only h in the steady-state. Taking these into account, the steady-state value of the real rate is:[4]

$$r=\left.\frac{\dot{h}}{h}\right|_{ss}+\rho=\theta \tag{17}$$

Here, the key variable that determines the real rate is θ, which is the productivity of human capital formation.[5] However, one should notice that human capital in this framework includes not only embodied skills but also disembodied technologies. Thus, θ in this model represents all types of knowledge production capabilities of a society.

D. Schumpeterian Growth Model with an Extension for Developing Countries

The Schumpeterian or R&D-based growth models of Aghion and Howitt (1992), Grossman and Helpman (1991), and Romer (1990) focus on R&D investments as the engine of growth. R&D investment increases innovation, raising total factor productivity (TFP). According to these models, the TFP growth rate g is expressed as a function of R&D intensity x.

$$g=F(x) \tag{18}$$

where the actual details of $F(x)$ vary across various models.

The optimization problem for the dynasty would be the same as the Ramsey model except that we now consider the growth rate of TFP explicitly. Here, the exogenous growth rate γ in Equation (8) is replaced by g, so that in the steady state we have:

$$r=F(x)+\rho \tag{19}$$

As for proxies for x, the theories proposed by Aghion and Howitt (1998), Dinopoulos and Thompson (1998), Peretto (1998), and Howitt

[4] The real interest rate adjusts to θ through a process of physical capital adjustment as in the Solow model which is not fully described here.

[5] Here, it is easy to see that the long run growth rate of the economy is $\theta-\rho$.

(1999) use R&D investment rates such as R&D expenditures as a percentage of GDP, which turn out to work well empirically according to Ha and Howitt (2005). In this framework, the R&D investment rate x is the key variable in the determination of the real rate, as R&D intensity determines the speed of knowledge creation and hence growth rates.

However, in developing countries, knowledge adoption would be as important as knowledge creation. Howitt (2000) and Acemoglu, Aghion, and Zilibotti (2002) develop this point by introducing the role of "distance to frontier." Specifically, the greater the technological distance to the frontier, the easier it is to imitate foreign technologies. All else being equal, as a developing country approaches the technological frontier, its growth rate must necessarily decline since it becomes increasingly difficult to imitate advanced technologies. This can be summarized in the following equation.

$$g=F(x;z)=F(x;A/A_{\max}) \tag{20}$$

where z is defined as the ratio between the level of TFP A and the level of the frontier's TFP $A_{\max}$.

In this framework, the real rate of interest is:

$$r=F(x;z)+\rho \tag{21}$$

Note that r is an increasing function of x and a decreasing function of z.

E. Schumpeterian Growth Model with Human Capital - A Unified Framework

Most R&D-based models do not take into account the role of human capital, although they all agree with Nelson and Phelps's (1966) arguments that human capital is crucial both in creation and dissemination of knowledge. Ha (2004a) proposes a dynamic general equilibrium model where human capital generates some sort of scale effects that jumpstart and increase innovation. Here, the engine of long-run growth is R&D and human capital stays at its steady-state level. This is consistent with the fact that humans are mortal and their human capital disappears upon death. However, human capital is still important in that its level determines the rate of innovation.

The salient features of this model come from the unique dynamics of human capital, which are consistent with the Mincerian approach. In this subsection, I would like to extend this model to incorporate demographic factors such as the birth rate and death rate, as well as technological distance to the frontier.

The Mincerian approach of human capital focuses on average years of schooling S, according to which human capital per person is defined as follows.

$$h = e^{\theta S} \tag{22}$$

where θ is interpreted as the rate of return to one additional year of schooling.

Now the dynamics of average years of schooling can be written as follows:

$$\dot{S} = (1 - \upsilon) - bS \tag{23}$$

where b is the birth rate. Note that it is the birth rate and not the population growth rate, that affects the depreciation of human capital per person. Only birth imposes a burden on the dynasty that continuously tries to maintain its average years of schooling, whereas death decreases the size of the population (denominator) as well as human capital (numerator).

From Equations (22) and (23), one can obtain the following dynamics of human capital accumulation.

$$\dot{h} = \theta(1 - \upsilon)h - bh \ln h \tag{24}$$

This is different from Lucas-type models in that the depreciation rate of human capital $b \ln h$ is increasing in h and not just a constant. This feature reflects the mortality of human beings and their human capital.

Applying these unique dynamics of human capital to a continuous time overlapping generations model yields the following optimization problem.

$$
\begin{aligned}
&\max \int_0^{\infty} \ln Ac \cdot L \cdot e^{-\rho t} dt \\
&s.t. \\
&\dot{a} = ra + wvh - c - (n+g)a \\
&\dot{h} = \theta(1-v)h - bh \ln h \\
&\dot{L} = nL = (b - \delta_{II})L \\
&\dot{A} = F(x;z)A
\end{aligned}
\tag{25}
$$

where A represents the level of TFP and δ_{II} is the death rate. And, $F(x;z)$ is assumed to be determined by firms, so that from the view point of the dynasty, TFP growth $\dot{A}/A$ is given. Note here that the variables are expressed in per effective worker terms.

The corresponding current-value Hamiltonian is:

$$
H = \ln Ac \cdot L + \mu_a\{ra + wvh - c - (n+g)a\} + \mu_h\{\theta(1-v)h - bh \ln h\} \tag{26}
$$

Now, the head of the dynasty chooses c and v so that the utility of the dynasty is maximized. The first order condition with respect to c is:

$$
\frac{dH}{dc} = \frac{L}{c} - \mu_a = 0 \Leftrightarrow \mu_a = \frac{L}{c} \tag{27}
$$

Combining this result and the first order condition with respect to v, we have:

$$
\frac{dH}{dv} = \mu_a wh - \mu_h \theta h = 0 \Leftrightarrow \mu_h = \mu_a \frac{w}{\theta} = \frac{wL}{\theta c} \tag{28}
$$

From this, the corresponding Euler equations are:[6]

$$
\begin{aligned}
\rho &= \frac{\dot{\mu}_a}{\mu_a} + \frac{dH/da}{\mu_a} = n - \frac{\dot{c}}{c} + r - n - g = r - g - \frac{\dot{c}}{c} \\
&= r - F(x;z) - \frac{\dot{c}}{c}
\end{aligned}
\tag{29}
$$

[6] The transversality condition $\lim \mu_{h,t} h_t e^{-\rho t} = \lim \mu_{a,t} a_t e^{-\rho t} = 0 \Leftrightarrow \rho > 0$ is satisfied.

$$\rho = \frac{\dot{\mu}_h}{\mu_h} + \frac{dH/dh}{\mu_h} = \frac{\dot{w}}{w} + n - \frac{\dot{c}}{c} + \frac{\mu_a}{\mu_h} wv + \theta(1-v) - b(1+\ln h)$$

$$= \frac{\dot{w}}{w} + n - \frac{\dot{c}}{c} + \theta - b(1+\ln h) = \frac{\dot{w}}{w} - \frac{\dot{c}}{c} + \theta - b\theta S - \delta_H \tag{30}$$

where Equation (30) makes use of the fact that $n = b - \delta_H$ and $\ln h = \theta S$. Combining Equations (29) and (30), we have:

$$r - F(x;z) = \frac{\dot{c}}{c} + \rho = \frac{\dot{w}}{w} + \theta - b\theta S - \delta_H$$

$$r = \frac{\dot{w}}{w} + F(x;z) + \theta(1-bS) - \delta_H \tag{31}$$

In the steady state where $\dot{w}/w = 0$,[7] the real rate is:

$$r = F(x;z) + \theta(1-bS) - \delta_H \tag{32}$$

In this model, the real interest rate is increasing in R&D intensity x, rate of return on one additional year of schooling θ for reasonable values of b and S, and decreasing in relative level of technology z, birth rate b, average years of schooling S, and death rate δ_H.

It is worth noting that demographic factors such as the birth rate and death rate play a substantial role in determining the real rate. By looking at the birth rate and death rate separately, one can obtain more information than just from population growth rates that show only the difference between the birth rate and death rate. For example, a 1% population growth rate can be generated from a "2% birth rate and 1% death rate" or "3% birth rate and 2% death rate" and so on. However, the two cases are totally different from the view point of human capital management: A lower death rate indicates a longer life expectancy, raising the return to human capital investment. In Equation (32), even with the same rate of population growth,

[7] In this model, w is defined as the productivity adjusted level of the wage rate, so that w itself is constant in the steady state in which the wage rate rises with TFP.

TABLE 1
LONG-RUN EQUILIBRIUM REAL RATES r FOR VARIOUS GROWTH MODELS

	r	Increasing in	Decreasing in
Solow model	$((n+\delta)/s)-\delta$	n, δ	s
Ramsey model	$(\gamma+\rho)$	(ρ)	-
Lucas model	θ	θ	-
Schumpeterian model	$F(x;z)+\rho$	x	z
Schumpeterian model with human capital	$F(x;z)+\theta(1-bS)-\delta_H$	x, θ	z, b, δ_H, S

Note: It is assumed that $bS<1$.

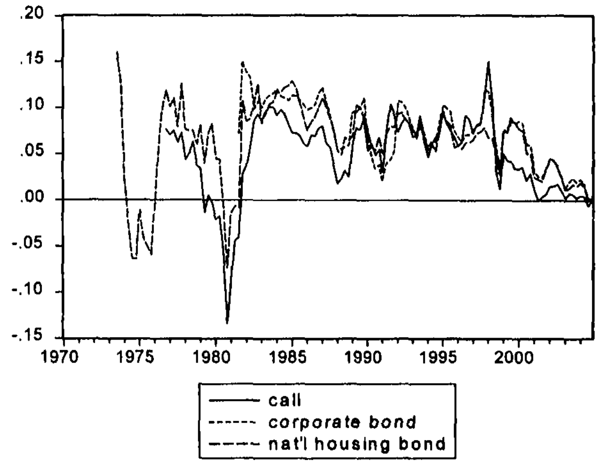
a "high birth rate and high death rate" implies a lower real rate than a "low birth rate and low death rate." Here, the simple fact that humans are mortal makes a lot of difference.

F. Summary

The determinants of the real interest rate can be summarized in Table 1. The key determinants of the real rate vary across models. In the Solow model, the savings rate or investment rate is the key variable, which, however, is still to be explained. The Ramsey model looks at the rate of time preference, which is difficult to measure. In the human capital-based model of Lucas, the speed of progress in embodied skills as well as disembodied knowledge is the key. In the Schumpeterian R&D-based models, R&D intensity is what determines the real interest rate in the long-run, together with the technological distance to the frontier in the cases of developing countries. Then, with regard to the unified model that incorporates human capital as well as R&D, the productivity of human capital and demographic factors also play an important role.

III. Measuring the Long-Run Equilibrium Real Interest Rate

Now we can compute the estimates of the long-run equilibrium or benchmark levels of the real rates by using the formulas summarized in Table 1. Before going into the details, let's take a look the trends in the real rate in Korea.



Sources: IFS database

FIGURE 1
REAL INTEREST RATES IN KOREA

A. Trends in Real Interest Rates

Figure 1 shows that Korea's real interest rate has been declining since 2000. The overnight call rates, corporate bond rates (3 years), and the national housing bond rates (5 years), which are converted into real rates by subtracting the inflation rate (CPI) from nominal rates, all show similar trends and approach zero by the end of 2004.

In order to see if there has been a structural break, I conducted Chow breakpoint tests for the first quarter of 2000 by constructing a simple regression model where real rates are regressed on a constant.[8]

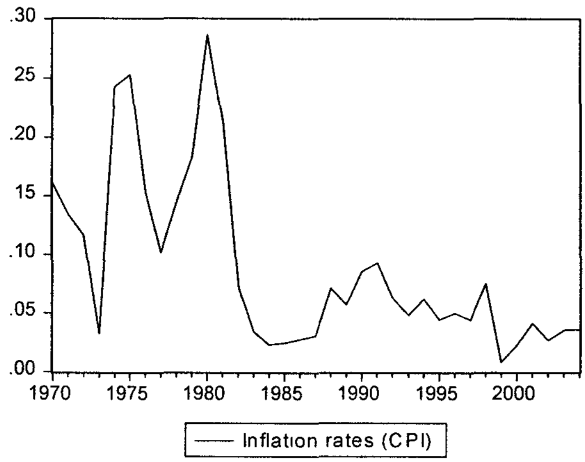
Table 2 summarizes the results, and one can easily see that there was indeed a structural break around 2000.[9] This confirms that the low interest rate regime is not just a byproduct of changes in inflation rates, which are depicted in Figure 2.

[8] Other breakpoints such as Q4 2000 or Q1 2001 show similar results.

[9] Augmented Dickey-Fuller tests suggest that the hypotheses that the call rates and corporate bond rates have unit roots are not rejected at the 10% significance level.

TABLE 2
RESULTS OF CHOW BREAKPOINT TESTS FOR 2000 Q1

National housing bond	*F*-statistic	13.62538	Probability	0.000333
	Log likelihood ratio	13.13598	Probability	0.000290
Corporate bond	*F*-statistic	64.66593	Probability	0.000000
	Log likelihood ratio	50.03876	Probability	0.000000
Call	*F*-statistic	21.82570	Probability	0.000008
	Log likelihood ratio	20.28436	Probability	0.000007

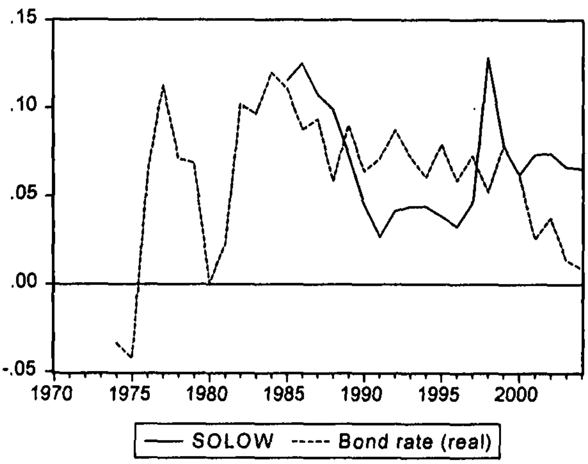


Source: IFS database

FIGURE 2
INFLATION RATE IN KOREA

B. *The Solow Model*

The long-run equilibrium real rate implied by the Solow model is $r=(n+\delta)/s-\delta$, which can be estimated by putting in the population growth rate n and investment rate s, with an assumption that the depreciation rate is constant. Here, n of year t is computed by subtracting the death rate of year t from the birth rate of year t-15, since the birth rate actually means the speed of entry into the labor force that can be proxied by the population of age 15 and over. Adjusting the level of r in such a way that the equilibrium level is



Source: Author's calculation and the IFS database

FIGURE 3
LONG-RUN EQUILIBRIUM REAL RATE IMPLIED BY THE SOLOW MODEL

TABLE 3
CHOW BREAKPOINT TEST FOR THE SOLOW MODEL (2000)

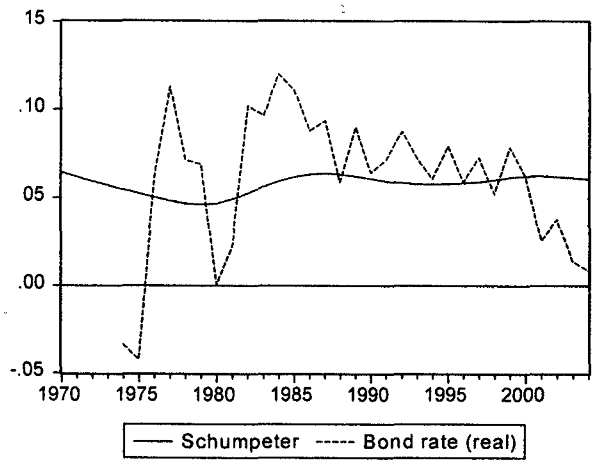
F-statistic	0.005389	Probability	0.942289
Log likelihood ratio	0.005988	Probability	0.938318

equal to the national housing bond rate in 2000,[10] the estimates are depicted in Figure 3. Note that the level of the estimates has been adjusted so that the equilibrium rate and actual rate are equal in 2000.[11]

Table 3 shows that the Chow breakpoint test suggests that there was no structural break around 2000.

These findings indicate that, according to the Solow model, (1) the equilibrium rate does not show any clue of a low interest rate regime, and (2) it seems that the actual real interest rate fell faster than the equilibrium rate.

[10] Other measures for the real rate yield similar results.
[11] This adjustment will be also applied to other models.



Source: Author's calculation and the IFS database

FIGURE 4

LONG-RUN EQUILIBRIUM REAL RATE IMPLIED BY SCHUMPETERIAN MODEL

C. The Ramsey Model and the Lucas Model

In the Ramsey model, the long-run equilibrium real interest rate is $\gamma + \rho$, which can be regarded as constant. As the Ramsey model does not offer any explanations for the determination of time preference rate ρ, this paper does not try to measure the long-run real rate implied by the Ramsey model, either.

In the Lucas model, the long-run rate is θ, which is the productivity of human capital formation. However, here human capital should be interpreted as something more than just skills; as it incorporates disembodied knowledge as well. Therefore, I will not try to measure the real rate of the Lucas model. Instead, this human capital flavor will be comprehensively added to the unified model, which we will see later in this paper.

D. The Extended Schumpeterian Model

According to the Schumpeterian or R&D-based models with an extension for developing countries, the long-run real rate is $F(x;z)+\rho$, where TFP growth is the key. The TFP growth rates, which have been

TABLE 4

CHOW BREAKPOINT TEST FOR THE SCHUMPETERIAN MODEL (2000)

F-statistic	3.623580	Probability	0.065718
Log likelihood ratio	3.646468	Probability	0.056188

smoothed by HP-filtering, together with the national housing bond rate are depicted in Figure 4. The TFP growth rates for the years between 1970 and 2000 are computed as in Ha (2004b), and those for post-2000 years are estimated using the model $g=F(x;z)=\lambda x^{\sigma}z^{-\psi}$, where the values for z and other parameters are estimated by Ha (2004b).

Here, a Chow breakpoint test in Table 4 shows that the equilibrium rate did not likely have a structural break around 2000.

These results indicates that, according to the Schumpeterian model, (1) there have been no regime shifts in the trends of the equilibrium real rate, and (2) the actual real rate fell faster than the equilibrium rate. These are similar to the findings in the Solow model.

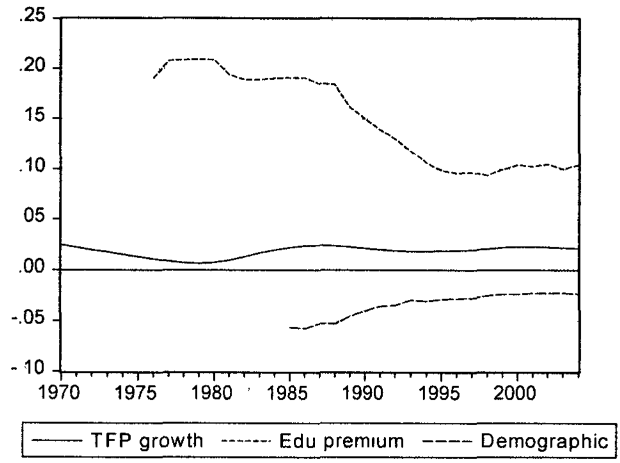
E. Schumpeterian Model with Human Capital - A Unified Model

In the unified Schumpeterian model with human capital, the long-run real interest rate is $F(x;z)+\theta(1-bS)-\delta_H$, which can be decomposed into the TFP growth rate $F(x,z)$, the productivity of human capital formation θ, and demographic factors $-(b\theta S+\delta_H)$.

Here, the TFP growth rates are defined in the same way as in the Schumpeterian model. The productivity of human capital formation, or education premium from the view point of the dynasty, is computed using the college premium data as follows. The college premium is defined as $w_{college}/w_{high}$ which is the average wage rate of college graduates divided by the average wage rate of high school graduates. As the wage rate would be proportional to the value of human capital, this ratio would be the same as the ratio between the human capital of college graduates and high school graduates. Applying the Mincerian framework of Equation (22), we have:

$$w_{college}/w_{high}=h_{S=16}/h_{S=12}=e^{16\theta}/e^{12\theta}=e^{4\theta} \tag{33}$$

from which θ can be computed as follows.



Source: Author's calculation

FIGURE 5
THREE FACTORS IN THE LONG-RUN EQUILIBRIUM REAL RATE

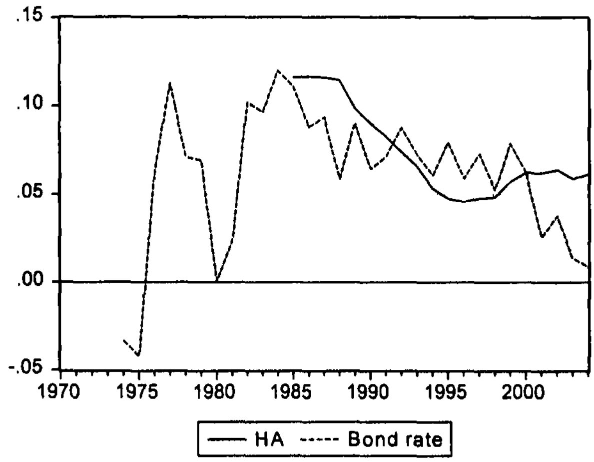
$$\theta = \frac{1}{4} \ln \frac{w_{college}}{w_{high}} \tag{34}$$

Using this result, the demographic factor $-(b\theta S + \delta_H)$ can easily be computed.[12] Note that the birth rate has a time lag of 15 years for the same reason as in the Solow model. The three factors that determine the long-run equilibrium real rate are depicted in Figure 5.

Putting those three factors together, we have the long-run equilibrium real rate as in Figure 6. And the results of Chow breakpoint tests for 2000 and 1992 are in Table 5. These show, according to the unified model, (1) there was a downward shift in the equilibrium rate in the 1990s, not in the post-2000 years, and (2) the actual real rate fell faster than the equilibrium rate.

Moreover, one can decompose the changes in equilibrium real rate for the years between 1985 and 2004. In Table 6, one can readily see that the contribution of the education premium has been the biggest, pushing down the real rate, while demographic factors have

[12] S is the weighted average of different education cohorts, whose data can be found in the National Statistical Office database. The death rate is also obtained from the same database.



Source: Author's calculation and IFS database

FIGURE 6
LONG-RUN EQUILIBRIUM REAL RATE IMPLIED BY THE UNIFIED MODEL (HA)

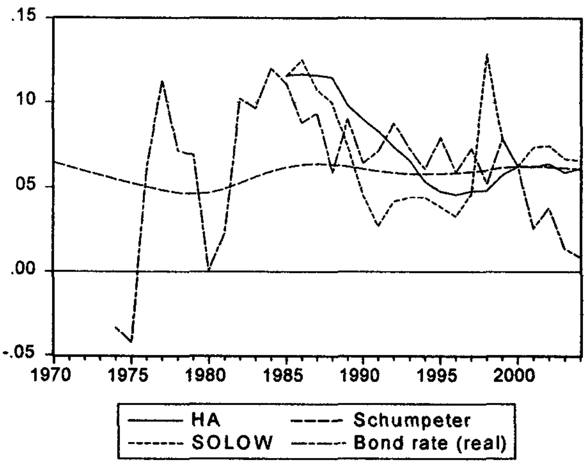
TABLE 5
CHOW BREAKPOINT TESTS FOR THE UNIFIED MODEL

2000	F-statistic	1.645545	Probability	0.215839
	Log likelihood ratio	1.749578	Probability	0.185930
1992	F-statistic	88.00724	Probability	0.000000
	Log likelihood ratio	35.46271	Probability	0.000000

TABLE 6
DECOMPOSING THE CHANGES IN EQUILIBRIUM RATE, 1985-2004

	Equilibrium Rate	TFP Growth	Education Premium	Demographic Factors
Changes from 1985 to 2004	-0.055	-0.001	-0.087	0.033
Contributions	100.0	1.7	158.6	-60.2

counteracted it. Note that a downward shift in the education premium reflects the overall decline in productivity of accumulated assets or



Source: Author's calculation and IFS database

FIGURE 7
LONG-RUN EQUILIBRIUM REAL RATES AND THE ACTUAL REAL RATE

TABLE 7
SUMMARY OF THE RESULTS

Model	Low Interest Rate Regime?	Gap between Actual Rate and Equilibrium Rate
Solow model	No	Widening
Schumpeterian model	No	Widening
Unified model	Yes, but in the 1990s	Widening

decreased profit opportunities, although there may have been other complexities affecting the wage structure.

Also, this framework explains the seemingly puzzling fact that low interest rates coexist with low investment rates, which is not reconciled in the Solow model.

IV. Conclusion

From what we have seen so far, we can say that (1) in all cases, the gap between the actual rate and the equilibrium rate seems to have been widening, and (2) estimates for long-run equilibrium rates indicate that there has been no significant regime shift that would have justified the low interest rate regime of the post-2000 period, although the unified model indicates that there was a downward regime shift in the real rate in the 1990s, mainly due to changes in the returns to human capital investment. Figure 7 and Table 7 summarize the results.

Then, what is the future course of the real rate in Korea? From the view point of the unified model, we need to look at the three factors.

First, TFP growth rates will not fall as long as Korea increases its R&D intensity. However, if Korea's TFP growth is too fast compared with advanced countries, some forces may pull down the growth rates and eroding the latecomer's advantage, which is not likely to happen in the near future.

Second, the productivity of human capital formation would improve as long as reforms on higher education continue in such a way that increases innovative capacity.

Third, the demographic factor would increase the real rate, as the birth rate and death rate all decrease. Low birth rates lower the burden of education, increasing the return on human capital and hence the real rate through an arbitrage process. At the same time, a low death rate increases life expectancy, pushing up the return on human capital, though this may be counteracted if longer life is not also attended by longer working years.

All in all, it is likely that the low interest rate regime, if it exists at all, will end in the future. It is also worth noting that one needs to look at the labor market and demographic factors more carefully, as those factors were found to affect the equilibrium rate substantially.

(Received 17 September 2005; Revised 4 November 2005)

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