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Abstract

Robust Contract under Knightian Uncertainties

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In this paper, we develop the canonical robust contract theory : Principal does not know exactly, or does not believe the basic components of agency relationship. He designs the optimal incentive contract based on the worst case for him. First we investigate situation where principal does believe mis-specification in characteristics of agent's utility function and evaluates contracts assuming agent with minimizing utility is working for him. We derive Innes(1990)'s debt contract is quite robust under this mis-specification and optimal. Second, we investigate the situation where the principal is now unaware of technology for which agent works. Using Hansen and Sargent(2011)'s argument, we derive that ambiguous information makes the contract similar to a contract which risk averse principal optimally designs. Last, We provide another possible approach to robust agency problem in Appendix.

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keywords : **Ambiguity aversion, Knightian uncertainty**
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1. Introduction

In economics, the theory of agency has been highlighted as the lens of viewing human activities in economy as the natural reaction to individual interest and incentive system. For example, people such as many CEOs manipulate the unobservable actions of workers through the contract function on observable performance measures and this contract gives some 'incentive' to work hard for many workers in firms. Principal-agent theory makes us to deal with this procedure in a mathematically tractable way and enlightens the ways various incentives affect human behaviors.

In general, theory of contract has been making extensive progress in dealing with some aspects especially. First it gives insights about how to give proper incentives to workers in firms using optimal contract. Also it deals with how to allocate jobs in organization, the method of designing efficient contingency claims in financial market, and how to allocate several tasks effectively to make firm's profit higher. Also as the 2008 global financial crisis hit the world and crisis has debilitated the global economy, the interest about proper incentive system and risk-management policy in wall-street firms and even local banks is soaring among the economics profession.

The classical contract theories have deep problems : It assumes there is given economic environment where agent meets principal, works for him and agent and principal regard this economic environment as their common knowledge in game-theoretical sense. For example, Holmstrom(1979) makes principal-agent problems with 3 components of economic environment such as $\{f(x|a), u(s), c(a)\}$[1] with each component regarded as purely exogenous.

However, in real world, there is little thing known about the stochastic output distribution as action of agents varies. Also if even principal does not know about his own utility function, then how is it possible to know his

1) The first component is stochastic output density representing working technology. The second and third are about agent's utility function and cost of effort.

worker's utility functional form? Even if there are some known facts about distribution and preference, principal may not believe those facts and he may try to contrive the feasible environment which he can believe.

There has been a long history in economics profession about how to deal with the uncertainty and especially Hansen and Sargent(2011), using the robust-control theory, derives how to make the optimal control which is quite robust under these kinds of uncertainties. Their logic is that economic agent contrives his own minimizing component which she does not know or believe under the so-called relative entropy constraint. When she chooses optimal control, she incorporates the contingent minimal environment which is under the uncertainty and find the 'robustly' optimal control. This approach makes possible that agents can deal with the worst-case scenario and because they do not know about exact distribution, agents might want to maximize the worst-case payoffs using robust-controls. We adapt and change this approach to canonical principal-agent problems and see how the fundamental results change.

The paper with bright ideas and mathematics, written by Carroll(2013), acknowledges the above fact but focuses on different kinds of uncertainty and using that ambiguity, he derives optimal simple contract under risk neutrality of an agent : linear contract. He acknowledges that principal in general may not know the feasible set of actions available to the agent. So principal is ambiguous about this set of feasible actions and robustly chooses the contract by maximizing his minimal payoff according to the available action set. However, in the model, this paper lets the agent in principal-agent relationship choose the functional form of distribution so that agent has more degree of freedom in choosing his action than in Holmstrom (1979). This makes a lot of mathematical proofs possible in their paper. Also, more importantly, it is more plausible that principal does not have information about utility function of an agent or output density, rather than agent's feasible action set.

Another paper by Shannon et al(2011) presents a principal-agent model in which the agent has imprecise beliefs using the incomplete preference system. The agent, when he chooses his optimal action, must compare alternative actions given the contract function. However in their model, he may compare some of feasible actions and cannot compare others so that when he cannot choose what is better option for him, he just chooses the outside option : the reservation utility. In this setting they derive that the bonus contract with only one-step bonus is optimal.

This paper consists of two parts. The next section starts with basic model. **The first section** is about the situation where principal doesn't know the exact form of agent's utility function in an agency model. **The second section** is about the situation where principal doesn't believe the specifed technology f and he himself thinks the worst case technology according to Hansen and Sargent(2007)'s multiplier preference.[2] Through these two parts, we find how to make exogenously given environment of agency somewhat 'endogenous' by coming up with the endogenous environment in which principal-agent relation begins. Paper ends with conclusion with some future research areas.

2. The Model

2.1. Analysis of Ambiguity-Averse Principal-agent Relationship : The utility case

The relationship consists of risk-neutral principal and risk-averse agent with utility $u(s)$. As in standard agency setting, the technology process $f(x|a)$ is common-knowledge to both principal and agent. However, in contrast to Holmstrom(1979), principal only knows that his employee is risk averse, but does not know what his utility's exact functional form is. In this paper only uncertainty behind this relationship is agent's preference structure, so we model the aversion of principal to this kind of

2) This preference is represented by adding up the relative entropy term. This kind of utility representation is aximatized through adding some assumptions. See Strzalecki(2011).

"ambiguity".

Principal thinks that the agent has the worst preference which may be harmful to his own payoff, and take consideration to this kind of ambiguity when designing the optimal incentive contract. Also, the timeline of this game is the same as Holmstrom(1979), Kim(1995) and other static contract literatures. To be specific, first, principal offers the contract to agent, and if agent signs on it, the agent inputs the best effort to maximize his own indirect utility given the contract. At the end of period, the stochastic output x is realized according to technology $f(x|a)$, and principal pays $s(x)$ to agent.

We assume that stochastic output x is confined to bounded support $[0, \overline{x}]$ and $f(0|a) = f(\overline{x}|a) = 0$, so that the *problems of infinity* do not arise in the model. We restrict that the distribution f has the full support so $f(x|a) > 0$ point-wisely for x. Also, as in other standard agency settings, we impose *Monotone Likelihood Ratio Property(MLRP) and First-order Stochastic Dominance* about f.[3)]

$$\frac{f_a(x|a)}{f(x|a)} \text{ is increasing in } \forall x \text{ and } F_a(x|a) \le 0 \text{ for } \forall x, \forall a$$

Also in many cases, we assume the limited-liability of agent : $s(x) \ge 0$, which is often necessary to make sure the existence of optimal contract in standard principal-agent problems.[4)] In addition to agent's limited-liability, we assume limited-liability of principal's side : $x - s(x) \ge 0$ as in Innes(1990). We may interpret agent's limited liability constraint as the real-world "minimum-wage" policy in labor market and principal's limited liability as the mechanism to prevent principal's "sabotaging activity" for output.

General contract function, the tool of incentive provision, is the function of stochastic output x by itself. We define the feasible contract by its

3) In fact, Monotone Likelihood Ratio Property(MLRP) implies FOSD conditions. See Milgrom(1981).

4) Theorists often call this property Mirrlees' 'Unpleasant theorem' because it can be possible that there is no optimal second-best contract in standard settings without agent's limited liability.

measurability with respect to Lebesgue-measure. In simple notation, if S is set of feasible contracts, then we denote it as follows.

$$S = \{s : X \to R \mid s(\bullet) \ is \ Lebesgue-measurable\}$$

Principal thinks that agent's utility function is not risk-loving, but does not know exactly. Or we interpret the situation as the agent tells to the principal about his utility function, but principal thinks there is some mis-specification in that preference so does not believe that information. By this logic, He evaluates incentive contracting according to worst-case utility of agent that minimizes principal's value function and so wants to design the contract *robust* to this kind of uncertainty.

(1) Benchmark Case : No Restriction on Agent's utility

In this case, principal's information about an agent's utility is that the utility function is not risk loving and he does not infer other kinds of properties of agent's utility. We can set-up principal's optimization program as follows. Optimizing in the following case is kind of convex-optimization, so we can formulate the *Lagrangian* and use the variation method as usual.

$$\max_{s(\bullet)}\min_{u(\bullet)}\int (x - s(x))f(x|a(u,s))dx \text{ subject to :}$$

$$\int u(s(x))f(x|a(u,s))dx - c(a(u,s)) \geq u_0 \qquad \text{(PC)}$$

$$a(u,s) \in \text{arg}max_{a'}\int u(s(x))f(x|a')dx - c(a') \qquad \text{(IC)}$$

$$0 \leq s'(x) \leq 1 \qquad \text{(MN)}$$

$$0 \leq s(x) \leq x, \ \forall x \in [0, \overline{x}] \qquad \text{(LL)}$$

$$u''(\bullet) \leq 0 \qquad \text{(RA)}$$

The $\max_{a(u)}$ operator can be imposed because if there exists multiple actions a satisfying (IC), principal may mandate the action which maximizes his own utility among them.

Now we look at each constraint and how constraints can be interpreted in this particular program. The first constraint (PC) is so-called *individual rationality* constraint. However in this model, in addition to participation constraint, it imposes some important restrictions on the agent's utility level : It is the constraint which settles agent's satisfaction level from $u(s)$ to some fixed level. Without this constraint, the minimizing utility of agent is that $u \simeq 0$ function, so very large payment is needed to make agent work for principal.

Also we impose Innes(1990)' assumption about incentive contract $s(x)$, $0 \leq s'(x) \leq 1$, so that the principal's payoff $x - s(x)$ is increasing function of x. This constraint implies that the higher agent's effort level, the expected payoff of principal becomes higher because of increasing property of $x - s(x)$. So given the contract, the principal wants agent to input more effort a.[5)]

Due to the monotonicity constraint, when principal gives any feasible contract $s = s(x)$, s has also the finite support $[\underline{s}, \overline{s}]$ likewise x has the finite support $[0, \overline{x}]$.

Theorem 1. In the above program, the fixed-wage contract survives as an optimal contract and an agent exert the minimal effort : $a^* = 0$

Proof. For any feasible contract of which $0 \leq s'(x) \leq 1$ holds, principal is made worse as the agent exerts less effort level due to the $MLRP$ of $f(x|a)$. So the minimizing utility of program is such that the agent with this utility exerts the least effort.

If we think of u_1 which is concave and $u_1(s) = b$, $\forall s \in [\underline{s}, \overline{s}]$, then agent does not 'feel' any incentive because his utility function is constant, as follows:

5) This can be proved by First-order stochastic domance(FOSD) of $f(x|a)$, supported by MLRP. The expected value of increasing function $x - s(x)$ becomes bigger as the agent inputs more effort a.

$$a(u,s) \in \arg max \int u(s(x))f(x|a')dx - c(a') = b\int f(x|a')dx - c(a) = b - c(a'),\ \therefore a^*(u,s) = 0$$

Because any feasible contract, with minimizing utility associated with that contract, induces no incentive, $a^* = 0$, principal's optimization program is to choose constant utility level b and contract. Because the principal gets worse-off as agent's general satisfaction level is low, the minimizing utility is that $u(s) = u_0 + c(0),\ \forall s,$ so the fixed-wage contract $s(x) = 0$ solves the following program because of limited liability constraint.

$$max_{s(\cdot)} min_b \int (x - s(x))f(x|0)dx \text{ subject to :}$$

$$\int u(s(x))f(x|0)dx - c(0) = b - c(0) \geq u_0$$

$$s(x) \geq 0,\ \forall x \in [0, \overline{x}]$$

Proof ended.

The basic intuition of the proof is that for any given feasible contract $s(x)$, we can find the constant utility u over the support of contract $s = s(x)$, with which the agent does not exert any effort. As a result, principal also does not have any incentive to give more than minimum wage $s_0 = 0$.

This theorem explains some widespread real-world phenomenon. In short-term agency relations or when principal does not investigate enough about his employee's preference, like in blue-color jobs, minimum wage is prevalent among principal-agent relationships because employer considers the worst case utility for his employee for robustness issue,

(2) Restriction on Marginal Utility of Agent

In the benchmark case as above, the principal regards that agent's utility function is not risk-loving, but does not infer any information about other characteristics of utility. In this section, we assume that principal thinks marginal utility of agent is bounded.

Too much high marginal utility of an agent implies agent feels naturally higher incentive to work given any contract $s(x)$. In contrast, too low marginal utility tells us that agent does not have enough incentive to work for principal. We exclude such extreme situation. Also we impose that utility level is such that it must be $u(0)$ is bounded below. This condition is surely important for existence of solution.

So we let the upper and lower bound of agent's marginal utility be $\infty > k_2 > k_1 > 0$ and we may assume that (k_1, k_2) are finite because the support of $x-space$ is bounded so that diminishing marginal utility $u'' \leq 0$ does not contradict with this finite assumption.

For example, $CRRA$ utility whose u' diverges to ∞ as $x \to 0^+$ may not be excluded in the feasible utility set because we restrict $x-space$ to be bounded below and above and not to include 0.

In such a case, principal's optimization scheme can be constructed as follows.

$$\max_{s(\cdot)} \min_{u(\cdot)} \int (x - s(x)) f(x \mid a(u,s)) dx \text{ subject to :}$$

$$\int u(s(x)) f(x \mid a(u,s)) dx - c(a(u,s)) \geq u_0 \qquad \text{(PC)}$$

$$a(u,s) \in \mathrm{argmax}_{a'} \int u(s(x)) f(x \mid a') dx - c(a') \qquad \text{(IC)}$$

$$0 \leq s'(x) \leq 1 \qquad \text{(MN)}$$

$$0 \leq s(x) \leq x, \ \forall x \in [0, \overline{x}] \qquad \text{(LL)}$$

$$k_1 \leq u'(s) \leq k_2 \qquad \text{(MU)}$$

$$u''(\cdot) \leq 0 \qquad \text{(RA)}$$

$$u(0) \geq p \qquad \text{(SC)}$$

Using the first-order approach of agency problem[6], we replace the (IC) constraint by following first order condition constraint with respect to the

6) We assume that the first-order approach is valid. Grossman and Hart(1983) and Rogerson(1985) show that MLRP and CDFC are sufficient for the validity of the first-order approach when the signal space is of one dimension. Jewitt(1988) finds less restrictive conditions for the validity of the first-order approach with the conditions on agent's utility. Recently, Conlon(2012) and Kim and Jung(2013) find the conditions justifying first-order approach in general cases for multi-dimensional signal case.

action a.

$$\max_{s(\cdot)} \min_{u(\cdot)} \int (x - s(x)) f(x|a(u,s)) dx \text{ subject to :}$$

$$\int u(s(x)) f(x|a(u,s)) dx - c(a(u,s)) \geq u_0 \qquad \text{(PC)}$$

$$\int u(s(x)) f_a(x|a(u,s)) dx - c'(a(u,s)) = 0 \qquad \text{(IC)'}$$

$$0 \leq s'(x) \leq 1 \qquad \text{(MN)}$$

$$0 \leq s(x) \leq x, \ \forall x \in [0, \overline{x}] \qquad \text{(LL)}$$

$$0 < k_1 \leq u'(s) \leq k_2 \qquad \text{(MU)}$$

$$u''(\bullet) \leq 0 \qquad \text{(RA)}$$

$$u(0) \geq p \qquad \text{(SC)}$$

Theorem 2. For any feasible contract that satisfies constraints of above program, there is an optimal contract which is debt contract[7]. and a principal-agent relation cannot obtain the first best outcome. In other words, uncertainty about agent's utility cannot bring the similar result to the case of risk-neutral agent of Harris and Raviv(1979)

Proof. The proof has 3 recursive steps for finding minimax solution for utility and contact.

Step 1. Inner minimization finding u-function given $s-$function.

For any feasible contract of which $0 \leq s'(x) \leq 1$ holds, we define the probability distribution on s-space as $g(s|a)$ such that $g(s|a)ds = f(x|a)dx$ with $g(\underline{s}|a) = g(\overline{s}|a) = 0$.[8] Then the (PC) and (IC) constraints also are transformed using the s-space representation.

7) This form of optimal contract is suggested by many seminal papers including Innes(1999), which formulates the simple second-best debt contract.

8) In fact, the composition of two absolutely continuous functions is not necessarily absolutely continuous. If f is Lipshitz continuous and u is absolutely continuous, it is known the $f \circ u$ is absolutely continuous within well-defined domain. We do not take into account these complicated issues for simplicity.

$$\int u(s(x))f(x|a)dx - c(a) \geq u_0 \quad \Rightarrow \quad \int u(s)g(s|a)ds - c(a) \geq u_0 \qquad \text{(PC)}$$

$$\int u(s(x))f_a(x|a)dx - c'(a) = 0 \quad \Rightarrow \quad \int u(s)g_a(s|a)ds - c'(a) = 0 \qquad \text{(IC)'}$$

Now, let the solution of second-step minimization be (u_0, a_0) and assume that $u_0'(s) > k_1$ holds for some positive Lebesgue measure on s-space. Then, there exists u_1 such that $u_1'(s) = k_1$ and it single crosses the utility function u_0 from above and satisfies $u_1(0) \geq p$ and the following level condition.

$$\int u_1(s)g(s|a_0)ds - c(a_0) = \int u_0(s)g(s|a_0)ds - c(a_0)$$

$$\rightarrow \int u_1(s)g(s|a_0)ds = \int u_0(s)g(s|a_0)ds, \ \int (u_1(s) - u_0(s))g(s|a_0)ds = 0$$

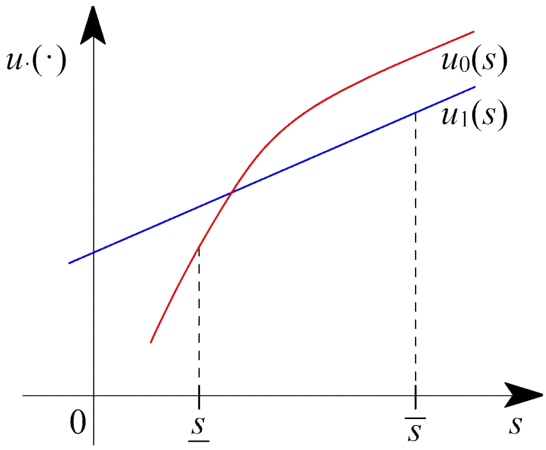

Figure 1. The utility function comparison

Because we assume the first-order approach is valid in this case, (u_0, a_0) satisfies (IC)'. By **Lemma 1** below, we finally get the inequality, which with a valid FOA implies the action a_1 under u_1 is smaller than a_0. Because of the monotonicity constraint $0 \leq s'(x) \leq 1, \forall x$, given the contract $s(x)$, smaller action $a_1 < a_0$ of agent yields smaller profit for principal, which means (u_0, a_0) is not the minimal solution of second-step minimization.

$$\int u_1(s)g_a(s|a_0)ds - c'(a_0) = \int u_0(s)g_a(s|a_0)ds - c'(a_0) + \int (u_1(s) - u_0(s))g_a(s|a)ds$$
$$= 0 + \int (u_1(s) - u_0(s))g_a(s|a)ds \leq 0$$

It is the contradiction, and (u_0, a_0) should be the case that $u_0'(s) = k_1, \forall s$. To summarize, principal robustly thinks that the agent who works for him is risk-neutral agent whose marginal utility is k_1 uniformly.

Step 2. Outer maximization formulation

For any feasible contract of which $0 \leq s'(x) \leq 1$ holds, we found the worst-case utility of agent to be risk-neutral and $u'(s) = k_1, \forall s$. Then we may assume $u(s) = k_1 s + p_s$ because the constant term may be dependent upon the given contract $s(x)$. For example, for an agent to participate in this relationship, his indirect utility given $s(x)$ should be bigger than u_0.

If we define a_s such that an agent with $u(s) = k_1 s + p_s$ chooses an action a_s given $s(x)$, for agent to participate, the following inequality holds for p_s.

$$p_s \geq p_0(s) \equiv u_0 + c(a_s) - k_1 \int s(x) f(x|a_s) dx$$

However, because principal becomes worse as the agent's general satisfaction level goes down, the minimizing utility has the constant term as low as possible, and this value is $\underline{p_s = p_0(s)}$. Now we formulate the outer maximization program.

$$\max_{s(\cdot),a} \int (x - s(x)) f(x|a) dx \text{ subject to :}$$

$$\int [k_1 s(x) + p_0(s)] f(x|a) dx - c(a) \geq u_0 \qquad \text{(PC)}$$

$$\int [k_1 s(x) + p_0(s)] f_a(x|a) dx - c'(a) = 0 \qquad \text{(IC)'}$$

$$0 \leq s'(x) \leq 1 \qquad \text{(MN)}$$

$$0 \leq s(x) \leq x, \ \forall x \in [0, \overline{x}] \qquad \text{(LL)}$$

Assume that the solution of the above program is $(s^0(x), a^0)$. Then participation constraint must hold because we let p_s be such that it makes participation constraint binding. In other words, we solve the optimization program without participation constraint first, and then can fine-tune the $p_0(s)$ to have the participation constraint binding.

So we ignore the participation constraint and find the conditions that $s^0(x)$ should satisfy. Then $s^0(x)$ should solve the following optimization as we ignore the (PC) and only consider (IC'), (MN), and (LL) :

$$\max_{s(\cdot),a}\int(x-s(x))f(x|a)dx \text{ subject to :}$$

$$\int s(x)f_a(x|a)dx-\frac{1}{k_1}c'(a)=0 \qquad \text{(IC)'}$$

$$0\le s'(x)\le 1,\ \forall x \qquad \text{(MN)}$$

$$0\le s(x)\le x,\ \forall x\in[0,\bar{x}] \qquad \text{(LL)}$$

We define new cost function $c_1(a)\equiv c(a)/k_1$ so that the optimization program above can be transformed into the following standard principal-agent model with monotonicity constraint and limited liability as in Innes(1990).

$$\max_{s(\cdot),a}\int(x-s(x))f(x|a)dx \text{ subject to :}$$

$$\int s(x)f_a(x|a)dx-c_1'(a)=0 \qquad \text{(IC)'}$$

$$0\le s'(x)\le 1 \qquad \text{(MN)}$$

$$0\le s(x)\le x,\ \forall x\in[0,\bar{x}] \qquad \text{(LL)}$$

This program is solved in Innes(1999)[9], and in that paper, he solves the second-best solution of above program with participation constraint using the debt contract. We can use the same strategy to prove that the form of debt contract is optimal even without the participation constraint. Then the optimal contract is of following form.

$$s(x)=\max[0,x-z]=x-z \text{ only for } x\ge z$$

9) Proposition 1 of Kim(1999) states the condition in which first-best bonus contract may exist.

The only remaining problem to solve is to find optimal value of z. Debt contract satisfies monotonicity and limited liability constraints so to find optimal z is equivalent to following optimization program. This program can be solved using general Kuhn-Tucker theorem.

$$\max \int^{z} xf(x|a)dx + z(1-F(x|a)) \text{ subject to :}$$

$$\int_{z}^{\bar{x}} (x-z)f_a(x|a)dx = c_1{}'(a) \qquad \text{(IC)}$$

Proof ended.

Lemma 1. Following inequality from the first part of the above theorem holds and (u_0, a_0) cannot be the minimizing solution.

$$\int (u_1(s)-u_0(s))g_a(s|a_0)ds \le 0$$

Proof Using the Monotone Likelihood Ratio Property, we define the new variable y which is score function of $f(x|a)$. Because contract $s(x)$ is monotonically increasing[10] and we already defined $g(s|a)$ so that it satisfies $g(s|a)ds = f(x|a)dx$, $g_a(s|a)ds = f_a(x|a)dx$ also holds.

$$y \equiv \frac{f_a}{f}(x|a_0) = \frac{g_a}{g}(s|a_0) \text{ increases as } x \text{ and } s = s(x) \text{ increases}$$

We then define the new function based on y-variable : $u_0(s) \equiv r_0(y)$, $u_1(s) \equiv r_1(y)$. Also as we did above, we define the probability distribution on $y-$space using transformation law.

$$g(s|a)ds = f(x|a)dx = h(y|a)dy.$$

10) Even if the contract is not strictly increasing, we can use the same argument without using s-space. We directly transform x-space representation to y-space representation and argue the same result.

Then, we can prove the Lemma using the method similar to Innes(1990).

$$\int (u_1(s)-u_0(s))g(s|a_0)ds = 0 \;\Rightarrow\; \int (r_1(y)-r_0(y))h(y|a_0)dy = 0 \quad (*)$$

$$\int (u_1(s)-u_0(s))g_a(s|a_0)ds = \int (u_1(s)-u_0(s))\frac{g_a}{g}(s|a_0)g(s|a_0)ds = \int (r_1(y)-r_0(y))yh(y|a_0)dy$$

$$\int (r_1(y)-r_0(y))yh(y|a_0)dy = y \cdot \int^y (r_1(y')-r_0(y'))h(y'|a_0)dy'|_{\underline{y}}^{\bar{y}} - \int\int^y (r_1(y')-r_0(y'))h(y'|a_0)dy'dy$$
$$= -\int\int^y (r_1(y')-r_0(y'))h(y'|a_0)dy'dy$$

We know, from single crossing property between $u_1(s)$ and $u_0(s)$, for (*) to be satisfied, it must be the case where $r_1(y)$ single crosses $r_0(y)$ from above and following is satisfied.

$$\int^y (r_1(y')-r_0(y'))h(y'|a_0)dy' \geq 0, \quad \forall y$$

$$\int (r_1(y)-r_0(y))yh(y|a_0)dy = -\int\int^y (r_1(y')-r_0(y'))h(y'|a_0)dy'dy \leq 0$$

$$\therefore \int (u_1(s)-u_0(s))g_a(s|a_0)ds = \int (r_1(y)-r_0(y))y\,h(y|a_0)dy \leq 0$$

Proof ended

The intuition for theorem 2 is clear and straightforward : We restrict that agent's marginal utility has both lower bound k_1 and upper bound k_2 point-wisely. This constraint can be interpreted as several distinct ways. First, we may think that even in the situation where principal does not have any information about his agent's utility functional form, he regards the agent as having some 'sense' of incentive to work hard so that his utility function is not the constant function. Or we might think that given Inada condition, support of $s-space$, and $x-space$ is small enough. Then agent's marginal utility may be bounded below and above in that region.

According to this theorem, given some contract $s(\cdot)$, minimizing utility of agent for principal is such that agent's sensitiveness to incentive is minimal, and minimal sense of incentive is represented by minimal marginal utility $u'(s)=k_1, \forall s$. Thus the linear utility, which says that agent is risk-neutral, makes possible that debt contract induces second best optimal incentive structure.

Classical moral hazard theory generally separates the case according to agent's utility : risk-neutral case and risk-averse case. And two cases must be analyzed very differently because of technical programming method. This theorem, with the concept of ambiguity aversion of principal, makes possible that two cases may be linked. So we derive the simple contract optimally : Debt contract. And this theorem shows quite strongly that Innes(1990)'s contract is robust against this kind of information uncertainty.

2.2. Analysis of Ambiguity-Averse Principal-agent : Technology case

This section deals with the situation where the stochastic technology $f(x|a)$ is not the common knowledge of both principal and agent, but only agent knows the true functional forms and other settings are the same with the standard agency frameworks.[11)]

Or we assume the situation in which there is some benchmark fact that production process follows the distribution $f(x|a)$. The principal, by years of interaction with agent, has got empirical distribution of production technology. The real problem is that the agent believes this fact but principal thinks that there might be some noisy measure distorting the benchmark density of $f(x|a)$. He thinks other possible parametrized density functions may describe the technology possibly and wants to protect against the distribution which minimizes his indirect utility.

We describe this situation by setting that principal is 'ambiguity-averse' and wants to design the optimal contract which is robust to this kind of Knightian uncertainty. As Hansen and Sargent(2007) noted, this principal comes up with his original distribution $g(x|a)$ and g represents the *worst-case* distribution for the principal around f-distribution. We model as g and f are more discrepant, he feels bad about this uncertainty.

11) For standard agency framework, see Holmstrom(1979), Grossman and Hart(1983), and Kim(1995).

All of these characteristics can be described well by using the Kullbeck-Leibler entropy measure between f and g to characterize the discrepancy between f and g. We assume that principal chooses the g distribution in the following entropy ball around f for each a.

$$G(a) \equiv \left\{ g(x|a) \geq 0 : \int g(x|a) \log \frac{g(x|a)}{f(x|a)} dx \leq \eta(a), \int g(x|a) dx = 1 \right\}$$

Then the principal's optimization program can be constructed using the fact that he chooses his own worst distribution g in $G(a)$ to minimize expected utility under any contract $s(x)$. Because this entropy constraint is always binding in the minimization step, we change the program and include the entropy value times *multiplier* θ in the objective function.

In general, we fix the value of $\eta(a)$ and find Lagrange multiplier θ endogenously by first order condition. In this problem however, we fix value of θ, solve first order condition, and substitute the solution to find the value of η. To be specific, as θ increases, we know that η generally decreases so principal is not much ambiguity averse. In contrast, as θ decreases, we know that corresponding η increases and this implies principal is much ambiguity averse so that chooses the minimizing g in the much bigger entropy ball around f. That means, we may interpret the fixed θ^{-1} as the 'measure of concern for robustness'.

Now, the two-step optimization procedure is represented as the following form.

$$\max_{s(x),a} \min_g \int (x - s(x)) g(x|a) dx + \theta \int g(x|a) \log \frac{g(x|a)}{f(x|a)} dx \text{ subject to :}$$

$$\int u(s(x)) f(x|a) dx - c(a) \geq u_0, \quad \int u(s(x)) f_a(x|a) dx - c'(a) = 0$$

$$\int g(x|a) dx = 1$$

Theorem 3. In this setting where principal has a constant ambiguity aversion to the technology uncertainty, the optimal contract is the *observationally equivalent* to the one in the case where principal has the constant absolute risk aversion θ^{-1}.

In other words, we cannot discriminate the situation where principal has θ^{-1} ambiguity aversion and θ^{-1} risk-aversion by seeing the form of contract.

Proof. This proof has 2 recursive steps for finding minimax solution for g and $s(x)$.

Step 1. This step is minimization step which finds g that minimizes the principal's welfare given the contract $s(x)$ and induced a. If we define the Lagrange multiplier on the normalization of g to be τ, the first-order condition for g is given below ;

$$x - s(x) + \theta(\log\frac{g(x|a)}{f(x|a)} + 1) + \tau = 0$$

$$\theta\log\frac{g(x|a)}{f(x|a)} = -(x - s(x)) - (\theta + \tau),\ \log\frac{g(x|a)}{f(x|a)} = -\theta^{-1}(x - s(x)) - \frac{\theta+\tau}{\theta}$$

$$\therefore g(x|a) = \frac{f(x|a)\exp[-\theta^{-1}(x - s(x))]}{\int f(x|a)\exp[-\theta^{-1}(x - s(x))]dx} = \frac{f(x|a)\exp[-\theta^{-1}(x - s(x))]}{E_f\ (\exp[-\theta^{-1}(x - s(x))])}$$

So we get minimizing distribution $g(x|a)$ for the principal given the value of θ^{-1}. In denominator of the above right-hand side, E_f means the integration using the original distribution f. Then we substitute this result to get the principal's value function V^{θ}, which he maximizes in the second step.

$$\log\frac{g(x|a)}{f(x|a)} = -\theta^{-1}(x - s(x)) - \log E_f(\exp[-\theta^{-1}(x - s(x))])$$

$$\theta\log\frac{g(x|a)}{f(x|a)} = -(x - s(x)) - \theta\log E_f(\exp[-\theta^{-1}(x - s(x))])$$

$$\theta\int g(x|a)\log\frac{g(x|a)}{f(x|a)}dx = -\int(x - s(x))g(x|a)dx - \theta\log E_f(\exp[-\theta^{-1}(x - s(x))])$$

$$V^{\theta}(s(x), a) = \int(x - s(x))g(x|a)dx - \int(x - s(x))g(x|a)dx - \theta\log E_f(\exp[-\theta^{-1}(x - s(x))])$$

$$\therefore\ V^{\theta}(s(x), a) = -\theta\log\int\exp(-\theta^{-1}(x - s(x)))f(x|a)dx$$

Step 2. This step is maximization step which finds optimal contract $s(x)$ and induced action a that maximize the principal's value function derived in above step. The program is simple.

$$\max_{s(x),a} V^{\theta}(s(x),a) = -\theta \log \int \exp(-\theta^{-1}(x-s(x)))f(x|a)dx \text{ subject to :}$$
$$\int u(s(x))f(x|a)dx - c(a) \ge u_0, \quad \int u(s(x))f_a(x|a)dx - c'(a) = 0$$

Because the logarithmic function is monotonically increasing, and $\theta > 0$, this optimization program can be made simpler by a monotone transformation which eliminates the logarithmic function. The changed program is given.

$$\max_{s(x),a} W^{\theta}(s(x),a) = -\theta \int \exp(-\theta^{-1}(x-s(x)))f(x|a)dx \text{ subject to :}$$
$$\int u(s(x))f(x|a)dx - c(a) \ge u_0, \quad \int u(s(x))f_a(x|a)dx - c'(a) = 0$$

This program is reproduced when the principal with constant absolute risk aversion θ^{-1} meets the agent and designs the optimal contract $s(x)$.

Proof Ended

The result is simple and straightforward. As the $x - s(x)$ gets bigger, the principal's worst case distribution g becomes smaller in those points because that would be the worst case. So principal, even with his risk-neutrality, can experience phenomena similar to the decreasing marginal utility in probability. These stories can be matched to the 'Murphy's Law' and we use term 'Statistical Murphy's Law' to explain this phenomenon.

To emphasize, we assumed constant ambiguity aversion θ^{-1} of principal and it is surprising that this principal uses the same incentive scheme with other principal with θ^{-1} constant risk aversion. The following Lemma 2 is crucial to prove the next result.

Lemma 2. If principal with utility function $G(\bullet)$ is risk-averse and agent is risk-neutral, in contrast with usual relationships, the only optimal contract is the fixed-rent contract as in Harris and Raviv(1979)

Proof Using the similar arguments to Holmstrom(1979), principal's optimization program is equivalent to find the saddle point of the following Lagrangian function.

$$\max_{s(x),a} L = \int G(x-s(x))f(x|a)dx + \lambda[\int u(s(x))f(x|a)dx - c(a)] + \mu[\int u(s(x))f_a(x|a)dx - c'(a)]$$

As in Holmstrom(1979), the first order conditions which optimal contract and action must satisfy are as follows : Using this we want to show that $\mu = 0$ at the saddle point.

$$G'(x-s(x)) = \lambda + \mu\frac{f_a(x|a)}{f(x|a)} \text{ with } G' : \text{decreasing function}$$

$$\int G(x-s(x))f_a(x|a)dx + \mu[\int u(s(x))f_{aa}(x|a)dx - c''(a)] = 0 \text{ where}$$

$$\int u(s(x))f_{aa}(x|a)dx < c''(a)$$

First, we assume $\mu < 0$ and show that there arises the contradiction. Let $r(x) \equiv x - s(x)$ and we define the constant r such that $G'(r) = \lambda$. We partition the $x-$space into X_+ and X_- by defining $X_+ = \{x | f_a(x|a) \geq 0\}$ and $X_- = \{x | f_a(x|a) < 0\}$ for an optimal action a. On X_+ and X_-, the following results hold and there exists the contradiction with the first-order conditions.

$$G'(r(x)) = \lambda + \mu\frac{f_a}{f}(x|a) \leq \lambda = G'(r) \rightarrow r(x) \geq r \text{ on } X_+$$

$$G'(r(x)) = \lambda + \mu\frac{f_a}{f}(x|a) > \lambda = G'(r) \rightarrow r(x) < r \text{ on } X_-$$

$$\therefore \int G(r(x))f_a(x|a)dx > \int G(r)f_a(x|a)dx = 0 \rightarrow \mu > 0 \text{ for the above condition :}$$

contradiction.

Second, we assume $\mu > 0$ and show that there arises the contradiction. In this case, on X_+ and X_-, following results hold and there exists the contradiction with the first-order conditions as in the other case.

$$G'(r(x)) = \lambda + \mu \frac{f_a}{f}(x|a) \geq \lambda = G'(r) \rightarrow r(x) \leq r \text{ on } X_+$$

$$G'(r(x)) = \lambda + \mu \frac{f_a}{f}(x|a) < \lambda = G'(r) \rightarrow r(x) > r \text{ on } X_-$$

$$\therefore \int G(r(x)) f_a(x|a)dx < \int G(r) f_a(x|a)dx = 0 \rightarrow \mu < 0 \text{ for the above condition :}$$

contradiction.

So we conclude that $\mu = 0$ at the optimum and the optimal contract $s(x) = x - G'^{-1}(\lambda)$ is of the same form as fixed-rent contract. We know that with risk neutral principal and agent, there are infinitely many optimal first-best contract. However, in this case, the first-best implementable contract is only Harris-Raviv's fixed-rent contract.

Proof Ended

With the case in **Lemma 2**, principal is risk-averse but an agent is risk-neutral. So risk sharing and incentive provision problem are aligned in the same direction : Agent takes all the risk not to give any risk to principal while feeling the maximal incentive to work. So first-best outcome can be achieved only through the fixed-rent contract. So we get the following corollary.

Corollary 1. In the theorem 3, if the agent is risk-neutral, the only optimal contract is the fixed-rent contract regardless of the value of robustness concern θ^{-1}.

So Harris-Raviv's fixed-rent contract is quite robust under this distributional uncertainty with multiplier preference of the principal. The principal is uncertain about the production process so delegates whole firm production to an agent to get rid of ambiguous decisions.

3. Conclusion

In this paper, we analyze two different case : the case where principal does not know or believe exact functional form of an agent's utility, and the case where principal does not know or believe the stochastic output density according to the action. The first case checks the robustness of the contract suggested by Innes(1990) and this contract is simple itself. We may interpret this as the reason people do not use the fine-tuning contract given by Holmstrom(1979). The second case deals with the case of uncertain output density. In adapting Hansen and Sargent(2011)'s multiplier preference framework, we obtain that the risk-aversion and ambiguity-aversion of a principal do yield the same contract function.

For future research, maybe it would be interesting to use this model's idea to the uncertainty asymmetry in financial intermediary market and investment banking industry, because in financial market, uncertainty or unknown risk is more important in aspects of risk-management and if we adapt this model, we may find more efficient way to robustly incentivize the workers under uncertainty of information. For these kinds of work to be possible, it should be interesting to research the dynamic robust agency model.

IV. Appendix : Saddle-function approach of robustness

In case 1 : benchmark model and restricted model, we solve the sequential $'\max-\min'$ program stepwisely, and in the inner minimization procedure, the monotonicity assumption $0 \leq s'(x) \leq 1$ is crucial to make clear that the principal wants the agent to work hard and less effort deteriorates principal's payoff.

We approach the problem in slightly different way in this section. In the case 1, to emphasize, firstly given the contract function $s(\bullet)$ we find the minimizing utility $u_s(\bullet)$ of agent in the interest of principal, and incorporating this utility function u_s, which depends on given contract function, we find the optimal contract function s. So each contract meets different utility function of agent that minimizes principal's payoff, and when principal chooses optimal contract, he considers this kind of contingency plan.

The principal thinks the agent's utility function follows some common properties. Given the feasible contract, he calculates the worst case utility in the specific set of utilities. We also here assumes that derivative of agent's utility is at least k_1 and less than k_2 :

$$k_2 \geq u'(s) \geq k_1, \forall s$$

In this appendix, we synthesize the two step optimization into saddle-function program. First we find the minimizing utility given the contract function s. In contrast with the case 1, we find the optimal contract s given the utility function of agent u. In other words, we find the contract function and utility function jointly with saddle relationship to each other. This program might be interpreted similarly as in the case 1 : principal may find the minimizing utility function of agent u_s given the contract s. However in contrast to the above section, we fix the utility u_s and find the optimal contract $s(u_s)$. And $s = s(u_s)$ ends the solving procedure. This is kind of saddle shape functional form and the solution of this program may be different from that of above two-step $'\max - \min'$ optimization.

One reader may argue that the solutions are the same across (Case 1) and (Appendix) because we found in the above theorem 2 that the minimizing utility is of the same functional form across any contract function satisfying the monotonicity constraint. We will check it using direct optimization.

In this case, we also assume the monotonicity constraint of contract s to find analytic solution but this constraint is not the crucial constraint because here we relax the problem with different optimization approach. It is the kind of difference between minimax strategy and Nash equilibrium strategy in non-cooperative game situation. Here we find the relevant 'Nash'-type equilibrium.

Also as we did above, we also assume so-called 'first-order approach' for mathematical tractability. The original optimization program(OP) is constructed as follows :

$$[max_{s(\cdot)} min_{u(\cdot)}] max_a \int (x - s(x)) f(x|a) dx \text{ subject to :}$$

$$\int u(s(x)) f(x|a) dx - c(a) \geq u_0$$

$$\int u(s(x)) f_a(x|a) dx - c'(a) = 0$$

$$k_1 \leq u'(s(x)) \leq k_2 \;\Rightarrow\; u'(s(x)) - k_1 \geq 0$$

To make clear this program to satisfy the sufficiency for Kuhn-Tucker theorem, we introduce the auxiliary function G that behaves as a convex function : $G(x) \geq 0$ when $x \geq 0$ and $G(x) < 0$ when $x < 0$. and $G'(x) > 0$ for $\forall x$ and $G''(x) > 0$ when $x \geq 0$. We may take $G(x) = x^3 + \epsilon x^2 + \epsilon x,\ \epsilon \to 0^+$ for example. Also, from above section, we are able to infer that $u'(s) \leq k_2, \forall s$ is not binding in the extremum.

Now we construct the convexified optimization program(COP) as follows:

$$[max_{s(\cdot)} min_{u(\cdot)}] max_a \int (x - s(x)) f(x|a) dx \text{ subject to :}$$

$$\int u(s(x))f(x|a)dx - c(a) \geq u_0$$

$$\int u(s(x))f_a(x|a)dx - c'(a) = 0$$

$$G(u'(s(x)) - k_1) \geq 0$$

To solve this program, we introduce the total Lagrangian L, collect the terms of utility, and use the variation method to find the extremum, or minimal path-dependence. We may start with defining the proper Lagrange multipliers on each of constraints.

First let Lagrange multiplier which is imposed on participation constraint and incentive constraint be λ, μ respectively. and let the point-wise Lagrange multiplier on the upward sloping constraint $\delta(x)$. We can construct the Lagrangian using these multipliers and do not impose limited-liability constraint, monotonicity constraint and concavity condition in constructing the Lagrange function, and only after solving first-order condition, we check the solution satisfies these constraint and characterize the solution.

As we discussed above, total Lagrangian function and terms about utility function is given here. Due to the Kuhn-Tucker condition, we know that multiplier δ, point-wisely, is non-negative : $\delta(x) \geq 0$, and by the complementarity slackness, $\delta(x)G(u'(s(x)) - k_1) = 0$ at the extremum solution. Lastly, we factorize $\delta(x) \equiv \delta(x|a)f(x|a)$ for mathematical convenience.12)

The Lagrangian function is constructed and in contrast with the proof of theorem 2, we attack the solution of optimization program more 'directly' with techniques of Kuhn Tucker theorem.

$$L = \int (x - s(x))f(x|a)dx + \lambda(\int u(s(x))f(x|a)dx - c(a) - u_0) + \mu(\int u(s(x))f_a(x|a)dx - c'(a))$$
$$+ \int \delta(x)G(u'(s(x)) - k_1)dx, \ \delta(x) \geq 0 \text{ for } \forall x, \ \lambda \geq 0$$

$$L^u = \lambda \int u(s(x))f(x|a)dx + \mu \int u(s(x))f_a(x|a)dx + \int \delta(x)G(u'(s(x)) - k_1)dx$$

12) Because we assumed $f-$distribution has the full-support, $f(x|a) > 0$ holds point-wisely on the domain.

Theorem 4. At extremum, for increasing optimal contract, $\delta(x) > 0,\ \forall x$. In other words, the upward-sloping constraint is binding and principal assumes that agent's utility function has the constant-slope point-wisely. So principal may assume agent has a linear utility in this case too as in **Theorem 2.**

Proof. Proof has 3 recursive steps for finding minimax solution for utility and contact.

Step 1. Now let's assume the solution of (COP) $s(x)$ satisfies $0 \le s'(x) \ge 0$, so $s = s(x)$ has the finite support $[\underline{s}, \overline{s}]$ likewise x has the finite support $[0, \overline{x}]$. We analyze the minimizing utility given this contract function first.

We assume we can change probability distribution from $f(x|a)$ to $s-$space distribution $g(s|a)$ such that $g(s|a)ds = f(x|a)dx$.[13] Also likelihood ratio g/f is well-defined. Then using the change of variable, we define $\epsilon(s|a) \equiv \delta(x^{-1}(s)|a)$.

$$\begin{aligned} L^u &= \lambda \int u(s(x)) f(x|a)dx + \mu \int u(s(x)) f_a(x|a)dx + \int \delta(x|a) G(u'(s(x)) - k_1) f(x|a)dx \\ &= \lambda \int u(s) g(s|a)ds + \mu \int u(s) g_a(s|a)ds + \int \epsilon(s|a) G(u'(s) - k_1) g(s|a)ds \end{aligned}$$

Because specified contract is monotonic, $g(\underline{s}|a) = g(\overline{s}|a) = 0$, we can think that L^u is the integral of functional $F(u(s), u'(s); s)$ which comprises $u(s), u'(s)$ and other functional form of s. we collect the terms and summarize the $u-$Lagrangian L^u.

$$\begin{aligned} L^u &= \lambda \int u(s) g(s|a)ds + \mu \int u(s) g_a(s|a)ds + \int \epsilon(s|a) G(u'(s) - k_1) g(s|a)ds \\ &= \int \{\lambda u(s) g(s|a) + \mu u(s) g_a(s|a) + \epsilon(s|a) G(u'(s) - k_1) g(s|a)\} ds = \int F(u(s), u'(s); s) ds \end{aligned}$$

Note $\lambda u(s) g(s|a) + \mu u(s) g_a(s|a) + \epsilon(s|a) G(u'(s) - k_1) g(s|a) = F(u(s), u'(s); s)$ and

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13) There can be the atom in the distribution of s or distribution function of s is not absolutely continuous. In that case, $dG(s|a)$ can be used instead of $g(s|a)ds$ without making any change in the arguments.

$g(\underline{s}|a) = g(\overline{s}|a) = 0$ for $\forall a$ so that $F(u(\underline{s}), u'(\underline{s}); \underline{s}) = F(u(\overline{s}), u'(\overline{s}); \overline{s}) = 0$ is satisfied.[14] This end-point condition is crucial for finding analytic solution using dynamic programming.

Also we calculate $F_{u'} = \epsilon(s|a) G'(u'(s) - k_1) g(s|a)$ because $g(\underline{s}|a) = g(\overline{s}|a) = 0$, $F_{u'}(\underline{s}) = F_{u'}(\overline{s}) = 0$ also holds. Then, what we have to do is to find the minimal path u of s that minimizes L_u. We assume differentiability of utility function except on the measure-zero region and let $u_*(s)$ is the solution. Then, for any given well-defined continuous mapping $\pi(s)$, the path $u_*(s) + \gamma\pi(s)$ achieves the minimum at $\gamma = 0$ necessarily. That means:

$$F(u_*(s) + \gamma\pi(s), u_*'(s) + \gamma\pi'(s); s) \equiv F(\gamma, s)$$

$$L^u(\gamma, \pi) = \int F(u_*(s) + \gamma\pi(s), u'_*(s) + \gamma\pi'(s); s) ds \geq L_u(\gamma = 0, \pi) = \int F(u_*(s), u'_*(s); s) ds$$

$$\frac{\partial}{\partial\gamma} L^u(\gamma, \pi) = \int (F_u(\gamma, s)\pi(s) + F_{u'}(\gamma, s)\pi'(s)) ds$$

And we separate the first-term and second term and use the integration-by-parts technique to second-term. By $F_{u'}(\gamma, \underline{s}) = F_{u'}(\gamma, \overline{s}) = 0$, we can eliminate the boundary values and just switch the steps of integration by this process.

$$\int F_{u'}(\gamma, s)\pi'(s) ds = F_{u'}(\gamma, s)\pi(s)|_{\underline{s}}^{\overline{s}} - \int \frac{d}{ds}(F_{u'}(\gamma, s))\pi(s) ds = -\int \frac{d}{ds}(F_{u'}(\gamma, s))\pi(s) ds$$

$$\frac{\partial}{\partial\gamma} L^u(\gamma, \pi) = \int (F_u(\gamma, s)\pi(s) + F_{u'}(\gamma, s)\pi'(s)) ds = \int [F_u(\gamma, s) - \frac{d}{ds} F_{u'}(\gamma, s)]\pi(s) ds$$

$$\therefore \frac{\partial L^u}{\partial\gamma}(0, \pi) = \int (F_u(0, s)\pi(s) + F_{u'}(0, s)\pi'(s)) ds = \int [F_u(0, s) - \frac{d}{ds} F_{u'}(0, s)]\pi(s) ds = 0, \ \forall \pi(s)$$

$$\therefore F_u(u_*(s), u'_*(s); s) = d_s F_{u'}(u_*(s), u'_*(s); s)$$

These analyses result in 'Euler-Lagrange equation' and we may use this necessary condition to justify our result. $F_u = \lambda g(s|a) + \mu g_a(s|a)$, and $F_{u'} = \epsilon(s|a) G'(u'(s) - k_1) g(s|a)$ so at the minimal solution, the following relation must hold.

14) In fact, we implicitly assume that Lagrange multiplier (λ, μ) is not divergent at the extremum solution.

$$F_u = \lambda g(s|a) + \mu g_a(s|a) = d_s[\epsilon(s|a)G'(u'(s) - k_1)g(s|a)] = d_s F_{u'}$$

So we summarize this result and move on to the second step.

$$\therefore\ \epsilon(s|a) = \frac{1}{G'(u'(s) - k_1)g(s|a)} \int^s (\lambda g(s'|a) + \mu g_a(s'|a))ds'$$
$$= \frac{1}{G'(u'(s) - k_1)g} \int^s (\lambda + \mu \frac{g_a}{g}(s'|a))g(s'|a)ds'$$

Step 2. Now, given that utility function is weakly concave[15], we find first-order condition in $s(x)$ of L. To be specific, let us use the original Lagrangian function L, not L^u to find the first-order condition for $s(x)$. And we may use Step 1 and 2's results jointly.

We can point-wisely differentiate $s(x)$ from $s(x)$'s measurability and differentiability and if the L function is linear in s, the coefficient should be zero to make the solution of program s not the corner solution.

$$-f(x|a) + \lambda u'(s(x))f(x|a) + \mu u'(s(x))f_a(x|a) + \delta(x|a)f(x|a)G'(u'(s(x)) - k_1)u''(s(x)) = 0$$
$$\therefore\ \frac{1 - \delta(x|a)G'(u'(s(x)) - k_1)u''(s(x))}{u'(s(x))} = \lambda + \mu \frac{f_a}{f}(x|a)$$

In this kind of optimal contract, by the assumed MLRP about technology f, if we get $\mu \geq 0$, (RHS) is the increasing function of x[16] so we may think that (LHS) should also be the increasing function of x. Anyway, we get that $(RHS) > 0$.

$$\frac{1 - \delta(x|a)G'(u'(s(x)) - k_1)u''(s(x))}{u'(s(x))} = \lambda + \mu \frac{f_a}{f}(x|a) > 0$$

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15) Actually, this weak concavity presumes that linear function can be approximated to be concave locally.

16) Several standard agency models implement the Rogerson(1985)'s double-relaxing analysis and obtain $\mu \neq 0$ to prove $\mu > 0$, using the fact that principal should give the agent an incentive to work hard. However, in our model, the logic like this cannot work and the sign of μ is not the central problem.

Step 3. Now, we use the Step 1 and 2's results to find the optimum of (COP). If the optimal contract is increasing, probability measure is changed from x-space to $s_0(x)$ and represented by Radon-Nikodym derivative $g(s|a)/f(x|a)$. So $g(s|a)ds = f(x|a)dx$ holds and likelihood ratio is unchanging.

$$\frac{f_a}{f}(x|a) = \frac{g_a}{g}(s|a),\ \forall s = s_0(x)$$

$$\begin{aligned}\epsilon(x|a) &= \frac{1}{G'(\bullet)g}\int^{s}(\lambda + \mu\frac{g_a}{g}(s'|a))g(s'|a)ds' \\ &= \frac{1}{G'(u'(s_0(x)) - k_1)g(s_0(x)|a)}\int^{x}(\lambda + \mu\frac{f_a}{f}(x'|a))f(x'|a)dx'\end{aligned}$$

And in the Step 2, contract $s_0(x)$ should satisfy the following optimum equality. Let us write down that equation and analyze joint extremum of utility and contract.

$$\frac{1 - \delta(x|a)G'(u'(s_0(x)) - k_1)u''(s(x))}{u'(s_0(x))} = \lambda + \mu\frac{f_a}{f}(x|a) > 0$$

At maximum-minimum saddle solution of convexified optimization program, it should be that both equality first-order conditions be satisfied. because the second equation implies that $\lambda + \mu(f_a/f)$ is positive almost everywhere, it should be that $\epsilon(x|a) > 0$: Upward-sloping constraint is binding here. Then, $u'(s(x)) = k_1,\ \forall x$ at the extremum (u, s) and the Lagrange multiplier should be that $\mu = 0$ to make interior solution.

$$\frac{1}{k_1} = \lambda + \mu\frac{f_a}{f}(x|a) > 0 \quad \therefore\ \mu = 0$$

In summary, principal robustly thinks that the agent with the minimal 'sense' of incentive works for him as above and we made this proof with mathematically tractable way.

Proof Ended

The results of theorem 2 and 4 are the same even though we approach the program in different aspect. It is because we get from proof of theorem 2 that the minimizing utility of agent, given any monotonic contract s, is linaer utility with minimal marginal utility value. The intuition for achieving the linear-minimizing utility of the agent is the same across theorem 2 and theorem 4 : Linear utility with minimal sensitiveness to incentive is minimizing the principal's payoff in this relationship because agent with that utility feels the minimal incentive to work so he puts the smaller effort level a. Also, as we know that the participation constraint is binding in general also, because if not, principal may assume uniformly lower-valued utility of agent to be robust against utility-uncertainty.

So in that case we get $\lambda > 0$. If we look at Step 2's optimal contract equation, and substitute the $u'(s(x)) = k_1, \forall x$, then as seen, (LHS) is constant and (RHS) is function of output x. To make these quantities equal, the Lagrange multipler μ should be zero intuitively.

From theorem 2 and 4 we can link this powerful result to well-known literature in Principal-agent model and especially, the case of risk-neutrality. Well-known models where agent has the linear utility imply that the principal need not care about the agent's risk sharing problem and can always induce the first-best action to the agent. Harris and Raviv(1979) firstly solves this problem without agent's limited liability and showed that the fixed rent contract $s(x) = x - k$ can induce the first-best action. Innes(1990) and Kim(1999) shows later that with limited liability of participant in relationship, the simple contract such as debt contract or linear-bonus contract may achieve also the first-best incentive allocation. The theorem 2 and 4 quite strongly support that these solutions are robust against principal's inawareness about agent's utility function when agent is risk-averse.

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요약(국문초록)

이 논문에서 우리는 강건계약에 대해 연구하였다. 일반적인 주인-대리인 관계에서 주인과 대리인이 가진 정보는 주어진 것으로, 그리고 대칭적인 정보를 가정하였지만 현실세계에서는 주인의 정보가 대리인에 비해 불확실한 경우, 그리고 대리인에 대해 많은 것을 알지 못하는 경우의 이 관계가 더 많다고 판단되었고 따라서 이러한 상황에서 최적 계약의 형태를 수학적으로 도출하고자 하였다. 먼저 주인이 대리인의 선호체계를 정확하게 알지 못하지만 그의 한계효용이 유계인 상황을 가정할 경우에 최적 계약의 형태를 도출하였고, 대리인이 일하는 기술 환경에 대해 주인이 완벽하게 믿지 못하는 경우의 최적 계약을 도출하였다. 두 경우 모두 최적 계약의 형태가 매우 단순하여 현실세계의 계약환경을 잘 나타내 준다고 판단하였다.

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keywords : Ambiguity aversion, Knightian uncertainty

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