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Formation of Social Class: A Network Theory Approach

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Formation of Social Class: A Network Theory Approach

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Abstract

This paper studies the underlying incentives of social class formation. I use the network model of favor exchange developed in Jackson et al. (2012), and extend it to more generalized case where each agent has their own capacity of value, which they can deliver through doing a favor. I verify that even in the heterogeneous value case, the network that is renegotiation-proof and robust to social contagion must take the form of social quilt suggested in Jackson et al. (2012), and the agents in robust network are not connected to those who have significantly lower value than his own value. This suggests that the social class is formed because of the people's incentive to not to connected with other people with lower social status

Keywords: Social Network, Social Class, Homophily

Contents

1 Introduction

Humans are known as social animals. That is they tend to form a society and interact with each other to survive and fulfill whatever they want. And throughout the history of human society – although some may have taken more subtle form than others – the classification of people by their wealth, social status, etc. has been present at any certain point of time and in any culture. Some social classes were explicit showing the social status and deciding what people could do. For instance, during the Joseon Dynasty of Korea, there were four different classes of people and altering one's class was not easy. On the other hand, other classes were more implicitly formed such as the Bourgeoisie in the middle age Europe, or any upper-class society in modern societies.

The role of these social classes on the behavior of the people has been an interesting question among many social scientists. Sociolinguists study how the language develops in different classes(Bernstein (1960)), some psychologists study the interrelation between the social class and mental illness(Hollingshead and Redlich (1958)). However, there is not much work done to explain the reason these social classes form in theory. There has been many explanations on how certain types of people face the same situation forming a class with people with similar characteristics, but the question of why people tend to connect and form a relation with the others of the similar social or economic status rather than with the others of different status has not been answered much yet.

Of course, to address the formation of social class properly, I will have to clarify what social class means, or at least to which concept I will refer as social class. The identity of social classes and the question asking whether they exist explicitly and have an effect on people have been an issue of controversy(See Bourdieu (1987), Wacquant (2013)). However, I will refrain from the argument and simply define the social class as a group of people being connected to each other who have similar level of social status.

In this paper, I aim to address a formal model of social network game and with the model try to explain the agents' tendency to sustain a link with other agents who have the same or similar social status. For this, I will start from the favor exchange model of Jackson et al. (2012), and extend the model to heterogeneous values setting, characterizing the robust networks, then lay out the results that relate to my question of interest.

1.1 Related literature

As I talk about the people bonding with others who have similar level of social status, this paper is related to the literature on *homophily*.

McPherson et al. (2001), sums up the works done by the sociologists that supports the existence of homophily in various human characteristics. According to the paper, people tend to show fondness to others with similar characteristics, such as ethnicity, gender, age, education, occupation. They also mention that homophily is also found between the ones who have similar belief, abilities, or even the position in network. They also study the sources of homophily, which are the geography, organizational focus, one's role in the society, *et cetera*.

Ruef et al. (2003) studied the group formation between the entrepreneurs and found out that homophily and the presence of prior network ties shows strong effect on group composition. Whereas the geographical constraint were the major source of social isolation, or exclusion from a group.

Currarini et al. (2009) developed a economic model of friendship formation where individuals have types and see the benefit from the types. Then they examine the data and find out three patterns of friendship formation: (1) larger groups tend to form more same type ties than smaller ones, (2) larger groups form more ties per capita, (3) all groups are biased towards the same type. Then they use the model developed to understand those three patterns.

2 Jackson's Model of Favor Exchange

This part of the paper will briefly demonstrate the key ideas of the model from Jackson et al. (2012), as I base my model upon it. I omit the parts that are irrelevant to my extension.

2.1 Setup

A network g consists of agents and the links between agents. $N = \{1, \cdots, n\}$ is the set of agents(or nodes as in the graph theoretical notation), and agents are connected (or not) to each other with links. If an agent i is connected to j, then the link between them is expressed as ij. Then it is easy to say that the network is a set of links with agents being properly defined. [1] For set of the agents who has at least one link in g, we will denote it as $V(g) \equiv \{i \mid ij \in g \text{ for some } j\}$

For every time period, a favor occurs with probability $p < \frac{1}{n(n-1)}$ from one agent to another agent. Whenever a favor occurs, the agent who has

[1] This definition of the network suggests some sort of equivalence class as the networks will now be considered only for the significant agents who has at least one link connected to them. For instance a network $g = \{12, 23\}$ with set of nodes $\{1, 2, 3\}$ will be considered equivalent to the network $g' = \{12, 23\}$with set of nodes $\{1, 2, 3, 4\}$

been asked a favor – from now on I will refer to this agent as *favorer* – decides whether to perform the favor or not. If the favor is performed the cost c occurs to him, and a value of favor v is given to the agent who asked the favor – whom I will call *favoree.* If not, the link between the two agents disappear. The agents discount their utility at a rate $\delta < 1$.

The timeline for the game at each time period is as follows

Stage 0 Initial network is given.

Stage 1 Each agents decide whether to keep the links that are connected to them.

Stage 2 A favor happens with probability p.

Stage 3 The favorer decides whether to follow through with the favor, and if he does, cost c to himself, and value v to the favoree occurs.

Note that in stage 1, each agent gets rid of the links that are useless, that is those that when the favor occurs either himself will not perform the favor or the agent on the other end will not perform.

2.2 Sustainability and renegotiation-proofness

For each link that an agent has, the expected stream of utility from the link is $\frac{\delta p}{1-\delta}(v - c)$. If this expected stream of utility exceeds the cost of performing a favor, then all links will be sustained, hence the authors assume $\frac{\delta p}{1-\delta}(v - c) < c$. In this setting, no single links can be sustained in the equilibrium, and there exists an integer m that satisfies $m\frac{\delta p}{1-\delta}(v-c) > c$, but $(m-1)\frac{\delta p}{1-\delta}(v-c) < c$, *i.e.* an agent will carry out the favor when asked if not doing so results in the loss of at least m links that he has. From this it is easy to see that a network with all agents,if they have any, having

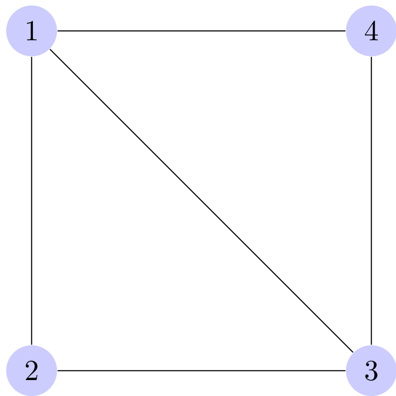

Figure 1: Example of sustainable but not renegotiation-proof network when $m = 2$

more than m links is sustainable in an equilibrium with all agents using *grim-trigger strategy*, where agents punish by cutting all the links with an agent if he fails to perform the favor that was asked. They

Definition. A network g is *sustainable* if all agents in the network have sufficient links so that they would

However, a question may be posed to the credibility of threat that agents will cut all the links to the failed favorer. For example, let the initial network given as figure 1, and $m = 2$. Then in this network, agents 1 and 3 has 3 links each, while 2 and 4 has 2 links. Therefore this network is sustainable in the equilibrium. However, if agent 1 asks agent 3 a favor and 3 decides not to deliver the favor, then the link 13 is immediately lost. Then, if the agents are to follow through with the punishment, 2 and 4 must cut the links with 3 at the stage 1 of following period. However, the resulting cycle is also sustainable, and it is more beneficial for the agents to sustain the network than to cut the links and collapse the network.

In this spirit, the authors introduce the concept of renegotiation-proof networks, which are sustainable in the equilibrium where agents decide to lose links only if it does not result in a network which is Pareto dominated by other network that is reachable. The definition is put into formal words as following.

Definition. A network g is *renegotiation-proof* iff beginning with g implies that there exists a pure strategy SPE such that

- on the equilibrium path g is always sustained
- in any subgame starting with some network g', if g'' is reached with some probability and then played in perpetuity in the continuation, then g'' is renegotiation proof, and there does not exist any $g''' \subset g'$ such that g''' is renegotiation proof and g''' Pareto dominates g''.

2.3 Robustness to social contagion

The authors then turn to a new criterion of network, *robustness to social contagion*. The concept of robustness comes from that the impact of a single agent not performing a favor must be localized, *i.e.* the loss of links due to an agent not delivering a favor must be confined to the links within the neighborhood of the agent. The formal definition of robust networks are given as below.

Definition. A network g is *robust to social contagion*, or simply *robust* if

- g is renegotiation-proof
- g is sustained as part of a pure strategy SPE such that in any subgame continuation from any renegotiation-proof $g' \subset g$ when $i \in g'$ does not perform the favor for j, then for any $g'' \subset g' - ij$ that is reached with some probability and then played in perpetuity, $\forall hl \in g' - g''$, $h, l \in N_i(g') \cup i$[2]

[2]Here, $N_i(g)$ is defined as $N_i(g) = \{j | ij \in g\}$, and called the neighborhood of i in g

Note that from the definition of robust networks, it naturally follows that all renegotiation-proof subnetworks of a robust network must also be robust, since the definition covers every subgame starting from any renegotiation-proof networks.

Then they fully characterize the robust networks with a concept of *social quilts*.

Definition. A network g is a social quilt if and only if g is union of $(m+1)$ complete networks and there is no cycle in the network involving more than $(m+1)$ nodes.

Theorem. *A network is robust against social contagion if and only if it is a social quilt.*

3 Model with Heterogeneous Values

In the original work of Jackson et al. (2012), the value and cost from a favor performed between any agents were all the same. But I pose a question of what would happen if the settings change to heterogeneous values and costs, *i.e.* when agent i asks j a favor and it is carried out, then cost c_{ij} occurs to j, and value v_{ij} is given to i.

3.1 Setup

As mentioned above, the values and costs are now heterogeneous but for the sake of analysis, I put some restrictions on them. First, the costs of performing a favor would now be a increasing function of the value it creates. This is natural as the favors generating a larger value should cost more. For instance, if one agent lends money to other agent (since this is a favor, let us assume that they lend it at no interest rate), then the cost, which would be the opportunity cost of the money lent, for the lender would be larger as the size of the loan grows. I will denote the cost of performing a favor when agent i askes j as $c_{ij} \equiv c(v_{ij})$ satisfying $\forall v > v', c(v) > c(v')$. Also, I assume that all values generated by a favor to be positive, and the cost to be strictly lower than the value it generates, *i.e.* $\forall i, j \in N, v_{ij} > 0$, and $\forall v > 0, c(v) < v$.

I also assume a condition to make sure that no single links are sustainable in the equilibrium.

$$\forall v > 0, \frac{\delta p}{1-\delta}(v - c(v)) < c(v) \tag{1}$$

, so that we could avoid the cases where a single link is sustainable by itself. If we assume this condition then at least one of the agent on the two ends of a link has an incentive to lose the link when they do not have any support, or another agent who has links with both of them. Since the probability of favor occurance p, and the discount factor δ does not play much role in our analysis, for the ease of writing I will denote $\rho = \frac{\delta p}{1-\delta}$.

I take the definition of renegotiation-proofness and robustness in the same manner it was described in homogeneous case. Figure 2 shows the example of robust network under the heterogeneous value setting. $v_{ij} = v_i v_j$ where v_i is the number written on the node. $c_{ij} = c(v_{ij}) = \frac{1}{3}$ and $\rho = .5$

3.2 Robust networks under heterogeneous value

Earlier, I have mentioned that every renegotiation-proof subnetwork of robust network is robust. This suggests the existence of a minimally robust network which completely breaks down whenever a link is lost.

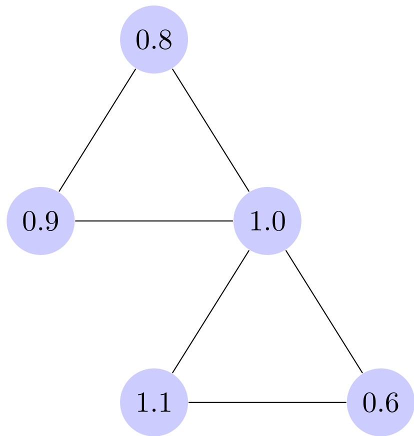

Figure 2: Example of robust network under heterogeneous settings:
$v_{ij} = v_i v_j$, $c(v) = .4v$, $\rho = .5$

Definition 1. A network g is an minimally robust network if g is robust and whenever a link $ij \in g$ is lost, all other links are subsequently lost along all equilibrium paths.

In other words, the only renegotiation-proof subnetwork of minimally robust network would be the null network. From this definition we can characterize the form of minimally robust network.

Lemma 1. A network g is an minimally robust network only if it is a complete network

Proof. Suppose g is an minimally robust network. Then from the definition of minimally robust network, for any agent i with a link in g, whenever i does not perform the favor that has been asked to him, all links in g must be subsequently lost along the equilibrium path. Then from the definition of robust network, all agents with a link in g must be a neighbor of i. This holds for any agent with a link in g, hence all agents must be a neighbor of all agents. Therefore g is a complete network. □

Lemma 1 suggests that all minimally robust networks must be of the form of complete network in which all agents have link connecting themselves to all the other agents in the network. However, being a complete network does not mean that it is minimally robust network. In many cases they may not even be sustainable.

Then using lemma 1, we can characterize the form of the robust networks under the heterogeneous setup. The proof of the proposition follows the method used in Jackson et al. (2012).

Proposition 1. A network is robust against social contagion only if it is social quilt, which can be partitioned into minimally robust networks, and every cycle[3] in the network is contained in only one of the partitions.

Proof. First, we start by noting that any robust network must contain a minimally robust subnetwork, since losing a link in robust network eventually results in a robust subnetwork, and repeating to remove a link from resulting robust network will trickle down to a minimally robust network and then null network.

Let g be a robust network. Suppose there is a minimally robust subnetwork $g_a \subset g$ with at most one node i connected to other nodes outside of g_a. Then any agent $j \neq i$ in g_a not performing a favor, must collapse g_a and continue on with $g - g_a$. Therefore $g - g_a$ is also robust. Continue on with this process must end up with null network, or some network $g' \subset g$ that does not contain a minimally robust subnetwork with at most one node connected to other nodes. If the process ends with null network, then the original network must have been the form of social quilt.

[3]A cycle is a subnetwork of a given network, and may be represented as a sequence of links in the network, such that adjacent links have one node in common, and the last link of the sequence shares a node with the first link of the sequence.

Suppose we ended up with g'. Then there must exist a link ij which is in a minimally robust subnetwork $g_b \subset g'$ and both i, j have a link outside of g_b. Then removing ij from the network would still leave a sustainable network, as both i, j have enough links to maintain the network. Repeat this process once for each minimally robust subnetworks until there is no minimally robust subnetwork left, and let the resulting network be $g'' \subset g'$. Note that g'' is still sustainable but does not contain any minimally robust subnetwork. Then there exists sustainable $g''' \subset g''$ such that any subnetwork of g''' except for itself is not sustainable. Then under any circumstances, agents who have link in g''' would carry out the favors that they are asked, therefore g''' is renegotiation-proof subnetwork of g, which implies the robustness of g'''. However, by the construction of g''' we know that g''' does not contain any minimally robust network, hence cannot be a robust network, which is contradiction. □

In other words, proposition 1 suggests that the robust networks under the heterogeneous values setting, also take the form of social quilt. However, unlike in the homogeneous case in which we have we do not know the size of each pieces of the quilt.

4 The Formation of Social Class

Until now I have examined the form of the networks that are robust under the heterogeneous value setting. Now, to exhibit my point, I will focus more on specific case in which each agent has a value v_i, a value generated whenever he decides to carry out a favor; in other words, $v_{ij} = v_j$[4]. For

[4]For those who might question this setup, the results for the cases when value depends only on favoree, or the cases when value depends equally on both agents shows similar result, but for the point I wish to make in this paper, I chose only to display the results

instance, it could be the amount of money he could lend, or it could be a true favor which he can make possible. In this sense, v_i can be interpreted as the social status such as wealth, or political power, etc.

Also I will assume that the cost function is linear, *i.e.* $c(v) = cv$ for some $c < 1$. Then from the assumption I made earlier, $\frac{\rho}{\rho+1} < c$. Then, just like in the homogeneous case, we can define an integer m, which satisfies

$$m\rho(v - cv) > cv > (m-1)\rho(v - cv). \tag{2}$$

Note that no matter how value is defined, the value of m is identical for every agent.

Before we move on, I give a short lemma proving that removing a link from a minimally robust network must leave the agent with the larger value no longer active in performing favors.

Lemma 2. Suppose the cost of conducting a favor is linear function of the value it generates, and the value generated only depends on the favorer. Then if g_m is a minimally robust network, $\forall ij \in g_m,\ s.t.\ v_i > v_j$,

$$\rho(\sum_{k\neq i} v_k - (n-1)cv_i) > cv_i, \tag{3}$$

$$\rho(\sum_{k\neq i,j} v_k - (n-2)cv_i) < cv_i, \tag{4}$$

Proof. Since g_m is robust, it must be sustainable. Therefore the inequality (3) holds naturally.

Now if for both i, j, $\rho(\sum_{k\neq i,j} v_k - (n-2)cv_i) > cv_i$, and $\rho(\sum_{k\neq i,j} v_k -$

under this setup.

$(n-2)cv_j) > cv_j$, then after the removal of ij, $g - \{ij\}$is still sustainable. Then there exist $g' \subset g - \{ij\}$ such that $g'$is sustainable and no other subnetwork of g' is sustainable. Then g' is sustainable in perpetuity, but this is contradiction to the fact that g_m is robust and therefore renegotiation-proof. Hence, at least one of the two inequalities must be reversed.

It is easy to show that , $\rho(\sum_{k \neq i,j} v_k - (n-2)cv_i) - cv_i - (\rho(\sum_{k \neq i,j} v_k - (n-2)cv_j) - cv_j) < 0$ since $v_i > v_j$. Therefore inequality (4) must hold □

Then using the lemma and assumptions of the model, we can show that once the parameters are fixed, hence m is fixed, the robust networks must take the form of *m-quilt*, a social quilt with each minimally robust network that constitutes the network being $m-$*clique* or $m+1$ complete network.

Proposition 2. Assume that values are heterogeneous and depends on the favorer, with linear cost function. Then, the size of minimally robust networks are fixed to an integer $m+1$ satisfying (2). Hence the robust networks are all in the form of m-quilt

Proof. Suppose g_m is a minimally robust network with size n. Then for any agent in g_m, (3) holds. If we sum it over all the agents in g_m, then we get

$$\rho\Big((n-1)\sum_{k \in V(g_m)} v_k \;-\; (n-1)c\sum_{k \in V(g_m)} v_k\Big) > c\sum_{k \in V(g_m)} v_k$$

which gives us $(n-1)\rho(1-c) > c$. This inequality, combined with (2), returns $n > m$

Now let $i = \arg\min_{k \in V(g_m)} v_k$, and $j = \arg\min_{k \in V(g_m)-\{i\}} v_k$. Then, from (4)

$$\rho(\sum_{k \neq i,j} v_k - (n-2)cv_j) < cv_j$$

However, $\forall k \neq i, j, v_k \geq v_j$. Therefore,

$$cv_j > \rho(\sum_{k\neq i,j} v_k - (n-2)cv_j) \geq (n-2)\rho(v_j - cv_j)$$

Which yields $c > (n-2)\rho(1-c)$, and combined with (2), $m > n-2$.

Therefore, $m < n < m+2$, and since m,n are both integers, $n = m+1$.

□

Also, lemma 2 gives a corollary that confines the range of the values in the same m-clique.

Corollary 1. Assume that values are heterogeneous and depends on the favorer, with linear cost function. Then no links in a robust network has negative expected stream of utility for both agents. In other words, $\forall ij \in g$ where g is robust, $v_i > cv_j$ and $v_j > cv_i$.

Proof. This follows naturally from subtracting (4) from (3). □

This corollary shows that the people will not be connected to the ones that are worth less than the cost of being connected to them. This result seems natural since this implies that no links with negative expected stream of utility will be sustained. And with some calculation, we can narrow down the range of the agents that can be connected in a robust network.

Proposition 3. For any agent i with the largest value, in a minimally robust network g_m,

- sum of the values of all the other agents must be in the range

$$\frac{m\rho}{c-(m-1)\rho+mc\rho}v_i > \sum_{k\neq i} v_k > \Big(\frac{c}{\rho}+mc\Big)v_i \tag{5}$$

- and $\forall i, j$, such that $v_i > v_j$

$$v_j > \Big(\frac{c}{\rho} + mc - (m-1)\Big) v_i > cv_i \tag{6}$$

Proof. The second inequality of (5) is directly driven from (3) and the fact that $n = m + 1$.

Also, for any agent $j \neq i$,

$$\rho\Big(\sum_{k \neq j} v_k - mcv_j\Big) > cv_j$$

holds. If we sum this over all $j \neq i$, we get

$$\rho\Big((m-1)\sum_{k \neq i} v_k + mv_i - mc\sum_{k \neq i}\Big) > c\sum_{k \neq i} v_k$$

and this gives the first inequality of (5).

If $i = \arg\max_{k \in V(g_m)} v_k$, *i.e.* i is the agent with the largest value, then from (5),

$$\forall j \neq i, \;\; (\frac{c}{\rho} + mc)v_i < \sum_{k \neq i} v_k \leq (m-1)v_i + v_j$$

This shows the maximum gap between the agents, hence we know that for any other agents, the gap must be smaller. Which gives the first inequality of (6)

Also from the assumption on m in (2),

$$c > (m-1)\rho(1-c)$$

which then can, after some change give

$$\left(\frac{c}{\rho} + mc - (m-1)\right) > c$$

and since $v_i > 0$, the second inequality of (6) holds. □

As we can see from the proposition, in robust networks, an agent is only connected to the ones that are *worth* being connected to. He tends to be connected to those who can provide a value through a favor that is in a certain range from what he can provide. It may seem that the cause of this separation between higher valued agent and lower valued agent is mutual. However if we think about the original network game, it is obvious that this disconnection between different level of agents is the result of higher valued agents not willing to connect to the lower valued agents.

Even when the high-value individual is connected with the agent with low-value one, since the summation of the values from other agents in the same clique must be in the certain range in a robust network, they would need a support from another high-value agent. which would make it less likely for an agent with lower value to connect with the agents with higher value.

We can also see how this boundary of connection changes as ρ or c changes. ρ reflects the patience, or how much agents value the future stream of utility; higher ρ means that they are more patient. And c reflects the efficiency of value generation; the higher the value of c, value added to the society by the favor gets lower. From (6), we can see that the boundary, or tolerance of higher-valued agent gets higher as he grows impatient and the value generation of favor becomes inefficient.

5 Conclusion

This paper shows that the underlying incentives to people forming a social class, or a group of similar social status, are that they do not wish to be connected to the ones with significantly lower social status. Agents are willing to be connected with the one with higher social status, but the link with the higher agent is less likely to be sustained. In a way, this may explain some of the social class formation; the class not just formed because of people connecting to those with similar status, but formed because of people avoiding connection with lower class.

Of course there are lots of drawbacks to this paper. As mentioned in Jackson et al. (2012), the portion of robust networks goes to zero as number of agents grow. And the results from this model heavily relies on some assumptions that may not be true in the real world. But still, I believe that this sheds some light on the way people form a social class.

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국 문 초 록

요약

이 논문은 사회계층의 형성에 있어 각각의 사람들에게 존재하는 유인에 대하여 연구한 논문이다. 이를 위해 Jackson et al.(2012)의 논문에서 사용된 호의 교환 모형(favor exchange model)을 네트워크상의 각각의 주체들이 들어줄 수 있는 부탁의 크기 및 그에 수반하는 비용들이 서로 다른 경우로 확장한다. 본 논문에서는 이러한 각 주체의 이질성을 가정한 경우에도 기본적으로 유지가능성 등 현실에 맞는 성질을 갖는 네트워크의 형태가 선행 논문에서 나오는 *social quilt*의 형태를 가지며, 이러한 네트워크 상의 주체들은 자신이 베풀 수 있는 호의의 크기에 비해 현저히 낮은 호의밖에 제공하지 못하는 주체들과는 연결고리가 끊어짐을 보인다. 이러한 결과는 인간 사회에서 계층이 형성되는 것은, 동류에 대한 이끌림 보다는, 자신에 비해 낮은 사회적 지위를 가진 사람과 연결되지 않으려는 유인 때문이라고 해석될 수 있다.

주요어: 사회계층, 네트워크, 동류애, 게임이론

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