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Tax Incidence in Dynamic Monopoly

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Abstract

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This paper compares the equilibria in dynamic monopoly model under diferent kinds and rates of taxes. We show that if profit tax is imposed, a monopolist bears all the burden of tax regardless of the rates of tax. In contrast, price tax might be shared by both the buyer and the seller and the ratio of tax burden between them depends on the type of buyer. We also show that tax incidece principle still holds which means that price and expected utilities for buyer and seller in the equilibrium do not depend on whether price tax is levied on consumer or seller. Although it is hard to generally verify the superiority of profit tax over price tax in terms of social welfare and consumer surplus, we prove it in the case when there are only two types of buyers regarding their willingness to pay for a product.

Keywords : dynamic monopoly, sequential bargaining, tax incidence, perfect bayesian equilibrium, profit tax, price tax

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1 Introduction

In economics, tax incidence is the analysis of the effects of tax policies on prices and the distribution of economic welfare. The key concept of tax incidence is that the statutory incidence is irrelevant to the economic incidence, so called tax incidence principle. For example, in competitive market, tax incidence is not dependent on where the revenue is collected, but on the price elasticity of demand and supply. Incidence analyses abound but the strategic aspect of economic interaction is mostly constrained. In most of them, the agents are supposed to be price takers, and thus, they behave as a solitary decision maker under the discipline of market. This is contrast with bargaining model in which market plays no role except for setting the stage for discussion. Although there are some papers analyzing the incidence of taxes under imperfect competition in static model, it is not sufficient for investigating the effects of tax on the behaviors of players strategically correlated with each other because sellers can only propose one shot take-it-or-leave-it offer. In this paper, I analyze tax incidence in sequential bargaining model in which the final outcome is determined entirely by the strategic interaction of the bargainers.

To show our results, we use sequential bargaining model of Fudenberg, Levine and Tirole[1]. It investigates a situation in which a seller with a constant production cost for a good and a buyer whose valuation for the product is private information bargain over the trade of one unit of the good. It is identical to Dynamic monopoly model without commitment for future prices for a product in Gul, Sonnenschein and Wilson[4]. They explain several structural properties of equilibrium such that buyers use reservation price strategies and conclude that the equilibrium path is generically unique if the lowest valuation buyer has is greater than the unit production cost. In our paper, this uniqueness is critical in that it helps us to directly compare tax incidence under different kinds and rates of taxes.

The main purpose of this paper is to study the incidence of taxes under various tax policies. We show that if profit tax is imposed, a monopolist bears all the burden of tax regardless of the rates of tax. In contrast, price tax might be shared by both the buyer and the seller and the ratio of tax burden between them depends on the type of buyer. We also show that tax incidece principle still holds which means that price and expected utilities for buyer and seller in the equilibrium do not depend on whether price tax is levied on consumer or seller. Although it is hard to generally verify the superiority of profit tax over price tax in terms of social welfare and consumer surplus, we prove it in the case when there are only two types of buyers regarding their willingness to pay for a product.

The paper is organized as follows: Section 2 describes the model which is acquired by applying the sequential bargaining model to the situation with the existence of tax. In section 3, I explain the relationships between the equilibrium sets under different rates and kinds of tax and show that tax incidence principle still holds. Also, I end the section by proving that the imposition of price tax reduces social welfare and consumer surplus more than that of profit tax at least in the case for two type buyers .

2 Model

To acquire results in this paper, we can use two different but essentially same models. The first one is sequential bargaining model from Fudenberg, Levine and Tirole(1985). A monopoly seller sells a durable good to a single buyer in periods $t \in \{1, 2, \cdots\}$. The seller's constant unit cost for the good is c and is commonly known. The buyer's value for the good $v \in V \subset [\underline{v}, \overline{v}]$ is private information which is randomly drawn from a commonly known cumulative distribution function $F(\cdot)$. Either $F(\underline{v}) > 0$ or F admits a strictly positive and continuous density at $\underline{v}$. If the buyer's lowest valuation is strictly greater

than unit cost,$\underline{v} > c$, we call the case the gap case and if not, $\underline{v} \leq c$, the case would be called the no-gap case. The seller and the buyer have the same discount factor δ which is positive and less than one.

At the start of any period, the seller specifies a taxed price $p_t \geq 0$ and the buyer decides whether to accept or reject the price. All actions are publicly observable. The game continues only if the buyer rejects the price. If the buyer never accepts an offered price, both the buyer's and the seller's utility are zero. A history is any finite sequence of the price decisions of the seller, and all the prices in the sequence are taxed prices. Seller's strategy is a function of any history to the next price if consumer still remains unaccepted. Buyer's strategy is a function of any history to his decision between accept or reject for the last proposed price under the assumption that he has not accpeted other prior proposed prices. We use the concept of *Perfect Bayesian Equilibrium*(PBE) which is a history-contingent sequence of the seller's offers p_t, the buyer's acceptance decisions, and updated beliefs about the buyer's value v, satisfying both that each player's actions are optimal given beliefs and the other's strategy and that beliefs are computed by given strategy profiles and Bayes' rule whenever possible, including off the equilibrium path.

The second model is dynamic monopoly model from Gul, Sonnenschein and Wilson(1986). The difference from the first model is that there is a unit measure of consumers and the concept of *Subgame Perfect Equilibrium*(SPE) is therefore used. To make two models equivalent, we use the regularity assumption "The equilibrium actions of each agent are constant on histories in which prices are the same and the sets of agents accepting at each point in time differ at most by sets of measure zero.". Under this assumption, this dynamic monopoly model is formally identical to the sequential bargaining model. Therefore, the results in our paper can be intrepreted in both models. However, the results in the case of positive production unit cost are better interpreted in the second model because they are closely related with the production cost for each period.

We investigate the effects of three kinds of taxes on society. Profit tax is a tax levied on the net profit of monopolist. Consumption tax is a price tax imposed on consumption and sales tax is a price tax levied on monopolist's revenue. In other words, profit tax and price tax can be imposed on the monopolist while only price tax can be levied on the consumer. All theses taxes are assumed to be ad-valorem taxes. Suppose that the consumer with value v buys the product at price p_t at the tth period. If profit tax τ is levied on the monopolist, his profit is $\delta^{t-1}(1-\tau)(p-c)$ and consumer's utility is $\delta^{t-1}(v-p)$. If price tax τ is imposed on the monopolist, his profit is $\delta^{t-1}((1-\tau)p-c)$ and consumer's utility is $\delta^{t-1}(v-p)$. If price tax τ is imposed on the consumer and p is taxed price, the monopolist's profit is $\delta^{t-1}(\frac{p}{1+\tau}-c)$ and consumer's utility is $\delta^{t-1}(v-p)$.

3 Results

In the following two subsections, we explain isomorphisms between the equilibrium sets under the different kinds of taxes. In mathematics, an isomorphism is a mathematical mapping that preserves the structure of the mapped entities, which is a stronger concept than bijection. By defining history as a sequence of proposed taxed prices, we impose each player' s strategy set to be equivalent under any kind of tax. We would define that the two equilibrium sets are isomorphic if they have precisely the same elements in terms of each player' s strategy and belief system implying that they have identical equilibrium path. In addition, the isomorphic sets are defined to be essentially same if the same element has the same relationship between taxed price and pre-tax price, which means that they bring about same expected utility for each player and identical expected tax revenue.

To prove the isomorphisms, we use the following logic. Suppose that there are two players and the strategy set for each player is respectively S_1 and S_2. If two pairs of

utility function for each player (U_1, U_2) and (U_1', U_2') ,$U_1, U_2, U_1', U_2' : S_1 \times S_2 \to R^+$satisfy the following conditions, the Nash equilibrium sets for two pairs of utility functions are isomorphic.

$$U_i(s_1, s_2) \geq (>) U_i(s_1', s_2) \Leftrightarrow U_i'(s_1, s_2) \geq (>) U_i'(s_1', s_2)$$

and

$$U_i(s_1, s_2) \geq (>) U_i(s_1, s_2') \Leftrightarrow U_i'(s_1, s_2) \geq (>) U_i'(s_1, s_2')$$

for all $i = 1, 2,\ s_1, s_1' \in S_1$ and $s_2, s_2' \in S_2$. This is because if the preference relation is preserved, the best response is preserved given the other player's strategy. This logic can be generalized for players more than two players and be applied to different concepts for equilibrium such as Perfect Bayesian equilibrium.

3.1 No Production Cost

The case in which the product has already been produced can be interpreted to have zero production cost. If $c = 0$, for the monopolist, the price tax τ is same as the profit tax τ and thus, the equilibrium set would be same for both taxes. Therefore, we only need to focus on the price tax. Let's first consider a consumption tax, the price tax levied on the consumer. The next lemma shows that after the imposition of the consumption tax, the seller bears all the burden of the tax in the equilibrium.

Lemma 3.1. *For any positive τ, the set of PBE when consumption tax is τ is isomorphic to the set of PBE when there is no consumption tax.*

Proof. Fix a PBE when consumption tax is τ $(s_B^\tau, s_S^\tau, \mu^\tau)$. s_B^τ is the equilibrium strategy of a buyer which assigns a history to a decision of whether to accept or reject. s_S^τ is a seller's equilibrium strategy assigning a history to a taxed price in the next period. $\mu^\tau$is a belief system satisfying Bayesian consistency with those strategies. Let's show that $(s_B^\tau, s_S^\tau, \mu^\tau)$ is a PBE when $\tau = 0$. First, after any history, the sequence of future proposed

taxed prices in both cases are equivalent due to the same seller's strategy. Therefore, it is evident that s_B^τ is still the buyer's optimal strategy for the seller's strategy s_S^τ when $\tau = 0$, and thus, μ^τ satisfies the Bayesian consistency. Now, it remains to prove that at each information set, s_S^τ specifies the seller's best response to s_B^τ when $\tau = 0$. Suppose that there is an information set h such that there exists another strategy $\tilde{s_S^0}$which gives more expected utility to the seller than s_S^τ conditional on arriving h. Then, when there exists an consumption tax τ, a strategy $\tilde{s_S^0}$ is better thans_S^τ for the seller conditional on ariving h . To prove that, let $u_S^{c,t}(s_B, s_S, \mu|h)$ be the seller's expected utility conditional on arriving h when a buyer and a seller's strategy is respectively $s_B$and s_S, the belief system is μ,and the size of consumption tax is t. When the seller's strategies are identical facing the two tax schedule, the buyer with a strategy s_B^τ responses in the same way, that is, with the same probability the buyer accepts the proposed price at each period after the history h. So, $u_S^{c,\tau}(s_B^\tau, \tilde{s_S^0}, \mu^\tau|h) = \frac{1}{1+\tau} u_S^{c,0}(s_B^\tau, \tilde{s_S^0}, \mu^\tau|h)$ and for the same reason, $u_S^{c,\tau}(s_B^\tau, s_S^\tau, \mu^\tau|h) = \frac{1}{1+\tau} u_S^{c,0}(s_B^\tau, s_S^\tau, \mu^\tau|h)$. However, it means that $u_S^{c,\tau}(s_B^\tau, \tilde{s_S^0}, \mu^\tau|h) = \frac{1}{1+\tau} u_S^{c,0}(s_B^\tau, \tilde{s_S^0}, \mu^\tau|h) > \frac{1}{1+\tau} u_S^{c,0}(s_B^\tau, s_S^\tau, \mu^\tau|h) = u_S^{c,\tau}(s_B^\tau, s_S^\tau, \mu^\tau|h)$ which contradicts the fact that $(s_B^\tau, s_S^\tau, \mu^\tau)$ is a PBE. For the opposite direction, fix a PBE when no consumption tax (s_B^0, s_S^0, μ^0) and let's prove that the PBE is also a PBE when the size of consumption tax is τ. For the same reason from the above, the optimality of the buyer's strategy is evident. To prove the optimality of the seller's strategy, as in the above case, suppose that there is an information set h such that there exists a better strategy $\tilde{s_S^\tau}$ for seller i.e. $u_S^{c,\tau}(s_B^0, \tilde{s_S^\tau}, \mu^0|h) > u_S^{c,\tau}(s_B^0, s_S^0, \mu^0|h)$. Then, we can also get contradiction from $u_S^{c,0}(s_B^0, \tilde{s_S^\tau}, \mu^0|h) = (1+\tau) u_S^{c,\tau}(s_B^0, \tilde{s_S^\tau}, \mu^0|h) > (1+\tau) u_S^{c,\tau}(s_B^0, s_S^0, \mu^0|h) = u_S^{c,0}(s_B^0, s_S^0, \mu^0|h)$ and the fact that (s_B^0, s_S^0, μ^0) is a PBE when there is no consumption tax. Therefore, we can prove the isomorphism between the set of PBE when consumption tax is τ and that when there is no consumption tax. $\square$

Now, instead of the consumption tax, consider the sales tax, the price tax levied on seller. With the same logic, we can easily prove similar isomorphism between the different sales taxes.

Lemma 3.2. *For any positive t less than 1, the set of PBE when sales tax is t is isomorphic to the set of PBE when there is no sales tax.*

Proof. The proof would be same as Lemma 3.1. considering $u_S^{s,t}(s_B, s_S, \mu|h)$, the seller's expected utility conditional on arriving h when a buyer and a seller's strategy is respectively $s_B$and s_S, the belief system is μ,and the size of sales tax is t, instead of $u_S^{c,t}(s_B, s_S, \mu|h)$ and the relation that $u_S^{s,\tau}(s_B, s_S, \mu|h) = (1-\tau)u_S^{s,0}(s_B, s_S, \mu|h)$ □

The two Lemmas show that in the either case that the consumer starts to pay the consumption tax or the case that the monopolist starts to pay the sales tax, the sequence of taxed price and of the probabilities that the monopolist accept at each period on the equilibrium path remain unchanged compared with the case of no tax. Actually, the case with no consumption tax is not different from no-sales-tax case. Therefore, we can get the following proposition from the above two lemmas.

Proposition 3.3. *For any $\tau \geq 0$ and $0 \leq t < 1$, the set of PBE when consumption tax is τ is isomorphic to the set of PBE when sales tax is t. Especially, if $t = \tau/(1+\tau)$, the two sets are essentially same.*

Proof. We can get the first argument directly from the above two lemmas. For the second argument, consider a PBE when the consumption tax is τ and the corresponding PBE when the sales tax is t which has the same pair of equilibirum strategies and belief system. This implies that the two PBE have the same equilibrium path in terms of the sequence of proposed taxed prices and the buyer's acceptance probability. On the other hand, if the taxed price is p, the pre-tax price is $\frac{p}{1+\tau}$ when the consumption tax is τ and is $(1-t)\cdot p$

when the sales tax is t. So, under the same taxed price, the consumption tax τ and the sales tax $\tau/(1+\tau)$ give the same pre-tax price and the same gap between taxed price and pre-tax price. Therefore, not only the expected utility for all types of buyer but also the expected utility of seller and the expected tax revenue are identical because they are all function of the sequence of taxed prices, pre-tax prices and buyer's accpetance probability on the equilibrium path. The opposite direction is also correct with the same logic. This finishes our proof. □

3.2 Positive Production Cost

When the production unit cost is positive, profit tax is different from price tax for the monopolist, and thus, the set of PBE would be different for two kinds of taxes. First, let's think about profit tax. In this case, we can also get similar isomorphism result as above.

Lemma 3.4. *For any positive τ, the set of PBE when profit tax is τ is isomorphic to the set of PBE when there is no profit tax.*

Proof. The proof would be same as Lemma 3.1. considering $u_S^{p,t}(s_B, s_S, \mu|h)$, the seller's expected utility conditional on arriving h when a buyer and a seller's strategy is respectively $s_B$and s_S, the belief system is μ,and the size of profit tax is t, instead of $u_S^{c,t}(s_B, s_S, \mu|h)$ and the relation that $u_S^{p,\tau}(s_B, s_S, \mu|h) = (1-\tau)u_S^{p,0}(s_B, s_S, \mu|h)$. □

This lemma implies that the profit tax burden of monopolist is not shared with buyer. In other words, the tax revenue is collected only at the expense of the seller's welfare while all types of the buyer's expected utility remain unchanged. The intuition is that because the imposition of profit tax does not affect the superiority among seller's different strategies if the buyer's strategy is same, the set of PBE remains unchanged. In other words, the change in profit tax is identical for monopolist to that in exchange rate, which does not give any effect on the seller's best response to the buyer's strategy.

Now, let's focus on price tax. We can get similar isomorphism between the set of PBE with the existence of tax and the equilibrium set without tax but the production cost should be adjusted.

Lemma 3.5. *For any positive τ, the set of PBE when the production cost is c and consumption tax is τ is isomorphic to the set of PBE when the production cost is $(1+\tau)\cdot c$ and there is no consumption tax.*

Proof. Fix a PBE $(s_B^{\tau,c}, s_S^{\tau,c}, \mu^{\tau,c})$ when the production cost is c and consumption tax is τ. Let's prove that it is also a PBE when the production cost is $(1+\tau)\cdot c$ and consumption tax is zero. First, because the buyer and seller's strategies are same, it is evident that the consistency of $\mu^{\tau,c}$ holds. Now, let's prove the optimality of the buyer's strategy. After any history the seller's same strategy would give the same sequence of proposed taxed price in the future and the buyer's utility is a function of his value, the accepted taxed price and the timing of the acceptance. Therefore, the optimality of $s_B^{\tau,c}$ in the case when the production cost is c and consumption tax is τ guarantees its optimality in the case when the production cost is $(1+\tau)\cdot c$ with no consumption cost. The only thing to remain to prove is the optimality of the seller's strategy. Suppose not. Then, there exists an information set h such that there is another strategy $s_S^{0,\widetilde{(1+\tau)c}}$ which gives more expected utility to the seller than $s_S^{\tau,c}$ conditional on arriving h. If this is true, when the production cost is c and consumption tax is τ, a strategy $s_S^{0,\widetilde{(1+\tau)c}}$ is better than $s_S^{\tau,c}$ for the seller conditional on ariving h . To prove that, let $u_S^{c,\tau,c}(s_B, s_S, \mu|h)$ be the seller's expected utility conditional on arriving h, given the strategy profile (s_B, s_S), the belief system μ,the consumption tax with size τ and the production cost c. Because the seller's strategies are the same for both taxes, the buyer with a strategy $s_B^{\tau,c}$ responses in the same way, that is, with the same probability the buyer accepts the proposed price at each period after the history h. So, considering the formula $(\frac{p}{1+\tau} - c)\cdot q = \frac{1}{1+\tau}(p-(1+\tau)c)\cdot q$, we can get $u_S^{c,\tau,c}(s_B^{\tau,c}, s_S^{0,\widetilde{(1+\tau)c}}, \mu^{\tau}|h) =$

$\frac{1}{1+\tau}u_S^{c,0,(1+\tau)c}(s_B^{\tau,c}, s_S^{0,\widetilde{(1+\tau)c}}, \mu^\tau|h)$ and $u_S^{c,\tau,c}(s_B^{\tau,c}, s_S^{\tau,c}, \mu^\tau|h) = \frac{1}{1+\tau}u_S^{c,0,(1+\tau)c}(s_B^{\tau,c}, s_S^{\tau,c}, \mu^\tau|h)$. However, it means that $u_S^{c,\tau,c}(s_B^{\tau,c}, s_S^{0,\widetilde{(1+\tau)c}}, \mu^\tau|h) = \frac{1}{1+\tau}u_S^{c,0,(1+\tau)c}(s_B^{\tau,c}, s_S^{0,\widetilde{(1+\tau)c}}, \mu^\tau|h) > \frac{1}{1+\tau}u_S^{c,0,(1+\tau)c}(s_B^{\tau,c}, s_S^{\tau,c}, \mu^\tau|h) = u_S^{c,\tau,c}(s_B^{\tau,c}, s_S^{\tau,c}, \mu^\tau|h)$. This is a contradiction because $(s_B^{\tau,c}, s_S^{\tau,c}, \mu^{\tau,c})$ is a PBE. By using the same logic, the opposite direction also can be proved. □

We can also get a similar isomorphism for the sales tax.

Lemma 3.6. *For any positive t less than 1, the set of PBE when sales tax is t and the production cost is c is isomorphic to the set of PBE when there is no sales tax and the production cost is $\frac{c}{1-t}$.*

Proof. The proof would be same as Lemma 3.5. Considering $u_S^{s,t,c}(s_B, s_S, \mu|h)$, the seller's expected utility conditional on arriving h , given the strategy profile (s_B, s_S), the belief system μ,the sales tax with size t and the production cost c, instead of $u_S^{c,t,c}(s_B, s_S, \mu|h)$ and the relation that $u_S^{s,t,c}(s_B, s_S, \mu|h) = (1-t)u_S^{s,0,\frac{c}{1-t}}(s_B, s_S, \mu|h)$. □

The intuition for the two above lemmas is as follows. While profit tax has the same effects both on revenue and cost for monopolists, consumption tax or sales tax affects only the revenue. If the proposed price and quantity sold at each period are the same, the revenue when the consumption tax is τ would be $\frac{1}{1+\tau}$ times as the revenue when the tax is zero, but the cost would be alike. This means that consumption tax τ has the same effect on monopolist as the simultaneous increase of production cost and the profit tax, respectively, from c to $(1+\tau)c$ and from zero to $\frac{\tau}{1+\tau}$. As seen above, profit tax does not affect the seller's optimal behavior, and thus, the set of PBE remains unchanged. This is why the unit cost should be changed to get the above isomorphism results. From the two lemmas, we can generalize Proposition 3.3..

Proposition 3.7. *For any $\tau \geq 0$ and $0 \leq t < 1$, the set of PBE when consumption tax is τ and the production cost is c is isomorphic to the set of PBE when sales tax is t and the*

production cost is $(1-t)(1+\tau)c$*. Especially, if* $t = \tau/(1+\tau)$*, the two sets are essentially same.*

Proof. From Lemma 3.5., the set of PBE when the consumption tax is τ and the unit cost is c is isomorphic to the set of PBE when the unit cost is $(1+\tau)c$ and there is no production cost, which is also isomorphic to the set of PBE when the sales tax is t and the unit cost is $(1-t)(1+\tau)c$ from Lemma 3.6. Therefore, we can get the first argument. For the second argument, consider a PBE when the consumption tax is τ and the production cost is c and the corresponding PBE when the sales tax is t and the production cost is $(1-t)(1+\tau)c$ which has the same pair of equilibirum strategies and belief system. This implies that the two PBE have the same equilibrium path in terms of the sequence of proposed taxed price and the buyer's acceptance probability. On the other hand, if the taxed price is p, the pre-tax price is $\frac{p}{1+\tau}$ when the consumption tax is τ and is $(1-t)\cdot p$ when the sales tax is t. Hence, under the same taxed price, if $t = \frac{\tau}{1+\tau}$, the consumption tax τ and the sales tax t give the same pre-tax price and the gap between taxed price and pre-tax price. Therefore, in addition to $(1-t)(1+\tau)c = c$, the two PBEs have the same expected utility of seller and the same expected tax revenue as well as the identical expected utility for all types of buyer. The opposite direction is also correct with the same logic. □

3.3 Tax Incidence

In analyzing the effect of the introducing tax on welfare, we would use the following uniqueness result from Fudenberg, Levine and Tirole(1985).

Corollary. *(Fudenberg, Levine and Tirole 1985; Gul, Sonnenschein, and Wilson 1986) In the gap case, a Perfect Bayesian Equilibrium(PBE) exists and is generically unique. Also, in the equilibrium, if the price is lower than in the past, the buyer's behavior is*

independent of previous prices.("stationarity" or"the strong cutoff rule property")

By using the uniqueness result and Proposition 3.3., and 3.7., we can show that tax incidence principal still holds in dynamic monopoly if cost is sufficiently smaller than the minimum possible value that buyer has.

- When the cost is zero, the introduction of consumption tax τ is equivalent to that of sales tax $t = \tau/(1+\tau)$ in terms of the buyer's utility, the monopolist's profit, tax revenue, and social welfare in expected sense.

- When the cost is positive, the introduction of consumption tax τ is equivalent to that of sales tax $t = \tau/(1+\tau)$ in terms of the buyer's utility, the monopolists's profit, tax revenue, and social welfare in expected sense if $(1+\tau)c < \underline{v}$.

In addition, we can know the allocation of tax burden between buyer and seller when the unit cost is zero or when the cost is positive and only the profit tax is levied on monopolist. From Lemma 3.1., 3.2., and 3.4., the next two results mean that for both cases the entire tax burden remains on the seller's shoulder.

- When the cost is zero, the introduction of price tax or profit tax does not change expected consumer surplus and social welfare but decreases monopolist's expected profit in comparison with the case without tax

- When the cost is positive, the introduction of profit tax does not change expected consumer surplus and social welfare but decreases monopolist's expected profit in comparison with the case without tax.

On the other hand, different from profit tax, price tax might be shared by both the buyer and the seller, and the ratio of tax burden between them depends on the type of buyer. In static model, social welfare and consumer surplus decrease after the introduction of

price tax. However, it is hard to prove that such results hold for any prior probability of types of buyer. The main reason is that the set of time consistent paths that monopolist can choose might be changed after the introduction of a price tax in dynamic monopoly. The time consistent path is represented by a sequence $\{p_n, q_n\}_{n \in N}$, a sequence of pairs of the proposed price and buyer's unconditional accpetance probability at the nth period, satisfying that the buyer's accpetance probability at each period does not violate his optimality and that after mth period it is still optimal for seller to propose $\{p_n\}_{n \geq m+1}$afterwards. Actually, the only thing that the monopolist can choose is q_1 because $\{p_n, q_n\}_{n \geq 2}$ should be chosen from the equilibrium set when the seller's posterior after the first period is given as a new prior belief for the types of buyer and p_1 should be chosen to satisfy that buying the product at p_1 now and waiting for the next period and buying at p_2 are indifferent for the buyer with valuation $F^{-1}(1 - q_1)$. Then the seller's optimization problem

$$max \sum \delta^{i-1}((1-t)p_i - c)q_i$$

$$\Leftrightarrow max \sum \delta^{i-1}(p_i - \frac{c}{1-t})q_i$$

$$s.t. \ \{p_i, q_i\} : \ a \ time \ consistent \ path$$

means that as t grows, the seller behaves as if cost increases from c to $\frac{c}{1-t}$ and cares more about decreasing $c \cdot \sum \delta^{i-1} q_i$, the expeced discounted sum of cost. This implies that seller sells with lower probability at the earlier periods to reduce the expected cost, which could drag bargaining periods longer and induce higher proposed price at each period. This would result in the decrease in social welfare and consumer surplus. When it is possible for committment or bargaining persists during at most two periods in the equilibrium in both cases, the case without any tax and the case with a certain size of price tax, the set of time consistent paths remains unchanged after the introduction of price tax and we can use this logic. If the choice set changes, however, this logic cannot be directly applied.

Therefore, it is hard to affirm that price tax would reduce social welfare. However, the argument seems related to the following note in Gul, Sonnenschein and Wilson(1986),” Increasing costs, and hence the necessity of spreading production over time, enable the monopolist to commit credibly to constrain the rate of supply offered in the near future.”. It can be shown in the following example of two type buyers which shows that social welfare and consumer surplus decreases.

Example. Suppose that the set of buyer's value V is $\{1, h+1\}$ and $Pr(v=1)=p$ and $Pr(v=h+1)=1-p$. Also, suppose that the size of sales tax is t and the production unit cost is c. We assume that $c<1-t$ to guarantee that even after the introduction of the sales tax the seller sells the product to the buyer whose value is 1. Considering the uniqueness of equilibrium path, it is okay to focus on finding the equilibrium path. At the last stage, the seller would propose 1 to clear the market. If the seller wants to charge the price higher than 1 to the buyer with value $h+1$, he should propose $1+h(1-\delta)$ for the second last period such that high type buyer is indifferent between buying at the price $1+h(1-\delta)$ and waiting for 1 at the nex period. Also, seller who wants to sell during more than two periods at least prefers to sell during the two periods than one priod. This is because according to Gul, Sonnenschein and Wilson(1986) or Fudenberg, Levine and Tirole(1985), the seller uses successive price skimming equilibrium strategy, which implies that the seller's posterior about the buyer's valuation is the prior truncated at some value. If the seller who knows the prior probability for the type of buyer thinks that price discrimination for more than two periods is optimal, the seller at a certain period with a certain posterior belief would optimally choose to sell during precisely two periods. Considering the fact that the seller initially can also choose the same strategy as he does when facing with the posterior, it is initially better for the seller to do price discrimination during two periods than sell at the price 1, the buyer's minimum valuation. Therefore, when there is no tax, the monopolist's equilibrium strategy is to sell the product during

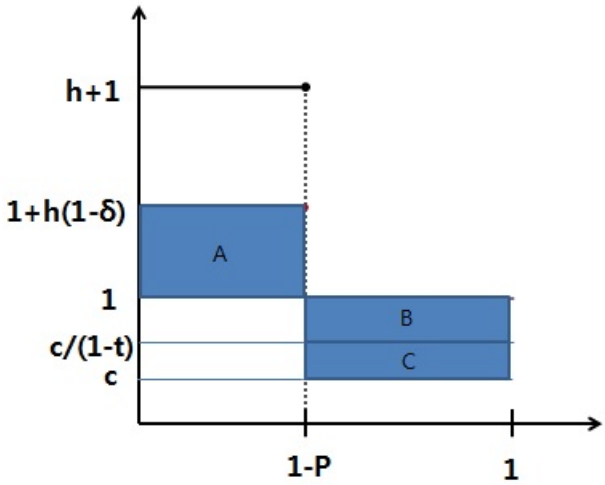

Figure 1:

more than the two stages if and only if

$$(1 + h(1 - \delta) - c)(1 - p) + \delta(1 - c)p \geq 1 - c$$

$$\Leftrightarrow p \geq \frac{h}{h + 1 - c}$$

The price at the first period $1 + h(1 - \delta)$ is the price which is indifferent from price 1 at the next period for the buyer with value $(1 + h)$. Similarly, when there is sales tax τ, the monopolist at least prefers to sell the product during more than the two stages if and only if

$$((1 - t)(1 + h(1 - \delta)) - c)(1 - p) + \delta(1 - t - c)p \geq (1 - t - c)$$

$$\Leftrightarrow p \geq \frac{h(1 - t)}{(h + 1)(1 - t) - c} > \frac{h}{h + 1 - c}$$

The two conditions can be explained by Figure 1. As explained above, for monopolist, the introduction of sales tax t is equivalent to the increase in cost from c to $\frac{c}{1-t}$. The area of A is the benefit of seller from proposing higher price to high type buyer at the first period which remains unchanged regardless of the size of cost. the areas of B and B+C are the costs from delaying the sales for low type buyer respectively when the cost is $\frac{c}{1-t}$

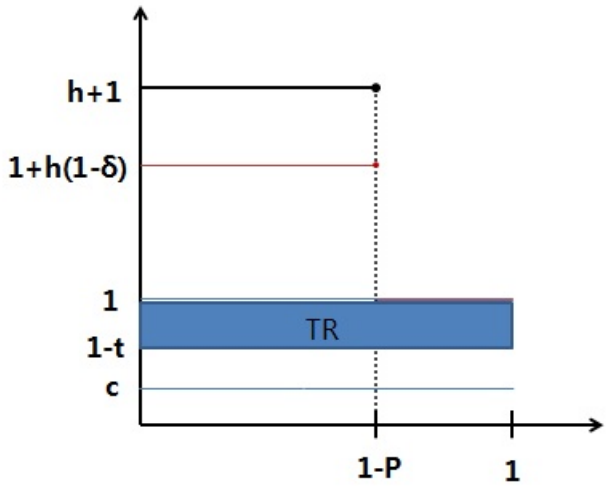

Figure 2:

and when it is c. Therefore, the lower bound of p which makes monopolist prefer to sell during at least more than two periods is bigger when there is no tax.

If $p > \frac{h(1-t)}{(h+1)(1-t)-c}$, the seller proposes 1 at the first period for both cases. Accordingly, the expected social welfare and the consumer surplus remains unchanged but the monopolist's expected profit decreases by expected tax revenue after the introduction of sales tax. It is shown in Figure 2.

If $\frac{h}{h+1-c} < p < \frac{h(1-t)}{(h+1)(1-t)-c}$, then in the case of sales tax t, the monopolist would charge more than $1 + h(1-\delta)$ to high type buyer during finite periods and 1 subsequently. In contrast, without sales tax, he charges 1 at the first period. The figure behind shows the case when the length of bargaining period is exactly 2. After the imposition of sales tax, social welfare decreases because of the longer bargaining period. For the high type buyer, his expected utility decreases because he accepts price higher than 1 which becomes much greater as h grows. However, for the low type buyer, although his utility decreases from the delayed purchasal, he still accpets the same price. On the other hand, the monopolist bears all the tax burden from the trade with low type buyer, while he would pass his burden of tax to high type buyer. If the pre tax price $(1-t)(1+h(1-\delta))$ is greater than 1, overshifting occurs. It is shown in Figure 3 and Figure 4.

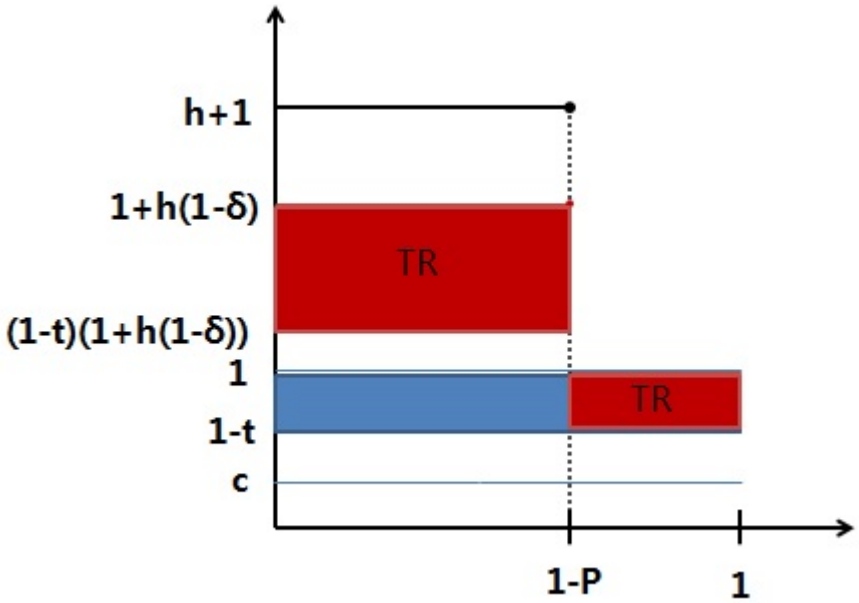

Figure 3:

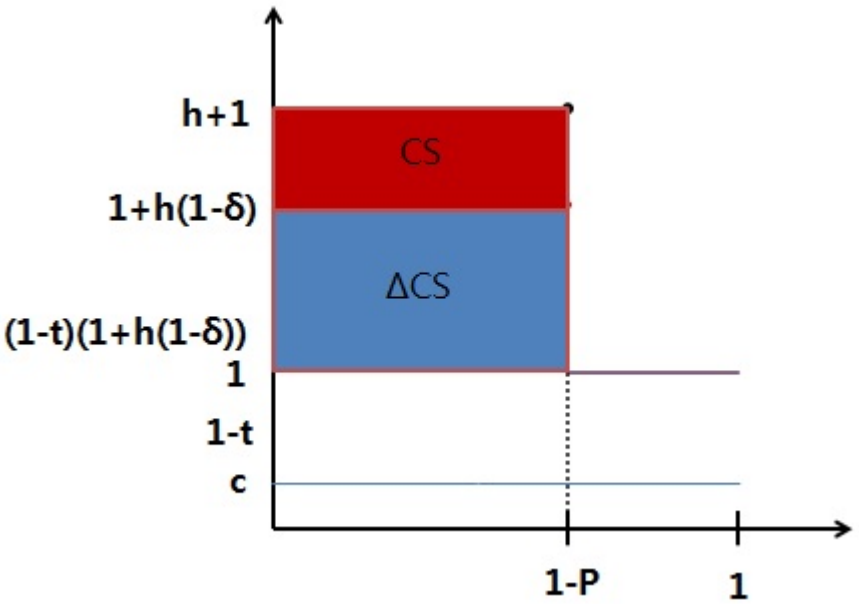

Figure 4:

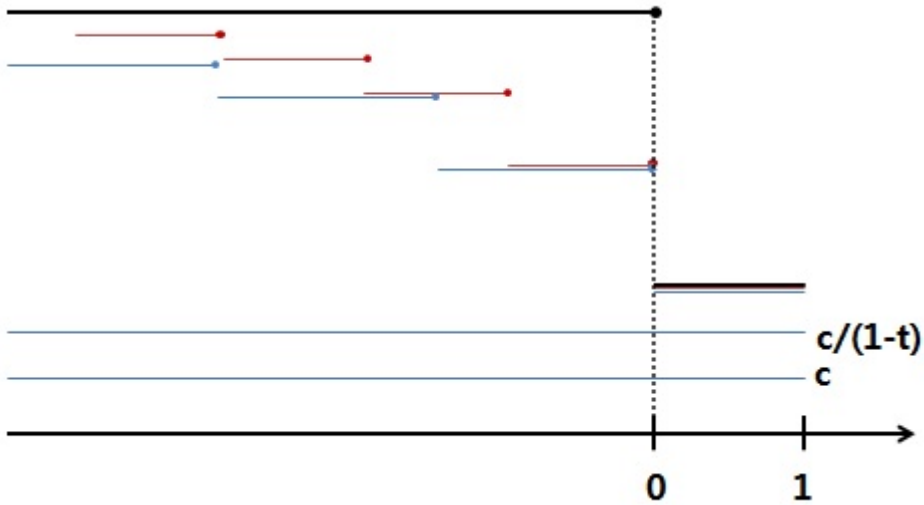

Figure 5:

The intuition is as follows. As seen above, after the introduction of sales tax t, the monopolist behaves as if the production cost increases from c to $\frac{c}{1-t}$. Therefore, it is worse for him to sell the product earlier due to discounting factor δ, and thus, buyer believes that the seller would spend more time to do bargaining. Therefore, seller can charge higher price to high type buyer and pass his burden to them.

Now, let's prove that social welfare and consumer surplus decrease or remain equal after the imposition of sales tax for every $0 \leq p \leq 1$. Before proving it, I want to point out some characteristics of reservation price strategy of buyer. First, when we express reservation price for each type of buyer on coordinate plane whose x axis is his type and y axis is his reservation price, it would be a step function. This is because we address the case with discrete type buyer. Next, for every $n \geq 1$, the gap between nth step and $(n+1)$th step would be same regardless the size of cost. This is because the height of $(n+1)$th step should be chosen such that high type buyer feel indifferent between accpeting it now and waiting for the next period price equal to the height of nth step. Figure 5 indicates these facts. To illustrate the proof more easily, the measure of low type buyer is fixed as 1 and that of high type buyer is $\frac{1-p}{p}$ in the figure. Remembering them,

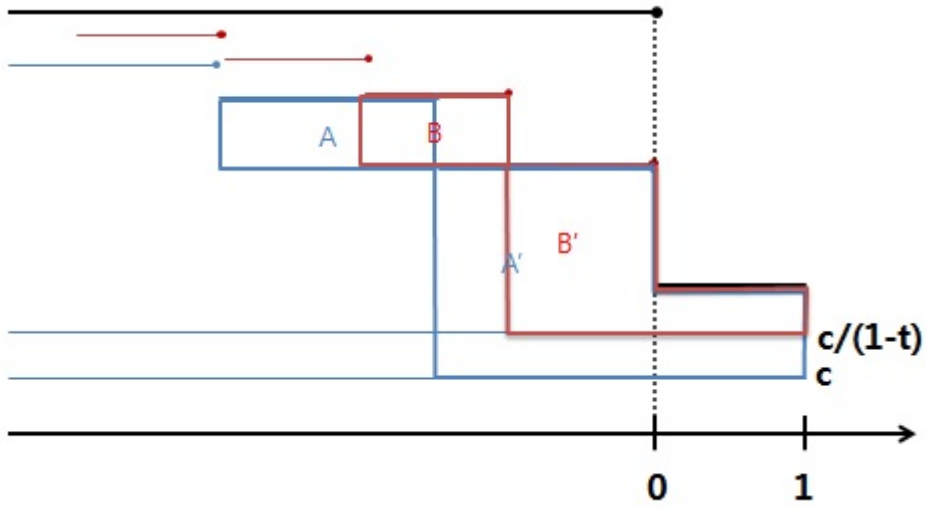

Figure 6:

we prove the argument by using induction for the width of nth step.

First, consider the width of second step. We can find out the logic for calculating the width of each step in figure 1. The width of second step when there is no tax or only profit tax is as $\frac{1}{p}$ times as p which satisfies the equality $A = (1-\delta)(B+C)$. This is because the critical point between the second step and the third step is chosen as the measure of high type buyer which allows the benefit from lengthening bargaining period from 1 to 2 to be equal to the cost from it. Similarly, we can get the width of second step when the size of tax is t from finding out the size of p satisfying the equality $A = (1-\delta)B$. Therefore, we can prove the argument when $n = 2$.

Now, Suppose that the width of nth step is greater when there is no tax for all $n \leq k$. Let's prove that it is also true when $n = k+1$ by using the logic above. From the inductive hypothesis and the fact that the gap between successive steps is independent on unit cost, the cost from extending the bargaining period by 1 is bigger when there is no tax. Therefore, after the introduction of tax, the width of $(k+1)$th step should be smaller to make the benefit and the cost equal. The figure 6 demonstrates this reasoning.

4 Conclusion and Future Study

In this paper, I analyze tax incidence in dynamic monopoly model, comparing the sets of PBE with the different kinds of taxes and finding out relations between them. Consequently, the paper shows that tax incidence principle still holds in this dynamic model with imperfect competition and incomplete information. For a future study, I wish to find out the condition of prior probability for the type of consumers in which price tax is worse than profit tax in terms of social welfare and consumer surplus. It is also possible that I would find a counterexample in which the introduction of price tax actually helps to increase the expected social weflare. Then, it would be surprising and interesting because it means that negative deadweight loss occurs as a result of the imposition of price tax. Furthermore, imposing a price tax is an easier way for the government to acquire tax revenue because of monitoring for the cost structure of firms to levy profit tax. Therefore, for the condition when price tax is worse than profit tax in terms of social welfare, I try to find out the rate of increase in price tax which has the same expected social welfare and tax revenue as a constant profit tax does.

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국문초록

조세귀착원리에 따르면 실질적인 세금 분담 비율은 법적으로 누가 세금을 부담하는지 와는 무관하다. 본 논문에서는 소비자의 지불용의를 정확히 모르는 독점기업이 순차적으로 가격을 제시하여 교섭하는 동태적 독점 상황에서, 다양한 조세 정책에 따른 실질적 조세 분담 비율을 비교해보고 조세귀착원리가 여전히 성립함을 보였다. 또한 가격세와 수익세 중 어떤 세금이 사회후생적으로 더 이로운지 논의해보고, 적어도 소비자의 예상되는 지불용의가 두 가지 뿐인 특수한 경우에는 수익세가 사회적으로 더 좋음을 보였다.

주요어 : 동태적 독점; 순차적 교섭; 조세 귀착; 완전 베이지안 균형; 가격세; 수익세.
학 번 : 2014-20213

Abstract

Tax Incidence in Dynamic Monopoly

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This paper compares the equilibria in dynamic monopoly model under diferent kinds and rates of taxes. We show that if profit tax is imposed, a monopolist bears all the burden of tax regardless of the rates of tax. In contrast, price tax might be shared by both the buyer and the seller and the ratio of tax burden between them depends on the type of buyer. We also show that tax incidece principle still holds which means that price and expected utilities for buyer and seller in the equilibrium do not depend on whether price tax is levied on consumer or seller. Although it is hard to generally verify the superiority of profit tax over price tax in terms of social welfare and consumer surplus, we prove it in the case when there are only two types of buyers regarding their willingness to pay for a product.

Keywords : dynamic monopoly, sequential bargaining, tax incidence, perfect bayesian equilibrium, profit tax, price tax

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1 Introduction

In economics, tax incidence is the analysis of the effects of tax policies on prices and the distribution of economic welfare. The key concept of tax incidence is that the statutory incidence is irrelevant to the economic incidence, so called tax incidence principle. For example, in competitive market, tax incidence is not dependent on where the revenue is collected, but on the price elasticity of demand and supply. Incidence analyses abound but the strategic aspect of economic interaction is mostly constrained. In most of them, the agents are supposed to be price takers, and thus, they behave as a solitary decision maker under the discipline of market. This is contrast with bargaining model in which market plays no role except for setting the stage for discussion. Although there are some papers analyzing the incidence of taxes under imperfect competition in static model, it is not sufficient for investigating the effects of tax on the behaviors of players strategically correlated with each other because sellers can only propose one shot take-it-or-leave-it offer. In this paper, I analyze tax incidence in sequential bargaining model in which the final outcome is determined entirely by the strategic interaction of the bargainers.

To show our results, we use sequential bargaining model of Fudenberg, Levine and Tirole[1]. It investigates a situation in which a seller with a constant production cost for a good and a buyer whose valuation for the product is private information bargain over the trade of one unit of the good. It is identical to Dynamic monopoly model without commitment for future prices for a product in Gul, Sonnenschein and Wilson[4]. They explain several structural properties of equilibrium such that buyers use reservation price strategies and conclude that the equilibrium path is generically unique if the lowest valuation buyer has is greater than the unit production cost. In our paper, this uniqueness is critical in that it helps us to directly compare tax incidence under different kinds and rates of taxes.

The main purpose of this paper is to study the incidence of taxes under various tax policies. We show that if profit tax is imposed, a monopolist bears all the burden of tax regardless of the rates of tax. In contrast, price tax might be shared by both the buyer and the seller and the ratio of tax burden between them depends on the type of buyer. We also show that tax incidece principle still holds which means that price and expected utilities for buyer and seller in the equilibrium do not depend on whether price tax is levied on consumer or seller. Although it is hard to generally verify the superiority of profit tax over price tax in terms of social welfare and consumer surplus, we prove it in the case when there are only two types of buyers regarding their willingness to pay for a product.

The paper is organized as follows: Section 2 describes the model which is acquired by applying the sequential bargaining model to the situation with the existence of tax. In section 3, I explain the relationships between the equilibrium sets under different rates and kinds of tax and show that tax incidence principle still holds. Also, I end the section by proving that the imposition of price tax reduces social welfare and consumer surplus more than that of profit tax at least in the case for two type buyers .

2 Model

To acquire results in this paper, we can use two different but essentially same models. The first one is sequential bargaining model from Fudenberg, Levine and Tirole(1985). A monopoly seller sells a durable good to a single buyer in periods $t \in \{1, 2, \cdots\}$. The seller's constant unit cost for the good is c and is commonly known. The buyer's value for the good $v \in V \subset [\underline{v}, \overline{v}]$ is private information which is randomly drawn from a commonly known cumulative distribution function $F(\cdot)$. Either $F(\underline{v}) > 0$ or F admits a strictly positive and continuous density at $\underline{v}$. If the buyer's lowest valuation is strictly greater

than unit cost,$\underline{v} > c$, we call the case the gap case and if not, $\underline{v} \leq c$, the case would be called the no-gap case. The seller and the buyer have the same discount factor δ which is positive and less than one.

At the start of any period, the seller specifies a taxed price $p_t \geq 0$ and the buyer decides whether to accept or reject the price. All actions are publicly observable. The game continues only if the buyer rejects the price. If the buyer never accepts an offered price, both the buyer's and the seller's utility are zero. A history is any finite sequence of the price decisions of the seller, and all the prices in the sequence are taxed prices. Seller's strategy is a function of any history to the next price if consumer still remains unaccepted. Buyer's strategy is a function of any history to his decision between accept or reject for the last proposed price under the assumption that he has not accpeted other prior proposed prices. We use the concept of *Perfect Bayesian Equilibrium*(PBE) which is a history-contingent sequence of the seller's offers p_t, the buyer's acceptance decisions, and updated beliefs about the buyer's value v, satisfying both that each player's actions are optimal given beliefs and the other's strategy and that beliefs are computed by given strategy profiles and Bayes' rule whenever possible, including off the equilibrium path.

The second model is dynamic monopoly model from Gul, Sonnenschein and Wilson(1986). The difference from the first model is that there is a unit measure of consumers and the concept of *Subgame Perfect Equilibrium*(SPE) is therefore used. To make two models equivalent, we use the regularity assumption "The equilibrium actions of each agent are constant on histories in which prices are the same and the sets of agents accepting at each point in time differ at most by sets of measure zero.". Under this assumption, this dynamic monopoly model is formally identical to the sequential bargaining model. Therefore, the results in our paper can be intrepreted in both models. However, the results in the case of positive production unit cost are better interpreted in the second model because they are closely related with the production cost for each period.

We investigate the effects of three kinds of taxes on society. Profit tax is a tax levied on the net profit of monopolist. Consumption tax is a price tax imposed on consumption and sales tax is a price tax levied on monopolist's revenue. In other words, profit tax and price tax can be imposed on the monopolist while only price tax can be levied on the consumer. All theses taxes are assumed to be ad-valorem taxes. Suppose that the consumer with value v buys the product at price p_t at the tth period. If profit tax τ is levied on the monopolist, his profit is $\delta^{t-1}(1-\tau)(p-c)$ and consumer's utility is $\delta^{t-1}(v-p)$. If price tax τ is imposed on the monopolist, his profit is $\delta^{t-1}((1-\tau)p-c)$ and consumer's utility is $\delta^{t-1}(v-p)$. If price tax τ is imposed on the consumer and p is taxed price, the monopolist's profit is $\delta^{t-1}(\frac{p}{1+\tau}-c)$ and consumer's utility is $\delta^{t-1}(v-p)$.

3 Results

In the following two subsections, we explain isomorphisms between the equilibrium sets under the different kinds of taxes. In mathematics, an isomorphism is a mathematical mapping that preserves the structure of the mapped entities, which is a stronger concept than bijection. By defining history as a sequence of proposed taxed prices, we impose each player' s strategy set to be equivalent under any kind of tax. We would define that the two equilibrium sets are isomorphic if they have precisely the same elements in terms of each player' s strategy and belief system implying that they have identical equilibrium path. In addition, the isomorphic sets are defined to be essentially same if the same element has the same relationship between taxed price and pre-tax price, which means that they bring about same expected utility for each player and identical expected tax revenue.

To prove the isomorphisms, we use the following logic. Suppose that there are two players and the strategy set for each player is respectively S_1 and S_2. If two pairs of

utility function for each player (U_1, U_2) and (U_1', U_2') ,$U_1, U_2, U_1', U_2' : S_1 \times S_2 \to R^+$satisfy the following conditions, the Nash equilibrium sets for two pairs of utility functions are isomorphic.

$$U_i(s_1, s_2) \geq (>) U_i(s_1', s_2) \Leftrightarrow U_i'(s_1, s_2) \geq (>) U_i'(s_1', s_2)$$

and

$$U_i(s_1, s_2) \geq (>) U_i(s_1, s_2') \Leftrightarrow U_i'(s_1, s_2) \geq (>) U_i'(s_1, s_2')$$

for all $i = 1, 2,\ s_1, s_1' \in S_1$ and $s_2, s_2' \in S_2$. This is because if the preference relation is preserved, the best response is preserved given the other player's strategy. This logic can be generalized for players more than two players and be applied to different concepts for equilibrium such as Perfect Bayesian equilibrium.

3.1 No Production Cost

The case in which the product has already been produced can be interpreted to have zero production cost. If $c = 0$, for the monopolist, the price tax τ is same as the profit tax τ and thus, the equilibrium set would be same for both taxes. Therefore, we only need to focus on the price tax. Let's first consider a consumption tax, the price tax levied on the consumer. The next lemma shows that after the imposition of the consumption tax, the seller bears all the burden of the tax in the equilibrium.

Lemma 3.1. *For any positive τ, the set of PBE when consumption tax is τ is isomorphic to the set of PBE when there is no consumption tax.*

Proof. Fix a PBE when consumption tax is τ $(s_B^\tau, s_S^\tau, \mu^\tau)$. s_B^τ is the equilibrium strategy of a buyer which assigns a history to a decision of whether to accept or reject. s_S^τ is a seller's equilibrium strategy assigning a history to a taxed price in the next period. $\mu^\tau$is a belief system satisfying Bayesian consistency with those strategies. Let's show that $(s_B^\tau, s_S^\tau, \mu^\tau)$ is a PBE when $\tau = 0$. First, after any history, the sequence of future proposed

taxed prices in both cases are equivalent due to the same seller's strategy. Therefore, it is evident that s_B^τ is still the buyer's optimal strategy for the seller's strategy s_S^τ when $\tau = 0$, and thus, μ^τ satisfies the Bayesian consistency. Now, it remains to prove that at each information set, s_S^τ specifies the seller's best response to s_B^τ when $\tau = 0$. Suppose that there is an information set h such that there exists another strategy $\tilde{s_S^0}$which gives more expected utility to the seller than s_S^τ conditional on arriving h. Then, when there exists an consumption tax τ, a strategy $\tilde{s_S^0}$ is better thans_S^τ for the seller conditional on ariving h . To prove that, let $u_S^{c,t}(s_B, s_S, \mu|h)$ be the seller's expected utility conditional on arriving h when a buyer and a seller's strategy is respectively $s_B$and s_S, the belief system is μ,and the size of consumption tax is t. When the seller's strategies are identical facing the two tax schedule, the buyer with a strategy s_B^τ responses in the same way, that is, with the same probability the buyer accepts the proposed price at each period after the history h. So, $u_S^{c,\tau}(s_B^\tau, \tilde{s_S^0}, \mu^\tau|h) = \frac{1}{1+\tau} u_S^{c,0}(s_B^\tau, \tilde{s_S^0}, \mu^\tau|h)$ and for the same reason, $u_S^{c,\tau}(s_B^\tau, s_S^\tau, \mu^\tau|h) = \frac{1}{1+\tau} u_S^{c,0}(s_B^\tau, s_S^\tau, \mu^\tau|h)$. However, it means that $u_S^{c,\tau}(s_B^\tau, \tilde{s_S^0}, \mu^\tau|h) = \frac{1}{1+\tau} u_S^{c,0}(s_B^\tau, \tilde{s_S^0}, \mu^\tau|h) > \frac{1}{1+\tau} u_S^{c,0}(s_B^\tau, s_S^\tau, \mu^\tau|h) = u_S^{c,\tau}(s_B^\tau, s_S^\tau, \mu^\tau|h)$ which contradicts the fact that $(s_B^\tau, s_S^\tau, \mu^\tau)$ is a PBE. For the opposite direction, fix a PBE when no consumption tax (s_B^0, s_S^0, μ^0) and let's prove that the PBE is also a PBE when the size of consumption tax is τ. For the same reason from the above, the optimality of the buyer's strategy is evident. To prove the optimality of the seller's strategy, as in the above case, suppose that there is an information set h such that there exists a better strategy $\tilde{s_S^\tau}$ for seller i.e. $u_S^{c,\tau}(s_B^0, \tilde{s_S^\tau}, \mu^0|h) > u_S^{c,\tau}(s_B^0, s_S^0, \mu^0|h)$. Then, we can also get contradiction from $u_S^{c,0}(s_B^0, \tilde{s_S^\tau}, \mu^0|h) = (1+\tau) u_S^{c,\tau}(s_B^0, \tilde{s_S^\tau}, \mu^0|h) > (1+\tau) u_S^{c,\tau}(s_B^0, s_S^0, \mu^0|h) = u_S^{c,0}(s_B^0, s_S^0, \mu^0|h)$ and the fact that (s_B^0, s_S^0, μ^0) is a PBE when there is no consumption tax. Therefore, we can prove the isomorphism between the set of PBE when consumption tax is τ and that when there is no consumption tax. $\square$

Now, instead of the consumption tax, consider the sales tax, the price tax levied on seller. With the same logic, we can easily prove similar isomorphism between the different sales taxes.

Lemma 3.2. *For any positive t less than 1, the set of PBE when sales tax is t is isomorphic to the set of PBE when there is no sales tax.*

Proof. The proof would be same as Lemma 3.1. considering $u_S^{s,t}(s_B, s_S, \mu|h)$, the seller's expected utility conditional on arriving h when a buyer and a seller's strategy is respectively $s_B$and s_S, the belief system is μ,and the size of sales tax is t, instead of $u_S^{c,t}(s_B, s_S, \mu|h)$ and the relation that $u_S^{s,\tau}(s_B, s_S, \mu|h) = (1-\tau)u_S^{s,0}(s_B, s_S, \mu|h)$ □

The two Lemmas show that in the either case that the consumer starts to pay the consumption tax or the case that the monopolist starts to pay the sales tax, the sequence of taxed price and of the probabilities that the monopolist accept at each period on the equilibrium path remain unchanged compared with the case of no tax. Actually, the case with no consumption tax is not different from no-sales-tax case. Therefore, we can get the following proposition from the above two lemmas.

Proposition 3.3. *For any $\tau \geq 0$ and $0 \leq t < 1$, the set of PBE when consumption tax is τ is isomorphic to the set of PBE when sales tax is t. Especially, if $t = \tau/(1+\tau)$, the two sets are essentially same.*

Proof. We can get the first argument directly from the above two lemmas. For the second argument, consider a PBE when the consumption tax is τ and the corresponding PBE when the sales tax is t which has the same pair of equilibirum strategies and belief system. This implies that the two PBE have the same equilibrium path in terms of the sequence of proposed taxed prices and the buyer's acceptance probability. On the other hand, if the taxed price is p, the pre-tax price is $\frac{p}{1+\tau}$ when the consumption tax is τ and is $(1-t)\cdot p$

when the sales tax is t. So, under the same taxed price, the consumption tax τ and the sales tax $\tau/(1+\tau)$ give the same pre-tax price and the same gap between taxed price and pre-tax price. Therefore, not only the expected utility for all types of buyer but also the expected utility of seller and the expected tax revenue are identical because they are all function of the sequence of taxed prices, pre-tax prices and buyer's accpetance probability on the equilibrium path. The opposite direction is also correct with the same logic. This finishes our proof. □

3.2 Positive Production Cost

When the production unit cost is positive, profit tax is different from price tax for the monopolist, and thus, the set of PBE would be different for two kinds of taxes. First, let's think about profit tax. In this case, we can also get similar isomorphism result as above.

Lemma 3.4. *For any positive τ, the set of PBE when profit tax is τ is isomorphic to the set of PBE when there is no profit tax.*

Proof. The proof would be same as Lemma 3.1. considering $u_S^{p,t}(s_B, s_S, \mu|h)$, the seller's expected utility conditional on arriving h when a buyer and a seller's strategy is respectively $s_B$and s_S, the belief system is μ,and the size of profit tax is t, instead of $u_S^{c,t}(s_B, s_S, \mu|h)$ and the relation that $u_S^{p,\tau}(s_B, s_S, \mu|h) = (1-\tau)u_S^{p,0}(s_B, s_S, \mu|h)$. □

This lemma implies that the profit tax burden of monopolist is not shared with buyer. In other words, the tax revenue is collected only at the expense of the seller's welfare while all types of the buyer's expected utility remain unchanged. The intuition is that because the imposition of profit tax does not affect the superiority among seller's different strategies if the buyer's strategy is same, the set of PBE remains unchanged. In other words, the change in profit tax is identical for monopolist to that in exchange rate, which does not give any effect on the seller's best response to the buyer's strategy.

Now, let's focus on price tax. We can get similar isomorphism between the set of PBE with the existence of tax and the equilibrium set without tax but the production cost should be adjusted.

Lemma 3.5. *For any positive τ, the set of PBE when the production cost is c and consumption tax is τ is isomorphic to the set of PBE when the production cost is $(1+\tau)\cdot c$ and there is no consumption tax.*

Proof. Fix a PBE $(s_B^{\tau,c}, s_S^{\tau,c}, \mu^{\tau,c})$ when the production cost is c and consumption tax is τ. Let's prove that it is also a PBE when the production cost is $(1+\tau)\cdot c$ and consumption tax is zero. First, because the buyer and seller's strategies are same, it is evident that the consistency of $\mu^{\tau,c}$ holds. Now, let's prove the optimality of the buyer's strategy. After any history the seller's same strategy would give the same sequence of proposed taxed price in the future and the buyer's utility is a function of his value, the accepted taxed price and the timing of the acceptance. Therefore, the optimality of $s_B^{\tau,c}$ in the case when the production cost is c and consumption tax is τ guarantees its optimality in the case when the production cost is $(1+\tau)\cdot c$ with no consumption cost. The only thing to remain to prove is the optimality of the seller's strategy. Suppose not. Then, there exists an information set h such that there is another strategy $s_S^{0,\widetilde{(1+\tau)c}}$ which gives more expected utility to the seller than $s_S^{\tau,c}$ conditional on arriving h. If this is true, when the production cost is c and consumption tax is τ, a strategy $s_S^{0,\widetilde{(1+\tau)c}}$ is better than $s_S^{\tau,c}$ for the seller conditional on ariving h . To prove that, let $u_S^{c,\tau,c}(s_B, s_S, \mu|h)$ be the seller's expected utility conditional on arriving h, given the strategy profile (s_B, s_S), the belief system μ,the consumption tax with size τ and the production cost c. Because the seller's strategies are the same for both taxes, the buyer with a strategy $s_B^{\tau,c}$ responses in the same way, that is, with the same probability the buyer accepts the proposed price at each period after the history h. So, considering the formula $(\frac{p}{1+\tau} - c)\cdot q = \frac{1}{1+\tau}(p-(1+\tau)c)\cdot q$, we can get $u_S^{c,\tau,c}(s_B^{\tau,c}, s_S^{0,\widetilde{(1+\tau)c}}, \mu^{\tau}|h) =$

$\frac{1}{1+\tau}u_S^{c,0,(1+\tau)c}(s_B^{\tau,c}, s_S^{0,\widetilde{(1+\tau)c}}, \mu^\tau|h)$ and $u_S^{c,\tau,c}(s_B^{\tau,c}, s_S^{\tau,c}, \mu^\tau|h) = \frac{1}{1+\tau}u_S^{c,0,(1+\tau)c}(s_B^{\tau,c}, s_S^{\tau,c}, \mu^\tau|h)$. However, it means that $u_S^{c,\tau,c}(s_B^{\tau,c}, s_S^{0,\widetilde{(1+\tau)c}}, \mu^\tau|h) = \frac{1}{1+\tau}u_S^{c,0,(1+\tau)c}(s_B^{\tau,c}, s_S^{0,\widetilde{(1+\tau)c}}, \mu^\tau|h) > \frac{1}{1+\tau}u_S^{c,0,(1+\tau)c}(s_B^{\tau,c}, s_S^{\tau,c}, \mu^\tau|h) = u_S^{c,\tau,c}(s_B^{\tau,c}, s_S^{\tau,c}, \mu^\tau|h)$. This is a contradiction because $(s_B^{\tau,c}, s_S^{\tau,c}, \mu^{\tau,c})$ is a PBE. By using the same logic, the opposite direction also can be proved. □

We can also get a similar isomorphism for the sales tax.

Lemma 3.6. *For any positive t less than 1, the set of PBE when sales tax is t and the production cost is c is isomorphic to the set of PBE when there is no sales tax and the production cost is $\frac{c}{1-t}$.*

Proof. The proof would be same as Lemma 3.5. Considering $u_S^{s,t,c}(s_B, s_S, \mu|h)$, the seller's expected utility conditional on arriving h , given the strategy profile (s_B, s_S), the belief system μ,the sales tax with size t and the production cost c, instead of $u_S^{c,t,c}(s_B, s_S, \mu|h)$ and the relation that $u_S^{s,t,c}(s_B, s_S, \mu|h) = (1-t)u_S^{s,0,\frac{c}{1-t}}(s_B, s_S, \mu|h)$. □

The intuition for the two above lemmas is as follows. While profit tax has the same effects both on revenue and cost for monopolists, consumption tax or sales tax affects only the revenue. If the proposed price and quantity sold at each period are the same, the revenue when the consumption tax is τ would be $\frac{1}{1+\tau}$ times as the revenue when the tax is zero, but the cost would be alike. This means that consumption tax τ has the same effect on monopolist as the simultaneous increase of production cost and the profit tax, respectively, from c to $(1+\tau)c$ and from zero to $\frac{\tau}{1+\tau}$. As seen above, profit tax does not affect the seller's optimal behavior, and thus, the set of PBE remains unchanged. This is why the unit cost should be changed to get the above isomorphism results. From the two lemmas, we can generalize Proposition 3.3..

Proposition 3.7. *For any $\tau \geq 0$ and $0 \leq t < 1$, the set of PBE when consumption tax is τ and the production cost is c is isomorphic to the set of PBE when sales tax is t and the*

production cost is $(1-t)(1+\tau)c$*. Especially, if* $t = \tau/(1+\tau)$*, the two sets are essentially same.*

Proof. From Lemma 3.5., the set of PBE when the consumption tax is τ and the unit cost is c is isomorphic to the set of PBE when the unit cost is $(1+\tau)c$ and there is no production cost, which is also isomorphic to the set of PBE when the sales tax is t and the unit cost is $(1-t)(1+\tau)c$ from Lemma 3.6. Therefore, we can get the first argument. For the second argument, consider a PBE when the consumption tax is τ and the production cost is c and the corresponding PBE when the sales tax is t and the production cost is $(1-t)(1+\tau)c$ which has the same pair of equilibirum strategies and belief system. This implies that the two PBE have the same equilibrium path in terms of the sequence of proposed taxed price and the buyer's acceptance probability. On the other hand, if the taxed price is p, the pre-tax price is $\frac{p}{1+\tau}$ when the consumption tax is τ and is $(1-t)\cdot p$ when the sales tax is t. Hence, under the same taxed price, if $t = \frac{\tau}{1+\tau}$, the consumption tax τ and the sales tax t give the same pre-tax price and the gap between taxed price and pre-tax price. Therefore, in addition to $(1-t)(1+\tau)c = c$, the two PBEs have the same expected utility of seller and the same expected tax revenue as well as the identical expected utility for all types of buyer. The opposite direction is also correct with the same logic. □

3.3 Tax Incidence

In analyzing the effect of the introducing tax on welfare, we would use the following uniqueness result from Fudenberg, Levine and Tirole(1985).

Corollary. *(Fudenberg, Levine and Tirole 1985; Gul, Sonnenschein, and Wilson 1986) In the gap case, a Perfect Bayesian Equilibrium(PBE) exists and is generically unique. Also, in the equilibrium, if the price is lower than in the past, the buyer's behavior is*

independent of previous prices.("stationarity" or"the strong cutoff rule property")

By using the uniqueness result and Proposition 3.3., and 3.7., we can show that tax incidence principal still holds in dynamic monopoly if cost is sufficiently smaller than the minimum possible value that buyer has.

- When the cost is zero, the introduction of consumption tax τ is equivalent to that of sales tax $t = \tau/(1+\tau)$ in terms of the buyer's utility, the monopolist's profit, tax revenue, and social welfare in expected sense.

- When the cost is positive, the introduction of consumption tax τ is equivalent to that of sales tax $t = \tau/(1+\tau)$ in terms of the buyer's utility, the monopolists's profit, tax revenue, and social welfare in expected sense if $(1+\tau)c < \underline{v}$.

In addition, we can know the allocation of tax burden between buyer and seller when the unit cost is zero or when the cost is positive and only the profit tax is levied on monopolist. From Lemma 3.1., 3.2., and 3.4., the next two results mean that for both cases the entire tax burden remains on the seller's shoulder.

- When the cost is zero, the introduction of price tax or profit tax does not change expected consumer surplus and social welfare but decreases monopolist's expected profit in comparison with the case without tax

- When the cost is positive, the introduction of profit tax does not change expected consumer surplus and social welfare but decreases monopolist's expected profit in comparison with the case without tax.

On the other hand, different from profit tax, price tax might be shared by both the buyer and the seller, and the ratio of tax burden between them depends on the type of buyer. In static model, social welfare and consumer surplus decrease after the introduction of

price tax. However, it is hard to prove that such results hold for any prior probability of types of buyer. The main reason is that the set of time consistent paths that monopolist can choose might be changed after the introduction of a price tax in dynamic monopoly. The time consistent path is represented by a sequence $\{p_n, q_n\}_{n \in N}$, a sequence of pairs of the proposed price and buyer's unconditional accpetance probability at the nth period, satisfying that the buyer's accpetance probability at each period does not violate his optimality and that after mth period it is still optimal for seller to propose $\{p_n\}_{n \geq m+1}$afterwards. Actually, the only thing that the monopolist can choose is q_1 because $\{p_n, q_n\}_{n \geq 2}$ should be chosen from the equilibrium set when the seller's posterior after the first period is given as a new prior belief for the types of buyer and p_1 should be chosen to satisfy that buying the product at p_1 now and waiting for the next period and buying at p_2 are indifferent for the buyer with valuation $F^{-1}(1 - q_1)$. Then the seller's optimization problem

$$max \sum \delta^{i-1}((1-t)p_i - c)q_i$$

$$\Leftrightarrow max \sum \delta^{i-1}(p_i - \frac{c}{1-t})q_i$$

$$s.t. \ \{p_i, q_i\} : \ a \ time \ consistent \ path$$

means that as t grows, the seller behaves as if cost increases from c to $\frac{c}{1-t}$ and cares more about decreasing $c \cdot \sum \delta^{i-1} q_i$, the expeced discounted sum of cost. This implies that seller sells with lower probability at the earlier periods to reduce the expected cost, which could drag bargaining periods longer and induce higher proposed price at each period. This would result in the decrease in social welfare and consumer surplus. When it is possible for committment or bargaining persists during at most two periods in the equilibrium in both cases, the case without any tax and the case with a certain size of price tax, the set of time consistent paths remains unchanged after the introduction of price tax and we can use this logic. If the choice set changes, however, this logic cannot be directly applied.

Therefore, it is hard to affirm that price tax would reduce social welfare. However, the argument seems related to the following note in Gul, Sonnenschein and Wilson(1986),” Increasing costs, and hence the necessity of spreading production over time, enable the monopolist to commit credibly to constrain the rate of supply offered in the near future.”. It can be shown in the following example of two type buyers which shows that social welfare and consumer surplus decreases.

Example. Suppose that the set of buyer's value V is $\{1, h+1\}$ and $Pr(v=1)=p$ and $Pr(v=h+1)=1-p$. Also, suppose that the size of sales tax is t and the production unit cost is c. We assume that $c<1-t$ to guarantee that even after the introduction of the sales tax the seller sells the product to the buyer whose value is 1. Considering the uniqueness of equilibrium path, it is okay to focus on finding the equilibrium path. At the last stage, the seller would propose 1 to clear the market. If the seller wants to charge the price higher than 1 to the buyer with value $h+1$, he should propose $1+h(1-\delta)$ for the second last period such that high type buyer is indifferent between buying at the price $1+h(1-\delta)$ and waiting for 1 at the nex period. Also, seller who wants to sell during more than two periods at least prefers to sell during the two periods than one priod. This is because according to Gul, Sonnenschein and Wilson(1986) or Fudenberg, Levine and Tirole(1985), the seller uses successive price skimming equilibrium strategy, which implies that the seller's posterior about the buyer's valuation is the prior truncated at some value. If the seller who knows the prior probability for the type of buyer thinks that price discrimination for more than two periods is optimal, the seller at a certain period with a certain posterior belief would optimally choose to sell during precisely two periods. Considering the fact that the seller initially can also choose the same strategy as he does when facing with the posterior, it is initially better for the seller to do price discrimination during two periods than sell at the price 1, the buyer's minimum valuation. Therefore, when there is no tax, the monopolist's equilibrium strategy is to sell the product during

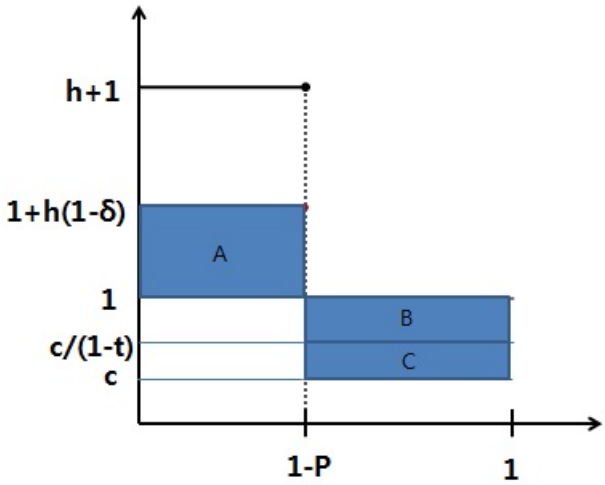

Figure 1:

more than the two stages if and only if

$$(1 + h(1 - \delta) - c)(1 - p) + \delta(1 - c)p \geq 1 - c$$

$$\Leftrightarrow p \geq \frac{h}{h + 1 - c}$$

The price at the first period $1 + h(1 - \delta)$ is the price which is indifferent from price 1 at the next period for the buyer with value $(1 + h)$. Similarly, when there is sales tax τ, the monopolist at least prefers to sell the product during more than the two stages if and only if

$$((1 - t)(1 + h(1 - \delta)) - c)(1 - p) + \delta(1 - t - c)p \geq (1 - t - c)$$

$$\Leftrightarrow p \geq \frac{h(1 - t)}{(h + 1)(1 - t) - c} > \frac{h}{h + 1 - c}$$

The two conditions can be explained by Figure 1. As explained above, for monopolist, the introduction of sales tax t is equivalent to the increase in cost from c to $\frac{c}{1-t}$. The area of A is the benefit of seller from proposing higher price to high type buyer at the first period which remains unchanged regardless of the size of cost. the areas of B and B+C are the costs from delaying the sales for low type buyer respectively when the cost is $\frac{c}{1-t}$

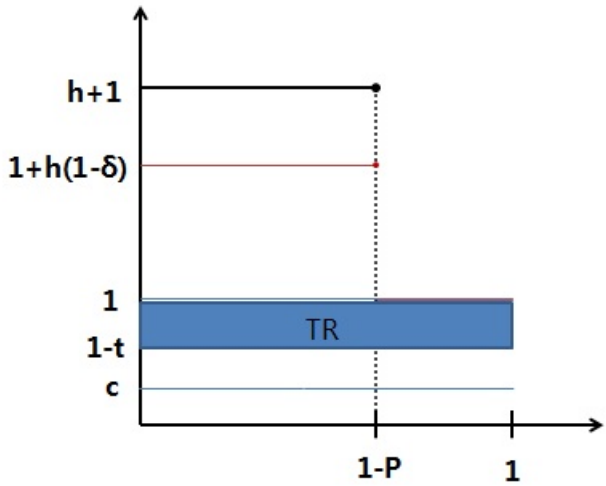

Figure 2:

and when it is c. Therefore, the lower bound of p which makes monopolist prefer to sell during at least more than two periods is bigger when there is no tax.

If $p > \frac{h(1-t)}{(h+1)(1-t)-c}$, the seller proposes 1 at the first period for both cases. Accordingly, the expected social welfare and the consumer surplus remains unchanged but the monopolist's expected profit decreases by expected tax revenue after the introduction of sales tax. It is shown in Figure 2.

If $\frac{h}{h+1-c} < p < \frac{h(1-t)}{(h+1)(1-t)-c}$, then in the case of sales tax t, the monopolist would charge more than $1 + h(1-\delta)$ to high type buyer during finite periods and 1 subsequently. In contrast, without sales tax, he charges 1 at the first period. The figure behind shows the case when the length of bargaining period is exactly 2. After the imposition of sales tax, social welfare decreases because of the longer bargaining period. For the high type buyer, his expected utility decreases because he accepts price higher than 1 which becomes much greater as h grows. However, for the low type buyer, although his utility decreases from the delayed purchasal, he still accpets the same price. On the other hand, the monopolist bears all the tax burden from the trade with low type buyer, while he would pass his burden of tax to high type buyer. If the pre tax price $(1-t)(1+h(1-\delta))$ is greater than 1, overshifting occurs. It is shown in Figure 3 and Figure 4.

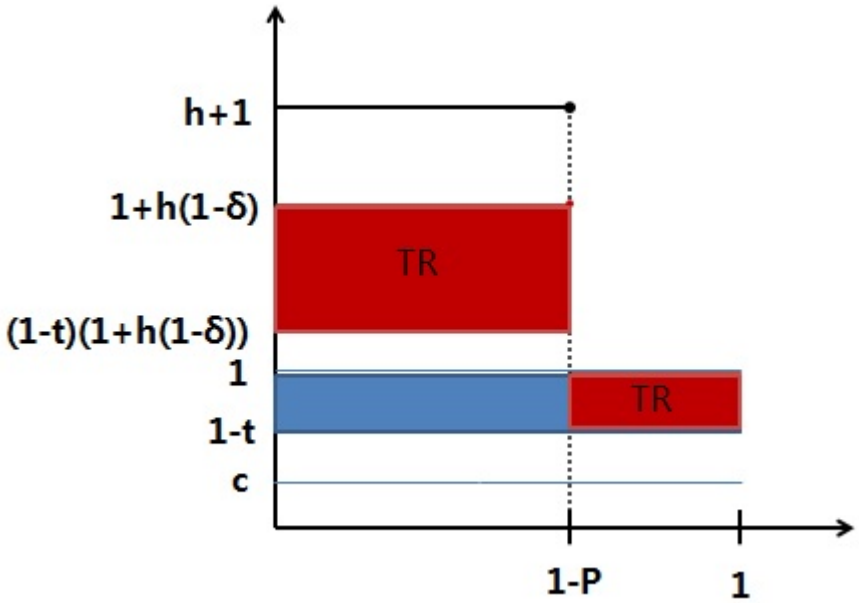

Figure 3:

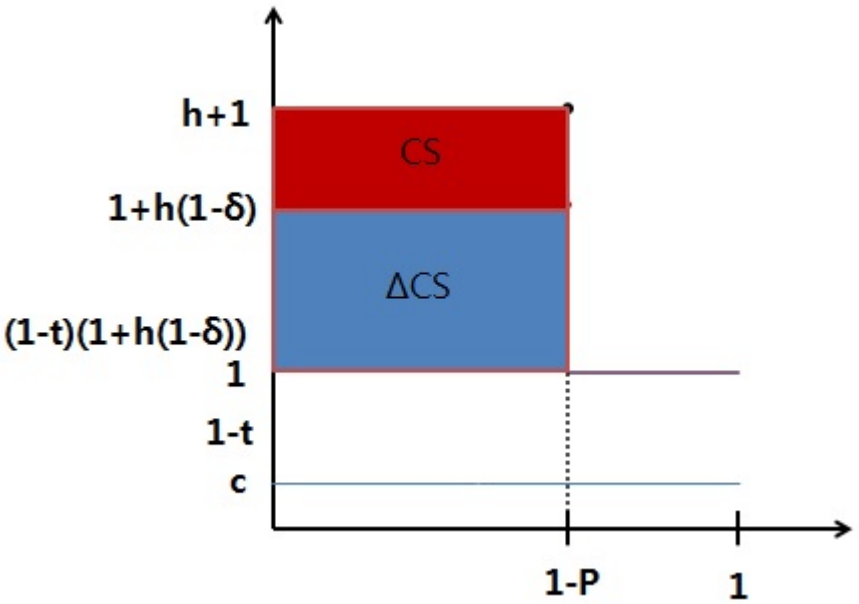

Figure 4:

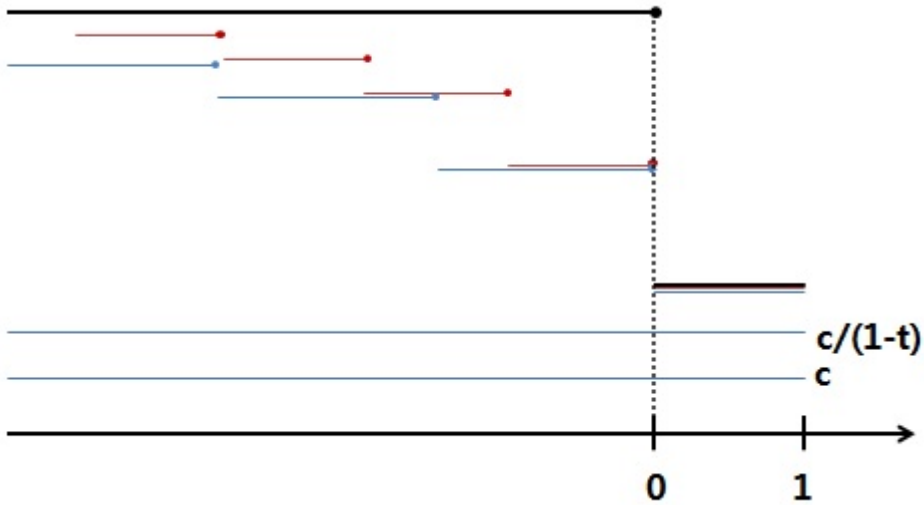

Figure 5:

The intuition is as follows. As seen above, after the introduction of sales tax t, the monopolist behaves as if the production cost increases from c to $\frac{c}{1-t}$. Therefore, it is worse for him to sell the product earlier due to discounting factor δ, and thus, buyer believes that the seller would spend more time to do bargaining. Therefore, seller can charge higher price to high type buyer and pass his burden to them.

Now, let's prove that social welfare and consumer surplus decrease or remain equal after the imposition of sales tax for every $0 \leq p \leq 1$. Before proving it, I want to point out some characteristics of reservation price strategy of buyer. First, when we express reservation price for each type of buyer on coordinate plane whose x axis is his type and y axis is his reservation price, it would be a step function. This is because we address the case with discrete type buyer. Next, for every $n \geq 1$, the gap between nth step and $(n+1)$th step would be same regardless the size of cost. This is because the height of $(n+1)$th step should be chosen such that high type buyer feel indifferent between accpeting it now and waiting for the next period price equal to the height of nth step. Figure 5 indicates these facts. To illustrate the proof more easily, the measure of low type buyer is fixed as 1 and that of high type buyer is $\frac{1-p}{p}$ in the figure. Remembering them,

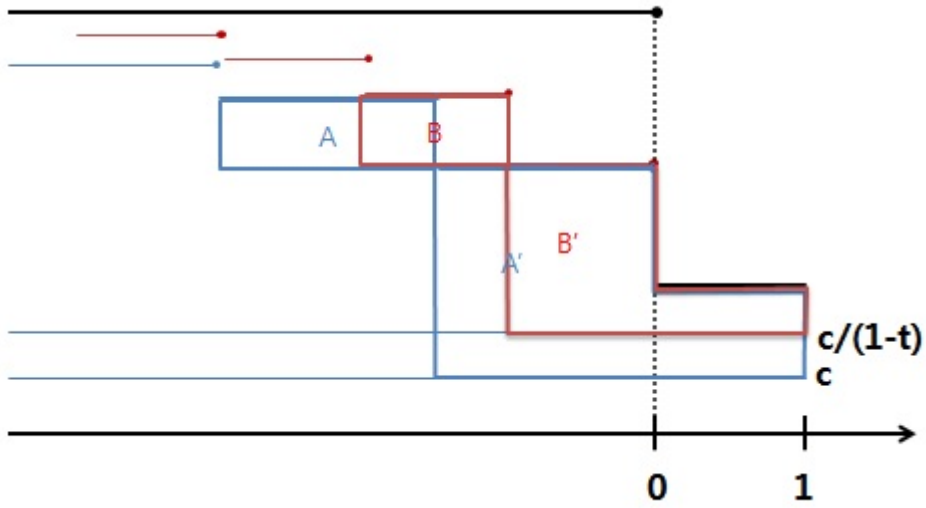

Figure 6:

we prove the argument by using induction for the width of nth step.

First, consider the width of second step. We can find out the logic for calculating the width of each step in figure 1. The width of second step when there is no tax or only profit tax is as $\frac{1}{p}$ times as p which satisfies the equality $A = (1-\delta)(B+C)$. This is because the critical point between the second step and the third step is chosen as the measure of high type buyer which allows the benefit from lengthening bargaining period from 1 to 2 to be equal to the cost from it. Similarly, we can get the width of second step when the size of tax is t from finding out the size of p satisfying the equality $A = (1-\delta)B$. Therefore, we can prove the argument when $n = 2$.

Now, Suppose that the width of nth step is greater when there is no tax for all $n \leq k$. Let's prove that it is also true when $n = k+1$ by using the logic above. From the inductive hypothesis and the fact that the gap between successive steps is independent on unit cost, the cost from extending the bargaining period by 1 is bigger when there is no tax. Therefore, after the introduction of tax, the width of $(k+1)$th step should be smaller to make the benefit and the cost equal. The figure 6 demonstrates this reasoning.

4 Conclusion and Future Study

In this paper, I analyze tax incidence in dynamic monopoly model, comparing the sets of PBE with the different kinds of taxes and finding out relations between them. Consequently, the paper shows that tax incidence principle still holds in this dynamic model with imperfect competition and incomplete information. For a future study, I wish to find out the condition of prior probability for the type of consumers in which price tax is worse than profit tax in terms of social welfare and consumer surplus. It is also possible that I would find a counterexample in which the introduction of price tax actually helps to increase the expected social weflare. Then, it would be surprising and interesting because it means that negative deadweight loss occurs as a result of the imposition of price tax. Furthermore, imposing a price tax is an easier way for the government to acquire tax revenue because of monitoring for the cost structure of firms to levy profit tax. Therefore, for the condition when price tax is worse than profit tax in terms of social welfare, I try to find out the rate of increase in price tax which has the same expected social welfare and tax revenue as a constant profit tax does.

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