Evidentiary Uncertainty and Negligence Rules

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This paper analyzes the effects of evidentiary uncertainty on people's incentives to abide by rules in the context of negligence rules in tort law. It demonstrates that i) the comparative negligence rule is not necessarily superior to the simple or the contributory negligence rule, ii) an application of lenient standards of due care under the contributory negligence rule than in comparative negligence rule would not necessarily restore social efficiency, and iii) a partial reduction of legal uncertainty would not necessarily improve social efficiency. These results contrast previous theoretical literature.

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I. Introduction

It is well known in tort literature that under perfect information, all three negligence rules - simple, contributory, and comparative negligence - generate socially efficient levels of care both by the injurer and the victim. It is also well known that once uncertainty, caused either by measurement error or vagueness of a statute, is introduced into this ideal world of perfect information, the efficiency proposition breaks down. (Haddock and Curran 1985; Calfee and Craswell 1984; Craswell and Calfee 1986; Cooter and Ulen 1986; Shavell 1987).

Since uncertainty is unavoidable in life, the following set of questions naturally arises:

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1) whether one negligence rule has efficiency advantage over the remaining two,
2) whether one can restore social efficiency by altering the legal standard of due care, and,
3) whether one can improve social efficiency by reducing the degree of uncertainty.

Regarding the first question, Haddock and Curran (1985) conjectured that the comparative-negligence rule might be better than the remaining two, and Cooter and Ulen (1986) developed this idea into a full-blown proposition that the comparative-negligence rule is superior to the other rules.

As to the second question, Calfee and Craswell (1984) suggested that, in order to correct any incentives to overcomply or under-comply induced by uncertainty, courts should make corresponding adjustments in legal standards or damages awarded. Following this line of thought, Edlin (1994) argued that the jury's tendency to be more lenient toward the plaintiff under the contributory than the comparative-negligence rule does in fact accomplishes the required adjustment, thereby playing an instrumental role in promoting efficiency.

Concerning the third question, calls for reduced uncertainty are too pervasive to list – many legal arrangements such as the "bright-line principle," the "void for vagueness" doctrine, the provision of a collegiate judgment system purport to improve the legal accuracy, despite the caveat (Craswell and Calfee 1986) that reduced uncertainty may backfire by inducing the injurer to take excessive precaution.

This paper reexamines these three issues in a general game-theoretic setting, and shows that those earlier writers' conclusions may not always hold true. In particular, this paper demonstrates that:

1) the comparative-negligence rule is *not* necessarily superior to the other rules,
2) jury leniency does *not* always improve the social efficiency under the contributory-negligence rule, and
3) in general, it is impossible to improve social efficiency by merely reducing the degree of legal uncertainty.

The key driving force behind these conclusions is that both the direction and the size of the deviations from the social optimum induced by uncertainty in each negligence regime are generally

indeterminate. And the change in equilibrium care to be brought about by the suggested remedial measures is also indeterminate. Therefore, unless one imposes strong restrictions on the way people behave, it is not, in general, possible to 1) rank the different negligence rules by their efficiency properties, 2) make an adjustment in legal standards to the different negligence rules, nor 3) reduce uncertainty in such a way that would guarantee an improvement of social efficiency across the board.

This paper is organized as follows. In section II, a model is set up to derive equilibrium solutions under different negligence regimes. In this model, the potential injurer and victim are likened to players in a liability shifting game. The equilibrium arises as a non-cooperative solution to the game, and the social optimum as the cooperative solution. As usual, the non-cooperative solution diverges from the cooperative one, creating suboptimality. In section III, it is demonstrated that both the magnitude and the direction of deviation of the noncooperative solution from the cooperative optimum cannot be predicted in advance, because of the many intervening factors such as externalities that one party inflicts upon the other party, and strategic substitutability or complementarity between the two players' precaution. This implies that the efficiency loss induced by uncertainty is much harder to cure than is suggested by earlier writers. Section IV provides some graphical illustration. Section V compares the results with those of earlier literature. Section VI concludes.

II. The Model

Let x and y denote the care level taken by a potential injurer and a victim, respectively. Care taking is costly and the unit cost of care is represented by u and v. Greater care reduces the expected accident loss, $A(x, y)$, $A_i<0$, $i=\{x, y\}$. A social planner wants to minimize the sum of the cost of care and the cost of expected accident loss,

$$\underset{x,\ y}{Min}\ J(x, y)=ux+vy+A(x, y). \tag{1}$$

The first order conditions (FOC) are

$$J_x(x^*, y^*) = u + A_x(x^*, y^*) = 0, \tag{2a}$$
$$J_y(x^*, y^*) = v + A_y(x^*, y^*) = 0, \tag{2b}$$

meaning that the marginal cost of precaution must be equated with the marginal reduction of expected accident loss at equilibrium. Let the optimum solution be denoted as x^* and y^*. For later reference, note that a negative A_{xy} implies that the injurer's care and the victim's care are complements in reducing the expected accident loss, while a positive A_{xy} implies that they are substitutes.

A. Various Negligence Rules and Equilibrium Care under Evidentiary Uncertainty

In considering various negligence rules, I assume that the court sets the legal standard of due care at the socially efficient level, x^* and y^*, and the damages at the level of actual loss. The party whose care level falls short of the legal standard will be deemed negligent or at fault.

The damages are apportioned differently depending on which negligence rule is applied. Under the simple negligence rule, the injurer is liable if he is negligent.[1] If the injurer is not negligent, the entire loss must be borne by the victim. Under the contributory negligence rule, the injurer is liable only when he is negligent *and* the victim is not negligent. Negligence on the part of the victim, no matter how minor, will result in a complete bar of damage compensation. The comparative negligence rule is identical to the contributory negligence rule except when both are negligent. In the latter case, the damages will be shared in some pre-stipulated manner.

It is now well known that in an ideal world of complete information, all three negligence rules generate an efficient outcome in which both the injurer and the victim exercise care at the socially optimal level. So there is no apparent reason to choose one negligence rule over the others as long as efficiency is concerned. This equivalence among different rules of negligence, however, breaks down when uncertainty is brought into the picture.

In order to see this, let's assume that x and y are observed with an error, e, which has a density function $f(e)$ and a cumulative

[1] I will arbitrarily call the injurer "he" and the victim "she."

distribution function $F(e)$. The party whose *observed* care level is less than the legal standard will now be deemed negligent. This implies that neither the plaintiff nor the defendant can be certain whether or not the court will determine that their precautionary behavior was sufficient to absolve one of them of fault. We may call this condition "evidentiary uncertainty."[2] The probability that the injurer will be found negligent is

$$\text{prob}\,(x+e<x^*)=\text{prob}\,(e<x^*-x)=F(x^*-x). \tag{3}$$

The victim's probability of being held negligent is similarly defined. For distinction, let $F(x^*-x)$ denote the probability of the injurer's being found negligent, and $G(y^*-y)$, the probability of the victim's being found negligent.

Each party's cost minimization problem under different negligence rules can now be summarized as follows.

1) Simple Negligence

$$I^s=ux+A(x,\ y)F(x^*-x)$$
$$V^s=vy+A(x,\ y)[1-F(x^*-x)]$$

2) Contributory Negligence

$$I^n=ux+A(x,\ y)F(x^*-x)[1-G(y^*-y)]$$
$$V^n=vy+A(x,\ y)\{1-F(x^*-x)[1-G(y^*-y)]\}$$

3) Comparative Negligence

$$I^m=ux+A(x,\ y)F(x^*-x)[1-G(y^*-y)r]$$
$$V^m=vy+A(x,\ y)\{1-F(x^*-x)[1-G(y^*-y)r]\}$$

where the superscripts s, n, and m stand for the simple, contributory and comparative negligence rules, respectively. The parameter r represents the proportion of the damages that the victim has to bear when both parties are found to be negligent, $0\leq r\leq 1$.[3]

[2] Even though we are mainly concerned with measurement error problem in this paper, the arguments developed below will be equally applicable to the case where parties have uncertainty about the standards of care applied by courts, or courts make mistakes in applying the standards.

[3] Different versions of the comparative negligence rule have different damage-sharing rules. For example, the traditional version of the US maritime rule has the damages to be split equally when both the plaintiff and the defendant are found negligent. In this case, $r=0.5$ and is fixed.

Note first that all the objective functions under each negligence rule can be obtained by assigning different values to r: If $r=0$, it will give rise to the simple negligence rule; if $r=1$, the contributory negligence rule; if $0<r<1$, the comparative negligence rule. Thus, we can use the comparative negligence rule as an all-encompassing representative model, and generalize the result by alternating the value of r. Rewrite the objective functions as[4]

$$I=ux+A(x,\ y)F(x^*-x)\phi(y,\ r) \tag{4a}$$
$$V=vy+A(x,\ y)[1-F(x^*-x)\phi(y,\ r)] \tag{4b}$$

where $\phi(y,\ r)=1-G(y^*-y)r,\ \ 0\leq r\leq 1$. Here, we have $\phi=1,\ \ \phi_y=0$ under the simple negligence rule and $\phi=1-G,\ \ \phi_y=g$ under the contributory negligence rule. Given these, $F\phi$ now stands for the probability that the injurer is held liable, weighted by the sharing parameter r, and $1-F\phi$ the probability that the victim is held liable.

Second, note that $J\equiv I^k+V^k$ for all $k=\{s,\ n,\ m\}$. Under any negligence rule, the social planner's objective function is simply the sum of the injurer's and the victim's objective functions. This is so because either injurer or victim must bear the loss once an accident occurs with no third party being affected. Therefore, we can presume that the social planner tries to minimize the joint cost, whereas the injurer and the victim each try to minimize his/her own individual cost, with no regard to the other party's cost. (For mnemonics, J stands for joint, and I and V for injurer and victim, respectively.)

The FOCs for the noncooperative solution are

$$I_x(x^k,\ y^k)=u+A_xF\phi-Af\phi=0 \tag{5a}$$
$$V_y(x^k,\ y^k)=v+A_y(1-F\phi)-AF\phi_y=0. \tag{5b}$$

The marginal cost of precaution is again equated with the marginal savings of expected liability. Unlike the cooperative solution,

More commonly, however, sharing rules divide the liability in proportion to fault, making r a function of x and y. Nonetheless, for simplicity of analysis, I consider only the case with fixed r. In the Appendix, I provide a brief extension to the case of variable r.

[4] Since functions in (4) subsume all three different negligence rules, the superscripts, s, n, m are spared.

however, the marginal savings here are composed of two terms (the second and the third term), with the second term capturing the accident loss curtailed by the increased precaution, and the third term, the liability passed onto the other party. In the non-cooperative game, whenever one player increases his/her level of precaution, it always generates these two mutually reinforcing effects. In the following, I will call the former as the *loss curtailing* (LC) effect of one party's precaution and the latter as the *liability shifting* (LS) effect.

Equations (5a) and (5b) are the reaction functions of the injurer and the victim, respectively. The Nash equilibrium is obtained by solving these two reaction functions simultaneously. Let this noncooperative solution be denoted as (x^k, y^k), $k=\{s, m, n\}$.

III. Properties of Equilibrium

In this section, I present some of properties of equilibrium under evidentiary uncertainty.

Proposition 1

In every negligence rule, the level of care taken by each party under the evidentiary uncertainty is sub-optimal, *i.e.*, $x^k \neq x^*$ and $y^k \neq y^*$, $k=\{s, m, n\}$.

Proof: Using the identity $J \equiv I + V$, rewrite the FOCs for the cooperative solution given in (2) as

$$J_x^* \equiv I_x^* + V_x^* = 0 \tag{6a}$$
$$J_y^* \equiv I_y^* + V_y^* = 0, \tag{6b}$$

where * attached to each expression implies that the function is evaluated at the social optimum point, (x^*, y^*). Equations (6) say that in order to reach the social optimum, each party must consider not only the effect of one's precaution on his/her own cost but also on the other party's cost as well. The FOCs for the non-cooperative solutions in (5), however, consider one's own cost only, thus failing to take into account the effect that one's precaution might have on the other party's cost. Therefore, unless these externality effects are zero ($V_x^* = I_y^* = 0$), the equilibrium levels

of care under evidentiary uncertainty, (x^k, y^k), necessarily deviate from (x^*, y^*).

Below, it will be shown that neither V_x^* nor I_y^* can be zero unless each player's LC effect of precaution exactly cancels out the LS effect, which is rather unlikely.

Now let's investigate the possible directions of the deviation from the social optimum. In particular, we are interested in examining whether overcompliance or undercompliance will take place as a result of evidentiary uncertainty. This is interesting because some researchers claim that there would be a general tendency that evidentiary uncertainty induces parties to overcomply (Cooter and Ulen 1986; Shavell 2003).

Taking a Taylor series expansion around (x^*, y^*) and evaluating at the other party's non-cooperative solution, we can rewrite the FOCs in (5) as

$$I_x(x^*, y^k) = I_x^* + I_{xy}'(y^k - y^*) = -V_x^* + I_{xy}'(y^k - y^*) \tag{7a}$$
$$V_y(x^k, y^*) = V_y^* + V_{yx}'(x^k - x^*) = -I_y^* + V_{yx}'(x^k - x^*), \tag{7b}$$

where $I_{ij}' = I_{ij}(x', y')$, $V_{ij}' = V_{ij}(x', y')$ and $x' = \alpha x^k + (1-\alpha)x^*$, $y' = \alpha y^k + (1-\alpha)y^*$, $0 < \alpha < 1$. The second equalities are obtained using (6). Here, a positive $I_x(x^*, y^k)$ implies the injurer's undercompliance, and a negative $I_x(x^*, y^k)$ overcompliance. The same relationship holds with regard to the victim's precaution.

Note that in each equation of (7), the right hand side of the first equality has two terms: The first term representing the deviation from the optimum at the socially optimal point, I_x^* and V_y^*, and the second term representing one party's reaction to the other party's deviation from the optimum. I will call the first term *the pure uncertainty effect*, and the second term *the feedback effect.* By inspecting the terms following the second equalities, we can see that the pure uncertainty effect is none other than the negative of the externality effects mentioned above. And the feedback effect is determined by the signs of the cross derivatives, I_{xy} and V_{yx}, or equivalently by whether x and y are viewed as *strategic* substitutes or complements.[5] If the injurer regards the victim's precaution as substitute, he will adjust his level of precaution to the opposite

[5] For the definition of strategic complements and substitutes, see Bulow *et al.* (1985) and Fudenberg and Tirole (1984).

direction of the victim's, and if he regards it as complement, to the same direction.

In the following, I will first examine the pure uncertainty effect and then the feedback effect, and show that 1) the pure uncertainty effect is composed of two opposing externality effects, the positive LC effect and the negative LS effect,[6] and the net effect is indeterminate, 2) the feedback effect is determined again by the interaction of two players' LC and LS effect, which generates four interaction terms, (LC by LC, LC by LS, LS by LC, and LS by LS), and is indeterminate.

A. The Pure Uncertainty Effect

Differentiating one party's objective function with respect to the other party's control variable and evaluating at (x^*, y^*) gives

$$V_x^* = A_x^*(1-F^*\phi^*) + A^* f^* \phi^* = (1-F^*\phi^*)\left[\frac{f^*\phi^*}{1-F^*\phi^*}A^* - u\right] \tag{8a}$$

$$I_y^* = A_y^* F^* \phi^* + A^* F^* \phi_y^* = F^*\phi^*\left[\frac{\phi_y^*}{\phi^*}A^* - v\right]. \tag{8b}$$

In each equation, the first term on the right hand side of the first equality is the LC effect and is negative; by increasing his/her precaution, each party reduces the accident loss itself, thus lowering the burden that the other party would bear (positive externality). The second term is positive and represents the LS effect; one party's incremental precaution increases the cost to the other party by raising the other party's probability of being held liable[7] (negative externality).

In short, whenever one party increases his/her precaution, it slices off the accident loss, which is beneficial for both. At the same time, however, it also has the effect of passing the liability to the other party, which is good for oneself but bad for the other

[6] As we will see later, the LC effect is numerically negative. Nonetheless I call it positive externality effect because one party's precaution helps the other party by reducing the cost that the other party has to bear.

[7] Or equivalently, by increasing the caretaker's probability of being exonerated.

party. The net of these two opposing forces then determines the overall effect of the pure uncertainty. This concurs with the general economic principle that the net negative externalities entail excessive activity, and the positive externalities insufficient activity.

Since there is no way of knowing a priori whether there will be net positive externalities or negative externalities, it is obvious that the pure uncertainty effect is indeterminate. However, we can figure out a few important factors that affect the direction of externality. Note from (2) above that $A_x^* = -u$, and $A_y^* = -v$ at the social optimum point. Replacing each and rewriting, we obtain the second equality in (8).

The first term in each bracket on the right hand side of the second equality represents the marginal gain that one party can enjoy by passing the liability to the other party and the second term the unit cost of care. Therefore the expressions show that one would overcomply if the liability shifting gain is greater than his/her unit cost of care, and undercomply if the gain is smaller than the unit cost. Examining each component of the terms, we can see also that;

1) Other things being equal, the cheaper one's unit cost of precaution is, one would more overcomply.
2) The larger A^*, the size of the accident loss at the optimum point or socially unavoidable accident loss, is, more likely is one to overcomply.[8] Obviously this is because the stake is high.
3) Assuming the error distribution is symmetric and uni-modal, the smaller the variance of the distribution is and therefore the larger the value of $f(0)$ or $g(0)$[9] is, one would more overcomply. If one can change one's fate significantly by undertaking a small additional care, one would surely take that extra care.

Summarizing what has been stated so far, we can establish the following proposition.

Proposition 2

The pure uncertainty effect depends on the relative size of the

[8] This implies that if the causation rule is strictly imposed and therefore the compensation damage is limited only to the excess of accident loss over the unavoidable, both injurer and victim would unvaryingly undercomply. See Kahan (1989).

[9] $f(x^* - x)$ and $g(y^* - y)$ evaluated at $x = x^*$ and $y = y^*$.

positive LC externality and the negative LS externality and the overall direction of the effect is in general indeterminate. One is more likely to overcomply, the cheaper the cost of care is, the bigger the stake one can pass onto the other party is, and the easier one can alter the probability of being exonerated from the liability.

An exception to this general indeterminacy is the case of the victim under the simple negligence rule. In this case, the victim does not have the leverage to exercise her LS effect ($\phi_y=0$) because her level of care does not matter under the simple negligence rule. Constrained to create only the beneficial LC effect, the victim is likely to take inadequate amount of care unless the feedback effect more than offset the pure uncertainty effect.

B. The Feedback Effects

By differentiating the FOCs in (5) with respect to the other party's precaution, we have

$$I_{xy}=A_{xy}F\phi+A_xF\phi_y-A_yf\phi-Af\phi_y \tag{9a}$$
$$V_{yx}=A_{yx}(1-F\phi)+A_yf\phi-A_xF\phi_y+Af\phi_y. \tag{9b}$$

A few things are worthy of note: First, every term except the first in I_{xy} is identical to the corresponding term in V_{yx} in magnitude, but opposite in sign. This is again because of the zero-sum nature of the game: Every additional gain enjoyed by the injurer by passing the buck becomes an additional burden to the victim, and *vice versa*.

Second, when we put aside these LS related terms, the sign of I_{xy} and V_{yx} is solely determined by the sign of A_{xy}, the technical complementarity or substitutability between x and y. This implies that any divergence between the technical and strategic complementarity is due to the LS effect.

Third, we have $I_{xy}+V_{xy}=A_{xy}$ in every negligence regime. As the LS effects are being completely netted out and the LC effects are being probabilistically divided between the two, we are left only with A_{xy} when we add them together. One immediate implication is that I_{xy} and V_{xy} are asymmetrical unlike the cooperative solution. In particular, $A_{ij}=0$ no longer guarantees the mutual independence

TABLE 1
THE SIGNS OF EACH INDIVIDUAL TERM

	LC by LC	LC by LS	LS by LC	LS by LS
I_{xy}	?	−	+	−
V_{yx}	?	−	+	+

between the injurer's precaution and the victim's precaution. Rather, $A_{xy}=0$ dictates $I_{xy}=-V_{yx}$ implying that if one party considers x and y as strategic complements, the other party should necessarily consider them substitutes.

If we examine the signs of each individual term, it can be summarized as those given in Table 1.

The first term of each equation represents the LC by LC effect and is indeterminate; it depends on the sign of A_{xy}, the technical substitutability or complementarity of each party's precaution. The second term (the LC by LS effect) is negative; each party's LC effect is strengthened by the other party's precaution because the other party's increased precaution raises the probability of one's liability, the probability that one will enjoy the loss diminution one has created. This makes each other's precaution strategically complementary. The third term is positive; the accident loss size has been made smaller by the other party's greater precaution and therefore shifting it onto the other party does not bestow that much benefit to the shifter.

Unlike the first three terms, the sign of the fourth term diverges; it is negative to the injurer, but positive to the victim. This disparate response to the other party's precaution has its roots in the fundamental asymmetry between the injurer and the victim; the injurer can pass the burden only when the victim is *non-negligent* whereas the victim can pass the buck only when the injurer is *negligent*. In other words, the victim's care increases $1-G$ (by decreasing G) and therefore magnifies the injurer's LS effect which works through F term in $F(1-G)$. From the viewpoint of the victim, however, the injurer's additional care reduces F and consequently shrinks $F(1-G)$ through which $(1-G)$ term can work. This built-in asymmetry makes injurer view the victim's care as strategic complements, but makes the victim consider the injurer's care as strategic substitutes.

Turning to the question of different negligence regime, the victim has no power to shift the liability ($\phi=1$ and $\phi_y=0$) under the simple negligence rule, but only to cut it small. This leaves only the first and the third term in (9a) and the first and second term in (9b). The A_{xy} term is common to both I_{xy} and V_{yx}. The remaining term is $A_y f$, which concerns the interaction between the injurer's LS effect and the victim's LC effect. This effect is positive to the injurer, but negative to the victim. This implies that, under the simple negligence regime, it is more likely for the injurer to consider the victim's care as a substitute for his care (a downward sloping reaction curve), and for the victim to view the injurer's care as a complement to her care (an upward sloping reaction curve).[10] This is definitely so when $A_{xy}=0$.

As the negligence regime tilts toward the contributory negligence rule, the strategic position taken by either party gradually switches toward the opposite direction, with the injurer's position heading from strategic substitutes toward complements, and the victim's from strategic complements toward the substitutes. As explained earlier, this comes from the fact that the victim's LS effect tends to work as complements to the injurer and the injurer's LS effect works as substitutes to the victim.

Summarizing we have,

Proposition 3

The feedback effect depends one whether one party's care works as strategic complements or substitutes to the other party's care, which in turn hinges on 1) the technical substitutability between the two, and 2) which negligence rule is in force. Assuming technical independence ($A_{xy}=0$), the injurer views the victim's care as strategic substitutes under the simple negligence rule but tends to view more as strategic complements under the contributory negligence rule. The opposite is true with the victim; she views the injurer's care as complements under the simple negligence rule but tends to view more as substitutes under the contributory negligence rule.

[10] Putting aside the first term, which reflects the LC effects of each party's care.

C. The Total Effect

Rearranging and solving (7) simultaneously gives,

$$\begin{bmatrix} I'_{xx} & I'_{xy} \\ V'_{yx} & V'_{yy} \end{bmatrix} \begin{bmatrix} x^k - x^* \\ y^k - y^* \end{bmatrix} = \begin{bmatrix} V_x^* \\ I_y^* \end{bmatrix},$$

$$x^k - x^* = \frac{V_x^* V'_{yy} - I_y^* I'_{xy}}{|H|}, \quad y^k - y^* = \frac{I'_{xx} I_y^* - V'_{yx} V_x^*}{|H|} \tag{10}$$

where $|H|$ is the determinant of the coefficient matrix. Assuming that the equilibrium is locally stable, we have $|H| > 0$.[11] Note that if $V_x^* = I_y^* = 0$, $x^k - x^* = y^k - y^* = 0$. This implies that the ultimate source of the deviations from the optimum is the pure uncertainty effect. If the pure uncertainty effects become non-zero, then the feedback effects start to propagate the initial deviations to each player.

To see how this system works, let's consider a simplest possible case; the simple negligence rule with $A_{xy} = 0$. Under the simple negligence rule, the victim has no leverage to pass the liability and therefore exerts only positive externality to the injurer, $I_y^* < 0$. Furthermore, the assumption of $A_{xy} = 0$ assures us that $I_{xy} > 0$ and $V_{yx} < 0$.

Under this hypothetical situation, the only unknown free term is V_x^*. Suppose it is positive. Then, according to equation (10), the injurer's equilibrium care will surely be excessive, but the victim's care is indeterminate. Alternatively, if V_x^* is negative, the victim's care will be definitely deficient, but injurer's care is indeterminate. The latter case is especially interesting because initially at the social optimum point, both the injurer and victim have incentives to undercomply but the victim only maintains the deficiency to the end. The injurer's initial undercompliance may end up with overcompliance because he sees the victim's care as strategic substitute and therefore react inversely to the victim's undercompliance.

[11] This condition is very much likely to be satisfied because 1) I_{xx} and V_{yy} are positive from the SOC, and 2) I_{xy} and V_{yx} tend to have the opposite sign.

IV. Graphical Illustration

In this section, I provide a few graphical illustrations to check some of the results obtained thus far. I assume that $A(x, y) = \bar{A} - 4x - 4y$, and a uniform distribution with the support $[-e, e]$ and that the cost of care function is given by x^2 and y^2, respectively.[12] This implies that $A_{xy} = 0$ and $F(x^* - x) = (x^* - x)/2e + 1/2$. In particular, I first set $\bar{A} = 20$, $e = 1/2$ and then allow small changes in $\bar{A}$ and e later. Apparently these assumptions are highly restrictive. But it still provides us with a few interesting implications. Trivially, the social optimum solution is attained at $x^* = y^* = 2$.

Figure 1 shows the equilibrium outcomes under various negligence rules, the simple negligence ($r = 0$), the comparative negligence ($r = .2$ and $.6$), and the contributory negligence rule ($r = 1$). In each panel, the solid curve (R_1) represents the injurer's reaction curve, while the dashed one (R_2) represents the victim's. The horizontal axis measures the level of the injurer's care, and the vertical axis, the victim's care. The equilibrium values of care, which are found at the cross of the two reaction curves, are given in the first row of Table 2.

Under the simple negligence rule with $A_{xy} = 0$, the injurer has a negative-sloped reaction curve, and the victim a positive-sloped reaction curve as explained earlier. At the intersection of R_1 and R_2, the equilibrium (x^s, y^s) occurs at (2.33, 1.67), that is, $x^s > x^*$ and $y^s < y^*$, implying that the injurer overcomplies and the victim undercomplies.

As the value of r increases, the slope of R_1 gets steeper and finally turns into a positive value at the equilibrium point under the contributory rule. On the other hand, the slope of R_2, which used to be positive, becomes negative. This confirms the predicted switch of each party's strategic position.

At the same time, the injurer's reaction curve shifts in, and the victim's curve shifts up as the value of r increases. These shifts are clearly shown in Figure 2, which combines all four panels. There, the shifts and the simultaneous changes in slope make the

[12] This example is from Edlin (1994). Being non-linear, the cost of care function here is slightly different from the one given in section II. Nothing substantive is changed, though.

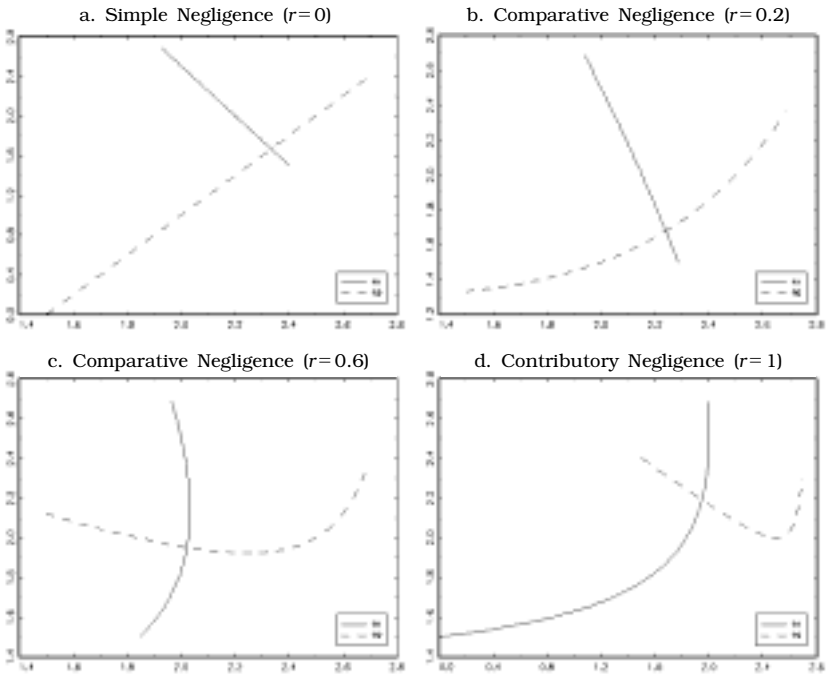

FIGURE 1
EQUILIBRIUM CARE UNDER DIFFERENT NEGLIGENCE RULES

TABLE 2
EQUILIBRIUM CARE LEVELS UNDER DIFFERENT VALUES OF $\bar{A}$

	$r=0$		$r=0.2$		$r=0.6$		$r=1.0$	
	x^s	y^s	x^m	y^m	x^m	y^m	x^n	y^n
$\bar{A}=20$	2.33	1.67	2.24	1.68	2.02	1.95	1.95	2.19
$\bar{A}=22$	2.44	1.88	2.39	1.86	2.21	2.03	2.15	2.25
$\bar{A}=24$	2.56	2.12	2.53	2.04	2.40	2.04	2.34	2.23

injurer's and the victim's reaction curves rotate clockwise. The shifts create a general tendency that the care level taken by the injurer decreases, from 2.33 to 2.24, 2.02, and finally to 1.95, and the care level taken by the victim increases, from 1.67 to 1.68, 1.95 and finally to 2.19, as the value of r increases. However, this general tendency is, sometimes obtruded by the concomitant slope changes. For example, the victim's care decreases from 1.88 to 1.86

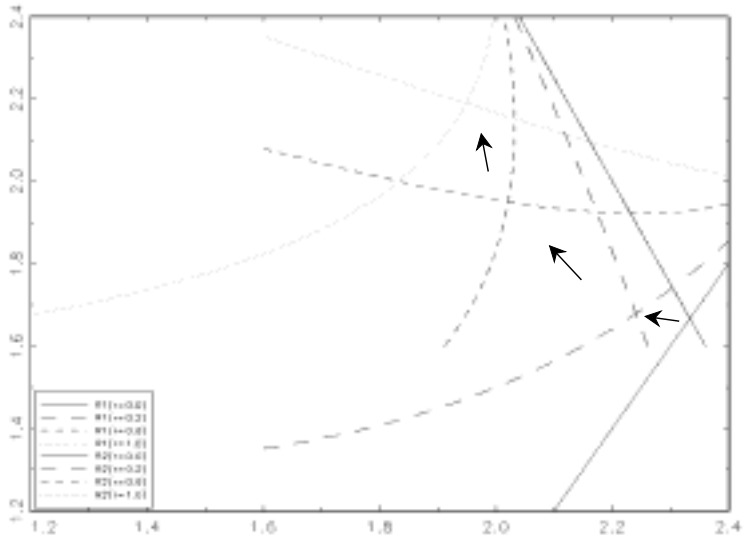

FIGURE 2
THE SHIFTS AND SLOPE CHANGES OF THE REACTION CURVES

as the value of r increases from 0 to 0.2 under $\bar{A}=22$ (see Table 2). In fact, the equilibrium path always contains an initial segment that declines southwestward before it climbs northwestward.

Figure 3 shows the effect of an increase in $\bar{A}$, from 20 to 22. It makes the injurer's reaction function shift outward and the victim's reaction curve shift upward, except for the victim's reaction curve under the simple negligence rule. It also makes the reaction curves rotate clockwise again.

Under the simple negligence rule, the victim's reaction curve stays put, but the injurer's reaction curve shifts out. This makes both the injurer's and the victim's care levels to increase from (2.33, 1.67) to (2.44, 1.89). This implies that if $\bar{A}$ is sufficiently large, both the injurer and the victim will overcomply. Similarly, a sufficiently small $\bar{A}$ would make both undercomply. This simple relationship between the equilibrium points and changes in $\bar{A}$ does not, however, extend to the case of other negligence rules under which the victim's reaction curve also shifts. For example, the victim's care under the contributory negligence rule declines from 2.25 to 2.23 as $\bar{A}$ increases from 22 to 24 (Table 2).

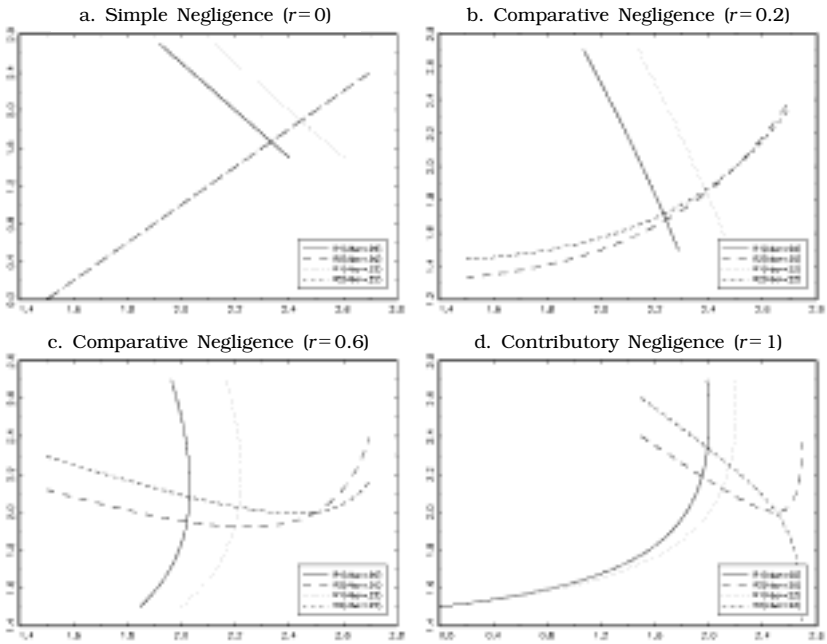

FIGURE 3

THE EFFECT OF $\bar{A}$ CHANGES ON EQUILIBRIUM CARE

TABLE 3

EFFECTS OF CHANGES IN THE MARGIN OF MEASUREMENT ERROR

	$r=0$		$r=0.2$		$r=0.6$		$r=1.0$	
	x^s	y^s	x^m	y^m	x^m	y^m	x^n	y^n
$e=0.3$	2.30	2.01	2.28	1.96	2.19	2.02	2.15	2.17
$e=0.5$	2.33	1.67	2.24	1.68	2.02	1.95	1.95	2.19
$e=1.0$	2.25	1.25	2.08	1.37	1.74	1.77	1.56	2.13

Figures 4 and 5 below show the effects of altering the margin of measurement error, first from $e=0.5$ to 1.0, and then from 0.3 to 0.5. The reactions differ depending on the initial level of uncertainty.

When the margin of error increases from 0.5 to 1.0, both the injurer's and the victim's equilibrium care decrease in every negligence regime, as shown by the arrows all pointing southwest. This is the result predicted initially by Craswell and Calfee (1986),

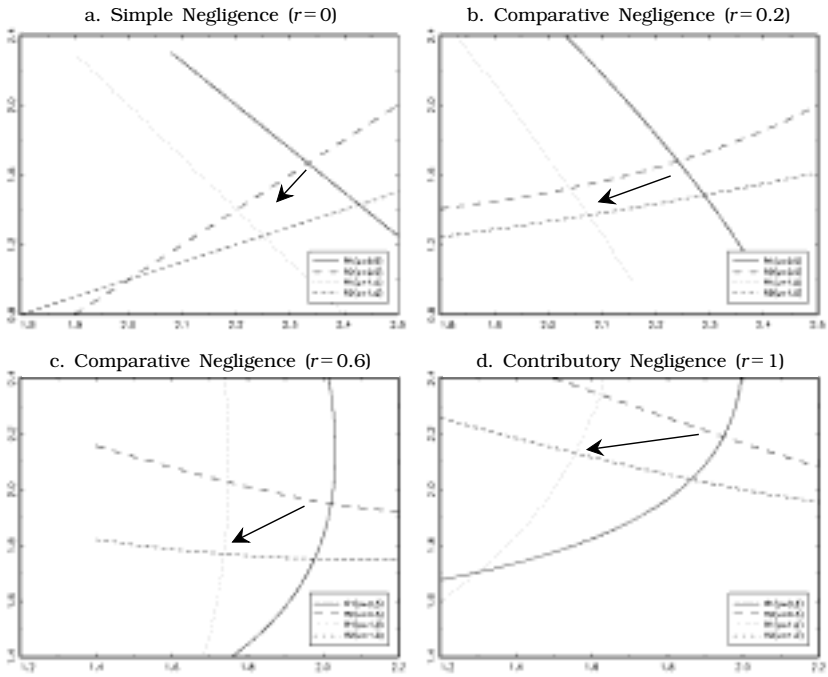

FIGURE 4

THE EFFECT OF INCREASED MARGIN OF MEASUREMENT ERROR (I)

and analyzed more formally later by Yoo (2004), who named them as "discounting effects." When the initial variance of legal uncertainty is relatively large, people tend to become lax and consequently take less care as variance further increases.

When the initial variance is relatively small, however, people may react by increasing their precaution in response to an increase in uncertainty; some people who have already secured a high-enough probability of getting exonerated by taking a sufficiently large amount of care may try to guard their expected good fortunes by expensing more care as the randomness increases. Their incentives are like those who purchase insurance against a small chance of bad outcome (thus termed the "insurance effect" in Yoo (2004)).[13]

[13] People who initially exerted a relatively low level of care, and are thus facing a high probability of bad outcome, may also increase their care levels in response to the increased variance in the hope that the increased

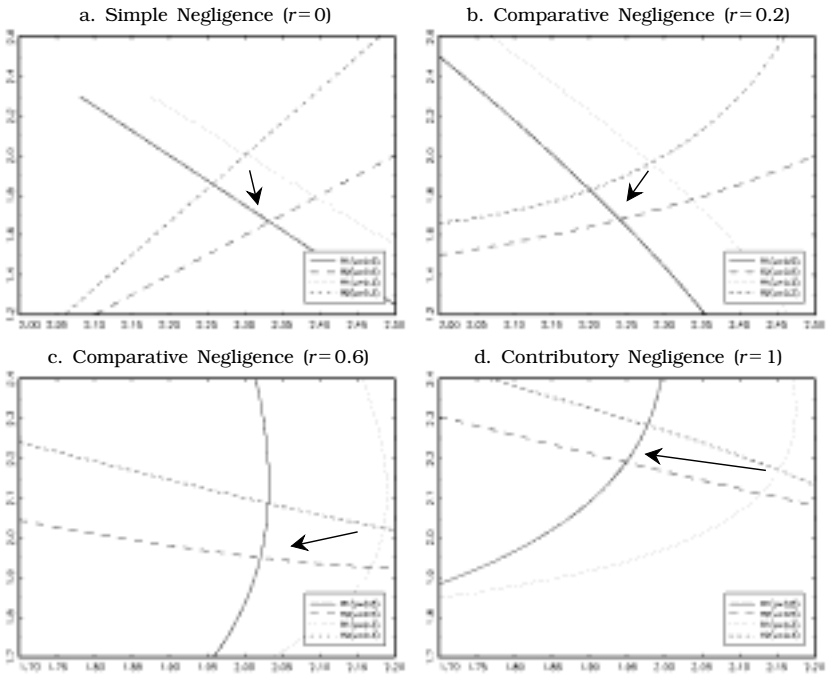

FIGURE 5
THE EFFECT OF INCREASED MARGIN OF MEASUREMENT ERROR (II)

This is what happens to the injurer under the simple negligence rule (southeast movement) and the victim under the contributory rule (northwest movement) in Figure 5. In response to the increased margin of error, from 0.3 to 0.5, the injurer under the simple negligence rule increases his care level from 2.30 to 2.33 (insurance effect), whereas the victim decreases her care level from 2.01 to 1.67 (discounting effect). Under the contributory negligence regime ($r = 1$), the injurer decreases his care level from 2.15 to 1.95, but the victim increases her care level from 2.17 to 2.19 as the variance increases.

randomness may bring them a luck by chance. They "gamble" in the face of increased risk. This "gambling" behavior, which can occur under a normal distribution, however, does not arise here because the assumed distribution is uniform. See Yoo (2004).

V. Comparison with Earlier Literature

A. Non-Superiority of the Comparative Negligence Rule[14]

The proposition that the comparative negligence rule is superior to the other two rules (Cooter and Ulen 1986) is founded on the following three building blocks: i) Under evidentiary uncertainty, both parties are induced to overcomply in every negligence rule, ii) the magnitude of overcompliance, however, can be ranked among the three different negligence rules in such a way that $x^n < x^m < x^s$ and $y^s < y^m < y^n$, and iii) assuming that both injurer and victim are symmetrically situated, the comparative negligence rule is superior to the other two because it entails the smallest sum of deviations from the optimum. Below, it is shown that the first two of these claims are not necessarily true and the third building block is not the proper criterion for the social efficiency.

First, we have already seen that the first building block is not true. Depending on the unit cost of care, the size of the expected accident loss, the variance of measurement error distribution, and the direction of technical and strategic complementarity, people may as well undercomply as overcomply.

Second, there exists no clear-cut ranking among (x^k, y^k), $k=\{s, n, m\}$ as is shown in Table 3. It is true that as r increases, there is a general tendency for the injurer's care to decrease and the victim's care to increase. This is because the injurer's reaction curve shifts in, and the victim's reaction curve shifts up as the value of r increases. Yet the concurrent change in slope of the reaction curves disturbs this monotonic relationship between the care levels and negligence regime, and thereby makes the final results ambiguous.

More formally, by differentiating (5) with respect to r, we have

$$\frac{\partial x^k}{\partial r} = \frac{1}{|H|}[V_{yy}(A_xF - Af)G - I_{xy}(Ag - A_yG)F] \tag{11a}$$

$$\frac{\partial y^k}{\partial r} = \frac{1}{|H|}[I_{xx}(Ag - A_yG)F - V_{yx}(A_xF - Af)G]. \tag{11b}$$

[14] Bar-Gill and Ben-Shahar (2003) makes similar argument. But they show the non-superiority through a numerical simulation, not by algebraic proof as is done in the present paper.

As expected, we cannot pin down the sign of the expressions because of the indeterminacy of I_{xy} and V_{yx}. What is interesting in this context is that if we set $I_{xy}=V_{yx}=0$ and thereby turn off the feedback effect, Equations (11) yield the claimed ranking.[15]

Lastly, and trivially, what is important for the social efficiency is the sum of the costs not the sum of the care deviations. Therefore one cannot rank the relative efficiency among various negligence rules simply based on the sum of deviations. To see this, denote J^* the minimized social cost at the optimum, and J the social cost at any other arbitrary levels of care. Then we have

$$J-J^* = X'HX. \tag{12}$$

where $X=\{x-x^*,\ y-y^*\}'$. This is positive definite by definition and the contour map of this function represents an ellipse centered at the origin. The direct implication is that there is no way of ascertaining the superiority or inferiority of any negligence rule simply by looking at the sum of the deviations. For example, point A in Figure 6 has a larger sum of deviations, but a smaller social cost than the point B. Therefore, point A is superior to point B from a social efficiency point of view.

B. Curing the Efficiency Loss by Changing the Legal Standards of Due Care

It is clear from the above discussion that changing the legal standards of care would yield ambiguous results in the equilibrium care levels. Worse, one does not even know whether there will be under- or over-compliance from the beginning.

The solution to this impasse, suggested by Edlin (1994), is, basically to reset the legal standards such that $V_x^*=I_y^*=0$ is assured in (8). It attempts to stamp out the externality at its source. Once the externality elements are expunged, one needs not worry about the unknown feedback effect. Furthermore, it appears that the goals are theoretically attainable because we have two equations

[15] In fact, when examining whether one party has incentives to over- or undercomply, Haddock and Curan (1985), Cooter and Ulen (1986) assumed that the other party will stick to the socially optimal level of care. By forcing the other party's deviation in (7) to vanish, however, they effectively rendered the sign and magnitude of the I_{xy} and V_{yz} terms irrelevant.

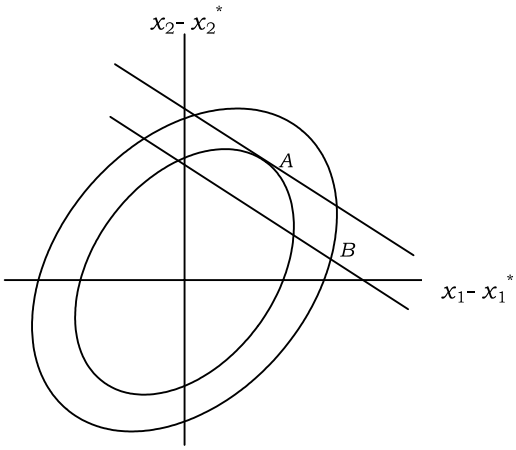

FIGURE 6
SUM OF DEVIATIONS AND SOCIAL EFFICIENCY

and two control parameters. Therefore, problems with the Edlin's proposal, if any, lie not in theoretical possibility but in practical applicability.

Imposition of the no-net-externality constraint on (8) transforms it into a rather complicated simultaneous equation system in two legal standards, $\bar{x}$ and $\bar{y}$, the solution to which may or may not exist. Edlin assumes a uniform distribution with $e=1/2$ and proposes a set of adjusted legal standard of due care as follows: When $r=1$, set $(\bar{x}, \bar{y})=(1.5, 1.5)$, and when $r=6/10$, $(\bar{x}, \bar{y})=(13/6, 13/6)$. Indeed, this new set of adjusted legal standards generates the desired social optimum. Based on this result, he interpreted that $\bar{x}$ and $\bar{y}$ are decreasing in r, that is, in the contributory negligence regime, the court has to impose a more lenient standard on both the injurer and the victim than in the comparative negligence regime.

One crucial problem with the Edlin's proposal is that it cannot be generalized to a non-uniform distribution. Implicit differentiation of (8) gives,

$$\frac{d\bar{x}}{dr} = \frac{(Af - A_x F)A(Gg' - g^2)}{(Af' - A_x f)(Ag' - A_y g)\phi}, \qquad \frac{d\bar{y}}{dr} = \frac{A_y G - Ag}{(Ag' - A_y g)r},$$

the signs of which cannot be determined because of the g' and f' terms, the curvature of the distribution function. Edlin gets around this problem by assuming a uniform distribution under which $g'=f'=0$.[16]

C. *Non-Curability of Efficiency Loss by Improving the Legal Accuracy*

As has been already suggested, it is in general impossible to correct the situation by partially reducing the measurement error or improving the legal accuracy. A reduced variance of the error distribution, being just another parameter subsumed in the model, will always generate ambiguous outcome because of the indeterminacy of the expected externality effects and the feedback effects.

As a matter of fact, there is one further complication that the authority has to face even in the case where only one party is involved. As is shown in Yoo (2004), a reduction in the variance of the error distribution may generate three different reactions - gambling, insurance as well as discount effects - depending on the initial size of the measurement error and the size of the damage to be awarded. These diverse responses to the improved accuracy in legal processes arise even under unilateral care, where strategic buck-passing incentives do not exist.

VI. Conclusion

Uncertainty, which is pervasive in every legal arena, inevitably creates social inefficiency. A vague legal statute may make some people overly chilled, causing excessive precaution. Uncertain punishment of crimes may induce some to engage in more crimes,

[16] Another problem, though not as critical as the first one, is that, even with the assumption of uniform distribution, the results tend to be very sensitive to the size of margin of error and easily set the adjusted standard well outside of the possible support of the distribution. In fact, this is what happens with the Edlin's numerical example. The adjusted set of the standard he proposed for the contributory negligence rule, $(\bar{x}, \bar{y})=(1.5, 1.5)$, is in fact equally effective for the comparative negligence rule as well. There is no need to provide a different set of (13/6, 13/6) for $r=6/10$ because for any value $\bar{x}\leq 1.5$, F is always zero in his example, and given $F=0$, $\bar{y}$ should always equal 1.5, independent of the value of r.

and others in defensive or protective activities.

In this paper, the effects of evidentiary uncertainty on social efficiency, and the probable effects of some of the suggested remedial measures, have been analyzed in the context of negligence rule in tort law.

Conclusions are largely dismal: No simple cure appears to exist to deal with the efficiency loss caused by the legal uncertainty. The comparative negligence rule, which has been hailed by some as *deus ex machina* to the evidentiary uncertainty problem, does not appear to be capable of solving the problem. Adjustment of legal standards of due care may not work, either, unless the measurement error distribution has a particular shape and deviations from the social optimum fall within a particular range. Still worse, a partial reduction of uncertainty through a decrease in the variance of the measurement error is not likely to be helpful because there still remains the possibility to make the situation worse. Reduced uncertainty creates divergent incentives for each party, *via* its discounting, gambling, and insurance effects depending on the initial degree of uncertainty.

A word on the assumption of the fixed r is in order. Apparently this assumption is highly restrictive and may seem to some to be unwarranted. Fortunately, however, this assumption is not as strong as it may at first appear because relaxation of it only strengthens the conclusion of the paper; the indeterminacy of the size and direction of the deviations from the social optimum. A detailed explanation is given in the Appendix. Here we provide only the summary of it. Firstly, if we let the damage sharing rule be a function of x and y, $\rho(x^*-x-e_x,\ y^*-y-e_y)$, rendering it a random variable, and its expected value as $r(x^*-x,\ y^*-y)$, we can prove that r_y is negative but r_x is indeterminate even with the assumption of $\rho_x>0$ and $\rho_y<0$. In short, by allowing the sharing rule to be a variable, we introduce an additional element of indeterminacy into the model.

Secondly, once we assume that $r_y<0$ and $r_x>0$ despite the aforementioned indeterminacy, every argument given earlier applies straightforwardly, except that we have one more element that works in a similar way as the LS effect, *i.e.*, one party's increased care shrinks the share of damages he/she has to bear when deemed negligent. We may call it the share reducing (SR) effect of one's care. In fact, the way how this SR effect works is so similar to the

LS effect, one may subsume it as a subset of the LS effect. All in all, the assumption of the fixed share is very much innocuous as far as the main conclusion of the paper is concerned.

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Appendix

Let $\rho(x^*-x-e_x,\ y^*-y-e_y)$ be the proportion of the accident loss that the victim bears under the comparative negligence rule. Since this formula is applicable only when both the injurer and the victim are found to be negligent, we have to impose restrictions such that $x^*-x-e_x>0$, $y^*-y-e_y>0$ or $e_x<x^*-x$, $e_y<y^*-y$ and $\rho(0,\ y^*-y-e_y)=1$, $\rho(x^*-x-e_x,\ 0)=0$. It is reasonable to assume that $\rho_x>0$ and $\rho_y<0$. Let the expected value of $\rho(x^*-x-e_x,\ y^*-y-e_y)$ be denoted as $r(x^*-x,\ y^*-y)$. Taking expectation over the relevant range, we have

$$r(x^*-x,\ y^*-y)=\int_{-\infty}^{x^*-x}\int_{-\infty}^{y^*-y}\rho(x^*-x-e_x,\ y^*-y-e_y)f(e_x)\,g(e_y)\,de_x\,de_y.$$

It has the properties of $r_y<0$ since

$$\begin{aligned} r_y(x^*-x,\ y^*-y)=&\int_{-\infty}^{x^*-x}\int_{-\infty}^{y^*-y}\rho_y(x^*-x-e_x,\ y^*-y-e_y)f(e_x)\,g(e_y)\,de_x\,de_y\\ &-\int_{-\infty}^{x^*-x}\rho(x^*-x-e_x,\ 0)f(e_x)\,g(y^*-y)\,de_x \end{aligned}$$

where the first term is negative and the second term drops out. The sign of r_x, however, becomes indeterminate because

$$\begin{aligned} r_x(x^*-x,\ y^*-y)=&\int_{-\infty}^{x^*-x}\int_{-\infty}^{y^*-y}\rho_x(x^*-x-e_x,\ y^*-y-e_y)f(e_x)\,g(e_y)\,de_x\,de_y\\ &-\int_{-\infty}^{y^*-y}\rho(0,\ y^*-y-e_y)f(x^*-x)g(e_y)\,de_y\\ =&\int_{-\infty}^{x^*-x}\int_{-\infty}^{y^*-y}\rho_x(x^*-x-e_x,\ y^*-y-e_y)f(e_x)g(e_y)\,de_x\,de_y\\ &-f(x^*-x)\,G(y^*-y) \end{aligned}$$

where the first term is positive and the second term is negative.

Reinterpreting $r(x^*-x,\ y^*-y)$ as the expected share, the FOC in

(5) can be rewritten

$$I_x(x^k,\ y^k)=u+A_xF\phi-Af\phi-AFGr_x=0 \tag{5a)'}$$

$$V_y(x^k,\ y^k)=v+A_y(1-\mathrm{F}\phi)-AF\phi_y+AFGr_y=\ 0. \tag{5b)'}$$

The second term represents the LC effect, the third term the LS effect, and the fourth term can be called as the share reducing (SR) effect of one party's care. The externality effect one party imposes on the other party's cost is given as

$$V_x^*=A_x^*(1-F^*\phi^*)+A^*f^*\phi^*+A^*F^*G^*r_x^* \tag{8a)'}$$

$$I_y^*=A_y^*F^*\phi^*+A^*F^*\phi_y^*-A^*F^*G^*r_y^*. \tag{8b)'}$$

The first term is negative, but the second and the third terms are positive. The feedback effect is given as

$$I_{xy}=A_{xy}F\phi+A_xF\phi_y-A_xFGr_y-A_yf\phi-Af\phi_y+AfGr_y-A_yFGr_x +AFgr_x-AFGr_{xy} \tag{9a)'}$$

$$V_{yx}=A_{yx}(1-F\phi)+A_yf\phi+A_yFGr_x-A_xF\phi_y+Af\phi_y-AFgr_x +A_xFGr_y-AfGr_y+AFGr_{yx} \tag{9b)'}$$

Since the LC, LS, and SR effects interact among themselves, there are now nine terms in each equation.

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