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Essays on Macroeconomics with Heterogeneous Agents

이질적 경제주체 기반의 거시경제학에 관한 에세이

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Abstract

This doctoral dissertation consists of the intersection of dynamic stochastic general equilibrium modeling, including heterogeneous agents, and the subjects of public finance and labor economics.

The first chapter explains a puzzling empirical phenomenon regarding fertility rate in the United States. Over the last few decades, high-income females have demonstrated a tendency to have more children in the U.S. At the same time, household income structure has changed, becoming more unequal and more favorable to females. However, these changes appear contradictory to the predictions found in classical fertility literature, which suggest that high-income women exhibit low fertility due to the high opportunity cost of raising children. To account for this puzzling empirical phenomenon, we offer a fertility choice model with preference heterogeneity on having children, which allows for a comparative advantage between employment outside the home and child-rearing. We highlight the composition effect of females who desire children newly entering the high-income group, while females less desirous of children exit as the income structure changes. Our model accounts for 55% of the observed variation in the complete fertility rate, while the comparable model without composition effect fails to explain the observation. We also decompose various income shocks and find that changes in skill premium represent the major factor behind the phenomenon.

The second chapter examines the quantitative effects of population aging driven by declining mortality and fertility rates. We also studies the macroeconomic effects of raising the mandatory retirement age in such an aging economy. When the mortality rate decreases, aggregate capital increases since individuals save more for a longer retirement. In contrast, an increase in aggregate labor input is negligible since lower mortality rates primarily affect those who are out of the labor force. When

the fertility rate decreases, both aggregate labor and capital inputs shrink radically because the aggregate population diminishes along with the working age population and aggregate savings plunges due to a downsized population. We analyze the effects of population aging when the mortality rate of all ages decreases by 1% each year and the population growth rate declines from 0.7% to 0.3%. A huge drop in aggregate labor input drags down aggregate output by about 15%. The pension system will run a big budget deficit with more retirees and a smaller number of workers. The government can alleviate the negative effects of population aging by raising the mandatory retirement age. When workers' retirements are postponed by either three or five years, both aggregate labor input and capital increase, and pension deficits are significantly reduced.

Keyword : Heterogeneous Agent, Fertility, Skill Premium, Gender Wage Gap, Population Aging, Extension of Retirement Age, Income Inequality

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Chapter 1

Do High-Income Females Have More Children?

Relaxing Homogeneous Preference and Composition Change

1 Introduction [1]

The female CEO of YouTube had her fifth child recently, and the occasion serves as an excellent counterexample of conventional wisdom, i.e., that high-income[2] women have fewer children. Of course she could be an extreme case, but according to the U.S. Census Bureau and the Panel Study of Income Dynamics (PSID), we can find similar trends of married women who are in the highest labor income quintile (top 20%) having more children over the last three decades (see Figure 1.2). This phenomenon has changed the relationship between female labor income and complete fertility[3] from the inverted slash shape (\) to the L-shape.

During the same period, household income structure has also changed dramatically. (i) The skill premium has increased, while (ii) the gender wage gap has fallen. In addition, (iii) income volatility has risen (Katz and Murphy (1992), Juhn et al. (1993), Goldin et al. (2006), Hong et al. (2015)). According to Heathcote et al. (2010), "Overall, the US wage structure has become more unequal,"(p. 3) and it has also changed to become more favorable to females.

In this situation, the increasing fertility trend of high-income women is quite

[1] This research was supported by Global PH.D Fellowship Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education (NRF-2015H1A2A1032085)

[2] We define female labor income as $wage \times hours$

[3] We define complete fertility as the number of children delivered by women ages 36-40. See Section 2 for more details.

puzzling if we simultaneously take classical fertility theories (CFTs) and recent changes in income structure into consideration. Recent examples in the extant fertility literature, for example, Becker (1960) and Croix and Doepke (2003), argue that a negative relationship between fertility and female labor income occurs, positing that a strong substitution effect reflecting the opportunity cost of having children outweighs a positive income effect.[4] Thus, when the wages of high-income females increase, as we have seen over last few decades, these theories predict that high-income females will reduce the number of children, which directly countermands the empirical observations.

Despite the discrepancy between recent empirical findings and existing theories, it is difficult to find related studies, with a few exceptions. Hazan and Zoabi (2014) found a similar phenomenon of highly educated females having more children than before. They introduced a time-outsourcing channel and argued that the relationship between female labor income and fertility has a U shape rather than inverted slash shape (see black solid and blue dashed lines in Panel (c) of Figure 1.1). Namely, there is a threshold for female income in which the income effect eventually outweighs the substitution effect. The researchers argue that their findings explain this puzzling observation. Additionally, Shang and Weinberg (2013) posited that increases in the use of personal services have reduced the burden of having children and may explain recent high fertility rates of female college graduates. Siegel (2016) argued that a husband spends more time working in the home as female wages increase. In this way, women maintain a steady fertility rate despite the rise in opportunity cost. All of these studies evince a common theme, having explored factors that reduce the magnitude of the substitution effect or, equivalently, the price of having children by introducing various time-outsourcing channels. We refer to the findings of these previous studies as the substitution-income effect approach (SIEA) for simplicity.

Contrary to this view, many CFTs have already argued that the substitution ef-

[4] See Jones and Tertilt (2008), Jones et al. (2010) for a more detailed explanation.

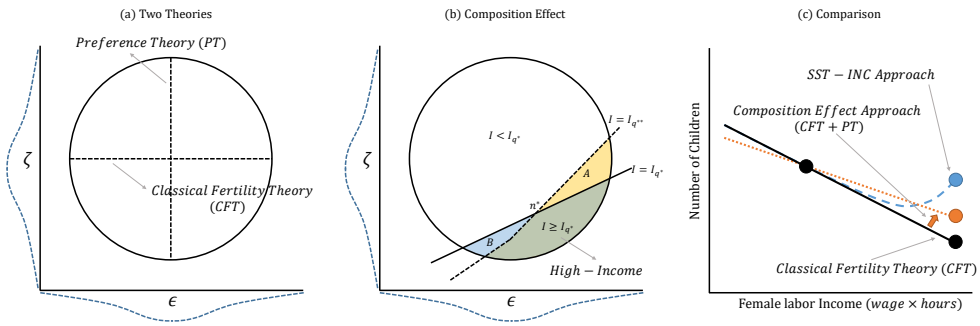

Figure 1.1: Classical Fertility Theory, Preference Theory, and Composition Effect

Note that ε means earning ability and ζ is preference on having children. In Panel (a), the horizontal line depicts Classical Fertility Theory (CFT) under fixed preference level while the vertical line illustrates Preference Theory. We harmonized theses two factors and augmented model. (circle)

fect is far greater than the income effect, even if we consider the time out-sourcing channel: for example, using a housekeeper and babysitting service (see Jones et al. (2010)). Therefore, questions remain regarding whether SIEA can exclusively explain observed phenomena quantitatively, although the arguments presented are theoretically acceptable. Additionally, we find that the estimated threshold income, which was arrived at using U.S. Census data (i.e., minimum point of U shape) is considerably high.[5] This means that a very small fraction (at most 2%) of the population should have a considerable number of children, which is incompatible with reality. Thus, we assert that there remains a significant element of this phenomenon which is unexplained by SIEA. [6]

Because existing approaches, including CFT and SIEA, cannot fully explain the recent puzzling phenomenon under reasonable settings, we require another explana-

[5]It was in the range of $83,866-$95,263 (adjusted by the 1999 dollar), as seen in Figure 1.17 and Table 1.9. The estimated fraction of people who are above the threshold was only 0.56% in 1990, and it had increased by 2.6% in 2010.

[6]Females in high-income families could use more medical treatment. Hazan and Zoabi (2014) demonstrated that the medical treatment does not have a significant effect on rises in fertility among highly educated women.

tion and an amplifying mechanism to supplement prior discussions. Toward this end, we suggest a new fertility choice model with a relaxed homogeneous preference regarding children, which had been assumed in most of the previous fertility literature. Even though this approach is relatively new in fertility studies related to economics[7], sociologists such as Hakim (2000, 2003) have already insisted upon the "preference theory." She has argued that preference per se is the most important factor in determining female behavior, such as fertility and workforce decisions, especially in modern, developed societies. We find that once we accept this view based on existing fertility theories, we are able to explain the puzzling empirical phenomenon without unrealistic assumptions regarding income and the substitution effect. Figure 1.1 helps to explain why the relaxation of homogeneous preference is important. As previously noted, ε stands for productivity (earning ability) and ζ represents preference for having children.[8] In summary, this study considers CFT (horizontal line)[9], adds a preference dimension (vertical line), and then harmonizes these two factors in one model together (circle). In this generalized environment, even under identical productivity levels, females can make different choices between employment outside the home and child-rearing following their comparative advantage. For example, given the same levels of productivity, a female who has a high preference for children chooses to have many babies and rear them, while a female who is less desirous of children decides to work more rather than have many children. Once we relax homogeneous preference, in response to the change in wage structure, it causes changes in the composition of (ε, ζ) in each income group via females' self-selection regarding labor supply and fertility. We call this variation the composition effect, and it serves

[7] Jones et al. (2010) suggests a static model, with a preference for heterogeneity in generating a negative relationship between female income and fertility under weak conditions. Gobbi (2013) adopts two types of preferences to explain the change in childless households in the U.S.

[8] These two characteristics are drawn from fixed distributions (blue dotted line), respectively.

[9] As CFT explains the effect of earning ability on fertility decisions under a fixed preference, we can depict it as a horizontal line on the $\varepsilon - \zeta$ plan, as shown in Panel (a) in Figure 1.1. Note that the slope of the iso-income curve becomes steeper when female wages increase as the role of productivity (ε) becomes more important in their labor and fertility decisions.

as the primary workhorse in this study. The reasoning is simple and straightforward. Suppose that the area under the curve $I = I_{q^*}$ (iso-income curve) in Panel (b) of Figure 1.1 indicates the highest income quintile (top 20%).[10] When the wage gap is small, the preference for having children (ζ) is relatively more important than earning ability (ε) in maximizing utility. Thus, females who have both high ζ and ε have many children and work less despite their high earning ability.(Area A in Panel (b)) This places them outside the high-income group. (outside the iso-income curve, $I = I_{q^*}$) Namely, ζ is more influential in determining female labor supply and fertility choice (equivalently, the labor income group), a finding which is consistent with the preference theory previously cited. In contrast, the slope of the iso-income curve becomes steeper ($I = I_{q^{**}}$) when the wage gap increases because high ζ and ε females (A in Panel (b)) now increase their labor supply due to the rise in forgone salary, which places them in the high-income group in the new economy. This brings the inflow of females who desire children (A in Panel (b)) into the high-income group while causing an outflow of females who are less desirous of children. (B in Panel (b)) Consequently, the average level of ζ of the highest income quintile increases following the change in wage structure. Thus, if the composition effect, which represents the influx of females desirous of children into the high-income group, outweighs the decrease in fertility due to the rise in opportunity cost, the average number of children in this group should increase.[11]

We can compare the distinct aspects of the new model with the composition

[10] Since higher ζ decreases female labor income as females reduce the number of children and increase labor supply while ε increases labor income, the iso-income curves that divide each income quintile should increase on the $\varepsilon - \zeta$ plan.

[11] Our model is also consistent with classical Roy (1951) and Borjas (1987) self-selection models. They explain the mechanism of job selection and immigration decisions for the agent who has two distinct skills. Similar to their logic, we also exploit the competitive advantage between ε and ζ in utility maximization. Furthermore, our study can be seen as an extension of abundant literature about preference heterogeneity, which has been investigated extensively in economics. For instance, Burtless and Hausman (1978) adopted preference heterogeneity to investigate the effect of taxation on the labor supply; this was done through the relative magnitude of income and substitution effect in different preference groups. Hausman (1982) and Blomquist (1983) offer other strong examples.

effect through Panel (c) of Figure 1.1. Both CFT and SIEA explain the movement along the fixed relationship between fertility and female labor income in response to the change in wage structure (black solid and blue dashed lines). On the other hand, the model with the composition effect induces the movement of the relationship itself from the black solid line to the orange dotted line. In summary, the mechanism suggests the possibility that the highest income group has more children in response to the unequal wage changes without unrealistic assumptions regarding the relative size of income and the substitution effect. We find that the model with the composition effect explains the changes in fertility rates across income groups at a rate of about 55% under reasonable calibration, while any other model without this mechanism fails to properly account for the phenomenon.

Based on our new model, we also assess the effect of individual income shocks and how they are related to household fertility choices. We consider three factors: changes in 1) skill premium, 2) gender wage gap, and 3) labor income volatility. Because these changes fundamentally coincide, it is difficult to explain their individual effect on fertility decisions in the statistical analysis. Thus, we propose a quantitative general equilibrium overlapping generations model (OLG) with abundant heterogeneity, and we not only analyze the total effect of these three factors, but also break down individual effects. We find that the change in skill premium is the most influential factor in generating the observed phenomena. As it potentiates a rise in skill premium, more young women decide to attend college, which causes more women desirous of children to be relocated to the college group in the new steady states. This results in rises in fertility rates of female college graduates who are concentrated in the high-income group. The change in the skill premium solely accounts for 50% of the variation in the data. The change in the gender wage gap intensifies the increase in the fertility rates of high-income females, although its explanatory power is not high overall. Finally, the variation in income volatility yields a distinct change in the reverse direction of the effects driven by the previous two factors. In this experiment,

females who are less desirous of children move to the college group in the new steady state as the wage volatility of the high school graduates increases more than that of the college group. This reduces the number of children of female college graduates, and it contributes in the prevention of an excessive rise in fertility levels among the college group in the model.

We derive two implications from the structural model. First, the composition effect, which represents the average preference for having children (ζ) among the high-income group, increases, while those of the other groups decrease. To investigate whether this theoretical derivation is supported by empirical evidence, following Miller et al. (2010), we adopt the question of the "ideal number of children" in a general social survey (GSS) as a proxy variable for the preference on children. We find that the observed change in the "ideal number of children" across income groups and the simulated results from the model demonstrate similar movement. The second implication is that the positive correlation between preference (ζ) and fertility decreases as the wage structure becomes more unequal. As two sides of the same coin, the negative relationship between preference and the female labor supply also decreases. We conduct empirical reduced-form regression analysis and find that these implications are supported by the evidence gathered from GSS data.

From a theoretical point of view, our model makes the following contributions. Because this study focuses on changes in income formation, such as skill premium and gender wage gap, it is essential to model the price variables with a sense of general equilibrium. As such, we made skill- and gender-biased technological changes in the firm side to generate the variation in income structure, as observed in the data. Toward this end, we adopted the home production model suggested by Greenwood et al. (2005) and Hazan and Zoabi (2014) and extended their theoretical framework to a general equilibrium dynamic problem. Based on this general equilibrium environment, we offer a firm micro foundation to explain why skill premium and gender wage gap changed instead of imposing exogenously chosen prices on the model.

This allows us to analyze the phenomena on a more rigorous basis. Another theoretical contribution of the study is that we adopt the concept of an incomplete financial market. Unlike Greenwood et al. (2005), which affected the work of Hazan and Zoabi (2014), our model explicitly considers uninsurable idiosyncratic productivity risk under an incomplete financial market in the spirit of Huggett (1996) and Aiyagari (1994). According to Heathcote et al. (2010) and Hong et al. (2015), income volatility in skilled and unskilled sectors has fluctuated over the last few decades. Our model examines the effect of the change in income volatility through the stochastic productivity process and the incomplete financial market.

This study is organized as follows: Section 2 explains the observation of the changes in complete fertility and household income structure over the last three decades. Section 3 suggests the relaxation of the homogeneous preference assumption and proposes a detailed mathematical model. Section 3 explains the calibration process, while Section 5 discusses the results of a benchmark case. In Section 6, we impose changes in three factors of the model and analyze the combined and individual effects. We derive theoretical implications from the model and provide supporting empirical evidence in Section 7. Finally, we conclude in Section 6.

2 Facts

In this section, we discuss the change in the complete fertility rate across income groups and factors that have affected household income structure over the last three decades.

2.1 Complete Fertility: Target Moments

We compute the complete fertility rate (CFR) across the female labor income quintile by using U.S Census and American Community Survey (ACS) data. We calculate the average number of children of married females whose ages are in the range of 36-

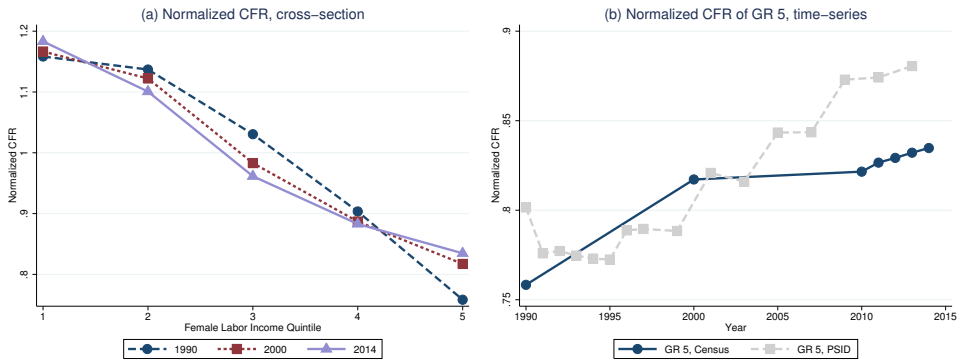

Figure 1.2: The Change in Complete Fertility

Source: IPUMS, U.S Census from 1970 to 2000 and American Community Survey from 2011 to 2014. PSID from 1990 to 2013. Note that we selected married white females whose age is in the range of 36-40 and married white males for this figure. Complete fertility is normalized by average complete fertility at each year to control for year-specific effects.

40.[12] We then normalize the CFR across the income groups by the total average CFR at each year to control for year-specific factors. Additionally, we define female labor income as $wage \times working\ hours$. Thus, this term, labor income, augments wages and the intensive and extensive margins of labor supply together. [13]

Panel (a) in Figure 1.2 shows the normalized CFR by female labor income quintiles from 1990 to 2014. We find that females in the highest quintile have increased

[12] According to Briley et al. (2017), "In the United States in 2010, over 85 % of period fertility resulted from individuals less than 34 years old, the youngest age observed in MIDUS (Human Fertility Database 2013). Additionally, 99 % of period fertility in the United States resulted from individuals under 41 years of age"

[13] Note that we fixed the female sample at age 36-40 to calculate the CFR in this exercise. However, it could be problematic if we compute the CFR by female labor income quintile although it is adequate to calculate the CFR itself. This is because the life-cycle income affects the CFR. This primarily depends on not only the present labor income at age 36-40, but also the sequence of household income during the whole period that females are fertile (henceforth, we call it fertile-lifetime) and after that. However, it is impossible to track down the full sequence of the labor income during the fertile-lifetime and after because of the characteristics of cross-sectional data of the U.S. Census. If we use panel data, such as the Panel Study of Income Dynamics (PSID), we can calculate the exact lifetime income and its dynamics, but we cannot figure out the change in the CFR, which is precisely due to the relatively small sample size. Because the primary purpose of this study is to examine the change in the CFR, we use the U.S. Census data to take advantage of the large sample size. Instead, we show that the female labor income at ages 36-40 represents fertile-lifetime labor income well by using PSID as a supplement in Appendix 9.5. Thus, we regard the labor income at age 36-40 as the representation of the fertile-lifetime labor income.

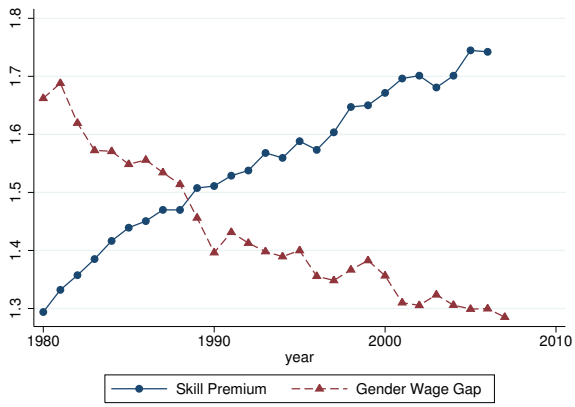

Figure 1.3: The Change in Skill Premium and Gender Wage Gap

Source: The U.S. Census Bureau calculations from Current Population Survey, Historical Income Table, Table P-38 and .Hong et al. (2015).

the CFR since the 1990s, and this has changed the shape of the relationship between female labor income and the CFR from the inverted slash(\) shape to the L-shaped curve. Panel (b) stands for the normalized CFR of the highest labor income quintile in the time-series manner. The normalized CFR in this group has increased gradually compared to the average(=1) at each year, and this is captured in both the U.S Census and PSID data. The CFR of the highest quintile was 76% of the average in 1990, but it rose to 84% in 2014. From these figures, we find that the fertility gap between the highest group and the average has been narrowed. That is, the growth rate of complete fertility among high-income females outweighs that of the average CFR. [14]

2.2 The Change in Income Structure: Input

During the last three decades, wage structure has also changed dramatically in the United States. The skill premium has risen, while the gender wage gap has narrowed

[14]Even though we empirically observe the decrease in the fertility gap between the high-income females and the average, existing literature fails to explain this trend. For example, Croix and Doepke (2003) expects that the fertility differential between the high- and low-income groups will increase as income inequality increases because the high-income group reduces the number of children due to the rise in their opportunity cost based on classical fertility theories.

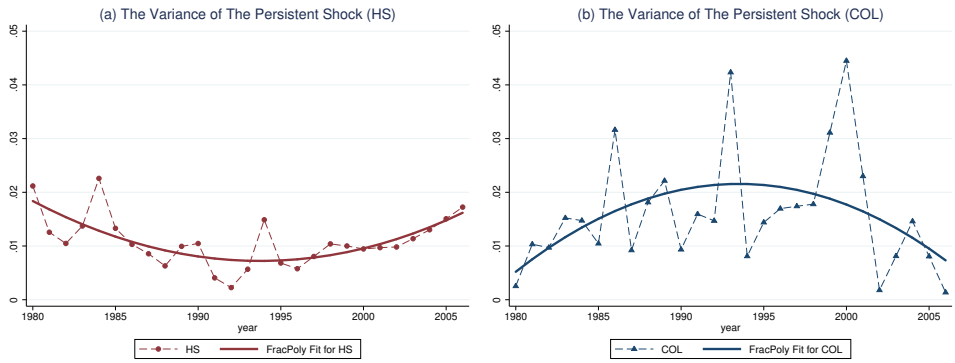

Figure 1.4: The Variance in the Persistent Shock by Educational Groups

The dashed lines show the raw values adopted from Hong et al. (2015), and the solid lines illustrate the fitted value by using a fractional polynomial fitting method to see the tendency.

(see Figure 1.3). The rise in the skill premium has gradually increased labor income inequality between educational groups (Katz et al. (1999)) and the narrowing gender wage gap has increased the female labor supply (Doepke (2015), Siegel (2016)). We also focus on labor income volatility. According to Heathcote et al. (2010) and Hong et al. (2015), income volatility has fluctuated over the last few decades, as depicted in Figure 1.4. Juhn et al. (1993) argue that increasing income volatility causes a rise in income dispersion and affects income inequality within the same educational group. Panel (a) of Figure 1.4 shows the variance in the persistent shock among high school graduates (HS), and Panel (b) shows the variance among college graduates (COL). We find that the variance in the persistent shock has increased gradually for high school graduates since 1990s, whereas college graduates have more fluctuated shocks, and this tendency decreased after the late 1990s.

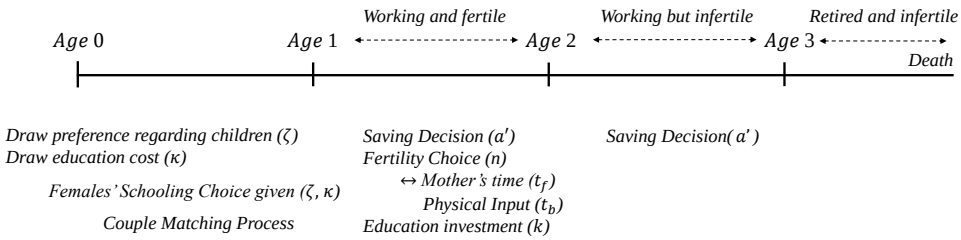

Figure 1.5: Time Horizon of the Model

3 The model

3.1 Preliminaries

Based on the concept suggested in the Introduction, the composition effect, we build a general equilibrium overlapping generations model with fertility choice and ex-ante preference heterogeneity toward having children. Following Greenwood et al. (2005) and Hazan and Zoabi (2014), we adopted a home production model regarding childbirth. We then extended it to a general equilibrium setting with a heterogeneous agent under the incomplete financial market in the spirit of Aiyagari (1994) and Huggett (1996).

Figure 1.5 illustrates the time horizon of the dynamic model. The decision unit of the economy is a single female in period zero, and after that, the household consisting of two distinct genders, a husband (m) and a wife (f). In period zero, a female draws her preference for having children (ζ) and her educational cost (κ) from the fixed distributions. After determining these characteristics, they choose whether or not to go to college based on their state variable, (ζ, κ). Once they have determined their educational level, it does not change through their entire life. Each female is matched to a male stochastically according to the matching probability depending on her educational level, and together they comprise a household. We assume that the marriage is an absorbing state so that divorce does not occur in the model. The one

period in the model is 20 years in reality, which is indexed by $i \in \{0,1,2,3\}$, and the ages of both spouses are in the same age category. At age one, a household is able to have children and work. If they have children in this period, they should spend both mother's time (t_f) and the physical resources (t_b) to take care of their children. In addition, they should input another resource to educate their children (k), which determines the quality of their children. At age two, households can work but they become infertile. They retire at the end of age two, and they die without any uncertainty at the end of age three.

3.2 Household Problem

3.2.1 Period One: Young Adult

$$V_1(\underbrace{a, \varepsilon_f, \varepsilon_m}_{variant}; \underbrace{\zeta, e_f, e_m}_{invariant}) = \max_{a',k,t_f,t_b} \ln(c) + \zeta \ln(n \cdot q(k)) + \beta \mathbb{E} V_2(a', w'_f, w'_m) \tag{1.1}$$

such that

$$c + \underbrace{w_f^e \varepsilon_f^e t_f + w_b \varepsilon_b t_b}_{child-rearing\,cost} + p_e \varepsilon_e k n + a' = w_m^e \varepsilon_m^e + w_f^e \varepsilon_f^e + (1+r)a \tag{1.2}$$

$$n = Z t_f^{\phi(\zeta)} t_b^{1-\phi(\zeta)} \in [0, \infty) \tag{1.3}$$

$$\phi(\zeta) = \min\left\{\max\left[\phi_b + \phi_{adj} \cdot \left(\zeta - \bar{\zeta}\right), 0\right], \bar{\phi}\right\} \tag{1.4}$$

$$q = k^{\theta} \tag{1.5}$$

$$w_b = \sum_{e,g} \mu_g^e w_g^e, \;\; p_e = \sum_g \mu_g w_g^h \tag{1.6}$$

$$a \geq 0, c \geq 0, t_f \in [0,1] \tag{1.7}$$

$$\varepsilon_{g,t}^e = \rho^e \varepsilon_{g,t-1}^e + \nu^e, \nu^e \sim \mathbf{N}\left(0, \sigma_{\nu,e}^2\right) \tag{1.8}$$

The dynamic model is constructed by using Bellman's equation. The above formula indicates the household problem in period one. In this period, households can supply the labor force and have babies. Households are comprised of two different genders, female (f) and male (m), and they decide the amount of savings (a'), educational investment for their children (k), mother's child-rearing time (t_f) or working time ($1-t_f$) equivalently, and another input for rearing children (t_b). Once they determine t_f and t_b, the number of children is determined through the home production function, $n(t_f, t_b, \zeta)$. We set home production as a form of the Cobb-Douglas function, which means that the mother's time, t_f, and child-rearing input, t_b are the substitutes, and the elasticity of substitution is equal to one.[15] The $\phi(\zeta)$ stands for the importance of the mother's time in the home production function, which depends on the ζ. Because this model does not include the labor decision in the utility function, we calculate it according to the residuals of the mother's time. Thus, $\phi(\zeta)$ represents the simple equipment used to generate a sufficient variation in the female labor supply across the labor income group. Following Becker (1960)'s quantity-quality choice approach, the amount of educational investment formulates children's quality via the quality function $q(k)$. Note that a household earns utility from $nq(k)$, which means the number and quality of children are substitutes for each other. In this model, households have ex-ante and ex-post heterogeneities in that they have different attitudes toward having children (ζ), assets (a), earning abilities or productivity (ε_f, ε_m), and education (e_f, e_m), which are considered the state variables. Equation 1.8 indicates the idiosyncratic productivity following the AR (1) process. Note that individuals have different productivity processes according to their educational level. Following Heathcote et al. (2010), we set the different genders in the same educational group and also follow the same stochastic process. w_b is the price of the child-rearing input, which is calculated as the weighted average of the wages of both genders and

[15] See Bar et al. (2015) for a more general version of the home production function. They suggest the nested CES home production function, and substitutability can be manipulated.

educations. ε_b indicates the price adjustment factor to match the data, but we can also interpret this value as an efficiency unit in the child-related goods sector. Similarly, p_e is the price of the educational investment, and ε_e is the price adjustment component, which is later calibrated endogenously.

The equivalent of the above problem is as follows.[16]

$$V_1(a, \varepsilon_f, \varepsilon_m; \zeta, e_f, e_m) = \max_{a', k, n} \ln(c) + \zeta \ln(n \cdot q(k)) + \beta \mathbb{E} V_2(a', \varepsilon'_f, \varepsilon'_m) \tag{1.9}$$

such that

$$c + p^*(n) + p_e \varepsilon_b k n + a' = w_m^e \varepsilon_m^e + w_f^e \varepsilon_f^e + (1+r)a \tag{1.10}$$

Where, $p^*(n)$ is the minimized child-rearing cost function specified below.

Cost Minimization

Households decide mother's time (t_f) and another input (t_b) for child rearing given the number of children. Thus, the minimization problem specified is as follows.

$$p^*(n) = \min_{t_w, t_b} w_f^e \varepsilon_f^e t_f + w_b \varepsilon_b t_b \tag{1.11}$$

such that

$$n = Z t_f^{\phi(\zeta)} t_b^{1-\phi(\zeta)} \tag{1.12}$$

The optimal choice of t_f and t_b are:

$$t_b^*(n; \varepsilon_f, e_f, \zeta) = \underbrace{\frac{1}{Z} \left(\frac{w_f^e \varepsilon_f^e}{w_b \varepsilon_b} \cdot \frac{1-\phi(\zeta)}{\phi(\zeta)} \right)^{\phi(\zeta)}}_{\equiv c_b\left(w_f^e \varepsilon_f^e / w_b \varepsilon_b, \zeta\right)} n \tag{1.13}$$

$$= c_b \left(w_f^e \varepsilon_f^e / w_b \varepsilon_b, \zeta \right) n \tag{1.14}$$

[16] As we use log utility function and adopt home production regarding the number of children as in Greenwood et al. (2005) and Hazan and Zoabi (2014), we can derive a closed-form relationship in equation 1.14, 1.16, and 1.18.

$$t_f^*(n;\varepsilon_f,e_f,\zeta) = \underbrace{\frac{1}{Z}\left(\frac{w_b\varepsilon_b}{w_f^e\varepsilon_f^e}\cdot\frac{\phi(\zeta)}{1-\phi(\zeta)}\right)^{1-\phi(\zeta)}}_{\equiv c_f\left(w_f^e\varepsilon_f^e/w_b\varepsilon_b,\zeta\right)} n \tag{1.15}$$

$$= c_f\left(w_f^e\varepsilon_f^e/w_b\varepsilon_b,\zeta\right)n \tag{1.16}$$

The relative unit wage, $w_f^e\varepsilon_f^e/w_b\varepsilon_b$, determines both t_b^* and t_f^* given n. If the mother's unit wage, $w_f^e\varepsilon_f^e$, is relatively higher than the price of the input for child rearing, $w_b\varepsilon_b$, which substitutes for mother's time, the amount of the optimized t_b increases while the amount of the mother's time, t_f^*, decreases.

Thus, the optimized cost is a linear function of the number of children, as follows.

$$p^*(n;w_f^e,\varepsilon_f^e,e_f,w_b,\zeta) = w_f^e\varepsilon_f^e t_f^* + w_b\varepsilon_b t_b^* \tag{1.17}$$

$$= \underbrace{p_n(w_f^e\varepsilon_f^e,w_b\varepsilon_b,\zeta)}_{\text{unit price of children}}\cdot n \tag{1.18}$$

where

$$p_n \equiv \frac{1}{Z}\left\{\left[\left(\frac{\phi(\zeta)}{1-\phi(\zeta)}\right)^{1-\phi(\zeta)} + \left(\frac{1-\phi(\zeta)}{\phi(\zeta)}\right)^{\phi(\zeta)}\right]\cdot w_b\varepsilon_b\left(\frac{w_f^e\varepsilon_f^e}{w_b\varepsilon_b}\right)^{\phi(\zeta)}\right\} \tag{1.19}$$

Proposition 1. *p_n is either an increasing or an inverted U-shape function of $\phi(\zeta)$ if $w_f^e\varepsilon_f^e/w_b\varepsilon_b$¿1.*

Definition 1. *Define $\bar{\phi}_p$, which satisfies that the unit price of children, p_n, rises in $\phi(\zeta)$ in the range of $\left[0,\bar{\phi}_p\right]$ and decreases after that.*

Assumption 1. *ζ satisfies that $\phi(\zeta) < \bar{\phi}_p$ and $w_f^e\varepsilon_f^e/w_b\varepsilon_b$¿1. It is equivalent that p_n is increasing function of $\phi(\zeta)$.*

Once we have minimized cost function, $p^* = p_n n$, we can determine the amount of the resource for education (k^*) by solving the utility maximization problem as in

equation 1.9.

$$k^* = \max\left\{0, \frac{\theta}{1-\theta}\frac{p_n(w_f^e\varepsilon_f^e, w_b\varepsilon_b, \zeta)}{p_e\varepsilon_e}\right\} \tag{1.20}$$

From the equation 1.20, we can see that educational investment, k^*, has a positive relationship with the unit price of children, p_n, and a negative connection with the price of the education, $p_e\varepsilon_e$. The efficiency parameter of the quality function, θ, has a positive partial effect on the k^*.

Given k^*, the optimal number of children, n^*, is determined jointly with the savings decision, a'^*. As shown in equation 1.21, this is constituted by the attitude on having children(ζ), household income without considering child care, the marginal price of children(p_n), and the amount of the educational investment per child($p_e\varepsilon_e k^*$).

$$n^* = \underbrace{\frac{\zeta}{1+\zeta}}_{\text{preference}(+)} \cdot \frac{\overbrace{w_f^e\varepsilon_f^e + w_m^e\varepsilon_m^e + (1+r)a - a'^*}^{\text{household income without children}(+)}}{\underbrace{p_n(w_f^e\varepsilon_f^e, w_b\varepsilon_b, \zeta) + p_e\varepsilon_e k^*}_{\text{marginal price for rearing children}(-)}} \tag{1.21}$$

We can derive the following propositions by using the above optimality conditions. Note that we do not consider the general equilibrium effect here to making the discussions more evident.

Proposition 2. *n^* is either an increasing or an inverted U-shape function of $\phi(\zeta)$.*

Definition 2. *Define $\bar{\phi}_n$, which satisfies that the optimal number of children, n^*, rises in $\phi(\zeta)$ in the range of $[0, \bar{\phi}_n]$ and decreases after that.*

Definition 3. *Define $\bar{\phi}$ which satisfies that $\bar{\phi} = \min(\bar{\phi}_n, \bar{\phi}_p) \in [0, 1]$*

Proposition 3. *If $0 \leq \phi(\zeta) < \bar{\phi}$, then $t_f^*(\cdot)$, $p_n(\cdot)$, $k^*(\cdot)$, and $n^*(\cdot)$ is monotonically increasing in ζ.*

As seen in the equation 1.4, we set the upper bound of the importance of mother's time, $\phi(\zeta)$, to $\bar{\phi}$. This restriction simplifies the model in that the optimality conditions

are monotonically increasing in ζ as stated in proposition 3.[17]

Proposition 4. *The unit price of children, p_n, is increasing and concave in the mother's unit wage, $w_f^e \varepsilon_f^e$.*

Proposition 5. *The optimal number of children, n^* is a U-shaped function of $w_f^e \varepsilon_f^e$, and $\exists \left(w_f^e \varepsilon_f^e\right)^*$ such that $\partial n^* / \partial \left(w_f^e \varepsilon_f^e\right) = 0 \; \forall \zeta$ satisfies $0 \leq \phi(\zeta) < \bar{\phi}_n$.*

Our model also includes the substitution-income effect mechanism, which has been studied in previous literature (Shang and Weinberg (2013), Hazan and Zoabi (2014), Siegel (2016).) As the unit wage of female labor increases, the unit price of children, p_n, also increases, which reflects females' opportunity cost. However, its marginal rate of increment decreases due to the substitutability, $\phi(\zeta)$, between t_f and t_b in the home production function (see proposition 4.) Due to this concavity, there exists a specific unit wage, $\left(w_f^e \varepsilon_f^e\right)^*$, which causes the income effect (numerator in equation 1.21) to eventually outweigh the substitution effect (denominator in equation 1.21). Note that this concavity is not necessary in our model including ex-ante preference heterogeneity to generate the puzzling phenomenon previously identified. We include this mechanism, however, to compare the results of the models with and without ex-ante preference heterogeneity under the reasonable calibration in Section 6.

Finally, households solve their savings problem, which should satisfy the intertemporal first-order condition, as shown below.

$$\frac{1}{c\left(a, \varepsilon_f, \varepsilon_m; \zeta, e_f, e_m\right)} = \beta \frac{\partial \mathbb{E} V_2(a', \varepsilon_f', \varepsilon_m')}{\partial a'} \tag{1.22}$$

[17] If we do not consider this restriction, there exist decreasing parts of the optimality conditions in proposition 3 as ζ increases are sufficient. This makes our result more complex and difficult to interpret, but the benefit from the generalization is negligible, as both results are similar.

3.2.2 Period Two: Middle Adult

In period two, households still participate in the labor market, but they are not fertile anymore. We also assume that their children leave their home in this period. Thus, the household problem does not have any variable related to children.

$$V_2(a, \varepsilon_f, \varepsilon_m; e_f, e_m) = \max_{a'} \ln(c) + \beta V_3(a') \tag{1.23}$$

$$= \max_{a'} \ln(c) + \beta \ln(s + (1+r)a') \tag{1.24}$$

such that

$$c + a' = w_m \varepsilon_m^e + w_f \varepsilon_f^e + (1+r)a \tag{1.25}$$

Following the standard backward induction method, the solution of the problem in period two is :

$$a'^*(a, \varepsilon_f, \varepsilon_m; e_f, e_m) = \frac{\beta}{1+\beta}\left[w_m \varepsilon_m^e + w_f \varepsilon_f^e + (1+r)a\right] \tag{1.26}$$

3.2.3 Period Three: Old

In the last period, households retire and spend their entire wealth because the model has neither the bequest motive nor stochastic death. Hence, the problem is reduced to a simple static setting, as shown below.

$$V_3(a; e_f, e_m) = \max_{a'} \ln(c) \tag{1.27}$$

such that

$$c + a' = (1+r)a \tag{1.28}$$

The policy function is:

$$a'^{*}(a; e_f, e_m) = 0 \tag{1.29}$$

3.2.4 Period Zero: Education Choice and Marriage

Before starting her adult life, an unmarried female decides her educational level. She compares the expected lifetime utilities when she graduates high school or college in period zero given the educational cost, κ, the attitude on having children, ζ, and the conditional matching probability, $\pi(\cdot)$. Let l be the high school and h be the college. $M(\cdot)$ indicates the expected lifetime utility at period zero, and $e(\cdot)$ is the educational decision function. Note that this model only considers the educational choice of females. The educational status of males is determined after the females' decision by the matching probability given exogenously. Once females make a decision about their educational level, it remains unchanged throughout their lives.

$$M(l_f; \zeta) = \pi(l_f, l_m) \cdot \mathbb{E}_{\varepsilon_f, \varepsilon_m}[V_1(l_f, l_m, \zeta)] + \pi(l_f, h_m) \cdot \mathbb{E}_{\varepsilon_f, \varepsilon_m}[V_1(l_f, h_m, \zeta)] \tag{1.30}$$

$$M(h_f; \zeta, \kappa) = \pi(h_f, l_m) \cdot \mathbb{E}_{\varepsilon_f, \varepsilon_m}[V_1(h_f, l_m, \zeta)] + \pi(h_f, h_m) \cdot \mathbb{E}_{\varepsilon_f, \varepsilon_m}[V_1(h_f, h_m, \zeta)] - \kappa \tag{1.31}$$

$M(l_f; \zeta)$ indicates the expected lifetime utility when a female chooses not to go to college (l_f) given her ζ. The first term shows the expected lifetime value when she marries a high school educated male given her educational level (l_f.) The second term indicates the expected utility if she marries a college educated male. $M(h; \zeta, \kappa)$ can be explained in the same way, but there is another argument, κ. When a female decides to attend college, she should pay the educational cost, which translates to disutility. Finally, a female makes an educational decision (e) based on each expected lifetime value as follows.

$$e(\kappa,\zeta) = \begin{cases} l_f & \text{if} M(l_f,\zeta) > M(h_f,\zeta,\kappa) \\ h_f & \text{otherwise} \end{cases} \tag{1.32}$$

where

$$\zeta \sim \mathbf{N}\left(\mu_\zeta, \sigma_\zeta^2\right) \tag{1.33}$$

$$\log\kappa \sim \mathbf{N}\left(\mu, \sigma_{\log\kappa}^2\right) \tag{1.34}$$

3.2.5 Firm

The firm has a standard Cobb-Douglas production function. The representative firm produces the goods by using aggregate capital, K, and aggregated efficiency labor, L. Similar to Katz and Murphy (1992), Heckman et al. (1998), Heathcote et al. (2010), Hong et al. (2015), we set the aggregate labor supply, L, which is aggregated via the CES function.

$$Y = A \cdot K^{\alpha} L^{1-\alpha} \tag{1.35}$$

$$L \equiv \left[\underbrace{\lambda_h^S \left(\underbrace{\lambda_f^G L_{f,h} + \lambda_m^G L_{m,h}}_{S} \right)^{\frac{\psi-1}{\psi}} + \lambda_l^S \left(\underbrace{\lambda_f^G L_{f,l} + \lambda_m^G L_{m,l}}_{U} \right)^{\frac{\psi-1}{\psi}}}_{H} \right]^{\frac{\psi}{\psi-1}} \tag{1.36}$$

where, $\lambda_f^G + \lambda_m^G = 1$ and $\lambda_h^S + \lambda_l^S = 1$.

The efficiency labor for both genders has a perfect substitution relationship, and $\left\{\lambda_f^G, \lambda_m^G\right\}$ denotes the average weights between gender-specific efficiency labor inputs. We define the aggregate efficiency labor of college and high school graduates as S (Skilled) and U (Unskilled), respectively, as shown in equation 1.36. We set the

elasticity of substitution between two types of efficiency labor to ψ and set weights of skilled and unskilled labor to $\left\{\lambda_f^S, \lambda_m^S\right\}$.

On the firm side, the demand for capital and each type of labor is determined by their productivity, as follows.

$$r+\delta = MPK = \alpha\left(\frac{L}{K}\right)^{1-\alpha} \tag{1.37}$$

$$w_{f,h} = A(1-\alpha)\lambda_h^S\lambda_f^G\left(\frac{K}{L}\right)^{\alpha}\left(\frac{L}{S}\right)^{\frac{1}{\psi}} \tag{1.38}$$

$$w_{m,h} = A(1-\alpha)\lambda_h^S\lambda_m^G\left(\frac{K}{L}\right)^{\alpha}\left(\frac{L}{S}\right)^{\frac{1}{\psi}} \tag{1.39}$$

$$w_{f,l} = A(1-\alpha)\lambda_l^S\lambda_f^G\left(\frac{K}{L}\right)^{\alpha}\left(\frac{L}{U}\right)^{\frac{1}{\psi}} \tag{1.40}$$

$$w_{m,l} = A(1-\alpha)\lambda_l^S\lambda_m^G\left(\frac{K}{L}\right)^{\alpha}\left(\frac{L}{U}\right)^{\frac{1}{\psi}} \tag{1.41}$$

Using these prices, we can derive the skill premium and the gender wage gap, as shown in equations 1.42 and 1.43. We will impose gender- and skill-biased technological change by manipulating the set of values, $\left\{\lambda_f^G, \lambda_m^G, \lambda_l^S, \lambda_h^S\right\}$ in Section 6 and analyze their effects on the fertility decision and macro variables.

$$GWG = \frac{w_m^e}{w_f^e} = \frac{\lambda_m^G}{\lambda_f^G} \tag{1.42}$$

$$SKP = \frac{w_g^h}{w_g^l} = \frac{\lambda_h^S}{\lambda_l^S}\left(\frac{U}{S}\right)^{\frac{1}{\psi}} \tag{1.43}$$

Note that the gender wage gap is determined by the ratio between aggregate weights, $\left\{\lambda_f^G, \lambda_m^G\right\}$. In the case of the skill premium, however, the ratio of augmented weights, w_g^h/w_g^l, not only affects the level of the skill premium, but also the ratio of U and S.

3.2.6 Aggregation

Individual decisions are aggregated using the measure of state variables. $\mathcal{S}$ indicates the state space, which is expressed as a Cartesian product of the asset space, $\mathcal{A} \in \mathbb{R}^+$, with two productivity spaces for each gender, $\mathcal{E}_g \in \mathbb{R}^+$, preference space, $\mathcal{Z} \in \mathbb{R}$, and two educational spaces for each gender, $\mathcal{D}_g \in [l,h]$. Let Σ_S be the $\sigma - algebra$ on $\mathcal{S}$ and define the measurable space $(\mathcal{S}, \Sigma_S)$. Note that μ_j is the measure of the households whose age is j, and $\psi_j(\cdot)$ stands for the measure of the states variables at age j.

$$s \in \mathcal{S} = \mathcal{A} \times \mathcal{E}_f \times \mathcal{E}_m \times \mathcal{Z} \times \mathcal{D}_f \times \mathcal{D}_m \tag{1.44}$$

$$\psi_{j+1}(\mathcal{S}) = \int_s Q(s,\mathcal{S})\, d\psi_j \; \forall \mathcal{S} \in \Sigma_S,\; j > 1 \tag{1.45}$$

where

$$Q(s,\mathcal{S}) = 1\left\{j+1, a_{j+1} \in \mathcal{A}, \zeta \in \mathcal{Z}, e_f \in \mathcal{D}_f, e_m \in \mathcal{D}_m\right\} \tag{1.46}$$

$$\times Pr\left\{\varepsilon_{f,j+1} \in \mathcal{E}_f, \varepsilon_{m,j+1} \in \mathcal{E}_m | \varepsilon_{f,j}, \varepsilon_{m,j}\right\} \tag{1.47}$$

K denotes the aggregate capital and S_g denotes aggregated skilled labor for specific gender g. U_g indicates the aggregated unskilled labor supply. These aggregates are determined at the levels satisfying the household's supply and firm's demand in the equilibrium, which creates a set of equilibrium prices.

$$K = \sum_j \mu_j \sum_{\varepsilon_f, \varepsilon_m, e_f, e_m} \int_{a \times \zeta} a d\psi(a, \varepsilon_f, \varepsilon_m; \zeta, e_f, e_m) \tag{1.48}$$

$$S_f = \sum_j \mu_j \sum_{\varepsilon_f, \varepsilon_m, e_m} \int_{a \times \zeta} (1 - t_f) \varepsilon_f^e d\psi(a, \varepsilon_f, \varepsilon_m; \zeta, e_f = h, e_m) \tag{1.49}$$

$$U_f = \sum_j \mu_j \sum_{\varepsilon_f, \varepsilon_m, e_m} \int_{a \times \zeta} (1 - t_f) \varepsilon_f^e d\psi(a, \varepsilon_f, \varepsilon_m; \zeta, e_f = l, e_m) \tag{1.50}$$

$$S_m = \sum_{j=1} \mu_j \sum_{\varepsilon_f, \varepsilon_m, e_m} \int_{a \times \zeta} \varepsilon_m^e d\psi(a, \varepsilon_f, \varepsilon_m; \zeta, e_f = h, e_m) \tag{1.51}$$

$$U_m = \sum_j \mu_j \sum_{\varepsilon_f, \varepsilon_m, e_m} \int_{a \times \zeta} \varepsilon_m^e d\psi(a, \varepsilon_f, \varepsilon_m; \zeta, e_f = l, e_m) \tag{1.52}$$

3.2.7 Recursive Stationary Equilibrium

We can define the recursive stationary equilibrium, as shown below, with the factors that we constructed in the previous sections.

Definition. *The recursive stationary equilibrium is the set of value functions,* $M(e)$ and $V_j(s)$*; policy functions* $e(\kappa, \zeta)$, $c_j(s)$, $a'_j(s)$,$l_{f,j}(s)$, $t_w(s)$, $t_b(s)$, $n(s)$, $k(s)$ *prices* r *and* w_g^e*; factor demand* K, S_f, S_m, U_f, U_m*; and distribution* ψ_j*such that:*

1. *The value functions and policy functions solve the household's utility maximization problem, given the prices and distribution of state variables.*

2. *Prices satisfy the firm's profit maximization problem*

3. *Markets are cleared*

4. *Distributions are consistent with individual behavior, as in equation 1.45.*

4 Matching the Model to U.S. Data

4.1 Parameters

External Parameters

In this subsection, we describe how the externally calibrated parameters are determined. Note that the values in this section are suggested in an annual unit. Table 1.1 represents the summary of the externally calibrated parameters. We set the population growth rate of 0.1% and the capital depreciation rate (δ) to 0.06 according to Heathcote et al. (2010). The capital share (α) is set to 0.36 following Aiyagari (1994), and the matching probability of female (π) is calculated from the 1990 U.S Census. We construct the wage structure following the AR (1) process by using the values estimated by Hong et al. (2015). They suggest the related parameters according to educational groups, which is compatible to our specification. The persistence parameter (ρ_{edu}) for the high school graduates is 0.986, and the value for the college graduates is 0.984. The variance of the persistent productivity shock ($\sigma^2_{w,edu}$) is 0.07 for the high school group and 0.18 for the college graduates. Note that we take a ±3-year window and calculate the average for smoothing values.

Internal Estimation

There are 12 parameters, which are estimated inside the model according to the simulated method of moments. First, the utility discount factor, β, is determined to match the capital-output (K/Y) ratio. As we have the ex-ante heterogeneous preference for having children and the educational cost, we must set the distribution of theses values. We adopt normal distribution with regard to both ζ and $\log\kappa$, respectively. We set the $\mu_{\log\kappa}$ and $\sigma^2_{\log\kappa}$ to match the average complete fertility rate and the childless ratio in 1990. We also determine $\mu_{\log\kappa}$ and $\sigma^2_{\log\kappa}$ to correspond to the female college graduate rate in 1990 and 2000, respectively. We now consider the value $\phi(\zeta)$, which

Externally Calibrated	Value	Explanation
Population growth rate	0.011	Heathcote et al. (2010)
Capital share (α)	0.36	Aiyagari (1994)
The elasticity of substitution between skilled and unskilled labor (ψ)	1.43	Katz and Murphy (1992)
Capital depreciation rate (δ)	0.06	Prescott (1986)
Matching probability of HS_f, $\pi(f_l, m_l)$	0.79	The U.S Census 1990
Matching probability of COL_f, $\pi(f_h, m_h)$	0.71	-
Persistence parameter for HS (ρ_{HS})	0.986	Hong et al. (2015)
Persistence parameter for COL (ρ_{COL})	0.984	-
Var of persistent shock for HS ($\sigma^2_{w,e=l}$)	0.07	Take the average on ± 3 years window
Var of persistent shock for COL ($\sigma^2_{w,e=h}$)	0.18	-

Table 1.1: Externally Calibrated Parameters

indicates the importance of the mother's time in the home production function. It has two parts, which are ϕ_b and $\phi_{adj}(\zeta - \bar{\zeta})$, as in equation 1.4. This value determines the child-rearing time of a mother or female labor supply equivalently. Thus, the first part, ϕ_b, is calibrated to match the average female labor supply, and the adjustment factor (ϕ_{adj}) that makes ϕ depend upon ζ is determined to generate the slope of the female labor supply across the income quintile. There are two price adjustment factors, ε_b and ε_e. These parameters adjust the price of the child-rearing input $(w_b\varepsilon_b)$ and the educational investment $(p_e\varepsilon_e)$ to match the child-rearing cost per child to household labor income ratio(=21%) and the educational investment per child to the household labor income ratio (=2%) following Lino (1996). We set θ in the quality function of children to account for the educational expenditure by income group. Finally, we determine the technological weight (λ^S_e) between skilled and unskilled labor to match the skill premium in 1990. Similarly, we set the gender weight (λ^G_g)) to generate the

Internal Calibration	Value	Target Moments
Utility discount factor(β)	0.99	$K/Y = 3.0$
μ_ζ	1.87	The average CFR
σ^2_ζ	2.00	The childless ratio, Chandra et al. (2013)
$\mu_{\log k}$	1.93	The college graduation rate in 1990
$\sigma^2_{\log \kappa}$	0.12	The college graduation rate in 2000
The Base factor of the importance of mother's time (ϕ_b)	0.32	The average female labor supply
The adjustment factor of the importance of mother's time (ϕ_{adj})	0.09	The slope of female labor supply
Price adjustment factor of child rearing materials (ε_b)	0.17	Child rearing cost to family labor income ratio per child (= 22%), Lino (1996)
Price adjustment factor of child education (ε_e)	0.25	Education investment to family labor income ratio per child (= 2%), Lino (1996)
Quality function parameter (θ)	0.04	The slope of education investment by income group, Lino (1996)
Skill bias factor (λ^S_{COL})	0.46	Target the skill premium in 1990 (=1.45)
Gender bias factor (λ^G_f)	0.42	Target the gender wage gap in 1990 (=1.39)

Table 1.2: Internally Calibrated Parameters

	CFR	$Childless$	K/Y	EI/LI	CC/LI	COL_f	SKP	GWG
$Data$	1.89	0.10	3.00	0.02	0.22	0.27	1.45	1.39
$Model$	1.88	0.10	3.00	0.02	0.22	0.27	1.45	1.39

Table 1.3: The Calibrated Moments

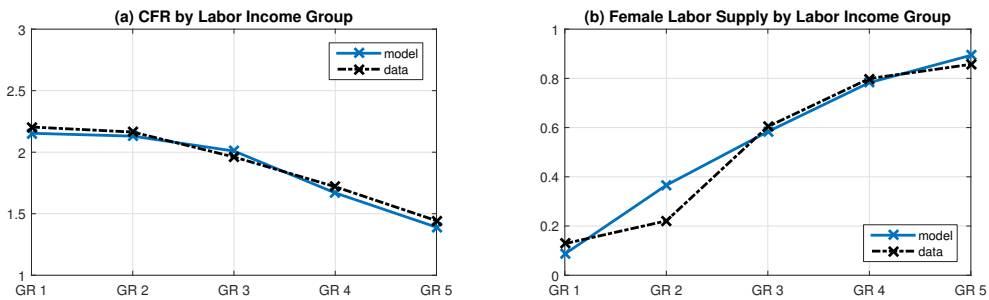

Figure 1.6: CFR and Female Labor Supply

gender wage gap in 1990.

5 The Result of Benchmark Model

5.1 Results

Table 1.3 is the result of the calibration. The benchmark model regenerates the target moments under reasonable parameterization. The model explains the CFR and childless household ratio well. Moreover, the benchmark model fits the data, including the capital-production ratio(K/Y), the educational investment-household income ratio (EI/LI), the child care cost-household income ratio (CC/LI), the fraction of college graduate of female (COL_f), skill premium (SKP), and gender wage gap (GWG), as presented in Table 1.3. In addition, the model explains the female labor supply, both on average and across income groups, as depicted in Panel (b) of Figure 1.6.

Panel (a) of Figure 1.6 shows the generated CFR across income group. By the construction of the model in which income quintile is determined endogenously, there is no direct parameter to calibrate the CFR across income group in this model. The

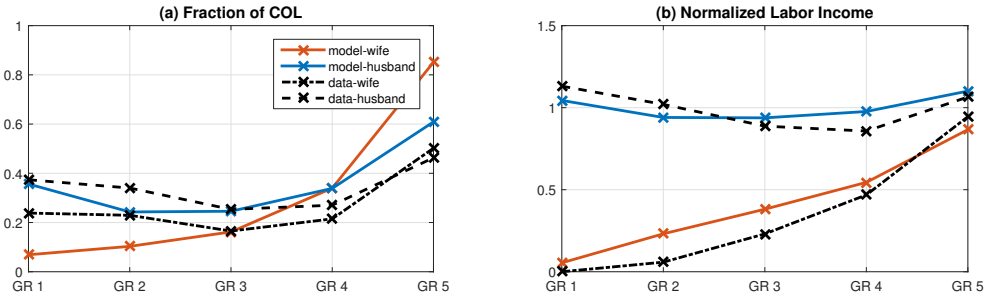

Figure 1.7: Robustness Check

generated CFR across income group, however, matches the data remarkably well, and we can surmise that the model is reasonably calibrated.

To further verify the validity of the model, we also investigate the fraction of college graduate and labor income generated by the model for both wife and husband, which is not calibrated intentionally. As shown in Panel (a) in Figure 1.7, the overall shapes of the fraction of college graduates for both genders are similar to the data. For males, the model-generated curve mimics the U shape of the fraction of college graduates. For females, the generated result depicts the key characteristics indicating that the highest income group has the highest fraction of college graduates and has a higher value than that of males. However, the generated moments are sorted by income group monotonically, while the data has a skewed U-shape curve. This result is attributable to the fact that our model does not have any labor market friction through the job searching and matching processes, and thus females can work as much as they want and earn labor income compatible to their productivity. In reality, however, there are many females in the low-income group because they cannot find jobs or work in part-time jobs even when they want to increase their hours of labor. Panel (b) in Figure 1.7 represents labor income across the female income group for both wife and husband. It is normalized by the average labor income of males. For the husband, the model generates a U-shape curve as observed in the data, and for the wife, the model mimics the monotonically increasing labor income curve. In sum, our

$\Delta\%$	$Bench$	SKP	GWG	VOL	$total$
$w_{f,l}$	-	-7.03	1.99	1.01	-4.17
$w_{f,h}$	-	3.83	2.10	-1.48	4.22
$w_{m,l}$	-	-7.03	-1.72	1.01	-7.27
$w_{m,h}$	-	3.83	-1.61	-1.48	0.86
$value$					
ζ_h/ζ_l	0.56	0.57	0.56	0.56	0.57
CFR	1.88	1.89	1.87	1.89	1.89
$\frac{CFR_h}{CFR_l}$	0.96	1.01	0.97	0.96	1.00

Table 1.4: The Change in Macroeconomic Variables

model not only accounts for the targeted moments well, but also explains important behaviors observed from the real data that are not calibrated directly. These results demonstrate the robustness of our model.

6 The Change in Wage Structure and Fertility Choice

In this section, we impose changes in the wage structure to the baseline model and examine whether the model can regenerate the empirical observation, which is the increase in the fertility of the highest income group. First, we analyze the case in which all factors that change the wage structure are implemented in the model. We then decompose the individual factors and take note of their effects, respectively.

6.1 Total Effect

6.1.1 The Measure of Explanatory Power

Before we study the effects driven by the change in household wage structure, we must define the measure specified blow. We use this measure to evaluate how the model explains the observed changes. We also exploit the measure when we decom-

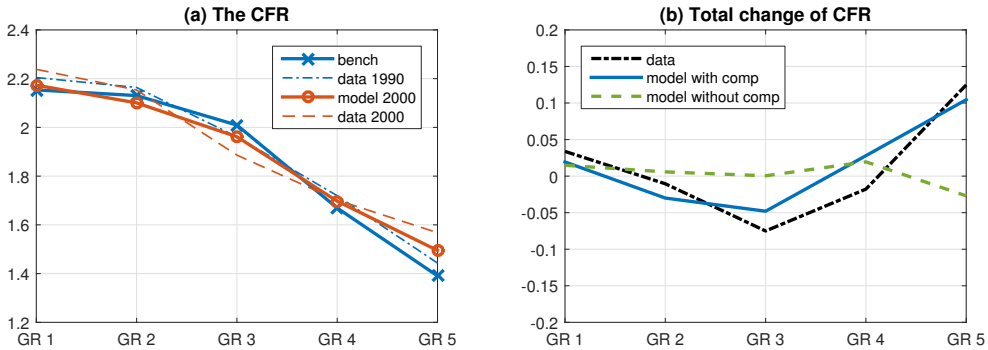

Figure 1.8: The Change in the CFR

pose the individual effects of each factor.

$$\Delta = \sum_{i=1}^{5} \left| n_{i,t+1}^{data} - n_{i,t}^{data} \right| \in [0, \infty) \tag{1.53}$$

$$U = \sum_{i=1}^{5} \left| \left(n_{i,t+1}^{data} - n_{i,t}^{data} \right) - \left(n_{i,t+1}^{model} - n_{i,t}^{model} \right) \right| \in (-\infty, \infty) \tag{1.54}$$

$$E(\%) = \frac{\Delta - U}{\Delta} \times 100 \in (-\infty, \infty) \tag{1.55}$$

The Δ measures the total fluctuation of complete fertility across income groups. We compute the absolute difference of the complete fertility between two periods at each income group and augment these values to construct total variation. The U indicates the unexplained fraction by the model. We then compute the explained fraction (E) according to the model as a residual of the unexplained portion, as in equation 1.55.

6.1.2 Total Effect

Figure 1.8 illustrates the changes in complete fertility when imposing three factors affecting household wage structure altogether. We adjust the parameter values corresponding to the economic situation in 2000. Panel (a) stands for complete fertility from the model and data in 1990 and 2000, respectively, and Panel (b) shows the

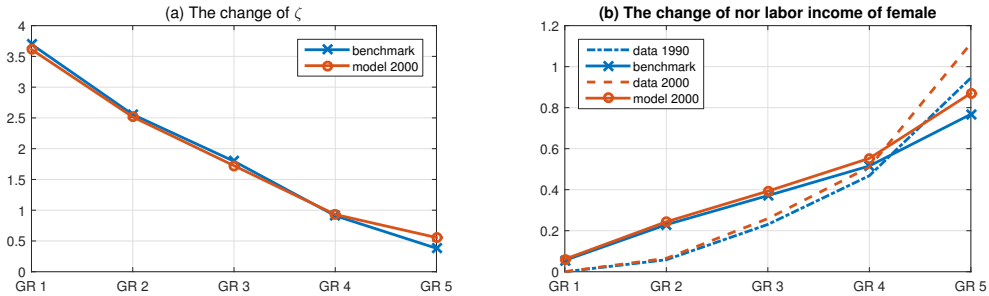

Figure 1.9: The Change in ζ and Female Labor Income

differences in complete fertility between theses two periods across the income group. Changes in the direction and level of complete fertility aligns with the real data, as shown in Panels (a) and (b). The model expects the rise in the CFR in the highest and lowest income groups and relative decreases in other income groups. The pattern of the adjustment of the CFR has a V shape across the income quintile, as shown in Panel (b), and the model regenerates both the V-shaped pattern and the level of changes well. The measure defined in the previous subsection indicates that the model explains 55% of the overall changes in the CFR. This result is meaningful when compared with the poor prediction of the model without the composition effect, as shown in Panel (b) of Figure 1.8. [18]Without the composition effect, the model predicted the decrease in the CFR in the highest income group, and it cannot explain the observed V-shaped variation overall. Based on this counterfactual experiment, we find that the composition effect has a significant role in accounting for the empirical change in the CFR.

Figure 1.9 shows the changes in the average preference for having children and

[18]We computed the result of the model indirectly without composition effect. First, we ran Local Polynomial Regression by using benchmark data.

$$n = \hat{f}\left(e_f, e_m, w_f^e \varepsilon_f^e, w_m^e \varepsilon_m^e, 1 - t_f\right) \tag{1.56}$$

Following this, we imposed a new data set $\left(e_f, e_m, w_f^e \varepsilon_f^e, w_m^e \varepsilon_m^e, 1 - t_f\right)$ from the model 2000 into estimated function, $\hat{f}$, and calculated the predicted number of children. Note that we don't need to consider assets here, as we assumed that the amount of the initial asset is zero.

female labor income across income groups. Due to the increase in the skill premium and the reduction of the gender wage gap, the wage for high productivity women increases. Consider females who have high productivity(ε_f) but work relatively less because they have a high preference for having children, and thus they will spend much time raising many children. As the wage structure changes in the new economy, these individuals are better off when they work more and have fewer children compared to the baseline economy. Thus, they increase labor supply via their utility maximization process and their relative position in terms of labor income rises, combined with the increase in both labor supply and wages. As explained earlier in Section 1, we call this change the composition effect, and it moves females who are desirous of children into the high-income group, while females who are less desirous of children exit this group. The shape of the average preference for children (ζ) across income groups becomes flat due to the composition effect, and this represents a major channel that results in the V-shaped CFR variation observed in reality, as shown in Figure 1.8.

6.2 Decomposition

6.2.1 Skill Premium (SKP)

We raised skill-biased technological progress by adjusting λ_g^S from 0.46 to 0.50 in the model. The rise in the skill premium reduces the wage of high school graduates, while the wage of college graduates increases for both females and males, as shown in Table 1.4. The fraction of female college graduates rises by 7.7% in response to the increase in the expected return from a college degree, caused not only by the increase in female wages, but also by the incline in the expected wage of her future husband through the assortative matching process. This causes the labor supply of high school graduates to decrease and those of college graduates to increase. The effect from the skill-biased technological progress on wages outweighs the effect driven by the

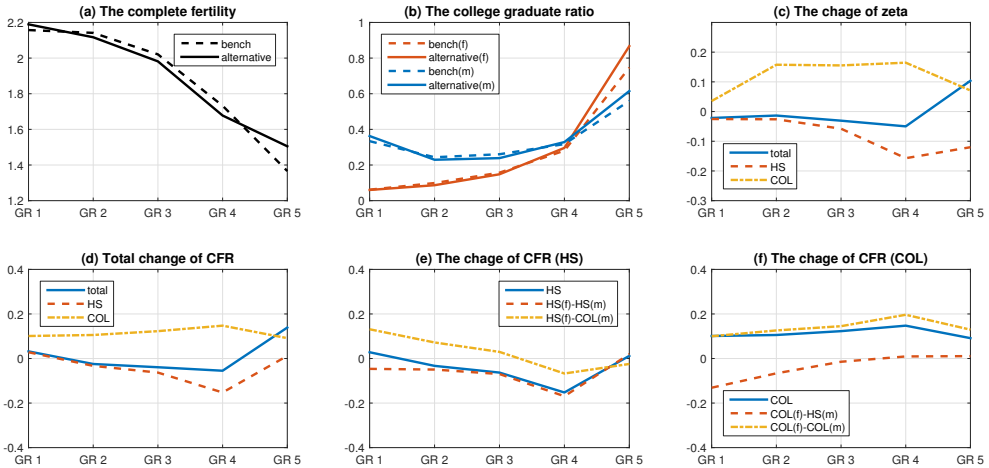

Figure 1.10: The Effect of the Skill Premium

change in labor supply among both educational groups. Hence, it causes wages of the low educational group to decrease and those of the high educational group to increase.

The effect of the change in skill premium on the CFR is determined by three factors: 1) the relative magnitude between the substitution and income effects of females, which are the traditional method of explanation, and 2) the pure income effect of the husband. In this study, we have another force, 3) the composition effect of ζ in each income group, derived from relaxing the homogeneous preference assumption. To see the effect of the change in skill premium, we analyze a new factor, the variation of ζ, first. Regarding the change in the ζ, there are two types of variation: 1) the change in ζ between the high school(HS) and college(COL) groups, and 2) the change within a specific educational group. Henceforth, we call the former between sector effect and the latter within sector effect for simplicity. We also call the augmented between and within sector effect the composition effect. In this experiment, the former has more considerable effect, as the change in skill premium facilitates the direct movement of individuals between educational groups. Thus, we focus on the between sector effect in this subsection. The expected lifetime value of attending

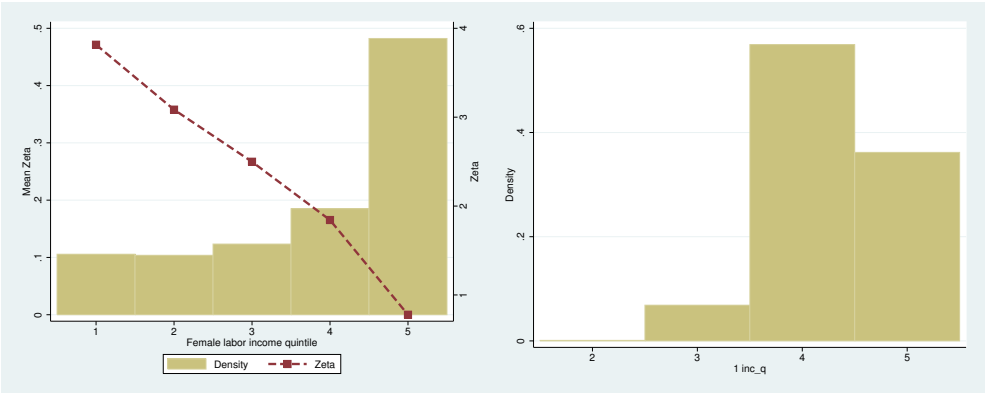

Figure 1.11: The Movement between HS and COL Sectors

college rises due to the increase in skill premium, and this causes people who have relatively higher ζ decide to participate in college more often in order to attain the benefit from the high skill premium. This causes the average ζ ratio between COL and HS, ζ_h/ζ_l to increase, as presented in Table 1.4. Figure 1.11 illustrates those individuals who have moved from HS in the benchmark to COL in the new steady state. About half of these people belong in the highest labor income quintile in the new economy, and this increases the college graduate ratio in GR 5 (Panel (b) of Figure 1.10). As more college graduates enter the highest income group and the wage of HS graduates decreases, high school graduates should be moved to a lower income group overall. Panel (b) of Figure 1.11 shows the result of the movement of high school graduates into the new steady state, who had previously been in GR 5 in the benchmark. Over 60% of people change their relative position to lower income groups. Thus, the average levels of ζ of HS at each income group decreases due to the influx of people who were in higher income groups in the benchmark as they have lower ζ on average. Panel (c) of Figure 1.10 shows the result of the change in ζ discussed above. The average ζ of the college graduates increases, and those of the high school graduates decreases in all income groups.

Panel (a) of Figure 1.10 illustrates the CFR across female labor income groups

in the benchmark and the alternative economy, and Panel (d) shows the difference in the CFRs between theses two steady states. We find that the gap between the CFRs has a V-shape curve (solid blue line in (d)) in this experiment. The difference in CFRs decreases in GR 2 to GR 4, and it rises in both GR 1 and 5. However, when we decompose this change by educational group, their movements are significantly different. The high school graduates reduce the overall CFR, while the college graduates increase the number of children. This difference is similar to the shape of the movement of ζ in Panel (c). Consider the COL group first. Since the majority of female college graduates have husbands who have attained the same educational level, they have the positive husband income effect due to the change in skill premium (yellow short dashed line in Panel (f)). Moreover, their average level of ζ increases due to the between sector effect. Thus, these two effects dominate the negative effect on the CFR caused by the rise in opportunity cost, and the overall CFR of the college group increases. For high school graduates, the negative income effect of the husbands (red dashed line in Panel (e)) and the decrease in average ζ cause them to have fewer children through the same mechanism. These changes in two educational groups increase the CFR ratio between the two educational groups from 0.96 to 1.01. This indicates that the CFR of highly educated women slightly exceeds the number of children of high school graduates. This implication from the model is compatible with recent empirical studies, such as Shang and Weinberg (2013) and Hazan and Zoabi (2014).

6.2.2 Gender Wage Gap (GWG)

We impose the gender-biased technological change by adjusting λ_e^G in the CES aggregator in equation 1.36. In this experiment, the gender wage gap (GWG) decreases from 1.39 to 1.35 as firms prefer to hire females more due to the gender-biased technological change. Although the same λ_e^S has effects on the wages of females in both educational groups, the absolute level of the rise in the wage of highly educated females is larger than those of female high school graduates on average. Thus, the

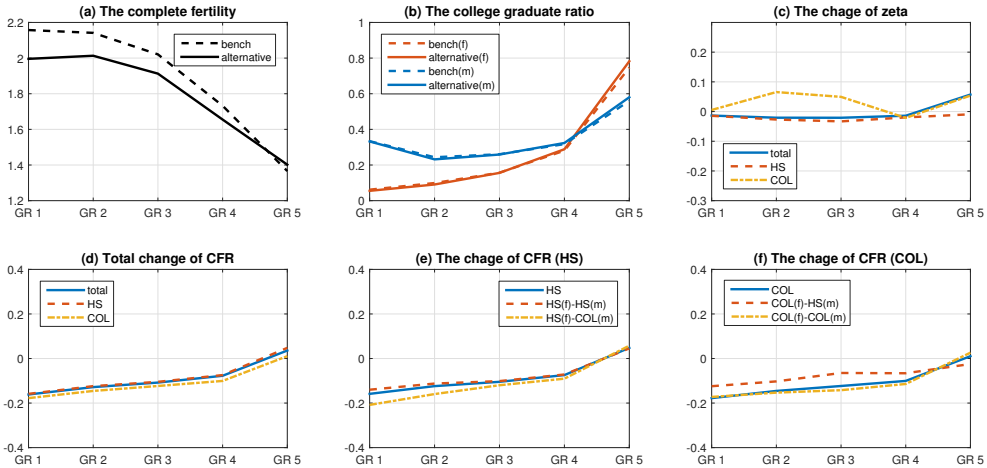

Figure 1.12: The Effect of the Gender Wage Gap

expected return from attending college increases relatively, and more women decide to participate in college. We compute two models with different values of λ_e^G. We call the case in which the parameter is matched to the gender wage gap in 2000 model (1) and the case in which there is no gender wage gap model (2). Note that we suggest the results of model (1) in Table 1.4 but illustrate the result of model (2) in Figure 1.12 since model (1) has a small variation in interest moments due to the minimal change in GWG. Panels (a) and (d) of Figure 1.12 depict the change in the CFR across the female labor income groups. We find that GR 1-4 decrease their CFR, while GR 5 has more children in the new steady state. To analyze the change in the CFR, we see the movement of the ζ, as shown in Panel (c). Similar to the SKP experiment in the previous section, the overall level of the ζ increases in COL and decreases in the HS group due to the between sector effect, which increases the average ζ ratio between the COL and HS groups. However, this effect is relatively small, as it is driven indirectly. In addition, we must note the within sector effect because the change in the GWG increases female wages regardless of educational group, which adjusts the labor choice of women who have a different combination of ζ and ε. Table 1.5 suggests the average ζ of the influx and outflux of GR 5 between two steady states. Note that

Within Sector Effect		
	$\zeta_{Others \to GR5}$ (Influx)	$\zeta_{GR5 \to Others}$ (Outflux)
HS	0.14	-0.39
COL	1.54	1.41

Table 1.5: Within Sector Effect of GWG

these females do not change their educational group, but their income quintiles are adjusted due to their different labor and fertility choices in response to the change in the gender wage gap. For HS, the average ζ of the influx has greater value, 0.14, than those of the outflux, -0.39. Thus, the net change in ζ of the HS is positive in the highest income quintile, despite the negative between sector effect. The GR 1-3 decrease the CFR as the husband's income represents a significant fraction of family income, which has a negative effect on income in this experiment. On the other hand, the CFR increases in GR 5 for both educational groups, since the composition effect dominates the negative income effect of the husband and the rise in opportunity cost of females.

6.2.3 Income Volatility (VOL)

In this section, we analyze the effect of the change in income volatility in both educational sectors. According to Hong et al. (2015) and Heathcote et al. (2010), income volatility has increased between 1990 and 2000 for both educational groups. Correspondingly, we adjust the variance term of the productivity process in equation 1.8 to generate the change in income volatility. Figure 1.13 depicts the transition of the distribution of the unit wage, $w_f^e \varepsilon_f^e$. The distributions of both the HS and COL groups become wider than before (Panel (a)), and this represents the rise in income volatility in the new economy. As shown in Panel (b), the distribution of the wage of the full sample becomes wider as well. We find that the wage processes of HS become more volatile than those of the college group, and this means that the high school group has

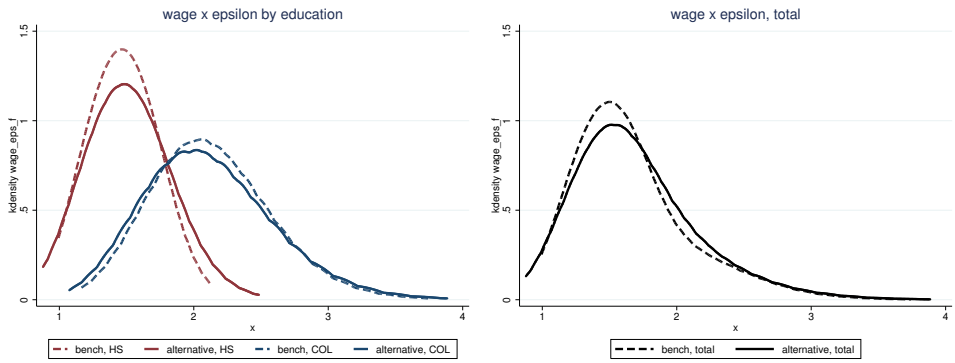

Figure 1.13: The Distribution of $w_f^e \varepsilon_f^e$ at Age 1

Note: We draw the distribution of the unit wage for both educational groups by using the kernel smoothing method for clear illustration.

a relatively larger change in the idiosyncratic productivity risk compared to its counterpart. Hence, the expected return of high school graduates decreases more, because they are required to have more precautionary savings and labor supply to prepare for future risks. As the COL sector becomes more attractive, although their idiosyncratic productivity risk rises as well, the female college ratio increases by 4.5%. This change decreases the skill premium by 2.5% as the labor supply of the skilled worker increases. Thus, the efficiency wage of the college group decreases, forcing the $w_f^e \varepsilon_f^e$ distribution of the COL to move to the left side (the solid line in Panel (a)), while that of HS moves to the right.

To see the change in the CFR, consider the variation of ζ first. We find that the transition of ζ between educational groups, and between sector effect, is distinct from the other two exercises conducted in the previous subsections. The high school graduates in the benchmark who have small ζ and thus work more move to the COL sector in the new economy, which is the reverse tendency in earlier experiments. In the HS group, the rise in income volatility has a larger negative impact on the people who work more hours, which has a small ζ on average. Hence, these types of people move to the COL sector in the new steady state, and thus they primarily determine the

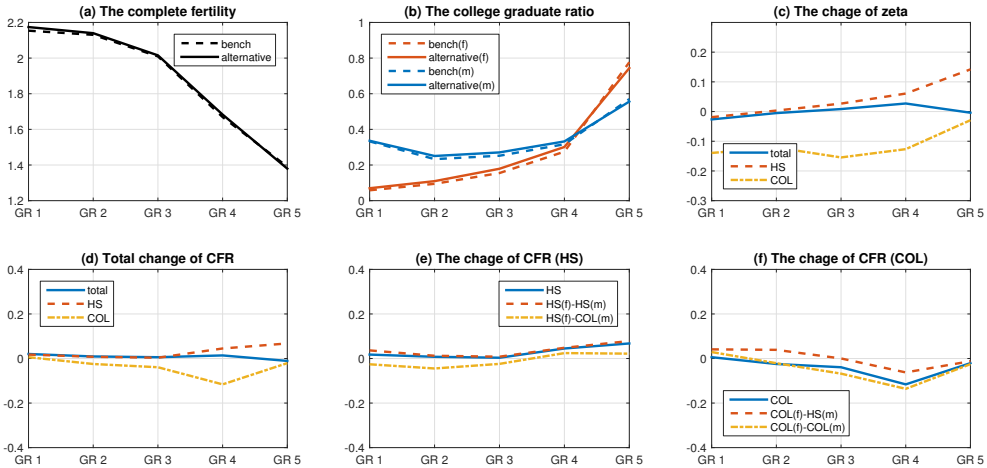

Figure 1.14: The Effect of the Income Volatility

between sector effect in this experiment. Table 1.6 shows the characteristics of this type of people. Note that we normalize the average ζ of COL in the benchmark to one to make the comparison easier. As the average $\zeta_{HS\to COL}$ $(= 0.96)$ is lower than those of the COL group $(= 1)$ in the benchmark, it causes the average ζ of the COL group to decrease in the new economy. In comparison, we suggest the $\zeta_{HS\to COL}$ in the SKP case as well. The average $\zeta_{HS\to COL}$ in the SKP experiment is larger than one, and this means that females who are desirous of children move into the COL group, which creates a positive between sector effect that is inverted in the direction of the VOL. The values in the quintile and labor supply columns identify the average labor income quintile and labor supply of individuals in the baseline model, who move from HS in the benchmark to COL in the new economy. As shown in this table, the people in the VOL case have a higher average labor income quintile and labor supply in the benchmark than those in the SKP case. This supports the argument that the change in income volatility causes people who have low ζ and work more to move to the COL group, which is a reverse tendency of previous SKP and GWG cases.

The change in income volatility causes the $w_f^e \varepsilon_f^e$ distribution to have fat tails, as more people have higher productivity than before (the reverse is also true), and

Between Sector Effect			
	$\zeta_{HS \to COL}$	Quintile	Labor supply
VOL	0.96	3.10	0.61
SKP	1.11	2.90	0.56

Within Sector Effect		
	$\zeta_{Others \to GR5}$ (Influx)	$\zeta_{GR5 \to Others}$ (Outflux)
HS	0.27	-0.04
COL	1.45	1.30

Table 1.6: Between and Within Sector Effects of VOL

Note: We normalize ζ of the college group in the benchmark to one and compute the relative values of ζ in the table.

this stimulates these people to work more, and vice versa. It derives from the within sector effect. For example, the average ζ of the high school group in GR 5 increases (dashed red line in Panel (c)) through this channel. Table 1.6 presents the within sector effect for both HS and COL groups. For high school graduates, the average ζ of the influx has a higher value than those of the outflux, and this causes the average ζ of the HS group to increase in GR5. Panels (a) and (d) indicate the change in the CFR. For female high school graduates, the CFR has little change in GR 1-4, but it increases in GR 5 due to the within sector effect and the positive husband income effect. College graduates decrease the CFR, especially in GR 4 and GR 5, because they have a negative between sector effect and a husband income effect. Note that females in GR 1 matched to high school graduated husbands increase in the CFR regardless of their educational level, as shown in (e) and (f), due to the income effect of their spouse. The change in income volatility has a distinct effect in that the CFR of the college graduated female decreases.

	(1)	(2)	(3)	(4)	(5)
SKP	✓	✓	✓		✓
GWG	✓	✓		✓	
VOL	✓		✓	✓	
$explained(\%)$	55.02	20.70	39.89	-12.66	50.51

Table 1.7: The Effects of Various Combinations

6.2.4 Relative Importance of Each Factor

In the previous sections, we investigated the total and individual effects of the three factors related to household wage structure. We will examine the relative importance of each factor to the variation of the CFR in this subsection. Table 1.7 demonstrates the explained fraction of the various combinations of the factors by using the measure defined in equation 1.55.

We found that the change of skill premium is the most influential factor to generate observed V-shaped variation of the CFR across income group. It solely explains 50.5% (model (5)) of the total variation observed from the data, and we found that the model without SKP (model (4)) fails to explain the phenomenon. The skill-biased technological change causes the increase in the skill premium, and it raises the CFR through the Composition Effect for GR 5 and husband income effect for GR 1.

The gender wage gap intensifies the rise in the CFR of the highest income group. The change in the gender wage gap causes high-income females to have more children due to the composition effect, especially the within sector effect, while the women in the low-income quintile decrease the CFR because of the negative income effect of their husbands. Thus, the shape of the change in the CFR monotonically increases the labor income quintile (Panel (d) of Figure 1.12).

Although the skill premium is the primary source for explaining the empirical observation, it is not sufficient to explain the movement of the CFR in the COL group. As seen in Panel (d), the change in the CFR in the college, group which was caused

by the SKP (red dashed line) has a positive value across all income groups, while the alteration of the CFR has negative values when we apply the change in income volatility in the model. Thus, these two forces offset each other and successfully generate the observed change in the CFR in the college group.

7 Implications

In this section, we study the two theoretical implications derived from the model and determine whether these are supported by empirical evidence. We examine the composition effect. Another implication is that the partial effects of the preference regarding children (ζ) on the CFR become weaker in the new economy when compared with those of the benchmark model.

7.1 The Composition Effect

The principal mechanism of our study is the composition effect. Through this channel, the average level of the ζ increases in the high-income group in the new economy, which has the larger wage gap. In the previous section, we found that this mechanism is important in generating the newly observed phenomenon indicating that high-income females increase the CFR theoretically. We will see the empirical evidence supporting the composition effect in this section. Toward this end, we use general social survey (GSS) data, which includes the question regarding the ideal number of children. Following Miller et al. (2010), we exploit this question as a proxy variable of the preference for having children, ζ, in the model.

So far, we have discussed our argument based on the female labor income quintiles. However, it is difficult to disentangle the wife's and husband's labor income when we use the GSS data, although it instead contains family income. Thus, we analyze the composition effect based on family income quintiles, rather than the female labor income in this section, due to limitations of the data. Note that this does not

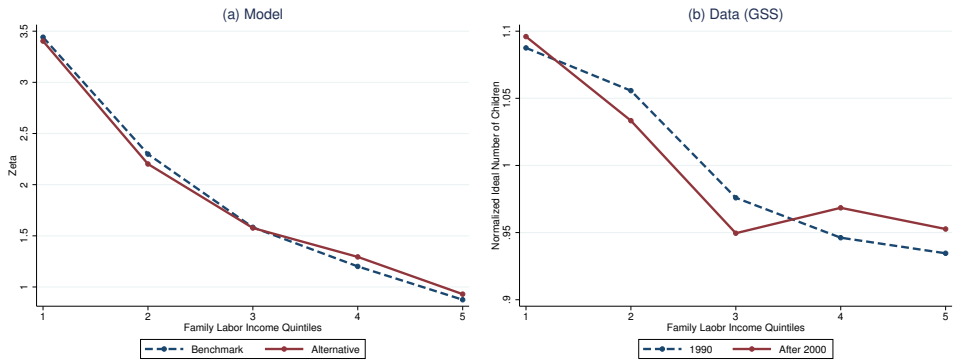

Figure 1.15: The Change in Preference Composition in the Model and Data
Source: General Social Survey (GSS)

cause serious problems, even if we view the composition effect based on household income, especially when we focus on high-income females. In our model, 61% of the highest household income group are females, who are in the highest female labor income group, and another 20% comes from the second highest group. Thus, we can see that the majority of females who constitute the highest family income quintile also belong to high female income groups. This allows us to analyze the composition effect of the highest female labor income group using family income instead.

Panel (a) of Figure 1.15 shows the average ζ across the household income quintile in the model. A dashed line indicates the result of the baseline, and a solid line depicts the average ζ in the new steady state applied to all income shocks. Similar to the composition change in ζ based on the female labor income quintiles, as shown in Figure 1.9, the average ζ increases in the high family income groups, while it decreases in low family income groups. Panel (b) shows the normalized ideal number of children across family income quintiles, which is calculated using GSS data. We normalize the ideal number of children by the average at each year to control for year-specific variations. The direction of the change in both the model and data are similar, and we find that the empirical evidence is compatible to the theoretical expectation

from the model.

7.2 The Effect of Preference on the CFR

7.2.1 Implication from the Model

Our model expects that the partial effect of ζ on the CFR decreases when household income structure becomes more unequal. For example, as the skill premium, the major factor of the model, increases, households place more weight on their earning ability ε rather than ζ in the process of utility maximization. It causes the effect of the ζ on the CFR to decline.

We compare the partial effect of the ζ in the two economies under control among other factors, such as the husband's income and the education of both spouses. We calculate the centile of the ζ and compute the percentage change in the CFR when one centile of ζ increases. Panel (a) of Figure 1.16 depicts the percentage change in the CFR when ζ increases from the 20th centile to 80th centile. A dashed line indicates the benchmark case, and the solid line illustrates the alternative economy, which has a larger wage gap due to the change in household wage structure. As seen in Panel (a), the partial effect of the ζ on the CFR in the alternative economy is smaller than that of the benchmark model. Thus, we can see that ζ has a smaller partial effect on the CFR in the new steady state.

Hakim (2000, 2003) argued that the preference of females is the most important factor when females make fertility and labor decisions in the modern developed economy. Our study, however, suggests the possibility that the effect of the preference on the CFR and female labor supply becomes weaker when the wage structure becomes more unequal.

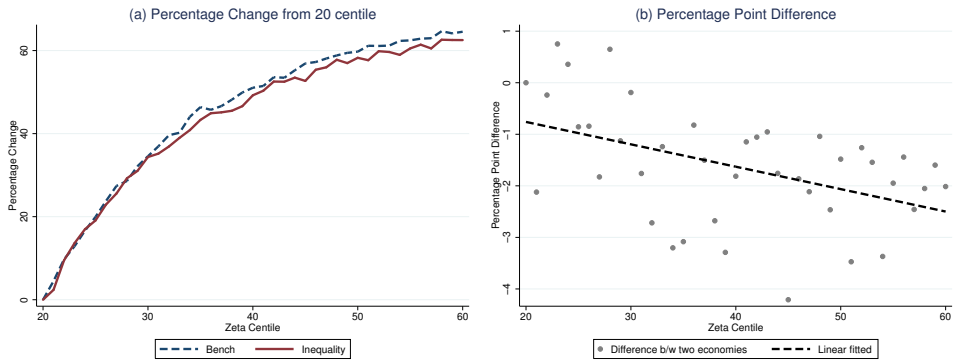

Figure 1.16: The Partial Effect of ζ on the CFR

Note: We compute the percentage change in the CFR when increasing a centile of ζ from the 20th centile. We conditioned for the husband's income and the education of both spouses.

7.2.2 Empirical Evidence

To examine the second implication, the decreasing partial effect of ζ on the CFR, we conduct reduced-form regression analysis with the following specification.

$$CFR = \beta_0 + \beta_1 Zeta + \beta_2 Hour_m + \beta_3 Hour_f \tag{1.57}$$

$$+\beta_4 Edu_m + \beta_5 Edu_f + \beta_6 X + \varepsilon \tag{1.58}$$

The dependent variable CFR means the complete fertility, and $Zeta$ is the ideal number of children, which is the proxy variable of the preference regarding children. We set $Hour_g$ to working hours for each gender g, and Edu_g indicates the educational level. X illustrates other covariates, such as race and rural dummy variables.

This specification, however, could have an endogeneity problem, as the complete fertility (CFR) may have a reverse effect on the ideal number of children ($Zeta$). Miller et al. (2010) studied whether the fertility preference of youth predicts later fertility outcomes by using National Longitudinal Survey (NLSY) 79 data. The re-

	1990s			After 2000		
	(1)	(2)	(3)	(4)	(5)	(6)
Zeta	0.571***	0.543***	0.534***	0.320***	0.323***	0.327***
	(0.11)	(0.11)	(0.11)	(0.08)	(0.08)	(0.08)
Working hrs: male	0.002	0.002	0.003	-0.003	-0.003	-0.003
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Working hrs: female	-0.010***	-0.011***	-0.010***	-0.008***	-0.009***	-0.009***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Education: male	-0.027	-0.026	-0.028	0.033	0.036	0.038*
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Education: female	-0.093***	-0.092***	-0.089***	-0.065***	-0.067***	-0.067***
	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)
Constant	4.649***	4.327***	4.304***	3.398***	3.142***	3.012***
	(0.86)	(0.86)	(0.88)	(0.62)	(0.63)	(0.64)
Control race		Yes	Yes		Yes	Yes
Control others			Yes			Yes
Observations	759	759	759	823	823	823

Source: General Social Surverys, The National Data Program for the Social Sciences
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses.

Table 1.8: The Results of Reduced-Form Regression

searchers used the "ideal number of children" surveyed in 1979 (age 15-22) and 1982 (age 18-25) as a proxy variable of the fertility preference, which is free from the endogeneity problem, and studied their effects on the actual number of children born in 2002 (age 38-45). They found that the effect of the fertility preference on complete fertility is positive and statistically significant. However, we must see one step further from their results to examine the change in the effect in fertility preference on the CFR by periods. Unfortunately, NLSY is not suitable for analyzing this tendency, as it does not include the "ideal number of children" question in NLSY 97. For this reason, we used GSS data, which has a cross-sectional structure to allow for the identification of changes in the effect of the preference, despite the endogeneity problem. However, under the weak regularity conditions, the estimates of β_1 are systematically biased and still have correct signs (see Appendix 9.6).

Table 1.8 shows the results of some specifications by using ordinary least squares regression. As a baseline, the model (1) is constructed with the preference on children and common covariates usually used in fertility literature. To be consistent with

many existing empirical works, female working hours and female educational levels have significant and negative coefficients. Additionally, we also find that *Zeta* has a significant and positive coefficient as the result of the previous empirical work, Miller et al. (2010). However, the coefficient of *Zeta* becomes smaller with the sample after 2000 compared to the result from the sample obtained in the 1990s. We interpret that the partial effect of *Zeta* decreases between two periods, and this result is comparable to the theoretical expectation of the model.

8 Conclusion

Why have high labor income females increased complete fertility over the last few decades, despite the rise in opportunity cost? This empirical observation seems contradictory to the expectation of existing fertility theories, which assume that the substitution effect is much stronger than the income effect under the homogeneous preference assumption. This study is an attempt to explain this puzzling question. Toward this end, we have suggested a new explanation for relaxing homogeneous preference and have demonstrated that the composition effect derived in this setting is essential in accounting for the phenomenon. The composition effect is associated with ordinary income and the substitution effect, as well as how they affect a household's fertility choice. As the wage gap rises, as seen in U.S. data, more people who would like to have children move to the college sector (between sector effect), and a significant percentage belong to the high-income group in the new steady state. Moreover, the within sector effect intensifies the increase in the average preference of the high-income group. Through these two composition effects, we find that females who are desirous of children enter the highest income quintile, while females less desirous of children exit the highest income quintile.

We also study the overall and individual effects of changes in: 1) skill premium, 2) gender wage gap, and 3) income volatility. The model explains 55% of the ob-

served variation, and we find that the change in skill premium is the most important factor in explaining the phenomena.

Finally, our model has two implications, driven by the relaxed unitary preference. First, the average preference for children increases in the high-income female group on average, while it decreases for others as the wage structure becomes more unequal, i.e., the composition effect. Another implication pertains to the effect of preference regarding children on the CFR, and how it decreases when the wage gap rises. This is attributable to the fact that earning ability becomes more of a viable factor than preference on having children in the utility maximization process. We conduct a reduced-form analysis by using "the ideal number of children" as a proxy variable for the preference and find that empirical evidence supports these theoretical implications.

9 Appendix

9.1 Related Literature: Strong Income Effect and Weak Substitution Effect

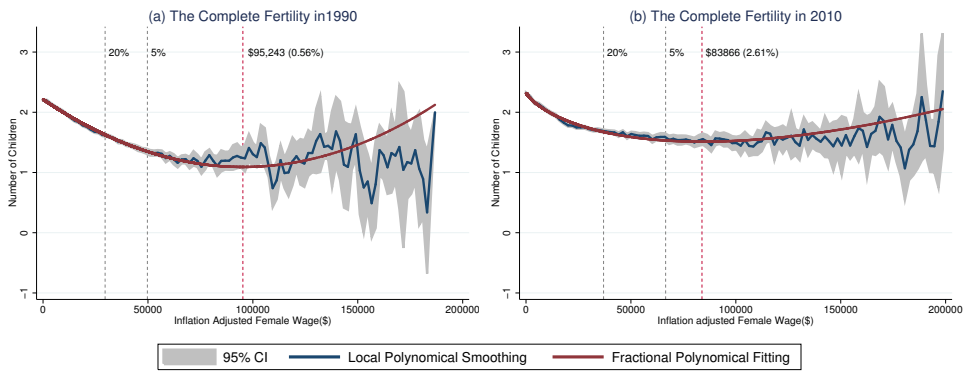

Figure 1.17: Female Labor Income and the CFR (1999 dollar)

	1990		2000		2010	
	Income	*Frac.*	*Income*	*Frac.*	*Income*	*Frac.*
Quadratic Fit	$95,263	0.56%	$84,688	1.89%	$86,321	2.34%
Frac. Poly Fit	$95,243	0.56%	$93,001	1.41%	$83,866	2.61%

Table 1.9: Estimated Threshold Female Income and the Fraction of the Population

Note: the unit of the female income is 1999 dollar. Fraction (Frac.) stands for the percentage of the females whose labor income is over the threshold income. (the minimum point of the U-shape curve in the text)

9.2 Cohort Analysis

Figure 1.18 shows how the fertility choice is changed through the lifetime between two cohorts whose ages are 36-40 at 1990 and 2011 respectively by using PSID data. Panel (a) depicts the lifetime path of the cumulative number of children of the income group 1-4 which is computed by the labor income at their age 36-40, and the panel

Figure 1.18: Number of Children through the Lifetime by Cohort

Source: PSID

(b) illustrates the lifetime path of childbirth for the group 5. We found that the rise in the CFR of the highest income group is mainly caused by the additional childbirth at the late thirties as shown in the panel (b). It means that the behavior at this age is important to explain the rise in the CFR of the highest income group. Thus, it can be another justification that the observation suggested in the text will not be seriously biased even we restricted the sample at the age 36-40 due to the limitation of the U.S census data. [19]

[19] We also found that the fertility timing is delayed from this figure. However, we focus on the CFR in this article, and this issue is beyond the range of the paper.

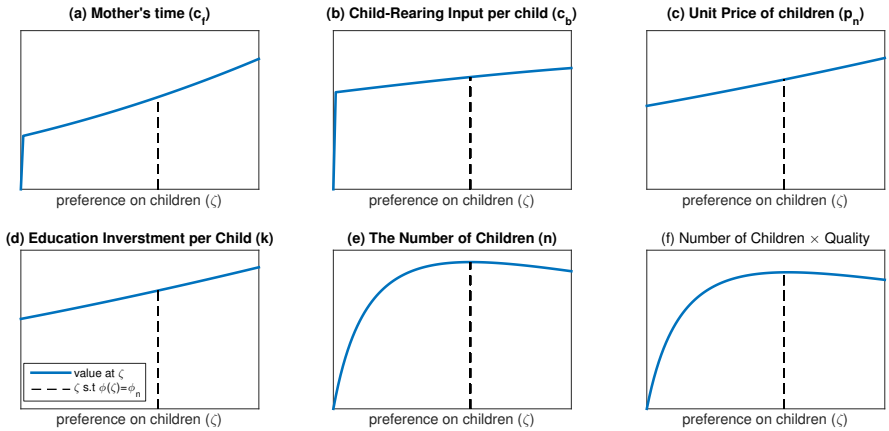

Figure 1.19: The Effect of ζ

9.3 Proof of the Propositions

In this Section, we prove the propositions suggested in the text. Figure 1.19 illustrates the relation between preference on children, ζ, and other endogenous variables.

Proposition 1.

Proof.

$$p_n = \frac{1}{Z}\left[\underbrace{\left(\frac{\phi(\zeta)}{1-\phi(\zeta)}\right)^{1-\phi(\zeta)}}_{A_p} + \underbrace{\left(\frac{1-\phi(\zeta)}{\phi(\zeta)}\right)^{\phi(\zeta)}}_{B_p}\right] \cdot \underbrace{w_b\varepsilon_b\left(\frac{w_f^e\varepsilon_f^e}{w_b\varepsilon_b}\right)^{\phi(\zeta)}}_{C_p} \tag{1.59}$$

$$\begin{aligned}\frac{\partial A_p}{\partial\phi(\zeta)} &= \left(\frac{1-\phi(\zeta)}{\phi(\zeta)}\right)^{\phi(\zeta)}\left[\frac{\left(-\frac{1-\phi(\zeta)}{\phi(\zeta)^2}-\frac{1}{\phi(\zeta)}\right)\phi(\zeta)^2}{1-\phi(\zeta)}+\log\left(\frac{1-\phi(\zeta)}{\phi(\zeta)}\right)\right]\\ &+\left(\frac{\phi(\zeta)}{1-\phi(\zeta)}\right)^{1-\phi(\zeta)}\left[\frac{(1-\phi(\zeta))^2\left(\frac{\phi(\zeta)}{(1-\phi(\zeta))^2}+\frac{1}{1-\phi(\zeta)}\right)}{\phi(\zeta)}-\log\left(\frac{\phi(\zeta)}{1-\phi(\zeta)}\right)\right]\end{aligned} \tag{1.60}$$

Since $\partial(A_p+B_p)/\partial\phi(\zeta)=0$ at $\phi(\zeta)$=0.5, (A_p+B_p) is increasing until $\phi(\zeta)=0.5$ and decreasing after that. Henceforth, we call this type of function as the inverted U-shaped function. Under the assumption that $w_f^e\varepsilon_f^e/w_b\varepsilon_b \geq 1$, C_p is monotonically increasing in $\phi(\zeta)$. Then, p_n is either a inverted U-shaped or a increasing function of $\phi(\zeta)$ □

Proposition 2.

Proof.

$$n^* = \underbrace{\frac{\zeta}{1+\zeta}}_{Z_n} \cdot \underbrace{\frac{\overbrace{w_f^e\varepsilon_f^e + w_m^e\varepsilon_m^e + (1+r)a - a'^*}^{A_n}}{p_n(w_f^e\varepsilon_f^e, w_b\varepsilon_b, \zeta) + p_e\varepsilon_e k^*}}_{B_n} \tag{1.61}$$

$$\frac{\partial n^*}{\partial\zeta} = \underbrace{\frac{1}{(1+\zeta)^2}\frac{A_n}{B_n(\zeta)}}_{+} - \underbrace{\frac{Z_n}{B_n(\zeta)^2}}_{+}\left(\underbrace{\frac{\partial a'^*}{\partial\zeta}B_n(\zeta)}_{\pm} + \underbrace{A_n\frac{\partial B_n}{\partial\zeta}}_{+}\right) \tag{1.62}$$

Suppose that there exists $\bar{\zeta}_n$ such that $\partial n^*/\partial\zeta = 0$. Then, following equation should be satisfied.

$$\underbrace{\frac{1}{(1+\zeta)^2}\frac{A_nB_n(\zeta)}{Z_n}}_{+} + \underbrace{B_n(\zeta)\left(\frac{-a'^*}{\partial\zeta}\right)}_{+} = \underbrace{A_n\frac{\partial B_n}{\partial\zeta}}_{+} \tag{1.63}$$

Notice that the sign of $\partial a'^*/\partial\zeta$ is pinned down to negative when $\partial n^*/\partial\zeta = 0$ through the intertemporal first-order condition as below.

$$\underbrace{\frac{1}{w_m^e\varepsilon_m^e + w_f^e\varepsilon_f^e + (1+r)a - p_nn^* - p_e\varepsilon_b k^*n^* - a'}}_{LHS} = \underbrace{\beta\frac{\partial\mathbb{E}V_2}{\partial a'}}_{RHS} \tag{1.64}$$

The left-hand-side (LHS) is increasing in a' and right-hand-side (RHS) is decreasing in a'. The equilibrium a' is determined at the cross point of LHS and RHS. Consider now we have $\bar{\zeta}_n - \varepsilon$ and it increases to $\bar{\zeta}_n$. Then, we have larger p_nn as p_n is increasing

function of ζ under Assumption 1, and n is not changed. We also have larger $p_e\varepsilon_b kn$, which is explained in the same way. Thus, LHS moves upside when we set x-axis to a', and the equilibrium a' should be decreasing in ζ. Namely, $\partial a'^*/\partial\zeta < 0$ around $\bar{\zeta}_n$.

In sum, if there exists $\bar{\zeta}_n$ as stated above then n^* is increasing until $\bar{\zeta}_n$ and decreasing after that. If not exists, $n^*$is increasing function of ζ. Let us define $\bar{\phi}_n$=$\phi\left(\bar{\zeta}_n\right)$. Then, above statement is equivalent that n^* is increasing until $\bar{\phi}_n$ and decreasing after that. □

Proposition 3.

Proof. P_n and n^* is proved directly by Proposition 1. and 2. As k^* is increasing function of p_n, k^* is also increasing in $\phi(\zeta)$.

$$p_n = \frac{1}{Z}\left[\underbrace{\left(\frac{\phi(\zeta)}{1-\phi(\zeta)}\right)^{1-\phi(\zeta)}}_{A_p} + \underbrace{\left(\frac{1-\phi(\zeta)}{\phi(\zeta)}\right)^{\phi(\zeta)}}_{B_p}\right] \cdot \underbrace{w_b\varepsilon_b\left(\frac{w_f^e\varepsilon_f^e}{w_b\varepsilon_b}\right)^{\phi(\zeta)}}_{C_p} \tag{1.65}$$

$$t_f^*(n;\varepsilon_f,e_f,\zeta) = \frac{1}{w_f^e\varepsilon_f^e}\cdot\frac{1}{Z}\underbrace{\left(\frac{\phi(\zeta)}{1-\phi(\zeta)}\right)^{1-\phi(\zeta)}}_{A_p}\underbrace{w_b\varepsilon_b\left(\frac{w_f^e\varepsilon_f^e}{w_b\varepsilon_b}\right)^{\phi(\zeta)}}_{C_p} n \tag{1.66}$$

Since the $\bar{\phi}_A$ such that $\partial A_p/\partial\phi(\zeta) = 0$ is larger than $\bar{\phi}_{A+B}$ such that $\partial\left(A_p+B_p\right)/\partial\phi(\zeta)$= 0 and C_p is the same monotone transformation, $t_f^* = \frac{1}{w_f^e\varepsilon_f^e}\cdot\frac{1}{Z}A_pC_p$ is increasing until $\bar{\phi} \equiv \min\left(\bar{\phi}_p,\bar{\phi}_n\right)$ under Assumption 1 and 2. □

Proposition 4.

Proof.

$$\frac{\partial p_n}{\partial\left(w_f^e\varepsilon_f^e\right)} = \frac{1}{Z}\left(A_p+B_p\right)\left(w_b\varepsilon_b\right)^{1-\phi(\zeta)}\phi\left(\zeta\right)\left(w_b\varepsilon_b\right)^{\phi(\zeta)-1} > 0 \tag{1.67}$$

$$\frac{\partial^2 p_n}{\partial \left(w_f^e \varepsilon_f^e\right)^2} < 0 \tag{1.68}$$

□

Proposition 5.

Proof. Since B_n is increasing and concave in in $w_f^e \varepsilon_f^e$ by Proposition 4 and A_n is linearly increasing in $w_f^e \varepsilon_f^e$, n^* is a U-shaped function of $w_f^e \varepsilon_f^e$. □

9.4 Toy Model: The Fertility Theory meats the Roy-Bojas Model

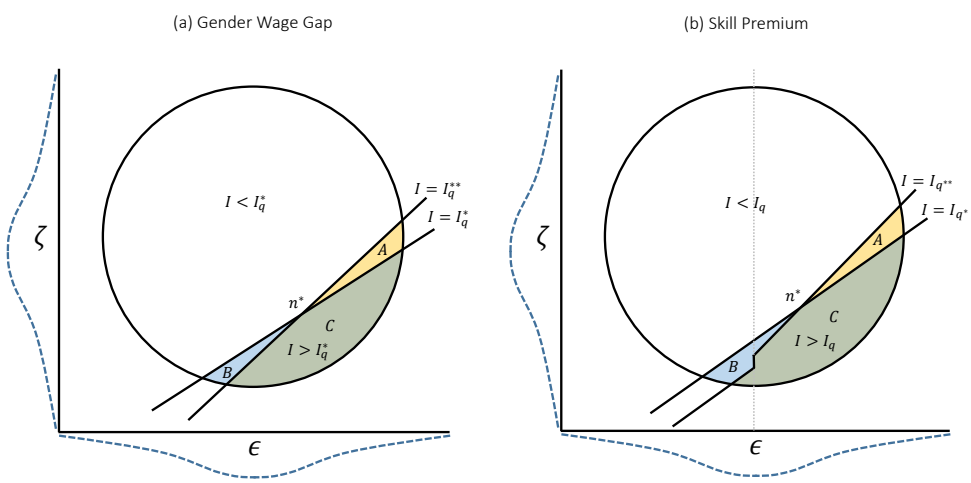

Figure 1.20: Toy Model

Consider a simple static problem as below. For simplicity, we only consider females in the toy model, and the number of children has positive real value or zero. A female earns utility from consumption (c) and their children (n). However, females have the different attitude on having children each other, which is captured by ζ. In budget constraint, w is wage for efficiency labor and ε means productivity. Having children reduces working hours by the amount of t per child. Eventually, females

earn their labor income, $w\varepsilon(1-tn)$, and we define it I. Notice that females have a distinct combination of (ζ, ε).

$$u = c + \zeta \log n \tag{1.69}$$

$$s.t \;\; c = w\varepsilon(1-tn) \equiv I \tag{1.70}$$

$$c \geq 0 \tag{1.71}$$

$$0 < tn \leq 1 \tag{1.72}$$

The optimal number of children is:

$$n^*(\zeta, \varepsilon) = \frac{\zeta}{w\varepsilon t} \tag{1.73}$$

Similarly to classical fertility theory, the optimal number of children decreases in the opportunity cost, or the price of children, $w\varepsilon t$. In addition, the number of children rises as the preference on having children, ζ, increases, which is a distinct aspect of this paper.

As we discuss the complete fertility rate by income quintiles, we need to define the iso-income curve that determines the income groups. We can derive this on the $\varepsilon - \zeta$ plan as below.

$$I = w\varepsilon(1 - tn^*(\zeta, \varepsilon)) \tag{1.74}$$

$$\zeta = w\varepsilon - I_q^* \tag{1.75}$$

We also define iso-n curve by using the optimal number of children. (Formula 1.73)

$$\zeta = n^* tw\varepsilon \tag{1.76}$$

Now suppose that I_{q^*} is the threshold income of the top 20%, i.e., group 5. Then,

the area below the iso-income curve, $I = I_{q^*}$ stands for the highest income quintile. The slope of the iso-income curve becomes steeper as following the equation 1.75 when the female wage rises or the wage of upper half of the ε increases. Note that the first illustrates narrowing gender wage gap (GWG) and the second one stands for increase in the skill premium (SKP). Panel (b) in Figure 1.20 shows the situation that the rise in skill premium occur. Let cross point of two iso-income curves, $I = I_{q^*}$ and $I = I_{q^{**}}$ be n^*. If $n^*$is small enough then we can show that the average number of children of the influx, A, is larger than those of the outflux, B.

9.5 Supplement Data

	GR 1	GR 2	GR 3	GR 4	GR 5
Average Quintiles from age 21 to 35	1.2	1.8	2.6	3.5	4.3

Table 1.10: Average Income Quintile from Age 21 to 35

Source: PSID

Note: We computed the female labor income quintile at each age category divided 5-years gap. We took average the quintile values of the group 21-25, 26-30, and 31-35.

9.6 Regularity Condition for Endogeneity Problem

9.6.1 Endogeneity Problem

Consider the case that the difference of the realized and true preference, $CFR - Zeta^*$, has positive effect on the ideal number of children, $Zeta$. We can specify this situation as in the Equation 1.77. Then we can see the effect of the contaminated $Zeta$ by imposing the realtion in Equation 1.77 into the original specification (Equation 1.79). Then we get the true specification with $Zeta^*$ and check the endogeneity problem. As shown in Equation 1.80, the level of bias is determined by A. If A has positive value

and larger then 1, the $\hat{\beta}_1$ and $\hat{\beta}_2$ are under estimated. If A has positive and smaller than 1, these coefficients are over estimated. Nevertheless, the sign of the coefficients is not changed in these two cases although the values are biased. However, if A is negative, the sign is flipped to opposite direction.

$$
\begin{aligned}
Zeta &= \alpha_0 + \alpha_1 (CFR - Zeta^*) + \alpha_2 Zeta^* + \eta && (1.77)\\
CFR &= \beta_0 + \beta_1 Zeta + \beta_2 X + \varepsilon && (1.78)\\
&= \beta_0 + \beta_1 [\alpha_0 + \alpha_1 (CFR - Zeta^*) + \alpha_2 Zeta^* + \eta] + \beta_2 X + \varepsilon && (1.79)\\
CFR &= \frac{\beta_0 + \alpha_0 \beta_1}{1 - \alpha_1 \beta_1} + \frac{\beta_1 (\alpha_2 - \alpha_1)}{1 - \alpha_1 \beta_1} Zeta^* + \frac{\beta_2}{1 - \alpha_1 \beta_1} X + \frac{\beta_1 \eta + \varepsilon}{1 - \alpha_1 \beta_1} && (1.80)\\
&= \beta_0^* + \beta_1^* Zeta^* + \beta_2^* X + \varepsilon^* && (1.81)
\end{aligned}
$$

9.6.2 Regularity Conditions

From the equation above, we found that if $(\alpha_2 - \alpha_1) / (1 - \alpha_1 \beta_1) > 0$ then the sign of β_2 and β_2^* are same each other. Thus we can derive following regularity conditions.

Condition 1. $\alpha_2 > \alpha_1$

Condition 2. $1 > \alpha_1 \beta_1$

Condition 1. indicates that the effect of true preference, $Zeta^*$ on observed preference is stronger than that of contamination or adjustment term, $CFR - Zeta^*$. In this context, we assume that Condition 1. hold.

Note that Miller et al. (2010) predicted that β_1^* has positive value, and we have positive estimate of β_1 in the text. Hence, we guess second regularity condition hold under reasonable environment.

Chapter 2

Population Aging and the Extension of Retirement Age

Quantitative Analysis using Overlapping Generation Model

1 Purpose and Motivation [1]

During the last few decades, population aging in developed countries is accelerating as 1) the average life expectancy increases, and 2) the population growth rate declines due to the deepening of low fertility rates. South Korea appears to be one of the best examples of an aging society. According to the National Statistical Office of Korea, the probability of surviving to aged 65 was 74.5% in 1990 and rose to 88.7% in 2010. Life expectancy also rose from 71.3 in 1990 to 80.8 in 2010. On the other hand, the average number of children decreased from 3.0 in 1990 to 2.4 in 2010 as the fertility rate continued its decline. Thus, the population growth rate dropped from 0.99% in 1990 to 0.46% in 2010. These two demographic changes have caused population aging, which has in turn reduced the economically active population as a percentage of the total population. For this reason, there have been active debates about the need to extend retirement age as one of the measures against the adverse effects of an aging population. The law extending retirement age was enacted in 2013 and was fully implemented in 2017.

The extension of retirement age has the advantage of enabling a society to use

[1] This chapter is based on the paper "인구고령화와 정년연장 연구: 세대 간 중첩모형(OLG)을 이용한 정량 분석" co-authored with Prof. Jay H. Hong, who is a dissertation committee member, and Dr. Taesu Kang. The original article was published in *경제분석* Vol 22 (2) in 2016. I translated and reconstructed the paper and added a new, detailed description. Note that the journal and two co-authors approved the use of the original article as a part of this dissertation.

high-level human capital accumulated through someone's lifetime. In addition, the government can substantially shorten the beneficiary period of the pension as the working period increases due to the extension of retirement age. Hence, the government can better secure the financial soundness of its social security system.

However, with regard to youth unemployment, which has recently become a controversial social problem, the extension of retirement age may have the disadvantage of displacing the employment of young people by restraining the recruitment demands of enterprises. Recent empirical studies also demonstrate that population aging is the cause of deepening income inequality. Specifically, low-income older wage earners and their income gaps are considered one of the main reasons for current high levels of inequality. In this study, we analyze whether an extension of retirement age can effectively achieve economic growth, fiscal soundness, and mitigation of income inequality by using a quantitative macroeconomic model composed of multiple generations based on the neo-classic framework.

We interpret previous extant studies on population aging as follows. A study conducted by Kim Yong-jin and Lee Chul-in (2013) theoretically demonstrated that an increase in the aged labor force due to an increase in average life expectancy results in a decrease in relative wages among the elderly. Kim Joo-young and Cho Jin-hwan's (2012) research analyzed the aging of the labor force across industry sectors. They demonstrated that aging of the traditional manufacturing sector was especially severe, having investigated overseas cases such as Germany and Japan. According to a study by Kim Dae-il (2010), as a result of the extension of the working period, the incentive for savings decreases, and capital input in the economy declines as a result. Further, a study conducted by Cho Jang-ok (2005) demonstrated that when population aging is accelerated, the burden of pension financing becomes colossal. He argued that the pension system should be improved to address changes in demographic structure.

In our study, we quantitatively analyze the effect of an aging population on the macro economy, as well as the impact of the retirement extension policy. This article

conducts a quantitative investigation of changes in the soundness of social security finances, along with variations in macroeconomic variables such as gross output and employment rate by age. Additionally, we compare welfare changes among different age groups before and after policy implementation.

This Chapter is comprised as follows. In Section 2, we construct the theoretical overlapping generations model to study the macroeconomic effects of population aging and discuss the specific settings and components of the theoretical model. In Section 3, we describe the process of calibrating the parameters by using empirical data. Section 4 explains the results of the model and examines the dynamic changes in macroeconomic variables caused by population aging. Section 5 studies the macroeconomic effects and welfare changes that occur when the retirement extension policy is implemented. We conclude the discussion in Section 6.

2 Model

To study the effects of the population aging, we propose the following Overlapping overlapping Generations model. There are infinitely many workers in the model economy by age, and the total number of workers at age one is normalized to one. We assume that individual workers at age i face an uncertain life expectancy due to the age-specific mortality risk(γ_i), and they can survive for the period I at maximum. That is, the probability of death is given as one in the last period of life. $(\gamma_I = 1)$

$$\mu_{i+1} = \frac{1-\gamma_i}{1+\lambda}\mu_i, \;\; \mu_1 = 1 \tag{2.1}$$

The total number of the population at each age is normalized to the mass of the entry population (μ_1) to prevent the divergence of the model due to population growth.

Workers are allowed to work until retirement age (R). After retirement ($i \geq R$), they will receive social security benefits and make consumption and savings deci-

sions by using these benefits for the remaining periods. We assume that individual workers at age one are born with different abilities in terms of labor productivity which follows a stochastic process after that. Additionally, the employment and unemployment statuses of workers are determined by ex-ante job separation and job finding probability, which are specified later. After the entry of individual workers into the labor market, they become distinct from each other according to their unique labor productivity and employment status. These distinct statuses cause differences in asset accumulation and labor income among workers. At a certain point, the model economy consists of people who are born at a different time and have distinct characteristics. Thus, the macro economy is constructed by aggregating all decisions by heterogeneous agents in the model economy.

We define average labor productivity by age as ε_i, and it gradually changes through people's lives as the economic agent ages. It reflects the variation in labor income, which changes through the life-cycle due to age, and human capital accumulation. In addition, we assume that individual workers of the same age have different idiosyncratic labor productivity(x) even though they share the same average age-specific labor productivity(ε_i). Idiosyncratic labor productivity fluctuates stochastically following the transition probability distribution $\Gamma(x'|x)$. Thus, the pre-tax labor income of a worker at a specific age is given as the multiplication of three factors: the economy's average wage (w), age-specific average productivity (ε_i), and idiosyncratic labor productivity (x).

The employment status of workers is determined by job separation and job finding probability depending on the age of the worker. The job separation rate of the employed is δ_i, while the job finding probability of the unemployed is θ_i. We can express the change in the employment rate by age as shown below.

$$e_{i+1} = (1-\delta_i)\, e_i + \theta_i\,(1-e_i) \tag{2.2}$$

The probability of the job separation and job finding rates are assumed to be different by age, as previously mentioned, reflecting the fact that the employment rate is different for each age group.

We assume that the output depends on the production function of the representative company, and it produces the final consumer goods by using the aggregate capital and labor input, which is the sum of the labor force of different ages. The production function has a standard Cobb-Douglas form, which is suggested as follows.

$$Y = zK^{\alpha}N^{1-\alpha} \tag{2.3}$$

α indicates the capital to output ratio, and z represents the total productivity of the economy. The labor forces of different ages are combined with the CES (constant elasticity of substitution) aggregator, and the sum of the total labor force is suggested as follows.

$$N = \left(\sum_i \chi_i N_i^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{2.4}$$

In this formula, ρ represents the elasticity of substitution among labor forces at different ages, and χ shows the proportion of labor input by age. Note that the sum of the labor input ratio is normalized to one ($\sum_i^{R-1} \chi_i = 1$). The representative enterprise combines the labor force of different ages, and together with the total capital, produces the final good.

2.1 Value Function

In this model economy, retirees and workers make savings and consumption decisions in the process of maximizing the value function specified below. We look at the retiree's problem first and account for the worker's problem after that.

2.1.1 The Retiree's Problem

The value function of a surviving retiree is as follows.

$$U_R^i(a) = \max_{c,a'} u(c) + \beta(1-\gamma_i) U_R^{i+1}(a') \tag{2.5}$$

$$c + a' = (1+r)a + \xi + (1-\tau)b \tag{2.6}$$

$$c \geq 0, \;\; a \geq 0 \tag{2.7}$$

Retirees make decisions on the consumption (c) and savings (a') given their state variable, asset (a). In the budget constraint, retirees receive capital income and a pension (ξ) for their remaining lifetime. We assume that there is no utility from inheritance to simplify the model. To be specific, the utility function increases with respect to consumption, and the agents have no utility after death. Hence, the inheritance incentive disappears for people who are in their last period, age I. However, note that if an economic agent whose age is less than I is dead, as their death probability is positive, an unintended inheritance (b) may occur. In this case, the government collects their assets and distributes them equally to the surviving agents in the economy. Here, τ indicates the inheritance tax rate.

2.1.2 The Employee's Problem

The value function of the workers before retirement is as follows.

$$V^i(a,x) = \max_{c,a'} u(c) + \beta(1-\gamma_i) E_{x'|x}\left[(1-\delta_i) V^{i+1}(a',x') + \delta_i U^{i+1}(a',x')\right] \tag{2.8}$$

$$c + a' = (1+r_{ss}) w\varepsilon_i x - T_i(a,x) + (1-\tau)b \tag{2.9}$$

$$c \geq 0, \;\; a \geq 0 \tag{2.10}$$

$T(\cdot)$ represents the income tax, and τ_{ss} denotes the social security tax rate levied during the employment period.

For the worker who will retire next period, the value function can be simplified as follows, with the same budget constraint as before.

$$V^i(a,x) = \max_{c,a'} u(c) + \beta(1-\gamma_i) U_R^{i+1}(a') \tag{2.11}$$

2.1.3 The Unemployed's Problem

In the same way, the value function of the unemployed is as follows.

$$U^i(a,x) = \max_{c,a'} u(c) + \beta(1-\gamma_i) E_{x'|x}\left[\theta_i V^{i+1}(a',x') + (1-\theta_i) U^{i+1}(a',x')\right] \tag{2.12}$$

$$c + a' = (1+r)a + \eta(w\varepsilon_i x) + (1-\tau)b \tag{2.13}$$

$$c \geq 0, \;\; a \geq 0 \tag{2.14}$$

η represents the unemployment benefit during the period of unemployment. This benefit includes the temporary labor income during the unemployment period, as a unit period in the model is a year in the reality.

Notice that the labor supply in this study is not determined by endogeneous choice of economic agent, but by the exogenous job separation(δ_i) and job finding rate (θ_i) corresponding to age-specific employment rates(e_i). If we internalize the labor supply in the model, the demographic change will affect the labor supply endogenously, and its effect on the macroeconomy will be analyzed more precisely. However, as a prerequisite for this, we need information on how the labor supply of each age responds to changes in demographic composition. In this paper, however, we simplify the labor supply decision in order to concentrate on changes in labor demand among enterprises as we are lacking in previous empirical studies on the

relationship between demographic change and labor supply. It is expected to be an interesting research topic if we construct the model including the endogenous labor supply and examine the differences from the current study.

When a worker dies, the asset is assumed to be transferred to the surviving agents via the government. Total assets transferred are given as follows, except for the inheritance tax deduction.

$$A = \sum_i \mu_i \int \gamma_i (1-\tau) a d\phi(a,x) \tag{2.15}$$

The government manages two fiscal accounts separately: the unemployment benefit and the pension for retirees. Equation 2.16 indicates how the former achieves a balanced budget. The tax revenue, left-hand side, consists of the sum of income tax and inheritance tax, while government spending, right-hand side, consists of public expenditures (G) and total unemployment benefits payments.

$$\sum_i \mu_i \int [e_i T_i(a,x) + \gamma_i \tau a] d\phi(a,x) = G + \sum_i \mu_i \int [(1-e_i)\eta(w\varepsilon_i x)] d\phi(a,x) \tag{2.16}$$

The left-hand side of pension account stands for the sum of social security tax paid by workers, and the right-hand side indicates the total payment of pensions to retirees.

$$\sum_i \mu_i \int [e_i \tau_{ss} w \varepsilon_i x] d\phi(a,x) = \sum_{i \geq R} \mu_i \int \xi \phi(a,x) \tag{2.17}$$

Notice that the pension account is modeled to achieve the balanced budget initially, but the changes in demographic structure create a gradual fiscal deficit. We will examine the soundness of the social security system in Section 4.

2.2 Equilibrium

Define state variables as age $i \in I$, employment status $\varepsilon_d \in E$, asset $a \in A$, and idiosyncratic productivity $x \in X$. Then we can depict state space as $S \equiv I \times E \times A \times X$. Suppose that $\sum_S$ indicates $\sigma - algebra$ of state space S and let $(S, \sum_S)$ is measurable space corresponding this setting. We illustrates specific state as $s \in S$, household measure as ϕ_t at time t regarding measurable space defined above, and its initial stationary distribution is ϕ_{ss1}. For convenience of description, we express the set of values for all ages by omitting superscripts, and the sequence of time is also depicted using braces. For example, $\{V_t(s)\}$ indicates the time sequence $\{V_{t=1}(s), V_{t=2}(s), \cdots\}$, and the element $V_t(s)$ means the set of values for all age such as $V_t^{i=1}(s), \cdots, V_t^{i=I}(s)$.

Suppose that the model economy is on the steady state initially. In the initial steady state, economic agents undergo unexpected population shocks in the first period. This shock can be expressed as a population sequence $\{\mu_t\}$ due to the change in the probability of death (γ) and population growth rate (λ). Since these changes took place in the first period, economic agents have a perfect foresight in the change of population composition. In this situation, we define the following as market equilibrium.

Definition. *Market equilibrium consists of the set of value functions, $\{V_t(s), U_t(s), U_{R,t}(s)\}$; policy functions $\{c_t(s), a_{t+1}(s)\}$; factor demand $\{L_t, K_t\}$; prices $\{w_t, r_t\}$; government policies ξ_{ss1}, $\{T_t\}$; household's distributions $\{\phi_t\}$, and it satisfies following conditions for all t.*

1. *The value functions and policy functions solve the household's utility maximization problem given prices $\{w_t, r_t\}$ and government policies ξ_{ss1}, $\{T_t(s)\}$.*
2. *Prices satisfy firm's profit maximization problem*
3. *Government policies satisfy government budget constraint*
4. *Markets are cleared*

(a) *Labor supply and demand at each age satisfy following condition*

$$N_i^d \equiv N_i = \mu_i \int e_i x d\phi_i(a,x) \equiv N_i^s \tag{2.18}$$

(b) *Capital supply and demand satisfy following condition*

$$K^d \equiv K = \sum_{i=1}^{I} \mu_i \int a d\phi_i(a,x) \equiv K^s \tag{2.19}$$

5. *Distributions are consistent with individual behavior*

$$\phi_{t+1}^{i+1} = \int_S Q_t d\phi_t^i \tag{2.20}$$

$$\textit{where } Q_t = (1-\gamma_{i,t}) \times 1\{i+1 \in I, a_{t+1}(s) \in A\} \times Pr\left[\varepsilon_{d,t+1}^{i+1} | \varepsilon_{d,t}^{i}\right] \times Pr\left[x_{t+1}^{i+1} | x_t^i\right]$$

In the fifth condition,$(1-\gamma_{i,t})$ is the survival probability and $1\{\cdot\}$ means the indicator function. $Pr\left[\varepsilon_{d,t+1}^{i+1} | \varepsilon_{d,t}^{i}\right]$ is the conditional probability of the employment state in the next period given current employment status. Lastly, $Pr\left[x_{t+1}^{i+1} | x_t^i\right]$ indicates the conditional probability of the productivity in the next period given current productivity.

We can define a steady state is a special case that value functions V_t, U_t, $U_{R,t}$; policy functions c_t and a_{t+1}; firm's factor demand L_t and K_t; prices w_t and r_t; government policy T_t; distribution ϕ_t are constant for all t.

3 Calibration

3.1 Analysis of the Population Aging Phenomenon

In this section, we quantify two factors that cause population aging: the increase in average life expectancy due to the improvement of medical technology, and the decline in population growth due to low fertility rates.

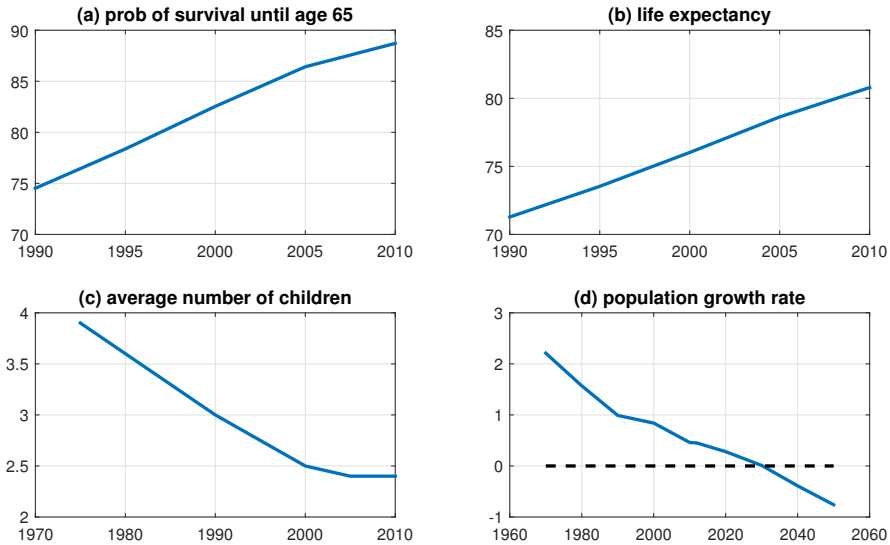

Figure 2.1: The Increase in Life Expectancy

Source: For (a), (b) and (d), Life Tables 2007 and 2010, National Statistical Office of Korea. (c) is Taken from Kim Honggi (2013).

3.1.1 Increase in Average Life Expectancy

Panel (a) of Figure 2.1 illustrates changes in mortality rates in Korea over the last two decades. The probability of survival from birth to aged 65 has gradually increased over the 20-year period. In 1990, the likelihood of survival to 65 years was 74.5%, while the rate of births in 2010 increased to 88.71%. In addition, the average life expectancy increased from 71.3 years in 1990 to 80.8 years in 2010. That is, it rose by about 10 years in the last 20 years.

In this paper, we use the age-specific mortality rates (γ_i) suggested in the 2006 life table from the National Statistical Office of Korea. In the initial steady state, the probability of survival to 65 years of age at aged 20 is 88%, and life expectancy is 79.3 years. In order to reflect the increase in life expectancy and the related demographic changes after the initial steady state, we assumed that the mortality rate by age (γ_i)

gradually decreases by 1% at every period for 80 years[2].

Under this setting, the population ages gradually. The probability of a 20-year-old adult surviving to aged 65 is 94.33%, and their average life expectancy is 86.6 years in the new steady state.

Panel (c) of Figure 2.1 depicts how population structure changes every 20 years from the initial steady state as the probability of survival gradually increases. The change in the demographic composition which normalized to the 20-year-old population size indicates that the effect of a decline in mortality is highly concentrated in the elderly after aged 60 since the proportion of the elderly population increases more than that of the younger generation.

3.1.2 Deepening low birth rate

Kim Honggi (2013) stated that the average number of children decreased from 3.0 in 1990 to 2.4 in 2010 due to the deepening of the low birth rate. It makes a sharp decline in the population growth rate in Korea as shown in panel (d) of Figure 2.1. According to the National Statistical Office's Future Population Estimates, the population growth rate dropped from 0.99% in 1990 to 0.46% in 2010, and after 2030, the population growth rate will turn negative, and the overall population will decrease.

In this paper, we set the population growth rate to 0.685% which is the average value for the ten years since 2000 in the initial steady state. We assumed that the population growth rate (λ) decreases to 0.3% in order to introduce the low fertility phenomena to the model, which is calculated by the average value of the next 20 years' estimates.

Panel (a) of Figure 2.2 shows the changes in demographic composition when the permanent change in the population growth rate is reflected in the model. The

[2]Suppose that the mortality rate of people whose age is i is $\gamma_{i,t}$ at time t. Then, we set the mortality rate of people whose age is i at time $t+1$ to $\gamma_{i,t+1} = \gamma_{i,t} \times 0.99$. This reduction in the mortality rate is conservatively determined so that life expectancy does not increase excessively. Based on the predictions of low income households among the scenarios proposed by Future Population Estimates (National Statistical Office, 2011), the life expectancy is set to 82 years after 30 years.

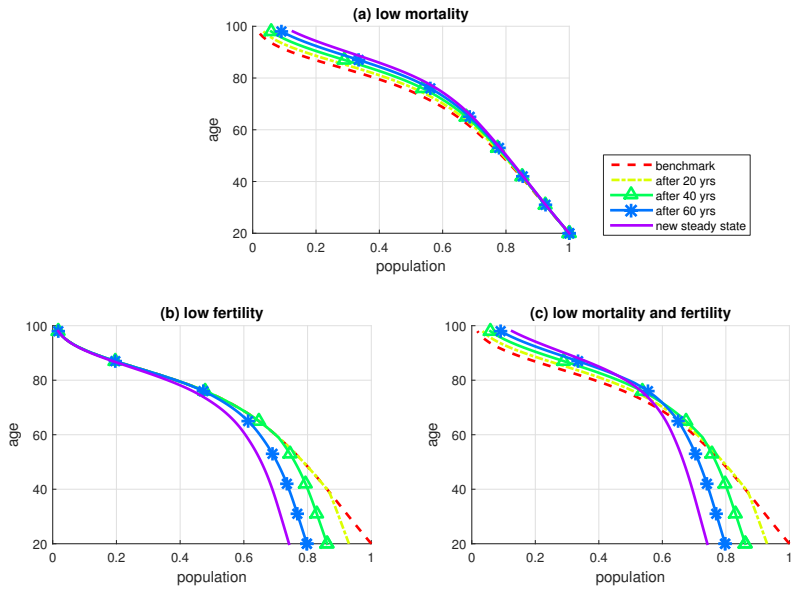

Figure 2.2: Population Structure: Low Motality, Low Fertility, and Both

Note: Change in population structure when 1) mortality, 2) fertility, and 3)both mortality and fertility decreases. Normalized by age 20 population of the initial steady state.

figure shows the variation of the population structure at every 20 years from the initial steady state. When compared the demographic structure between the initial and new steady state, we can see that the proportion of the economically active population is considerably reduced compared to the elderly.

3.2 Quantification of the model

We assume that workers are born and enter the labor market at aged 20 and survive up to aged 99 at maximum. ($I = 99$) The retirement age (R) is set at 57, and in Chapter 5, we will extend it by three years, focusing on the retirement age extension law, which is fully implemented in 2017. We also analyze the governmental policy that extends the retirement age by eight years (to aged 65) to more closely examine the effect of the retirement age extension.

We accept that the probability of the entry and exit of workers into the labor mar-

ket can vary according to age. Therefore, we set the job separation (δ_i) and finding rate (θ_i) differently at each age based on a study by Kim Hye Won (2008) that calculated relevant values by using the Korean Labor and Income Panel Study (KLIPS) data. In this process, we divide the age groups at every five years and compute the average job separation and finding rate of each age group. We also construct the five-year average employment rate, which is matched to the actual employment rate of males in 2006.

We assume that the a worker's idiosyncratic productivity (x) follows the Markov process, $\Gamma(x'|x)$. The stochastic process is approximated by Tauchen (1986) method. We equally divide the productivity domain between $-4\sigma_{y1}$ and $4\sigma_{y1}$ to create 17 grid points and set the maximum labor productivity to $6\sigma_{y1}$. According to Huggett (1996), we set σ_{y1} to 0.38 and build the AR(1) Markov process as shown in equation 2.21. Atkinson et al. (1992) estimated that γ_p is 0.96 and σ_ε^2 is 0.045 which is a variance of ε_t following $N(0, \sigma_{\varepsilon^2})$.

$$y_t - \bar{y}_t = \gamma_p (y_{t-1} - \bar{y}_{t-1}) + \varepsilon_t \tag{2.21}$$

To calculate the average productivity of workers by age (ε_i), we use the 2006 Survey Report on Labor Conditions by Employment Type conducted by the Ministry of Employment and Labor. Regarding the elasticity of substitution among different labor demands by age, there is a broad range of estimates. For U.S data, Borjas (2003) estimated the elasticity of substitution at 3.5, Card and Lemieux (2001) at 5, and Ottaviano and Peri (2012) at 7. Manacorda et al. (2012), who analyzed the UK data, argued that it was about 10. D'Amuri et al. (2010), using German data, estimated elasticity at 3.1. Since the range of estimated elasticity is about 3.1-10, it demonstrates a substitute relationship between intergenerational labor demands. Therefore, in this paper, we adopt the estimate of Card and Lemieux (2001), which is within the range of the values presented above. Note that this means we accept some degree of

substitution among intergenerational labor demands.

The labor input ratios (χ_i) at different ages are set to reflect the average productivity (ε_i) by age, i.e., the average labor income by age. For the convenience of calculation, we assume that the labor forces in the same five-year age group are perfect substitutes for each other. In the same way, we calculate the labor input ratio.

As explained in Section 2, government finance is divided into 1) the unemployment benefit portion, and 2) pension for retirees. These two accounts are set to achieve a balanced budget in the initial steady state. The former consists of government spending (G) and unemployment allowance. The size of government spending is set at 26% of gross product at initial equilibrium. This is the average value of government purchases from 2005 to 2012, and it is calculated based on data from the National Statistical Office. The unemployment benefit (η) is set to be 50% of the age-based (after-tax) average labor income, reflecting the actual unemployment benefit as well as earnings from a temporary job during the unemployment period because the time unit is a year in the model.

In the process of transferring the assets of the deceased to the surviving agents, the government imposed the inheritance tax and assumed that it is levied at the rate of 50%. This is a simple device to overcome the problem of tax revenues that are too small in the model economy, as we assume there is no bestowal or planned inheritance.

The government levies a fixed-rate tax on labor income, and we determine this tax rate endogenously to achieve a balanced budget in the initial steady state. The computed initial labor income tax rate is 1.53%. Regarding pension financing, the government imposes additional social security tax on labor income as well as labor income tax. The social security tax rate (τ_{ss}) is set at 9%, which is the effective national pension burden rate in Korea. The amount of the national pension is also calculated to achieve a balanced budget of pension finances in the initial steady state. The amount of pension receipt (ξ) in the initial equilibrium is 20.92% of the average

	Benchmark	Low Mortality
Total Population	100	109.63
Population under 60	71.45	72.27
Elderly Population Ratio	28.55%	34.08%
Output	100	106.33
Capital	100	116.77
Output per capita	100	96.99
Employment	62.49	63.03
Employment under 60	87.5%	87.21%
Labor	100	100.88

Table 2.1: The Change in Macroeconomic Variables in Low Mortality Economy

labor income.

The other parameters are determined as follows. The utility function of the workers and retirees in the economy is assumed to be CRRA type, $u(c) = \frac{c^{1-\sigma_u}}{1-\sigma_u}$ and is set at σ_u = 1.5. The capital input ratio (α) is set at 0.36, considering the average of the capital income ratio from 2005 to 2013. The time discount factor (β) is set at 0.9678 in the initial steady state to achieve an empirical capital-output ratio ($K/Y = 3.0$).

4 Results

In this chapter, by quantifying the factors of 1) decreasing mortality rate, and 2) decreasing population growth rate, we examine the macroeconomic effects both in a new steady state and on the transition path. In addition, we study the combined effects of the two factors on macroeconomic variables.

4.1 Effect of Reduction in Mortality Rate

4.1.1 Changes in Macroeconomic Variables

As can be seen in panel (a) of Figure 2.2, an increase in life expectancy leads to a rise in the proportion of older people in the economy. Compared with the initial economy,

the proportion of elderly people over 60 years of age greatly increases from 28.6% to 34.1%. The total number of the population normalized by the population in the initial steady state increases by about 9.6%. The population under 60 years of age increases by about 1%, while the population over 60 years of age rises by more than 30%. This is because even though the decline in mortality rates by age is uniformly applied, it is more effective in reducing the mortality rate of older age groups, which have relatively high mortality rates in the initial steady state. Note that the population growth calculated here indicates only an increase in the population through the reduction of mortality except for the effect of the natural population growth rate (n).

As shown in Table 2.1, the decrease in mortality rates demonstrates a slight increase in the population under 60 years of age, indicating that total employment also increases by 0.9%, from 62.5 to 63.0. The total amount of capital also rises because the agents increase precautionary savings in response to a prolonged post-retirement period. In particular, those in their 50s who were nearing retirement age and the older and elderly, ages 60-90, increase their capital accumulation compared to the initial economy, but the younger age group shows little variation. In addition, the effects of mortality declines are concentrated on the middle-aged and the elderly, whose savings rate is high on average. Because they constitute a larger fraction of the population due to the demographic change, the amount of capital increases even if we assume that the savings of individual workers and retirees remains unchanged. These two effects increase the amount of capital by about 16.8% compared with the value of initial equilibrium.

Gross output increased by 6.3%, which is derived from a combination of a 16.8% increase in capital and a 0.9% rise in labor. As the total population grows by about 10% while gross output increases by 6.3%, output per capita decreases from 100 to 97. This is because most of the effect driven by the decline in mortality is focused on the elderly, which results in little change in the mass of the economically active population under 57 years of age.

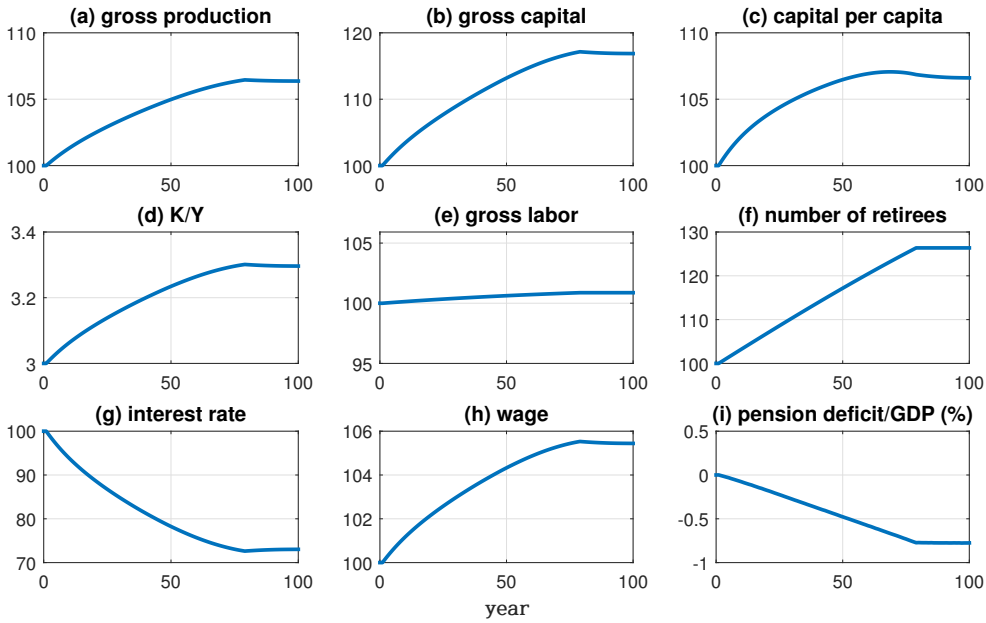

Figure 2.3: The Transition Dynamics of Low Mortality Economy

Figure 2.3 shows the dynamic changes in the macroeconomic variables over 100 years with a declining mortality rate. The number of retirees continues to increase, as shown in the sixth figure (f), but there is little change in total labor, as seen in the fifth figure (e), because the effect of the mortality decline is concentrated on retirees. The second (b) and third (c) figures show the increase in total capital and capital per capita in the economy. As the amount of capital increases, the real interest rate decreases and the average wage of total labor increases slightly. The rise in capital and labor inputs leads to an increase in output. The growth in the number of retirees and the number of pension recipients cause a deficit in pension financing (figure (i)). In the pension account that was in the balance at the beginning, the amount of pension payments by worker has not changed much, but the gross pension receipt considerably increases, resulting in a fiscal deficit of 0.78% of total production.

Figure 2.4 shows the variation in the employment rate by age due to the decrease in mortality. The employment rate ($e_{i,t}$) of age i at time t is determined to be in-

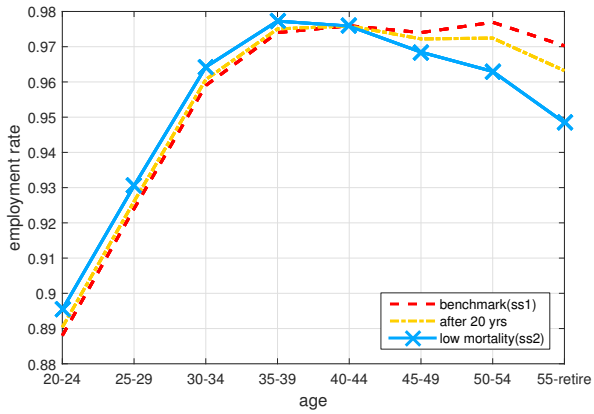

Figure 2.4: The Change in Employment Rate in Low Mortality Economy

versely proportional to the population proportion ($\mu_{i,t}$) given the average productivity (ε_i) and employment weight (χ_i). We assume that the total employment rate remains constant. As the decrease in mortality has a stronger effect on the older age group, the proportion of the 40s and 50s in the economically active population becomes relatively larger than before. As a result, the ratio of the employment rate (e_i/e_1) based on the 20 to 24-year-old employment rate (e_1) will be significantly lower after the 40s are reached. In order to keep the total employment rate constant, the trajectory of the employment rate by age should be moved upward while maintaining the ratio of the employment rate (e_i/e_1). As a result, the new employment rate by age is derived as the solid line in Figure 2.4 in the new steady state. Therefore, in the new equilibrium, the employment rate of young people before the age of 35-39 slightly increases compared with that of the initial equilibrium, but the employment rate after aged 40 decreases. We can explain this with the following intuition. The reduction in mortality leads to an increase in the total amount of capital in the economy, which in turn increases labor demand from the firm side. The rise in labor demand causes the employment rate of the younger generation to increase slightly, as there is little change in the population. On the other hand, the employment rate of the elderly decreases despite the rise in labor demand due to the larger increase in the employable

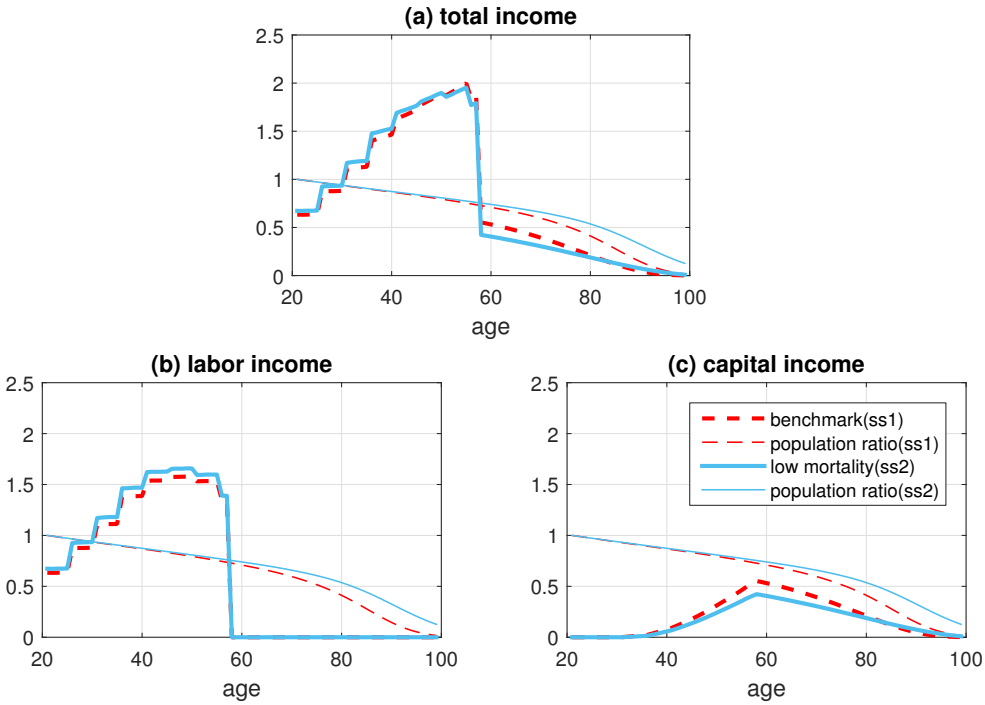

Figure 2.5: The Income Structure in Low Mortality Economy

population, which reduces the employment rate.

4.1.2 Changes in Income Inequality

Next, we examine how income inequality changes due to reductions in mortality. To illustrate the Gini coefficient, we briefly explain Figure 2.5 first. The thick red dotted line shows the values in the initial steady state, and the thick blue solid line indicates the income through the life cycle in the declining mortality economy, i.e.,

	Benchmark	Low Mortality
Total Income	0.352	0.415
Labor Income	0.269	0.267
Capital Income	0.521	0.509

Table 2.2: Gini index in Low Mortality Economy

the new steady state. The thin dotted and solid lines indicate how the demographic composition of each age group changes when normalizing a population of the 20s to one. This demonstrates that the proportion of the elderly population is increasing due to the declining mortality rate.

When calculating the Gini coefficient of labor income, we include the unemployed. However, households after retirement are excluded from the calculation, as they are not included in the economically active population by definition. As can be seen in Table 2.2, the Gini coefficient of labor income decreases slightly from 0.269 to 0.267 as the mortality rate decreases. The proportion of the middle-aged population with high labor income on average increases more than the younger population with low labor income. Thus, while the proportion of middle- and high-income workers increases, the share of the lower income earners decreases relatively. As a result, labor income inequality is reduced by moving the Lorenz curve in Figure 2.6 upward. However, since most of the demographic changes are concentrated on the age group after retirement, the shift of the Lorenz curve is very small, and therefore the Gini coefficient of labor income also decreases slightly.

As the probability of death decreased, the Gini coefficient of capital income fell from 0.521 to 0.509. The change in the proportion of middle-aged and elderly people who have more assets increases more than the younger population with fewer assets. In addition, as shown in the cumulative density function (CDF) in Figure 2.7, the capital income gap between the middle- and old-aged with substantial assets and the young with little assets decreases as the interest rate declines. Eventually, these two effects combined and cause the Lorenz curves to move upward. As a result, capital income inequality decreases. Inequality in gross income increases from 0.352 to 0.415, even though labor income and capital income Gini coefficients decreased. Because this may appear to be a contradiction at first glance, we are required to analyze it more carefully. To study this result, we divide the distribution of total income shown in the first diagram in Figure 2.5 into three levels. 1) The lowest-

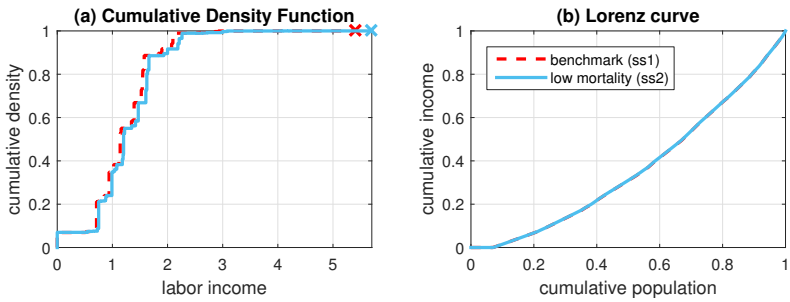

Figure 2.6: CDF and Lorenz Curve of Labor Income in Low Mortality Economy

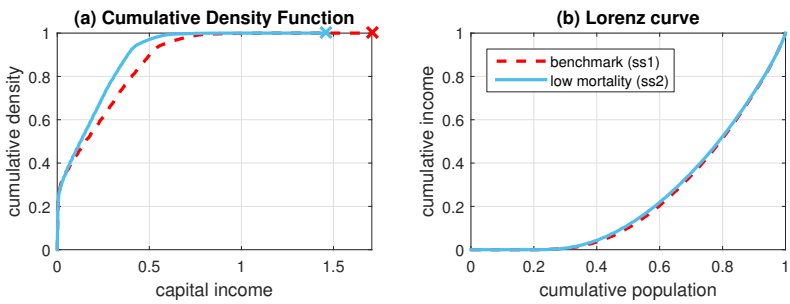

Figure 2.7: CDF and Lorenz curve of Capital Income in Low Mortality Economy

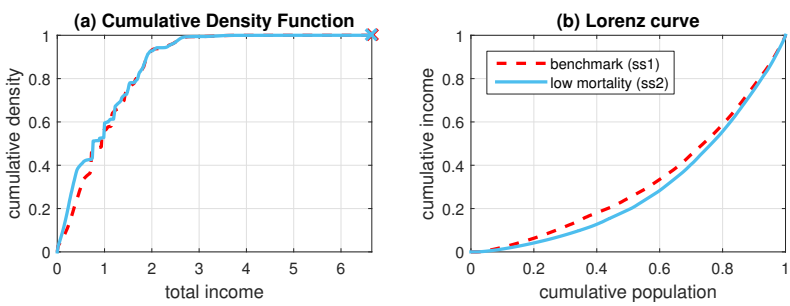

Figure 2.8: CDF and Lorenz Curve of Total Income in Low Mortality Economy

income group is made up of retired people aged 57 and older who earn only capital income. 2) The middle-income group consists of young people around the age of 20-40 with low capital and labor income. 3) The last group consists of middle-aged people around 40-56 who earn both high capital and high labor income. The change in gross income inequality is determined by the relative demographic composition of these three groups and the change in interest rates and wages. First, let us look at the aspect of the population composition ratio. As population aging progresses, the change in the proportion of the elderly who have the lowest total income increases the greatest. As can be seen from the CDF in Figure 2.8, the percentage of low-income households whose income is less than 0.7 increases from 36% to 43%. In other words, the proportion of the lowest income earners increases due to the change in demographic composition in the form of the decrease in mortality rate, which leads to an increase in gross income inequality. Second, because of the decline in the interest rate, the income of retired people who have only capital income declines more than that of other age groups who have both labor and capital income. It shifts the Lorenz curve downward, which in turn leads to higher income inequality.

4.2 Effects of Reduction in Fertility Rate

4.2.1 Changes in Macroeconomic Variables

This section examines the macroeconomic effects of the decrease in population growth due to declining fertility rates. This demographic change causes population aging, as it directly reduces the proportion of young people. The proportion of the elderly population aged 60 and older increases greatly from 28.6% in the initial equilibrium to 31.3% in the new steady state. The total number of the population normalized by the population in the initial steady state decreases by about 17%, and the population under 60 years of age declines by 20%. Overall employment declines by about 21% as the economically active population decreases significantly.

	Benchmark	Low Fertility
Total Population	100	82.95
Population under 60	71.45	57.01
Elderly Population Ratio	28.55%	31.27%
Output	100	80.32
Capital	100	82.27
Output per capita	100	96.83
Employment	62.49	49.51
Employment under 60	87.5%	86.84%
Labor	100	79.25

Table 2.3: The Change in Macroeconomic Variables in Low Fertility Economy

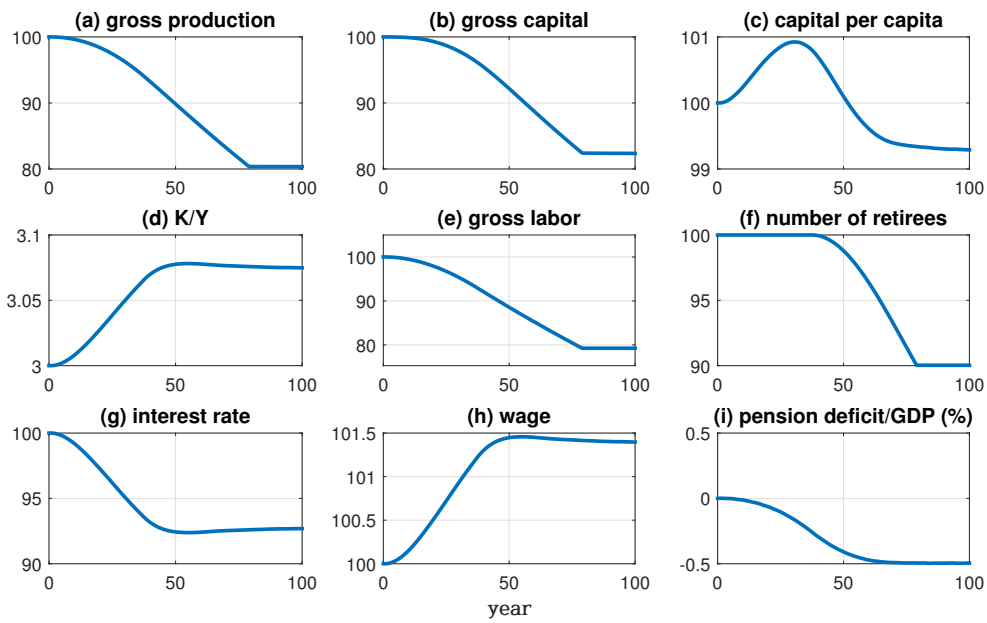

Figure 2.9: The Transition Dynamics in Low Fertility Economy

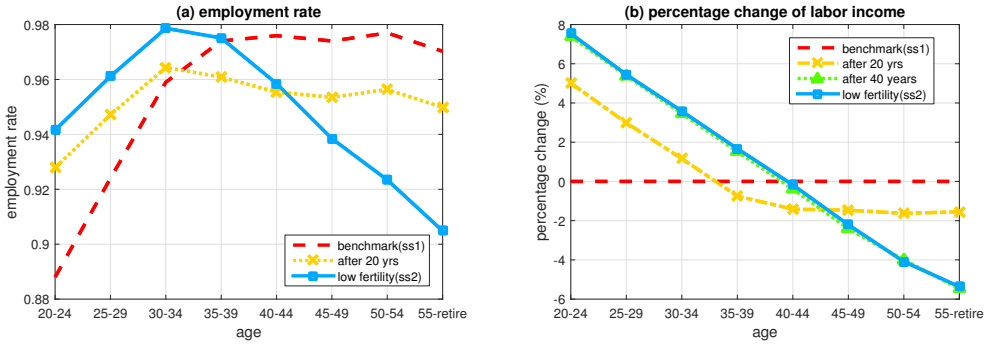

Figure 2.10: The Change in Employment Rate in Low Fertility Economy

As in Table 2.3, the reduction in the fertility rate leads to a decline in total employment and total labor input by 21%. We observe an even decrease in employment across all age groups before retirement. The total amount of capital is also reduced by about 18%, and the reduction in labor and capital input results in a 20% drop in output. In sum, the decline in the economically active population results in decreases in both labor input and capital accumulation, in which leads to a decline in gross production in the model economy. Output per capita drops by about 3%, which is due to the aging of the population. We next examine employment levels and employment rates by age. We found that the employment level decreases evenly across all age groups as firms decrease labor demand due to the decline of gross capital in the economy. The employment rate of young people, however, increases from 89% to 94%, while that of the elderly decreases from 97% to 91%.

Figure 2.9 shows the effect of the decline in population growth on the dynamic changes in macroeconomic variables. As we have seen, the decrease in population growth leads to a reduction in the mass of economically active people, which gradually reduces total labor input. However, the number of retirees has remained at its initial level for 40 years because the decline in fertility rates does not affect this number at the beginning of the transition path. After 40 years, the cohort who is newly born in the first period on the transition path begins to retire, and thus the number

of retirees decreases gradually from this point forward. For this reason, the pension deficit increases for 40 years after the low fertility shock occurs, leaving the fiscal deficit at about 0.5% of gross domestic product.

The graph on the left in Figure 2.10 shows the dynamic change in the employment rate by age as the birth rate falls. The declining fertility rate leads to a decrease in the population of young and middle-aged people, with relatively modest variation in the elderly population. For this reason, the young and middle-aged become more important in that firms demand a combined labor force of various age groups via the CES aggregator. This change in population causes a decline in the proportion of the employment rate (e_i/e_1) based on the 20 to 24-year-old employment rate at all ages, as was the case in the declining mortality economy in the previous section. In order to keep the total employment rate constant, the trajectory of the employment rate by age should move upward, resulting in the employment rate as shown in the solid line in Figure 2.10. In summary, the employment rates of young and middle-aged people rise, while the employment rate of the elderly falls. In other words, the decline in fertility rates leads to a decrease in the capital of the economy as a whole, and in response, representative firms reduce labor demand for all ages. However, for young people, the reduction of population is greater than the decrease in labor demand, which leads to a rise in the employment rate in the new steady state. On the other hand, among the middle-aged, the employment rate decreases because the decline in the population is relatively small and is not sufficient to outweigh the decrease in labor demand.

The changes in average labor income by age effectively illustrate this result. The right-hand side of Figure 2.10 shows the change in average labor income by age with a declining birth rate after the initial state. The labor income of young people increases by 8%. This is due to the combined effects of both the increase in the employment rate of young people and the rise of the average wage. On the other hand, workers over 40 years of age, especially those who are close to retirement,

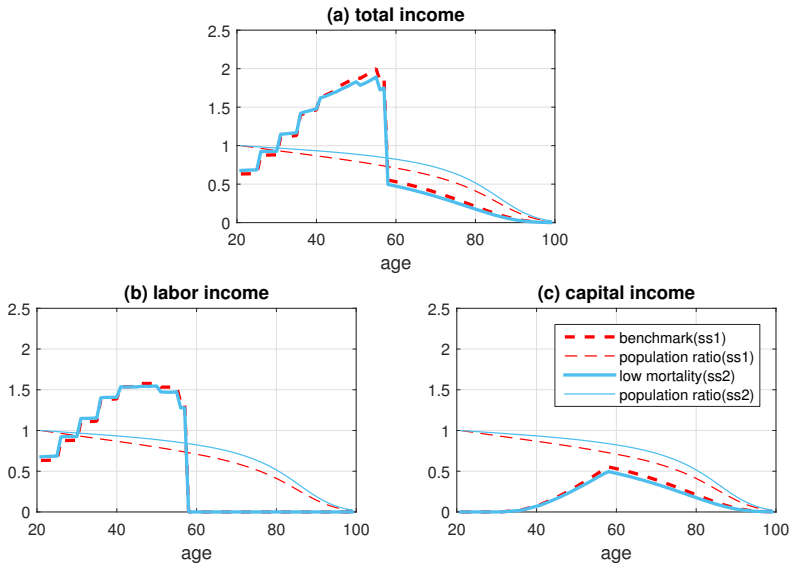

Figure 2.11: The Income Structure in Low Fertility Economy

	Benchmark	Low Fertility
Total Income	0.352	0.380
Labor Income	0.269	0.257
Capital Income	0.521	0.505

Table 2.4: Gini Index in Low Fertility Economy

have lower labor income that has fallen by about 6% compared to the initial level, despite rising average wages. This is primarily due to the decline in the employment rate of these age groups.

4.2.2 Changes in Income Inequality

The Gini coefficient of labor income declines slightly from 0.269 to 0.257. The change in the proportion of the elderly population is bigger than that of the younger population when the population growth rate decreases. Thus, we can apply the same logic as suggested in the declining mortality economy again. As the population growth rate decreases, the proportion of younger workers who have a low labor income on

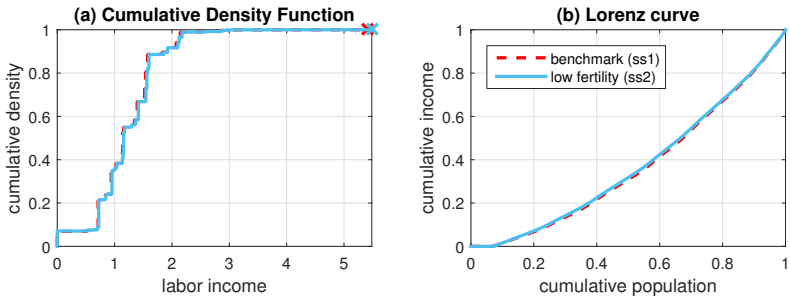

Figure 2.12: CDF and Lorenz Curve of Labor Income in Low Fertility Economy

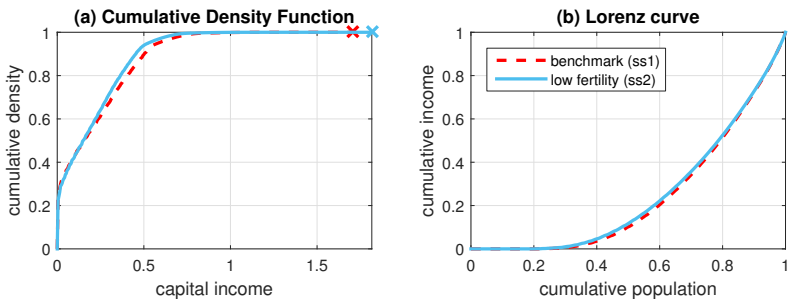

Figure 2.13: CDF and Lorenz Curve of Capital Income in Low Fertility Economy

average becomes smaller. On the other hand, the proportion of middle-aged workers with high labor income on average increases. Thus, as shown in the right-hand side of Figure 2.12, the Lorenz curve shifts upward, and the Gini coefficient for labor income decreases.

Capital income inequality also declines from 0.521 to 0.505 as the share of middle-aged and elderly people who hold more assets than the younger population increases. This reduces the inequality of asset holdings by shifting the Lorenz curve upward in Figure 2.13, similar to the previous discussion.

If the fertility rate declines, then the Gini coefficient for gross income increases from 0.352 to 0.380. As the proportion of the retirees who have the lowest gross income increases, it moves the Lorenz curve downward, and the Gini coefficient converges to a higher value than those of the initial steady state.

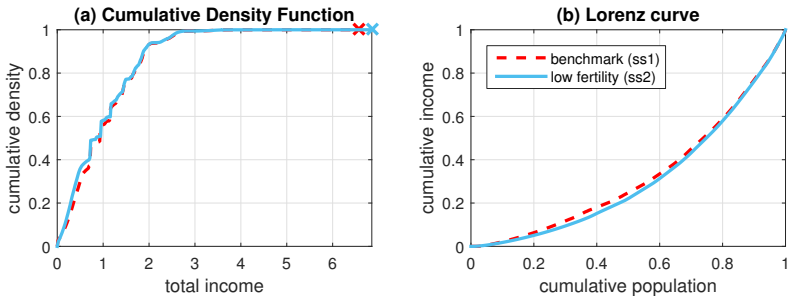

Figure 2.14: CDF and Lorenz Curve of Total Income in Low Fertility Economy

	Benchmark	Low Mor./Fer.
Total Population	100	91.98
Population under 60	71.45	57.69
Elderly Population Ratio	28.55%	37.28%
Output	100	85.47
Capital	100	96.31
Output per capita	100	92.93
Employment	62.49	49.94
Employment under 60	87.5%	86.6%
Labor	100	79.93

Table 2.5: The Change in Macroeconomic Variables in Low Mortality and Fertility Economy

4.3 Effects of Both Declining Mortality and Fertility Rate

4.3.1 Changes in Macroeconomic Variables

This Section examines the impact of population aging on the change in macroeconomic variables when mortality and fertility declines occur simultaneously. As shown in panel (b) of Figure 2.2, the mortality rate increases the life expectancy of the elderly population, and it causes the proportion of the elderly population to increase. On the other hand, the decline in fertility rates reduces the share of the economically active population even more rapidly.

As previously analyzed, the reduction in mortality rates slightly increases the

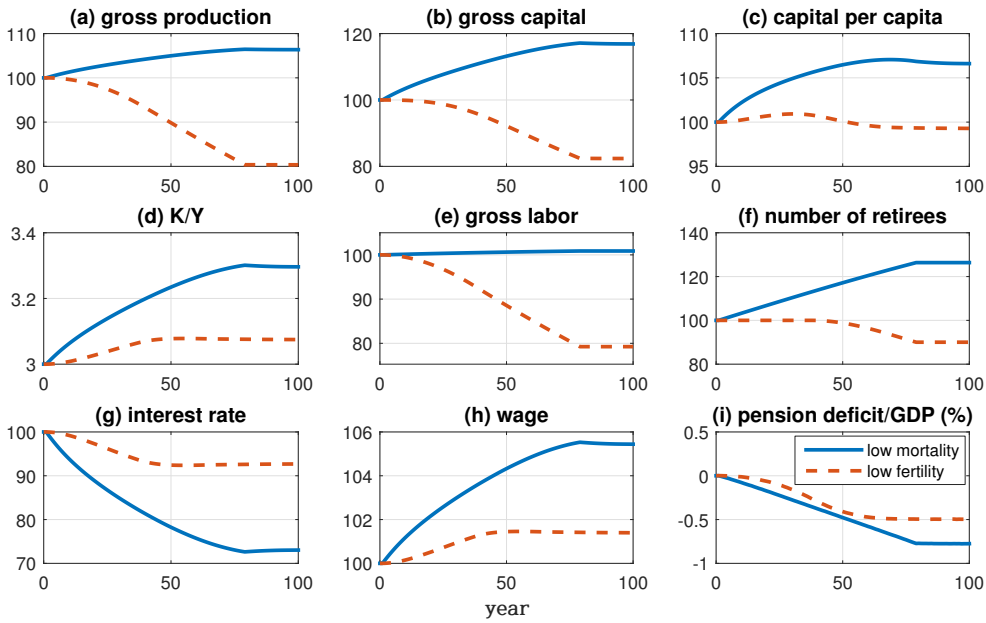

Figure 2.15: The Decomposition of Transition Dynamics of Two Economies

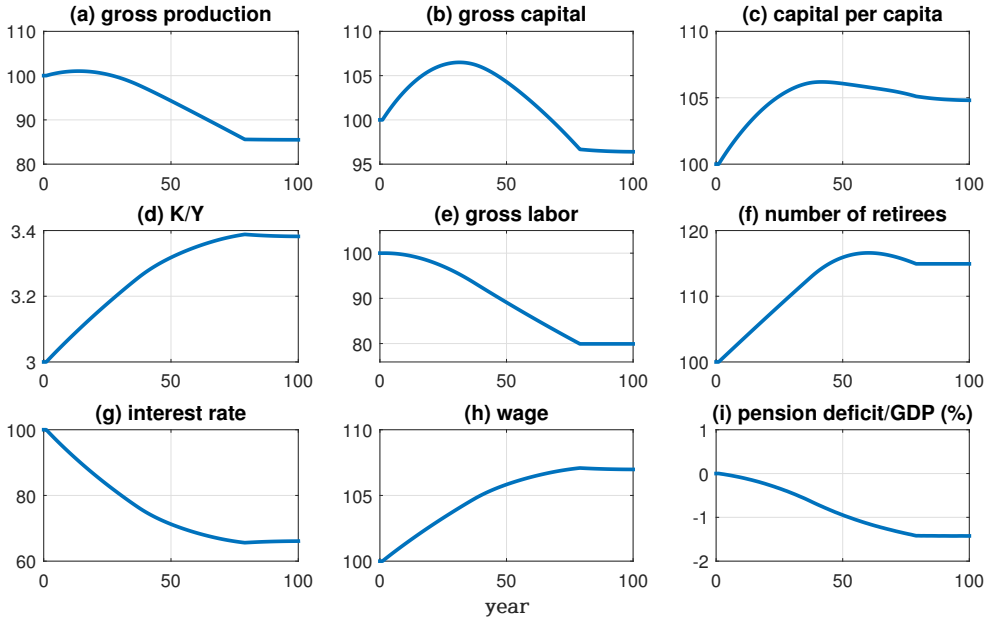

Figure 2.16: The Transition Dynamics in Low Mortality and Fertility Economy

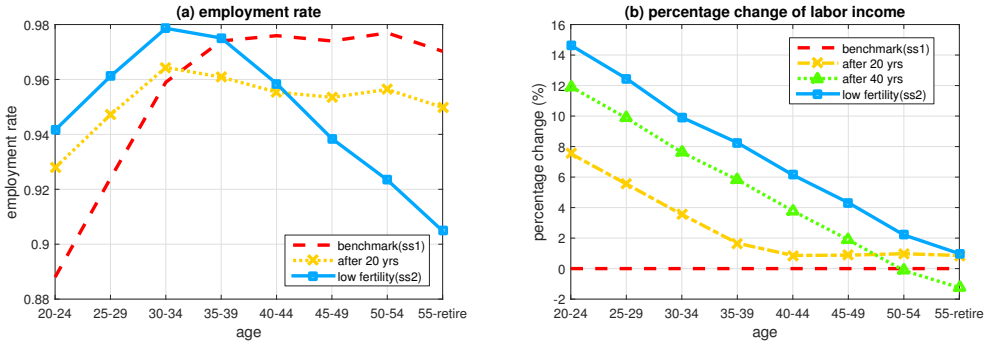

Figure 2.17: The Change in Employment Rate in Low Mortality and Fertility Economy

economically active population and the amount of capital, resulting in an increase in gross output to a relatively small extent. On the other hand, declining fertility rates significantly reduce the working population and thus capital accumulation. As a result, gross output drops by about 20%. We found that the decline in population growth due to low fertility rates has a significant impact on labor, capital and production, while the positive effect of reduced mortality on gross production is relatively small.

As depicted in Figure 2.15, mortality declines have little impact on labor input, but the number of retirees increases significantly. A decrease in fertility rates significantly reduces the labor force but does not cause a change in the number of retirees in the early stage of the transition path. Next, we examine changes in capital. A decreased mortality rate increases life expectancy and thus increases savings to address a prolonged retirement period. It also increases the proportion of the middle-aged population who have relatively high savings on average. These two effects are combined to increase capital accumulation in a low mortality economy. On the other hand, if the fertility rate declines, total capital is greatly reduced due to the population decline. When the mortality rate decreases, total output does not change much, but if the fertility rate decreases, gross production significantly declines as the economi-

cally active population is reduced.

In both the declining mortality and fertility economies, the effect of the latter is more significant. As shown in Table 2.5, total labor input decreases by about 20%, and output declines by about 15%. The proportion of the elderly population aged 60 and over increases more than 37%, and this indicates that the demographic structure is aging. The per capita gross product is also reduced by more than 7%. Thus, we can see that the overall size of the economy is reduced in terms of both level and per capita unit. Figure 2.16 suggests the dynamic changes in macroeconomic variables. As described above, the effects of decreasing fertility rates are more pronounced. Regarding capital accumulation, both demographic changes offset each other until the early 40 years on the transition path, but a declining population becomes more prominent in the end. Thus, it reduces the size of capital accumulation in the new steady state. In the government's pension financing, the two distinct demographic changes commonly exacerbate pension finance, although the reasons are different for each case. In the long run, the deficit in pension financing is about 1.43% of the GDP in the initial steady state.

The graph on the left side of Figure 2.17 shows the dynamic change in the employment rate by age in both the declining mortality and fertility economies. The employment rate is changing in a manner that is similar to the case of a declining fertility economy. The employment rate of young people increases while the employment rate of the middle-aged decreases because of the relatively significant reduction in the proportion of the young population.

The graph on the right side of Figure 2.17 shows the dynamic change in average labor income by age. Labor income increases by about 13% for the younger generation but decreases about 2.5% for the older generation. The average wage level in both the declining mortality and fertility economies rises by about 7% compared to the initial level, and this affects the labor income in the new equilibrium. When mortality declines and population growth rates decrease simultaneously, the decline in

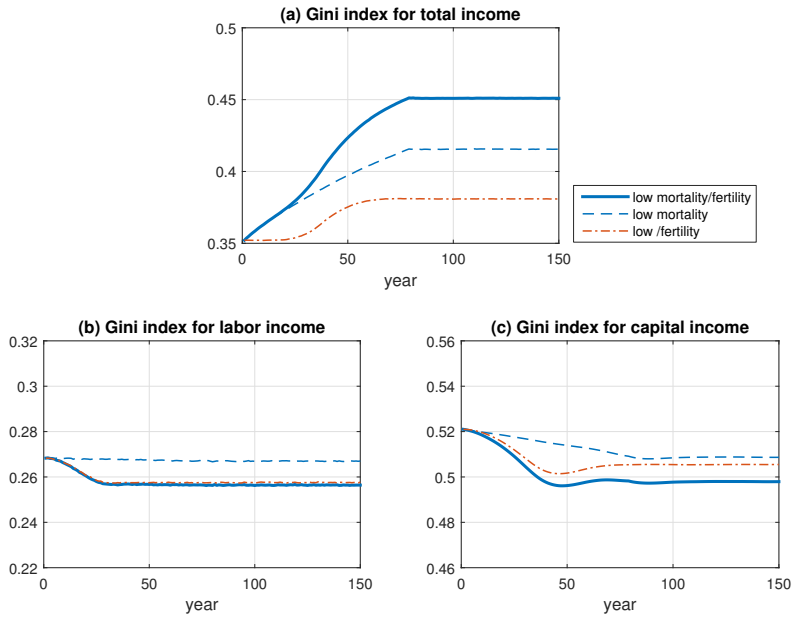

Figure 2.18: The Transition Dynamics of Gini Index in Low Mortality and Fertility Economy

	Benchmamrk	Low Mortality/Fertility
Total Income	0.352	0.451
Labor Income	0.269	0.256
Capital Income	0.521	0.498

Table 2.6: Gini Index in Low Mortality and Fertility Economy

the proportion of young people and the rise in the proportion of older people is faster and larger, as we have seen.

4.3.2 Changes in Income Inequality

As illustrated in Figure 2.18, the Gini coefficient of labor income and capital income declines more than the previous case, as the two demographic changes are combined. The Gini coefficient of gross income also increases further. As we can see in Table 2.6, labor income inequality decreases from 0.269 to 0.256, and the capital income

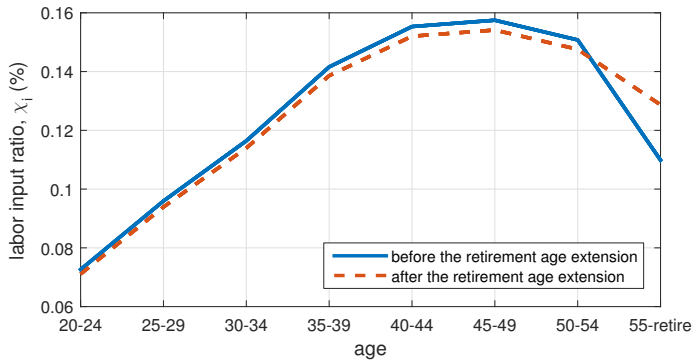

Figure 2.19: The Change in The Labor Input Ratio

Gini coefficient decreases from 0.521 to 0.498. Finally, gross income inequality rises from 0.352 to 0.451 due to the rapid increase in the proportion of older people, who have the lowest total income on average.

5 The Effect of the Retirement Age Extension

5.1 Extension of Retirement Age to 60 Years of Age

In this chapter, we analyze the effects of the policy that extends the retirement age from 57 to 60 on macroeconomic variables. Figure 2.19 shows the change in the labor input ratio before and after the retirement age extension. In the model economy, the representative firm produces final products by combining labor forces of different age groups (five-year unit). Therefore, if the retirement age is extended, the labor input ratio between 55-59 years should be adjusted. In this paper, we assume that the employment rate of workers aged 55 and older is maintained at the pre–retirement extension level.[3]

Table 2.7 shows the macroeconomic effects of a three-year extension of the re-

[3]In the absence of an increase in labor demand when there are increases in the labor supply due to the retirement age extension, the real wage of older workers may fall. In this paper, we assume that the increase in labor supply due to the governmental policy is accompanied by the labor demand of the firms, and the relative real wage of older workers is maintained at the pre–retirement level.

	Benchmark	Low Mor./Fer.	Ret. Age 60	Ret. Age 65
Total Population	100	91.98	91.98	91.98
Population under 60	71.45	57.69	57.69	57.69
Elderly Population Ratio	28.55%	37.28%	37.28%	37.28%
Output	100	85.47	91.55	100.63
Capital	100	96.31	99.64	103.28
Output per capita	100	92.93	99.54	109.40
Employment	62.49	49.94	53.33	58.70
Labor	100	79.93	87.28	99.17
Welfare at age 55	-36.23	-49.73	-48.28	-44.60
Welfare at age 20	-56.51	-58.76	-57.55	-55.92

Table 2.7: The Change in Macroeconomic Variables with The Retirement Age Extension

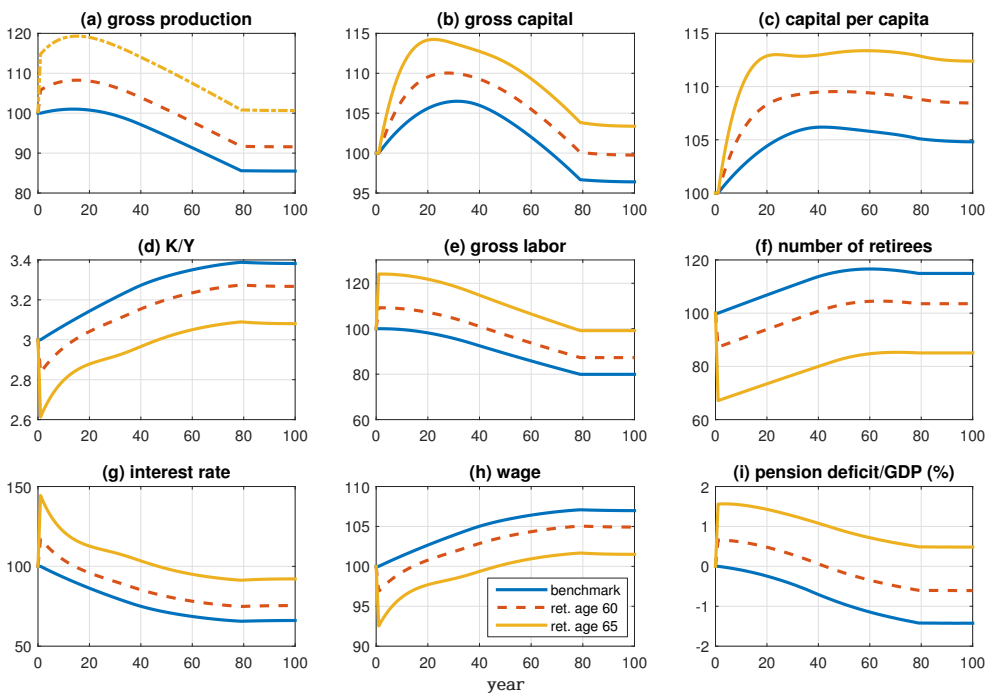

Figure 2.20: The Transition Dynamics with The Retirement Age Extension

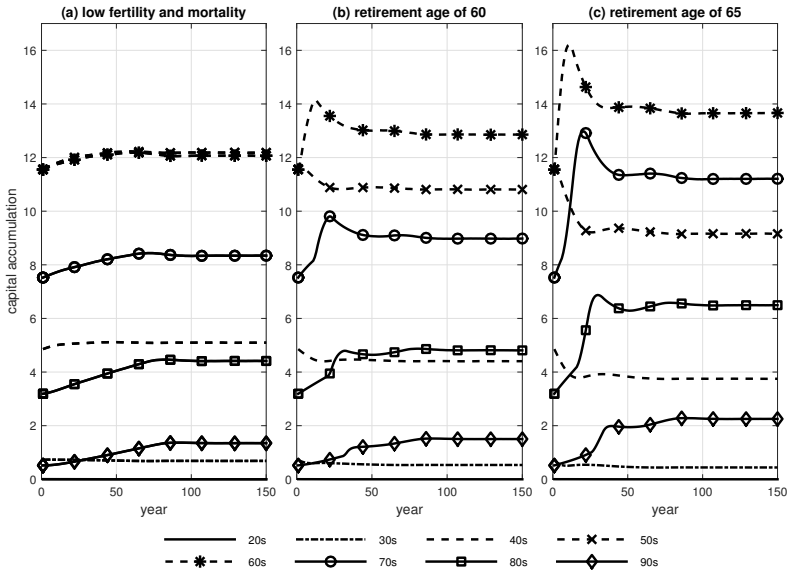

Figure 2.21: The Transition Dynamics of Capital with The Retirement Age Extension

tirement age from 57 to 60 years. As a result of the extension of retirement age, the employment rate of the economy increases from 86.6% to 92.4%, and total employment rises from 49.9 to 53.3, about 6.8%. Gross labor input increases by 9.2%, from 79.9 to 87.3, and total output rises by 7.1%, from 85.5 to 91.6.

Figure 2.20 reveals the dynamic change in macroeconomic variables. The number of retirees decreases considerably due to the retirement age extension (panel (f)) compared with both the declining mortality and fertility economies. We found that the fiscal deficit in pension financing decreases from -1.4% to -0.6% of initial output (panel (i)). The increase in the social security revenue and the reduction of the social security payment due to the prolonged working periods are the cause of this variation between the two economies.

Since the governmental policy delays the retirement age by three years, this increases capital accumulation as well as gross labor in the model economy. While the retirement age extension increases total capital, its effects on individual savings differ according to age, as shown in Figure 2.21. The policy changes the savings tendency

of the 40-70 age group as it brings about three additional working years. First, the 40s and 50s, who are pre-retirement, save less to address their remaining lifetimes. On the other hand, those aged 60 and older who have already retired, increase their savings when the extension policy is implemented. This demonstrates that the additional labor income increases the savings and consumption after retirement and it raises the welfare of retirees through current and future consumption. In the economy as a whole, the increase in savings among those in their 60s-80s dominates the decrease in the capital accumulation of the younger age group. Thus, total capital accumulation increases by about 3.5%. Through the increase of capital and labor input, the output also eventually increases by 7.1%.

The extension of retirement age affects the welfare of individual workers. The welfare of those aged 55-60 increases because the policy increases their labor income, also raising current and future consumption again. However, this paper considers only welfare through consumption and does not include the disutility from labor hours. Therefore, the welfare effect presented in the article may be on the upper bounds that can be achieved through a neo-classical setting. The welfare of 20-year-old workers also increases slightly. Comparing the economy before and after the retirement age extension, workers who newly enter the labor market prefer to implement the retirement age extension policy, but the effect is relatively small. It can be understood as the result of two different forces offsetting each other. The extension of retirement age lowers the average wage through the increase in the labor supply, which diminishes the welfare of the younger generation. On the other hand, the present value of lifetime labor income increases due to the longer working periods. As these two opposite effects are combined, the variation of young people's welfare is relatively small.

5.2 Extension of Retirement Age to 65 Years of Age

In this section, we further analyze the effect of an eight-year extension of the retirement age, from 57 to 65 years of age. To analyze the effect of such a long retirement extension rigorously, it is necessary to additionally estimate the age-specific labor productivity of older workers aged 57 to 65. In this paper, however, we assume that workers aged 57 and older have labor productivity and employment rates at ages 55-57 for simplicity. Under this circumstance, the employment rate increases from 49.9 in both the declining mortality and fertility economies to 58.7 after implementing the extension policy. If we look at total production, it was 85.5 in the aged society without the retirement age extension, whereas gross production increased by 17.7%, to 100.6, when the retirement age was extended to 65.

In addition, the welfare effect is significant for 55-year-old workers who are close to retirement. As shown in Figure 2.20, the extension policy delays the commencement of pension receipt by eight years, which improves the pension deficit and finally turns it to the positive in the new steady state. Additionally, labor input increases by about 24% due to the eight-year extension of the retirement age.

However, it is important to note that if labor productivity after the age of 60 is significantly lower than that of the 50s, then the effective increase in labor input through the extension policy is likely to be much lower than 24%, as calculated above. In addition, although it is assumed that the employment rate of people over aged 60 is the same as the level of the late 50s, we need more accurate empirical research on the demand for the labor force over 60 years of age. If the demand for aged workers is significantly different from those in their late 50s, the employment rate of those in their 60s may be considerably lower despite the extension of the retirement age. In this case, the effect of the extension of retirement age on gross output will be greatly reduced.

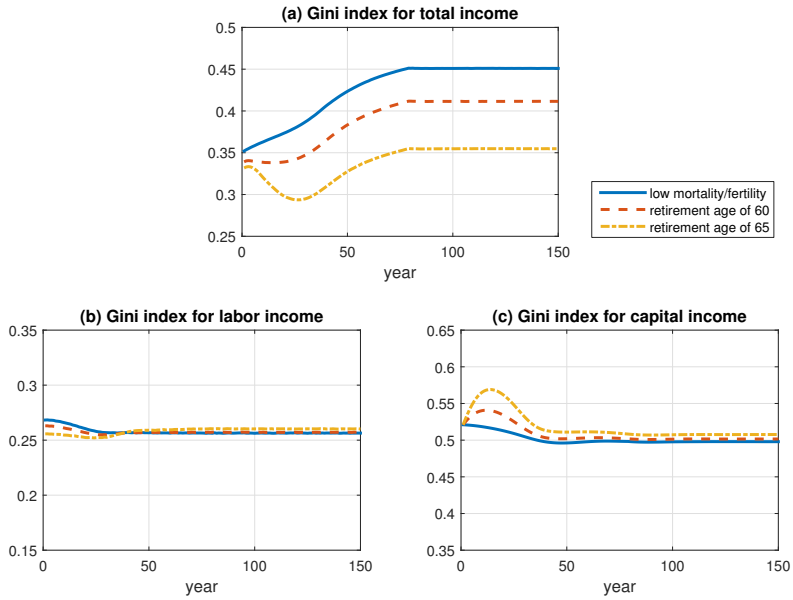

Figure 2.22: The Transition Dynamics of Gini Index with Retirement Age Extension

	Benchmark	Low Mor./Fer.	Ret. Aget 60	Ret. Age 65
Total Income	0.352	0.451	0.411	0.354
Labor Income	0.269	0.256	0.257	0.260
Capital Income	0.521	0.498	0.501	0.507

Table 2.8: Gini Index and The Retirement Age Extension

5.3 Changes in Income Inequality with Retirement Age Extension

Table 2.8 compares the Gini coefficients when the retirement age is extended to either 60 or 65 years of age in the new steady state. Additionally, Figure 2.22 shows the dynamic change of the Gini coefficient on the transition path. Henceforth, we will explain the results based on the case of the 65-year-old retirement age extension, as the effect of the policy is greater and more evident.

As in the second graph in Figure 2.22, the Gini coefficient for labor income decreases slightly at the beginning of the retirement age extension. This is because the proportion of people who have high labor income increases as the agents around

retirement age remain in the labor market longer. However, from about aged 40 onward, a reversal phenomenon occurs as the Gini coefficient of labor income becomes slightly higher than the value of the model without the retirement age extension. This is because the employment rate of people aged 57-65 gradually drops as the population composition changes. In other words, those who were previously excluded from the calculation of labor income inequality due to retirement are now included in the sample as the retirement age is extended. However, many of them are the unemployed without labor income, and this makes labor income inequality worse in the end. As a result, the labor income Gini coefficient, which was 0.256 in the economy without the extension policy, rises to 0.257 when the retirement age becomes 60, and to 0.260 when the retirement age is extended to 65.

The capital Gini coefficient rises in the early period after implementing the extension policy and then decreases again. The change in capital accumulation among the generations causes the rise of the capital income inequality at the beginning. When the retirement age is extended and thus the life income increases, the savings burden for remaining lives after retirement is reduced. As depicted in Figure 2.21, the pre-retirement age reduces the savings level after the extension of retirement age. On the other hand, those who benefit from the retirement extension policy will save a significant portion of their increased labor income. In addition, older people after retirement will increase their capital stock in response to higher interest rates. As a result, their savings will increase significantly, leading to an increase in gross capital in the macro economy. As the gap of capital accumulation between generations rises, the capital Gini coefficient in the early periods after enacting the extension policy increases.

The Gini coefficient for gross income declines for about the initial 30 years after the extension of retirement age and increases again after that. A decrease in the Gini index in the early periods occurs because 1) the proportion of the middle-aged whose labor income is high increases (demographic change), and 2) retirees

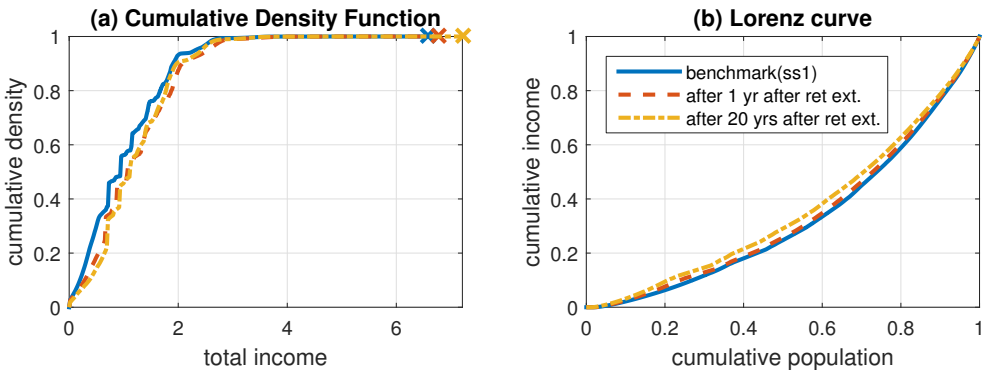

Figure 2.23: CDF and Lorenz Curve of Total Income with Retirement Age Extension

who have the lowest gross income have more capital income due to the rise in the interest rate (general equilibrium effect). Let us take a closer look at Figure 2.23. The left-hand side of this figure represents the CDF for gross income, and the right-hand side illustrates the Lorenz curve at the initial steady state, the first period after the retirement age extension, and the after 20-year periods. The CDF shifts to the right side because the proportion of people who earn high gross incomes increases due to the extension of retirement age, and the total income of older people after retirement rises because of the increase in the interest rate. These changes in gross income reduce the Gini coefficient by moving the Lorenz curve upward.

6 Conclusion

In this paper, we quantitatively analyzed the economic effects of population aging due to 1) a reduction in the mortality rate, and 2) a decrease in the fertility rate by constructing the overlapping generations model with a heterogeneous agent. In addition, we studied the effect of the extension of retirement age on the overall macroeconomic variables, such as gross production and income inequality.

In the case of declining mortality rates, gross production increased by about

6.3% as precautionary savings rose due to prolonged life expectancy and demographic changes in the form of increases in the proportion of elderly who have high savings on average. However, gross production per capita decreases as the effect of an increase in the elderly population outweighs that of the increase in gross production. Labor income inequality declined due to the increase in the proportion of middle-aged workers with a high labor income on average. Capital income inequality also declined as the share of middle-aged and elderly people with high capital holdings increased. Finally, the Gini coefficient of total income increased as the proportion of the elderly who have the lowest total income increased.

In the economy of declining fertility rates, labor input decreased greatly due to the decline in the economically active population. The decrease in the younger workforce reduced the employment rate of the elderly due to the substitute relationship among them. Accordingly, the total amount of capital in the economy decreased, resulting in a 20% reduction in total output. Even if the fertility rate declines, the proportion of the elderly became higher than that of young people, so labor and capital income inequality decreases while the Gini coefficient of gross income increases.

In an aging economy where these two effects are simultaneously combined, the effect of a decreasing fertility rate overwhelms the effect of a decreasing the mortality rate, and thus the gross output is reduced by about 15%. The increase in the number of pensioners due to demographic changes and the decrease in pension revenues because of the decline in the youth population have resulted in a fiscal deficit of 1.4% of gross production. As the two demographic changes progressed, the decline in the proportion of young people and the rise in the share of older people became faster and larger, so that the Gini coefficient of labor income and capital income declined further, and the level of income inequality as a share of total income further increased.

The extension of retirement age policy, which delayed the retirement age by three years from 57 to 60, increased labor input in the model economy as a whole and increased savings and, thus, total capital input. As a result, output increased by

7.1% compared to the level before the extension policy. The pension deficit was significantly eased. Let us now examine changes in income inequality. As the proportion of high-income workers increases due to the extension of retirement age, the income inequality declines in the early stages of policy implementation. However, in the new steady state, the employment rate of the beneficiaries of the retirement extension policy declines, resulting in a reversal of the Gini coefficient, which is slightly higher than that of the reference model. Capital Gini inequality rises due to the increase in capital accumulation among the elderly population, and the Gini coefficient of total income is calculated to decrease. This is because the elderly population, who has the lowest total income on average, has larger income than before due to the increase in capital income.

In this study, labor supply is not determined by the endogenous choice of the economic agent, but by the exogenous age-specific employment rate driven by changes in demographic composition. If labor supply is internalized, a more detailed analysis will be possible, because changes in the population structure affect the labor supply, and this again changes macro variables. However, in the absence of previous research on how the labor supply of each age responds to changes in the demographic structure, this study focuses more on changes on the labor demand of the firm side. We believe that an analysis of the model economy including the endogenous labor supply in this basic model, as well as an examination of the differences in the results when compared with this study, would prove an interesting research topic in the future.

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국 문 초 록

이질적 경제주체 기반의 거시경제학에 관한 에세이

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본 박사학위 논문은 이질적 경제주체가 포함된 동태확률 일반균형모형을 기반으로 재정학과 노동경제학에 관한 주제들을 다루고 있다

본 논문의 첫 번째 장은 1990년대 이래로 미국에서 관측되는 고소득 여성들의 출산율 퍼즐에 관해 설명하고 있다. 지난 2 30년 동안 미국의 노동소득 상위 20%에 해당하는 기혼여성들의 완결 출산율은 꾸준히 상승해 왔으며, 이와 동시에 가구의 소득구조는 점차 불평등하고 또한 여성에게 호의적으로 변화하였다. 이러한 소득구조의 변화는 1) 학력 간 임금 격차의 증가, 2) 성별 임금 격차의 감소, 그리고 3) 임금의 변동성 증대가 모두 어우러져 나타난 것이다. 하지만 이러한 변화 아래서 고소득 여성들의 출산율이 상승하는 현상은 기존의 출산이론으로는 잘 설명되지 않는 것이다. 기존의 이론은 여성의 임금이 증가하는 경우 기회비용의 상승으로 인하여 자녀의 수가 줄어든다고 설명하고 있지만 현실에서는 고소득 여성들의 출산율이 오히려 상승하고 있기 때문이다. 이러한 이론과 새로운 현상 간의 괴리를 해소하기 위해 본 연구에서는 기존 대다수의 문헌이 가정하고 있는 동질적 선호체계를 완화하여 가구가 자녀에 대한 다양한 선호를 가질 수 있는 보다 일반화된 모형을 제시하였다. 이러한 일반화된 설정은 가구가 효용을 극대화하는 과정에서 여성의 노동공급과 자녀 양육을 선택하는 데 있어 비교우위 개념을 적용할 수 있게

해 준다. 이로 인하여 임금구조가 변화하였을 때 자녀에 대한 더 높은 선호를 가진 여성들이 고소득 그룹으로 이동하게 되는 구성 효과가 발생하게 되며, 이에 따라 기존 이론으로는 포착해 낼 수 없었던 고소득 여성들의 출산율 상승 현상을 효과적으로 설명할 수 있었다. 또한, 본 연구에서는 앞서 언급한 개별 소득변화 요인들을 구조모형을 통해 분해하여 살펴보고 있다. 그 결과 학력 간 임금 격차의 증가가 고소득 여성들의 완결 출산율을 상승시키는 데 있어 가장 중요한 역할을 한다는 것을 발견할 수 있었다.

본 논문의 두 번째 장은 사망률 감소와 출산율 감소에서 기인한 고령화 현상이 거시경제에 미치는 영향을 연구하고 있다. 또한, 고령화가 진행되는 상황에서 정년 연장정책을 시행하는 경우 어떠한 효과가 발생하는지 살펴보고 있다. 사망률 감소로 인해 고령화가 진행되는 경우, 길어진 수명에 대비하기 위해 예비적 저축이 증가하게 되고 이 때문에 경제 내의 총 자본 양이 상승하게 된다. 반면 총 노동의 경우 그 증가 폭이 미미한데, 이는 사망률 감소의 효과가 경제활동인구에 포함되지 않는 은퇴 이후의 고령층에 집중되기 때문이다. 출산율 감소로 인해 고령화가 발생하는 경우 경제 내의 총노동과 자본 양이 급격하게 감소하는 것으로 나타났다. 이는 출산율 감소로 인해 경제활동 인구가 크게 감소하고 이에 따라 총저축량 역시 감소하기 때문이다. 이 두 가지 요인이 중첩되어 나타나는 고령화 경제의 경우 고령화 현상이 더욱 빠르고 심각하게 발생하게 되며, 모의실험 결과 총노동과 자본의 감소로 인하여 총생산의 약 15%가 감소하고 연금적자 역시 총생산의 1.4% 수준에 이를 것으로 계산되었다. 고령화가 진행되는 상황에서 정년을 3년 및 5년 연장하는 경우 고령화가 경제에 미치는 부정적인 영향이 상당 부분 완화될 수 있음을 수량적으로 계산하여 보이고 있다.

주요어 : 이질적 경제주체, 출산, 학력별 임금격차, 성별 임금격차, 고령화, 정년연장, 소득 불평등

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