공학석사 학위논문

Order Quantity Determination Under Limited Demand Information

제한적 수요 정보하에서의 주문량 결정법

2018 년 2 월

서울대학교 대학원

산업공학과

최 서 현

Abstract

Order Quantity Determination Under Limited Demand Information

Seohyeon Choi
M.S. in Industrial Engineering
The Graduate School
Seoul National University

This study shows the investigation of the maximum entropy approach for determining the order quantity of a single period newsvendor model under demand uncertainty. The characteristic of maximum entropy distribution is explored, i.e., forms of the estimated probability density function on different supports and availability of approximation methods. Comparisons among the solutions from the maximum entropy method, the solutions from the well-known distribution free approach and the optimal order quantities were conducted under the various demand distributions in the exponential family with identical means and variances. Additionally, this study extended the analysis to the situation where the higher order moment information is available.

Keywords: demand uncertainty, distribution free approach, maximum entropy method, newsvendor model

Student Number: 2016-21126

Contents

List of Tables

List of Figures

Chapter 1. Introduction

The newsvendor problem is one of the classical problems in the field of inventory management. This model has attracted significant attention from academic researchers due to its broad applicability. The objective of the newsvendor problem is to determine the expected-profit-maximizing order quantity under stochastic demand, assuming that any remaining inventory is sold at a reduced price or discarded at the end of each period. Since the newsvendor model can be widely utilized at retail and manufacturing environments where the profit is susceptible to the order quantity, various applications of the model have been suggested in different manners: variation in objectives, periods, pricing policies, and states of information regarding the probability distribution of the demand (Ismail & Louderback, 1979; Khouja, 1996; Hill, 1997).

Although it is easy to solve the newsvendor problem with full information about the demand, it is difficult to apply this model to real-world situations where the decision makers do not have full information about the demand. For real-world applications, uncertainty-immunized ordering rules should be examined. Estimating the demand distribution and finding the bound for the profit are viable approaches to the newsvendor problem with limited information. For example, Scarf (1958) and Gallego and Moon (1993) presented the bound for the profit by deriving the order quantity that maximizes profit against the worst-case distribution in the closed-form assuming only mean and variance of the demand is given. By utilizing the information about the mean and variance of the demand, Andersson et al. (2013) used maximum entropy distribution estimation method in order to determine the order quantity. Perakis and Roels (2008) analyzed the performance of the minmax distribution free (DF) approach and the order quantity determination based on the maximum regret under several types of demand distributions. Few researchers focused on the use of maximum entropy (ME) method in the realm of inventory management. Andersson et al. (2013) investigated the performance of the order quantity determination based on the maximum entropy distributions, but they did not consider the use of moment information whose order is higher than two. Maglaras and Eren (2015) proposed a demand estimation procedure based on the maximum entropy distributions for the revenue management problem, but they did not pay much attention to the characteristics of maximum entropy distributions.

In this study, two ordering rules for the single period newsvendor under demand uncertainty was considered: the ME method and the min-max DF approach. The analyses on the estimation performance and the characteristics of these two methods under various demand distributions from exponential family were conducted assuming that only the mean and the variance of the demand distribution are known. This study also considers the extension of estimation methods incorporating higher-order moment information into the estimation procedure. Improvement in the maximum entropy prediction was observed using the third moment information in addition to the first and second moment information. In addition, an enhanced lower bound for the expected profit was found using the third and fourth moment information. Furthermore, this study examined a simplified form of the maximum entropy distribution when the mean and the variance of the demand are available. This simplification allows a faster search procedure by greatly reducing the search space. An attempt was made to find the appropriate finite support that can provide the approximate solution for the semi-finite case.

The remainder of this paper is organized as follows. In the next chapter, the basic newsvendor problem and the different states of demand information are introduced. In Chapter 3, the characteristics of the DF approach and the ME method for the newsvendor problem under limited demand information are examined. Also, the solution methods were extended to the other information states where higher moment information is available. In Chapter 4, numerical analyses are presented regarding simplification of the ME method and the prediction performances of DF and ME approaches. Finally, Chapter 5 presents the conclusion for this study and a discussion of the implications of the results.

Chapter 2. The Newsvendor Problem

2.1 The Classical Single Period Newsvendor Problem

In the classical newsvendor model, the order quantity that can maximize the expected profit should be found with the assumption that there is a single opportunity for stocking the single-item inventory.

D	quantity demanded, a random variable
$f(D)$	the probability density function of D
p	selling price per unit
c	purchasing cost per unit
s	salvage value per unit
Q	order quantity, a decision variable
r	$= (p-c)/(p-s)$, a critical ratio

An order quantity (Q) is determined at the beginning of the period under the assumption that the demand (D) is stochastic. The order quantity is not restricted by the supplier since the capacity of the supplier is assumed to be unconstrained. D is randomly distributed having probability density function $f(D)$. $F(D)$ denotes the cumulative distribution function of the demand. Q items are purchased at a fixed price c and then sold at a fixed price p. The Profit is

$$\pi = \begin{cases} pD + s(Q-D) - cQ, & if \;\; Q \geq D \\ (p-c)Q, & if \;\; Q < D. \end{cases} \tag{1}$$

In this model, the order quantity which maximizes the expected profit is found because the actual profit is not observable before the demand is realized.

$$E(\pi) = p \cdot E[min(Q, D)] + s \cdot E[(Q - D)^+] - cQ \qquad (2)$$

$$E(\pi) = (p - s)\mu - (c - s)Q - (p - s)E[(D - Q)^+] \qquad (3)$$

If the probability distribution of demand is known, optimal order quantity Q^* is obtainable as follows:

$$Q^* = F^{-1}(r) \qquad (4)$$

2.2 The State of Demand Information

Researchers have presented analyses on the newsvendor problem assuming different information states. This study focused on identifying the strategy for the order quantity determination assuming that some probability distribution parameters are known, not the type of demand distribution itself. Scarf (1958) presented a DF approach for the single period newsvendor problem assuming that the decision maker only knows the mean and variance of the demand distribution. Andersson et al. (2013) adopted maximum entropy based approach for the same assumptions as used by Scarf (1958). This study provides analysis on these two solution approaches. Different from Andersson et al. (2013), Scarf (1958), and Gallego & Moon (1993), (i) this study provides the extensive analysis on the characteristics of maximum entropy distributions; and (ii) examined the inclusion of higher order information in the problem-solving process.

- Information State I_n

Moments of demand distributions are known in this state of information. The type of probability distribution is unknown. Let n denote the highest order of available moment information. In the state of I_n, $\{\mu_1, \mu_2, \dots, \mu_n\}$ is available where

$$\mu_n = \mathrm{E}(X^n).$$

Information is usually given in the form of mean, variance, skewness, kurtosis, etc. Moment can be generated from this information as follows.

$$\begin{aligned}
E(X) &= \mu \\
E(X^2) &= \mu^2 + \sigma^2 \\
E(X^3) &= \alpha\sigma^3 + 3\mu\sigma^2 + \mu^3 \\
E(X^4) &= \beta\sigma^4 + 4\mu\alpha\sigma^3 + 6\mu^2\sigma^2 + \mu^4
\end{aligned}$$

μ: $mean$ σ^2: $variance$ α: $skewness$ β: kurtosis

Chapter 3. Ordering Rules under Limited Information

In real life, obtaining information about the probability distribution of the demand is not easy. There exist some order quantity determination methods that can be used under limited demand information.

The method of assuming normal distribution is widely used, but it does not always guarantee good results. The DF approach is one way to maximize profits over the worst-possible demand distribution among the probability distributions having the same mean and variance. Scarf (1958) proposed this method which was reviewed by Gallego and Moon (1993) after a few decades. The ME method provides the entropy-maximizing probability distribution under the given information. This method is not yet widely used in operations research areas such as order quantity determination. This study applied and analyzed these methodologies and attempted to extend them to situations where additional information is observable.

3.1 Distribution Free Approach

3.1.1 Original Distribution Free Approach under I_2

Scarf (1958) derived a closed-form solution for the expected profit maximizing order quantity (Q^{DF}) against the worst-possible distribution under I_2. Gallego and Moon (1993) presented a simplified proof for the derivation of the solution in closed-form. They also provided expected profit for the worst-possible demand distribution which can be a lower bound for the expected profit with I_2 information.

$$E[(D-Q)^+] \leq \frac{[\sigma^2 + (Q-\mu)^2]^{1/2} - (Q-\mu)}{2} \tag{5}$$

$$(p-s)\mu - (c-s)Q - (p-s)\frac{[\sigma^2 + (Q-\mu)^2]^{1/2} - (Q-\mu)}{2} \tag{6}$$

$$\begin{aligned} E[\pi(Q)] \geq (p-s)\mu - (c-s)Q \\ - (p-s)\frac{[\sigma^2 + (Q-\mu)^2]^{1/2} - (Q-\mu)}{2} \end{aligned} \tag{7}$$

$$LB_1 = (p-s)\mu - (c-s)Q - (p-s)\frac{[\sigma^2 + (Q-\mu)^2]^{1/2} - (Q-\mu)}{2} \tag{8}$$

The optimal order quantity against the worst-possible demand distribution (Q^{DF}) is given by:

$$Q^{DF} = \mu + \frac{\sigma}{2}\left(\frac{2r-1}{[r(1-r)]^{1/2}}\right) \tag{9}$$

3.1.2 Distribution Free Approach under $I_{(4)}$

Informed of higher order moments, a lower bound LB_2 is available. The third and fourth moments can be represented using additional information:

α skewness of the demand distribution

β kurtosis of the demand distribution

Proposition 1

$$E[\pi(Q)] \geq \quad (p-s)\mu - (c-s)Q$$
$$-\frac{(p-s)}{2}\left([(Q-\mu)^4 + 6\sigma^2(Q-\mu)^2 - 4\alpha\sigma^3(Q-\mu) + \beta\sigma^4]^{\frac{1}{4}} - (Q-\mu)\right)$$
$$= LB_2$$

Proof

$$E|D-Q| \leq E[|D-Q|^4]^{\frac{1}{4}} = E[(D-Q)^4]^{\frac{1}{4}}$$
$$= E[D^4 - 4D^3Q + 6D^2Q^2 - 4DQ^3 + Q^4]^{\frac{1}{4}}$$
$$= [Q^4 - 4\mu Q^3 + 6Q^2(\mu^2+\sigma^2) - 4E[D^3]Q + E[D^4]]^{\frac{1}{4}}$$
$$= [(Q-\mu)^4 + 6\sigma^2(Q-\mu)^2 - 4\alpha\sigma^3(Q-\mu) + \beta\sigma^4]^{\frac{1}{4}}$$

The order quantity minimizing LB_2 is the distribution free order quantity under I_4. Although a closed-form solution for this minimization problem is not obtainable, LB_2 can be a tighter lower bound for the expected profit under certain circumstances.

Proposition 2

$$LB_2 \geq LB_1$$
$$for \ \forall Q \in \left[\left(\mu + \frac{\alpha\sigma}{2}\right) - \frac{\sqrt{\alpha^2-\beta+1}}{2}\sigma, \left(\mu + \frac{\alpha\sigma}{2}\right) + \frac{\sqrt{\alpha^2-\beta+1}}{2}\sigma\right]$$
$$when \ \alpha^2 - \beta + 1 > 0$$

Proof

$$LB_2 - LB_1 =$$

$$(p-s)\mu - (c-s)$$

$$-(p-s)\frac{[\mathrm{Q}^4 - 4\mu \mathrm{Q}^3 + 6\mathrm{Q}^2(\mu^2+\sigma^2) - 4E[\mathrm{D}^3]Q + E[\mathrm{D}^4]]^{\frac{1}{4}} - (Q-\mu)}{2}$$

$$-(p-s)\mu + (c-s)Q + (p-s)\frac{[\sigma^2 + (Q-\mu)^2]^{\frac{1}{2}} - (Q-\mu)}{2}$$

$$= (p-s)\frac{[\sigma^2 + (Q-\mu)^2]^{\frac{1}{2}} - [(Q-\mu)^4 + 6\sigma^2(Q-\mu)^2 - 4\alpha\sigma^3(Q-\mu) + \beta\sigma^4]^{\frac{1}{4}}}{2}$$

$LB_2 \geq LB_1$ if:

$$[\sigma^2 + (Q-\mu)^2]^{\frac{1}{2}} \geq [(Q-\mu)^4 + 6\sigma^2(Q-\mu)^2 - 4\alpha\sigma^3(Q-\mu) + \beta\sigma^4]^{\frac{1}{4}}$$

$$[\sigma^2 + (Q-\mu)^2]^{\frac{1}{2}} = [(Q-\mu)^4 + 2(Q-\mu)^2\sigma^2 + \sigma^4]^{1/4}$$

$$\{(Q-\mu)^4 + 2(Q-\mu)^2\sigma^2 + \sigma^4\} - \{(Q-\mu)^4 + 6\sigma^2(Q-\mu)^2 - 4\alpha\sigma^3(Q-\mu) + \beta\sigma^4\}$$

$$= -4\sigma^2(Q-\mu)^2 + 4\alpha\sigma^3(Q-\mu) - (\beta-1)\sigma^4$$

If $\alpha^2 - \beta + 1 > 0$, $Q \in \left[\left(\mu + \frac{\alpha\sigma}{2}\right) - \frac{\sqrt{\alpha^2-\beta+1}}{2}\sigma, \left(\mu + \frac{\alpha\sigma}{2}\right) + \frac{\sqrt{\alpha^2-\beta+1}}{2}\sigma\right]$

satisfies the following inequality.

$$4\sigma^2(Q-\mu)^2 - 4\alpha\sigma^3(Q-\mu) + (\beta-1)\sigma^4 \leq 0$$

As $\frac{\sqrt{\alpha^2-\beta+1}}{2}\sigma$ increases, the length of the interval for a Q value that satisfies $LB_2 \geq LB_1$ gets longer. In other words, LB_2 can be a stronger bound for the demand distribution having a large $\frac{\sqrt{\alpha^2-\beta+1}}{2}\sigma$.

3.2 Maximum Entropy Method

3.2.1 Maximum Entropy Distributions

A well-established approach when selecting a probability distribution subject to known moments is to choose the distribution that has the maximum entropy. The maximum entropy distribution, as it is the least informative given the available information, is a good choice for a prior distribution. The entropy of a continuous distribution (differential entropy) on a support S is defined by:

$$\int_S -f(x)\ln[f(x)]dx \tag{11}$$

An entropy-maximizing distribution can be found by solving the following maximization problem.

$$\max_f \int_S -f(x)\ln[f(x)]dx$$

$$\text{s.t.} \quad \int_S f(x)dx = 1$$

$$\mathrm{E}\left(X^i\right) = \mu_i \quad \mathrm{i} = 1,2,\dots,\mathrm{n}$$

The formal solution f^* for the problem is obtained using Lagrange multipliers:

$$L(f,\lambda) = \int_S -f(x)\log[f(X)]\,dx - \lambda_0\left(\int_S f(x)dx - 1\right) - \sum_{i=1}^{n}\lambda_i\left(\int_S x^i f(x)dx - \mu_i\right) \tag{12}$$

$$\frac{\partial L(f,\lambda)}{\partial f} = -\log[f(x)] - 1 - \lambda_0 - \sum_{i=1}^{n}\lambda_i\, x^i \tag{13}$$

$$\hat{f}(x) = e^{-1-\lambda_0{}^* - \sum_{i=1}^{n}\lambda_i{}^* x^i} \tag{14}$$

The order quantity based on ME method is:

$$Q^{ME} = \hat{F}^{-1}(r). \tag{15}$$

Classical examples of entropy-maximizing distributions, among others listed in Perakis and Roels(2008), include the uniform distribution, when only the finite support of the distribution is known; the exponential distribution, when the distribution is known to has a certain mean on R_+; and the normal distribution, when the distribution has known mean and variance on R.

3.2.2 Infinite Support R

Under I_2 with infinite support R, there exists an analytic solution for the maximum entropy distribution: the normal distribution. Therefore, the ME method is not useful in this setting. The order quantities obtained from the ME method are the same as the quantities obtained assuming that the demand distribution is normal distribution. In addition, a significant amount of density lies in the negative range if the variance is relatively large. This type of probability distribution is not suitable for the probability distribution of demand, which never shows negative values.

3.2.3 Semi-infinite Support R_+

Maximum entropy distributions with semi-finite support R_+ under I_2 is applicable in the field inventory management assuming the mean and variance of the demand distribution are known. The solution, however, cannot be obtained analytically. Hence, the maximum entropy distribution is found numerically searching the solution space. In addition, maximum entropy distribution does not always exist (Mead and Papanicolaou, 1984). Under I_2 on R_+, maximum entropy distribution exists only when $0 < \sigma^2 < \mu^2$. This existence condition may restrict the application of ME approach under I_2 on operations research area.

$$\hat{f}(x) = e^{-1-\lambda_0-\lambda_1 x-\lambda_2 x^2} \tag{16}$$

Under I_2 on the semi-infinite support R_+, finding the maximum entropy distribution can be simplified by reducing the number of dimensions of the search space. The problem searching for Lagrangian multiplier $\lambda = [\lambda_0, \lambda_1, \lambda_2]$ can be reduced to the problem searching for $\lambda_2 \in (0, \frac{1}{2\sigma^2})$. In Chapter 4, the estimation method for λ_2 is discussed. By using the approximate values, the search process can be eliminated, which enables easier application of the ME method.

Proposition 3

$$\hat{f}(x) = e^{-1-\lambda_0-\lambda_1 x-\lambda_2 x^2}$$

$$= \frac{1-2\lambda_2\sigma^2}{\mu} e^{\frac{2\lambda_2(\mu^2+\sigma^2)-1}{\mu}x-\lambda_2 x^2}$$

$$\lambda_2 \in \left(0, \frac{1}{2\sigma^2}\right)$$

Proof

$$\int_0^\infty e^{-1-\lambda_0-\lambda_1 x-\lambda_2 x^2} = 1$$

$$\int_0^\infty x \cdot e^{-1-\lambda_0-\lambda_1 x-\lambda_2 x^2} = \mu$$

$$\int_0^\infty x^2 \cdot e^{-1-\lambda_0-\lambda_1 x-\lambda_2 x^2} = \mu^2 + \sigma^2$$

By substituting x with $t - \frac{\lambda_1}{2\lambda_2}$, the expressions above are transformed into the simple form below.

$$\int_{\frac{\lambda_1}{2\lambda_2}}^\infty e^{-\lambda_2 t^2} = e^{1+\lambda_0-\frac{\lambda_1{}^2}{4\lambda_2}}$$

$$\int_{\frac{\lambda_1}{2\lambda_2}}^\infty t \cdot e^{-\lambda_2 t^2} = \frac{1}{2\lambda_2} \cdot e^{-\frac{\lambda_1{}^2}{4\lambda_2}} = (\mu + \frac{\lambda_1}{2\lambda_2}) \cdot e^{1+\lambda_0-\frac{\lambda_1{}^2}{4\lambda_2}}$$

$$\int_{\frac{\lambda_1}{2\lambda_2}}^{\infty}\left(t-\frac{\lambda_1}{2\lambda_2}\right)^2 e^{-\lambda_2 t^2} = e^{-\frac{\lambda_1{}^2}{4\lambda_2}}\left(\frac{\lambda_1}{4\lambda_2}+e^{1+\lambda_0}\left(\frac{1}{2\lambda_2}-\frac{\lambda_1}{\lambda_2}-\frac{\lambda_1{}^2}{4\lambda_2}\right)\right)$$

$$= (\mu^2+\sigma^2)\cdot e^{1+\lambda_0-\frac{\lambda_1{}^2}{4\lambda_2}}$$

$$\therefore\ \lambda_1 = \frac{1-2\lambda_2(\mu^2+\sigma^2)}{\mu} \quad \text{and} \quad e^{1+\lambda_0} = \frac{1}{2\lambda_2\mu+\lambda_1}.$$

Figure 1 shows the probability density functions of maximum entropy distributions having the same mean value. If the standard deviation is relatively small, the maximum entropy distribution shows symmetry. As the standard deviation increased, the maximum entropy distributions were skewed to the left.

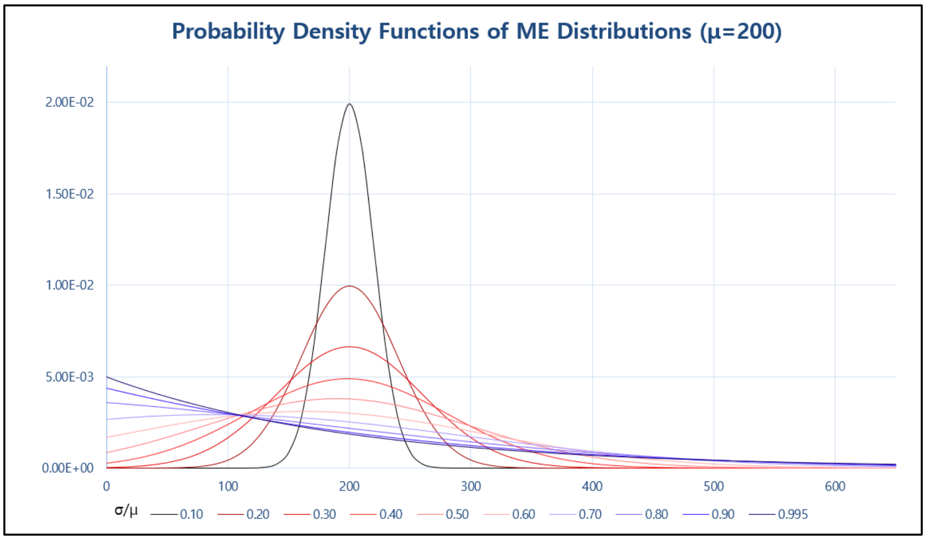

Figure 1 Maximum Entropy Distributions

Chapter 4. Numerical Analysis

4.1 Maximum Entropy Method under I_n

4.1.1 Estimating λ_2 for Semi-infinite Cases under I_2

The maximum entropy distributions were found for some sets of means and variances. The ratio 0.1 was excluded from this analysis for feasible solutions for it were not found.

$$\mu \in \quad \{200, \ldots, 1000\}$$

$$\sigma/\mu \in \quad \{0.2, \ldots 0.9\}$$

The best fitting exponential curves ($y = a \cdot e^{b \cdot \sigma/\mu}$) were found for the sets of data using the least square method. Figure 2 shows the fitted curve and the λ_2 value for each ratio when the mean is 200.

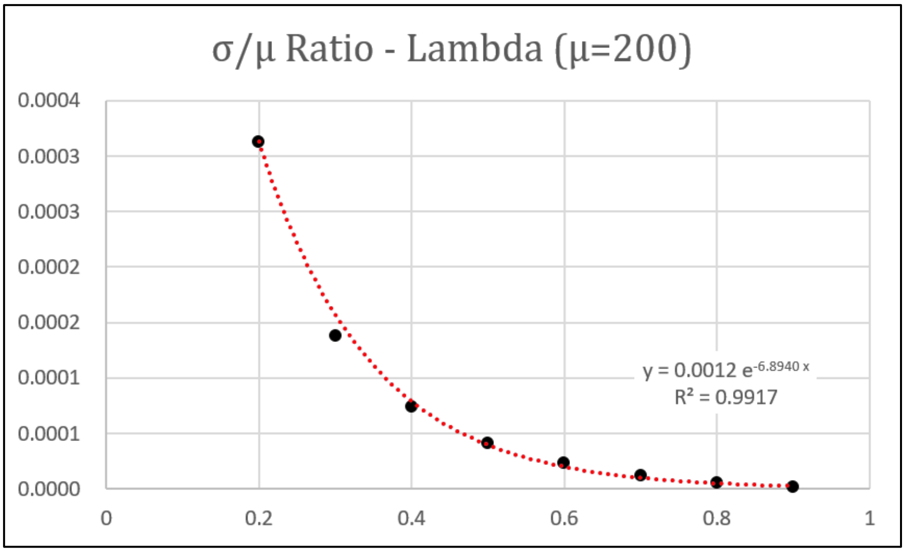

Figure 2 Estimation of $\boldsymbol{\lambda_2}$ using an exponential curve

Table 1 shows the coefficients and the R^2 values for the sets of data.

Table 1 Exponential curve estimation

mean	a	b	R^2
200	1.2417E-03	-6.893964	>0.99
300	5.5189E-04	-6.893964	>0.99
400	3.1044E-04	-6.893964	>0.99
500	1.9865E-04	-6.894732	>0.99
600	1.3816E-04	-6.895079	>0.99
700	1.0137E-04	-6.893964	>0.99
800	7.7609E-05	-6.893964	>0.99
900	6.1321E-05	-6.893964	>0.99
1000	4.9670E-05	-6.893964	>0.99

Figure 3 shows the values of the coefficient a for the sets of means. Each value seemed to be inversely proportional to the square of the mean value.

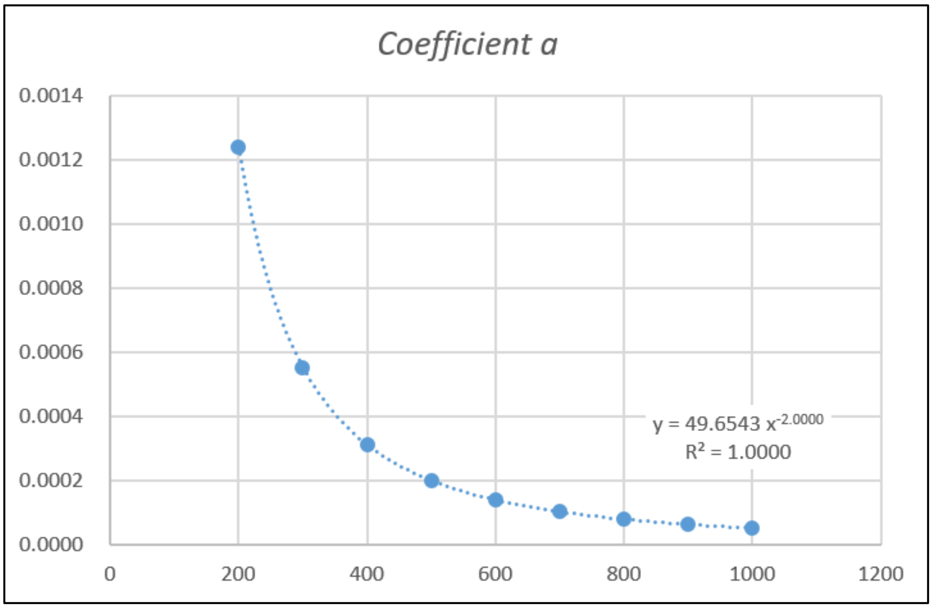

Figure 3 Mean and coefficient a values

From these observations, a function $\hat{h}$ for λ_2 was estimated.

$$\widehat{\lambda_2} = \hat{h}(\mu, \sigma) = \frac{49.6543}{\mu^2} \cdot e^{-6.8940 \cdot \sigma/\mu}$$

4.1.2 Upper-bound for Finite Cases

It is commonly assumed that the demand distributes along the semi-finite support R_+. For the simplification of the prediction procedure using ME method, however, assumption that the demand does not excess a certain level can be adopted. Let $\mu + k \cdot \sigma$ be the upper-bound for the demand. In this setting, the maximum entropy distribution is found on the finite support $[Q,\ \mu + k \cdot \sigma]$. The appropriate level of *k* is discussed in this section. Starting from *k=3*, *k* increased by 1 at each iteration until the λ converged. The search procedure terminates if

$$\frac{\|\lambda_i - \lambda_{i-1}\|}{\|\lambda_{i-1}\|} < \varepsilon.$$

- Under I_2

Under this information state, the search was conducted with the tolerance $\varepsilon = 10^{-5}$. Table 2 shows the value of k when λ converged for each set. The ratio 0.1 was excluded from this analysis for feasible solutions were not found. For each mean, k showed a similar tendency that k value increases as the mean-variance ratio grew larger.

Table 2 k values for sets of data

mean	std	σ/μ ratio	k
200	40	0.2	6
200	60	0.3	6
200	80	0.4	7
200	100	0.5	7
200	120	0.6	8
200	140	0.7	8
200	160	0.8	10
200	180	0.9	13
300	60	0.2	6
300	90	0.3	6
300	120	0.4	7
300	150	0.5	7
300	180	0.6	8
300	210	0.7	8
300	240	0.8	10
300	270	0.9	13
400	80	0.2	6
400	120	0.3	6
400	160	0.4	7
400	200	0.5	7
400	240	0.6	8
400	280	0.7	8
400	320	0.8	10
400	360	0.9	13
500	100	0.2	6
500	150	0.3	6
500	200	0.4	7
500	250	0.5	7
500	300	0.6	8
500	350	0.7	8

500	400	0.8	10
500	450	0.9	13

- Under I_3

Under this information state, the search was conducted with the tolerance $\varepsilon = 10^{-3}$. When a negative skewness was given, k showed a small value. For the sets with a positive skewness, k showed decreasing pattern as the mean-variance ratio increased.

Table 3 Upper-bounds for $\boldsymbol{I_3}$

mean	std	σ/μ ratio	skewness	k	$\mu + k \cdot \sigma$
200	40	0.2	-2	4	360
200	40	0.2	-1	4	360
200	40	0.2	1	63	2720
200	40	0.2	2	69	2960
200	60	0.3	-2	4	440
200	60	0.3	-1	4	440
200	60	0.3	1	43	2780
200	60	0.3	2	50	3200
200	80	0.4	-2	4	520
200	80	0.4	-1	4	520
200	80	0.4	1	32	2760
200	80	0.4	2	39	3320
200	100	0.5	-2	-	-
200	100	0.5	-1	4	600
200	100	0.5	1	25	2700
200	100	0.5	2	32	3400
200	40	0.2	3	75	3200
200	60	0.3	3	56	3560
200	80	0.4	3	45	3800
200	100	0.5	3	32	3400

300	60	0.2	3	74	4740
300	90	0.3	3	55	5250
300	120	0.4	3	44	5580
300	150	0.5	3	37	5850
300	210	0.7	3	28	6180
400	80	0.2	3	73	6240
400	120	0.3	3	54	6880
400	160	0.4	3	43	7280
400	200	0.5	3	36	7600
400	280	0.7	3	27	7960

When the skewness was positive, an estimation of the k value was conducted. The form of the function was determined through data exploration. The least square method was used for the fitting.

$$\hat{k} = \frac{12.3039}{\sigma/\mu} - \frac{\mu}{82.9226} + 5.2346 \cdot skewness$$

This value was used in the analyses presented in the section 4.2.2.

4.2 Prediction Based on I_n

In this section, prediction performance of the order quantity determination methods reviewed are evaluated under I_n on R_+. The comparison between the order quantities and the expected profit values obtained under limited information and the optimal values was conducted assuming that the real demand has specific probability distributions: the normal distribution, the gamma distribution, and the Weibull distribution. When analyzing the gap between the optimal values and the estimated ones, the range of the critical ratio($\frac{p-c}{p-s}$) is limited to the value between 0.2 and 0.8.

	Optimal	DF	ME
Order Quantity	$Q^* = F^{-1}(r)$	$Q^{DF} = \mu + \frac{\sigma}{2}\frac{2r-1}{[r(1-r)]^{1/2}}$	$Q^{ME} = \hat{F}^{-1}(r)$

r= $\frac{C_u}{C_u+C_0} = \frac{p-c}{p-s}$

F: *cumulative distribution function of demand*

4.2.1 Comparison Under I_2

- Normal Distribution

If the real demand distribution is symmetric, the gap between the optimal value and the estimated one is relatively small for both methods.

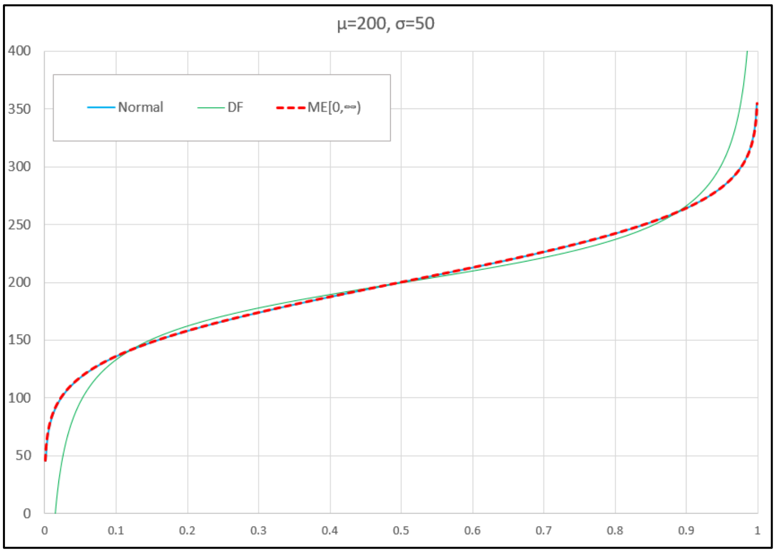

Figure 4 Q for normal distribution, $\mu = 200, \sigma = 50$

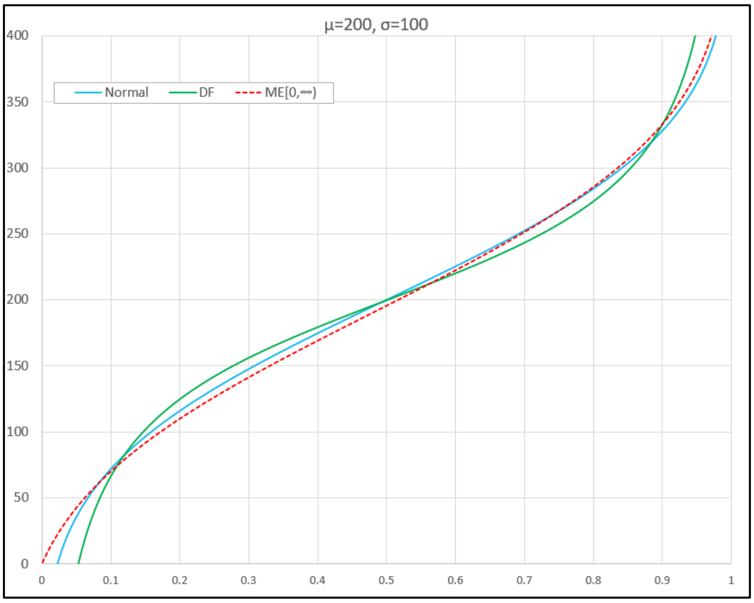

Figure 5 Q for normal distribution, $\mu = 200, \sigma = 100$

The gap between the predicted order quantities and the optimal values does not exceed 10 %. When it comes to expected profit, the difference was no more than 1%. Table 4 shows the gap between the optimal order quantity and the predicted order quantity. Table 5 shows the gap between the optimal expected profit and the expected profit using both prediction methods

Table 4 The optimal and predicted order quantities-normal

mean	std	method	Avg.gap(%)	Max.gap(%)	Min.gap(%)
200	20	ME	<0.0001	<0.0001	<0.0001
		DF	0.6351	1.0507	<0.0001
	50	ME	0.0032	0.0085	<0.0001
		DF	1.6134	2.9854	<0.0001
	100	ME	2.3593	5.1258	<0.0001
		DF	3.4330	7.9207	<0.0001

Table 5 The optimal and the predicted expected profit-normal

mean	std	method	Avg.gap(%)	Max.gap(%)	Min.gap(%)
200	20	ME	<0.0001	<0.0001	<0.0001
		DF	0.0234	0.0739	<0.0001
	50	ME	<0.0001	<0.0001	<0.0001
		DF	0.0710	0.2407	<0.0001
	100	ME	0.2250	0.9408	<0.0001
		DF	0.1137	0.3776	<0.0001

- Gamma Distribution

If the real demand follows a gamma distribution, the order quantity using ME falls within 15% from the optimal order quantity. The order quantity using the distribution free method, however, shows relatively large dispersion away from the optimal value since $Q^{DF} = \mu + \frac{\sigma}{2}\left(\frac{2r-1}{[r(1-r)]^{1/2}}\right)$ is symmetric to $(0.5,\ \mu)$. The gap between the optimal and the predicted expected profit using this approach reached up to 9.15 %. When the demand distribution shows a large skew, the maximum value of the gap between the optimal and the estimated value increased even for the ME method. When it comes to the expected profit, the gap was no more than 5% when ME method was used. Table 6 shows the gap between the optimal and the predicted order quantity. Table 7 shows the gap between the optimal expected profit and the expected profit using prediction methods.

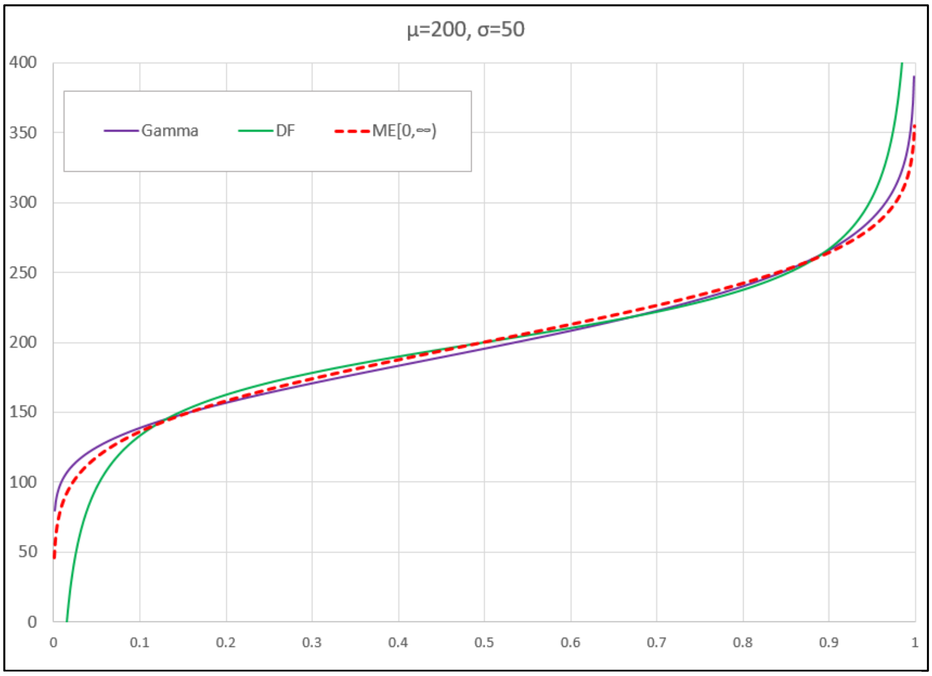

Figure 6 Q for gamma distribution, $\mu = 200, \sigma = 50$

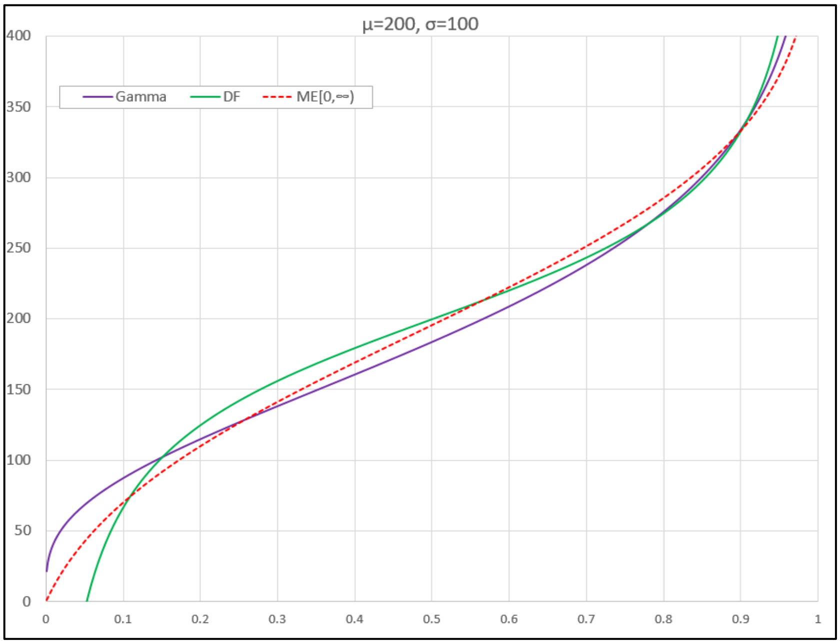

Figure 7 Q for gamma distribution, $\mu = 200, \sigma = 100$

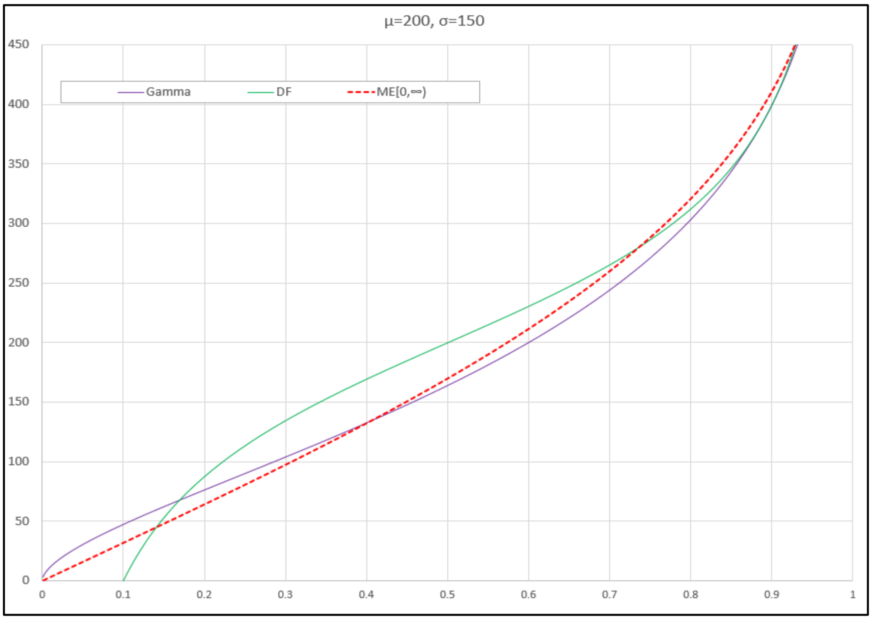

Figure 8 Q for gamma distribution, $\mu = 200, \sigma = 150$

Table 6 The optimal and the predicted order quantity-gamma

mean	std	method	Avg.gap(%)	Max.gap(%)	Min.gap(%)
200	20	ME	0.2621	0.3344	0.0901
		DF	0.6743	1.2269	0.0019
	50	ME	1.6548	2.1225	0.4656
		DF	2.2102	4.1811	0.0032
	100	ME	4.8153	6.6241	<0.0001
		DF	7.8680	13.1625	0.0092
	150	ME	5.4267	15.7757	<0.0001
		DF	19.1742	29.8161	2.9396

Table 7 The optimal and the predicted expected profit-gamma

mean	std	method	Avg.gap(%)	Max.gap(%)	Min.gap(%)
200	20	ME	0.0032	0.0052	<0.0001
		DF	0.0321	0.1018	<0.0001
	50	ME	0.0571	0.0936	<0.0001
		DF	0.1717	0.4822	<0.0001
	100	ME	0.2782	0.4770	<0.0001
		DF	1.0744	2.5209	<0.0001
	150	ME	0.3716	2.8413	<0.0001
		DF	4.1956	9.1534	0.0487

- Weibull Distribution

If the real demand follows a Weibull distribution, the results are similar to those of gamma-distributed demand case. The gap between the order quantity using maximum entropy method and the optimal lies within 10% while the order quantity using the distribution free method shows a large dispersion from the optimal value. Table 8 shows the gap between the optimal order quantity and the predicted order quantity. Table 9 shows the gap between the optimal expected profit and the expected profit using prediction methods.

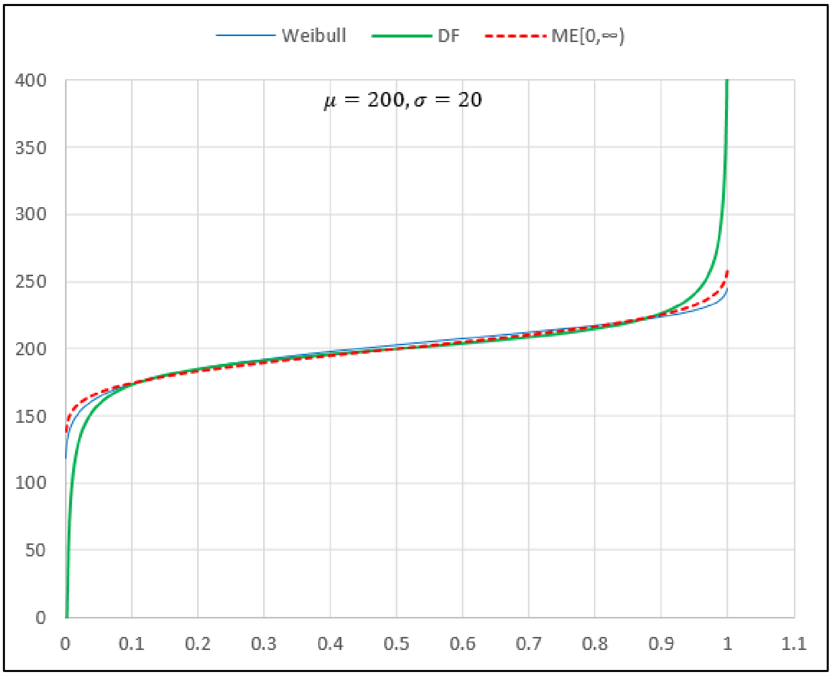

Figure 9 Q for Weibull distribution, $\mu = 200, \sigma = 20$

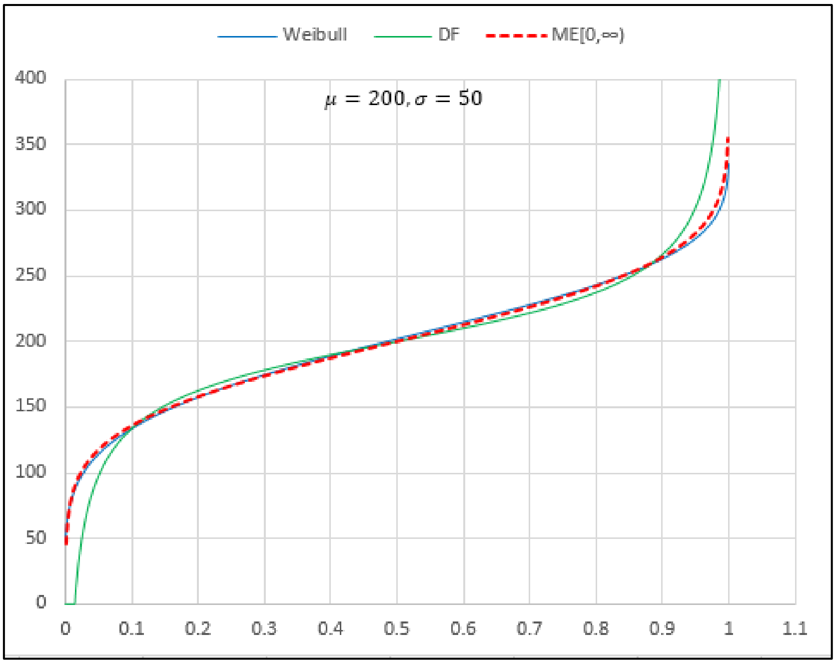

Figure 10 Q for Weibull distribution, $\mu = 200, \sigma = 50$

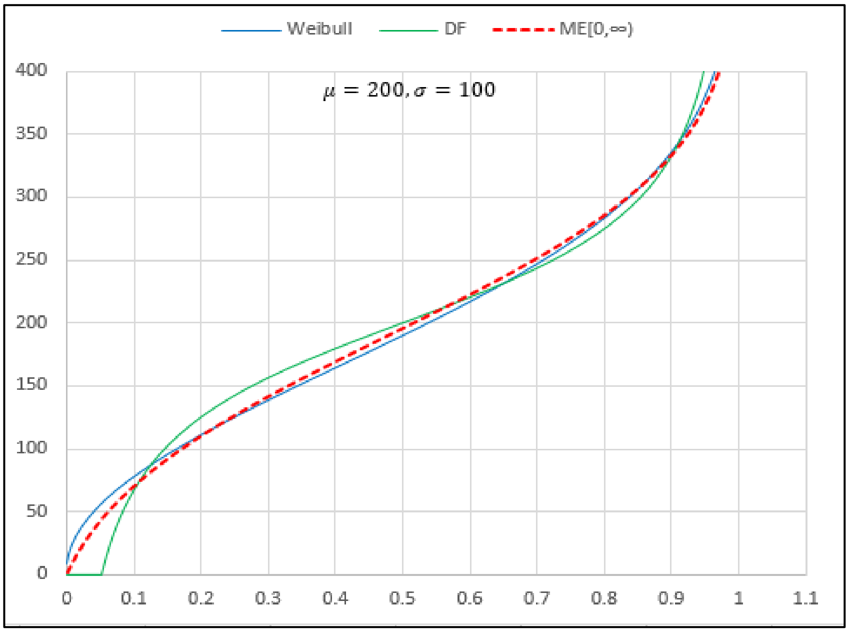

Figure 11 Q for Weibull distribution, $\mu = 200, \sigma = 100$

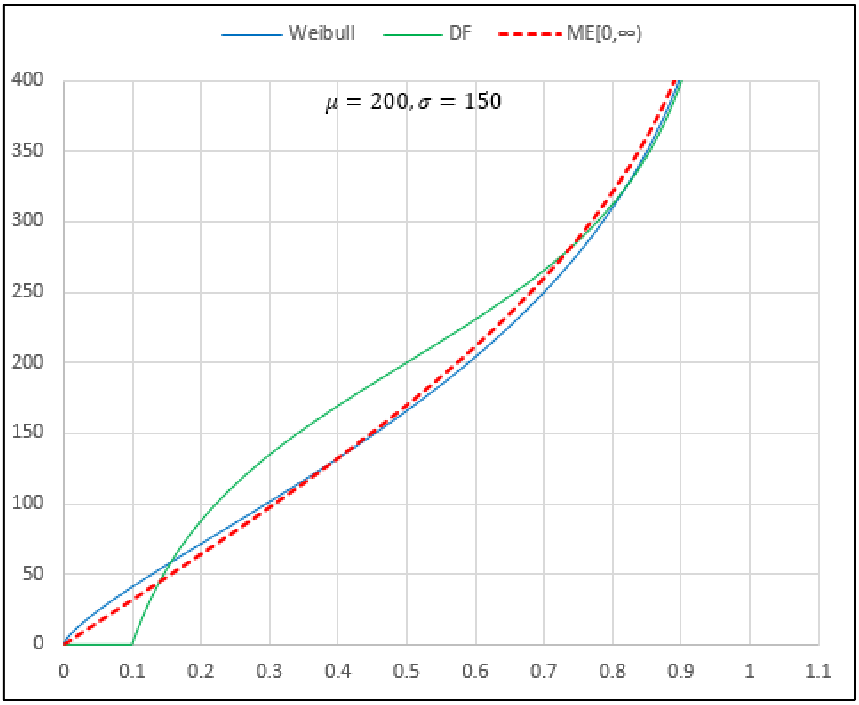

Figure 12 Q for Weibull distribution, $\mu = 200, \sigma = 150$

Table 8 The optimal and the predicted order quantity-Weibull

mean	std	method	Avg.gap(%)	Max.gap(%)	Min.gap(%)
200	20	ME	0.9385	1.2473	0.0488
		DF	0.9471	1.5061	0.0020
	50	ME	0.7524	1.0436	0.0013
		DF	1.8854	3.2144	0.0040
	100	ME	2.2422	3.1339	0.0005
		DF	6.5122	13.9995	0.0026
	150	ME	3.2775	10.5385	0.0093
		DF	18.9926	32.6461	0.6827

Table 9 The optimal and the predicted expected profit-Weibull

mean	std	method	Avg.gap(%)	Max.gap(%)	Min.gap(%)
200	20	ME	0.0465	0.0774	<0.0001
		DF	0.0459	0.0883	<0.0001
	50	ME	0.0122	0.0209	<0.0001
		DF	0.0834	0.2654	<0.0001
	100	ME	0.0655	0.1186	<0.0001
		DF	0.8724	2.7719	<0.0001
	150	ME	0.1465	1.1566	<0.0001
		DF	4.5012	10.8392	0.0028

4.2.2 Maximum Entropy Method under I_3

In this section, maximum entropy distributions based on I_3 information in order to analyze the effect of integrating additional information into the prediction procedure: skewness or third moment.

ME2 and ME3 in Table 10-13 denote the ME method based on I_2 and I_3, respectively. ME3 provided better solutions than ME2 did, on average, for every set of data. As the skewness of the real demand distribution increased, solutions of ME3 grew closer to the optimal value.

- Gamma distribution

If the real demand distribution follows a gamma distribution, the skewness increases as the variance increases. ME3 provided better protection for the data with large skewness.

Table 10 The optimal and predicted order quantity-gamma (I_3)

mean	std	skewness	method	Avg.gap(%)	Max.gap(%)	Min.gap(%)
200	20	0.2	ME2	0.2621	0.3344	0.0901
			ME3	0.2300	0.2934	0.0845
	50	0.5	ME2	1.6548	2.1225	0.4656
			ME3	1.4557	1.8679	0.5093
	100	1.0	ME2	4.8153	6.6241	<0.0001
			ME3	4.3927	6.0267	0.0078
	160	1.6	ME2	5.0385	15.6543	0.0002
			ME3	4.3346	12.7857	0.0106

Table 11 The optimal and predicted expected profit-gamma (I_3)

mean	std	skewness	method	Avg.gap(%)	Max.gap(%)	Min.gap(%)
200	20	0.2	ME2	0.0032	0.0052	<0.0001
			ME3	0.0025	0.0040	0.0001
	50	0.5	ME2	0.0571	0.0936	<0.0001
			ME3	0.0448	0.0737	0.0028
	100	1.0	ME2	0.2782	0.4770	<0.0001
			ME3	0.2366	0.4192	<0.0001
	160	1.6	ME2	0.3423	2.6733	<0.0001
			ME3	0.2283	1.7844	<0.0001

- Weibull distribution

If the real demand follows a Weibull distribution, the data set includes the cases with negative skewness. Because maximum entropy distributions under I_2 are positive-skewed or almost symmetric, ME2 showed larger dispersion from the optimal compared to the other cases having same mean and variance even when the variance is relatively small. Figure 13 shows the optimal order quantities under Weibull distribution, and estimated order quantities under I_2 and I_3.

Table 12 The optimal and predicted order quantity-Weibull (I_3)

mean	std	skewness	method	Avg.gap(%)	Max.gap(%)	Min.gap(%)
200	20	-0.7155	ME2	0.9385	1.2473	0.0488
			ME3	0.5277	0.6973	0.1071
	50	-0.1853	ME2	0.7524	1.0436	0.0013
			ME3	0.3266	0.7591	0.0004
	100	0.5664	ME2	2.2422	3.1339	0.0005
			ME3	1.9023	2.7381	0.2147
	160	1.4152	ME2	3.2775	10.5385	0.0093
			ME3	2.6065	7.9977	0.0089

Table 13 The optimal and predicted expected profit-Weibull (I_3)

mean	std	skewness	method	Avg.gap(%)	Max.gap(%)	Min.gap(%)
200	20	-0.7155	ME2	0.0465	0.0774	<0.0001
			ME3	0.0139	0.0230	0.0007
	50	-0.1853	ME2	0.0122	0.0209	<0.0001
			ME3	0.00026	0.0145	<0.0001
	100	0.5664	ME2	0.0655	0.1186	<0.0001
			ME3	0.0498	0.0946	0.0003
	160	1.4152	ME2	0.1465	1.1566	<0.0001
			ME3	0.0827	0.6661	<0.0001

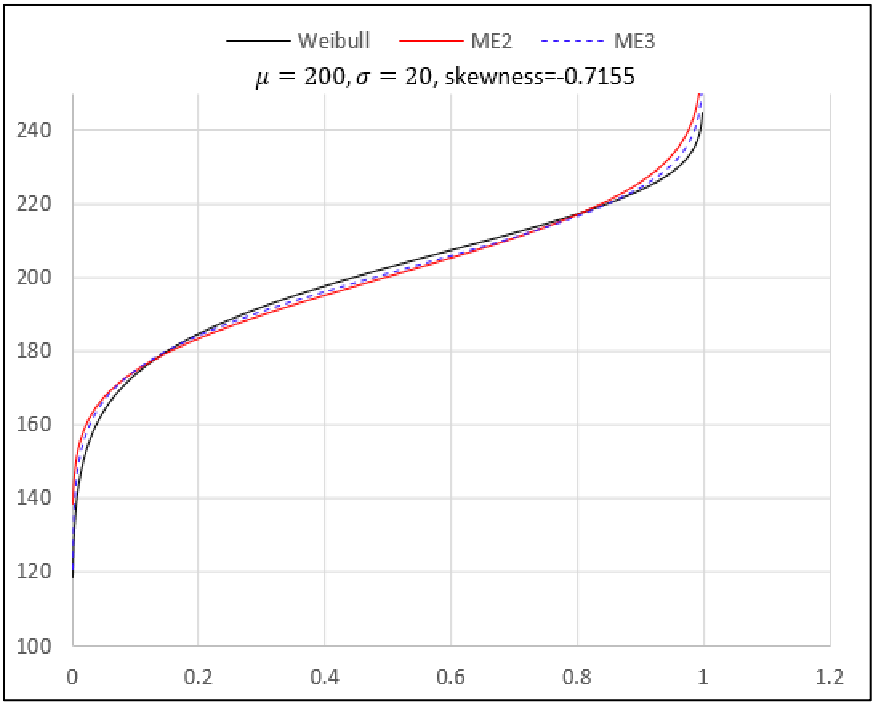

Figure 13 Q for Weibull Distribution with negative **skewness**

Chapter 5. Conclusion

Determining order quantity under limited demand information is not an easy task. Researchers have tried to find ordering rules which can provide robust protection against demand uncertainty. This study examined two existing ordering rules for the single period newsvendor problem: the ME method and the min-max DF approach. This study also extended the analysis to a situation where the higher order moment information is available to enhance the quality of the solutions.

When the information about the first two moments is available, comparisons among the solutions from the ME method and DF approach were conducted under the various demand distributions in the exponential family. Solutions from the DF approach diverged for very small or large critical ratios. For the ME method, if the variance-mean ratio was small, the predicted distributions showed symmetric forms. Predicted distributions were left-skewed when the variance-mean ratio was large. These findings imply that utilizing skewness information can improve the solutions for the same problem. By extending the order quantity determination methods to the situations where the additional moment information is available, improvement in maximum entropy prediction was achieved. In addition, a novel lower bound for the expected profit of the newsvendor problem was found. This lower bound can be a better bound under certain conditions.

Additionally, this study provides an extensive analysis on the characteristic of ME approach. The simplified form of the maximum entropy distribution under the information of mean and variance was investigated. This simplified form enables faster solution procedure by reducing the number of dimensions of the search space for the Lagrangian multipliers. As part of this attempt to speed up the procedure, an analysis on the appropriate finite support that can replace the semi-infinite support was also conducted.

Bibliography

Andersson, Y., Jörnsten, K., Nonås, S.L., Sandal1, L. and Ubøe, J., A maximum entropy approach to the newsvendor problem with partial information, *European Journal of Operational Research*, Vol.288, No.1, p.190-200, 2013.

Gallego, G., and Moon, I., The distribution free newsboy problem: review and extensions, *Journal of Operational Research Society*, Vol.44, No.8, p. 825-834, 1993.

Hill, R.M., Applying Bayesian methodology with a uniform prior to the single period inventory model, *European Journal of Operational Research*, Vol.98, No.3, p.555-562, 1997.

Ismail, B., and Louderback, J., Optimizing and satisficing in stochastic cost-volume-profit analysis, *Decision Science*, Vol.10, No.2, p.205-217, 1979.

Khouja, M., A note on the newsboy problem with an emergency supply option, *Journal of Operational Research Society*, Vol.47, No.12, p.1530-1534, 1996.

Maglaras, C., Eren, S., A maximum entropy joint demand estimation and capacity control policy, *Production and Operations Management*, Vol.24, No.3, p.438-450, 2015.

Mead, L.R., Papanicolaou, N., Maximum entropy in the problem of moments, *Journal of Mathematical Physics*, Vol 25, No.8, p.2404-2417, 1984.

Perakis, G., Roels, G., Regret in the newsvendor model with partial information, *Operations Research*, Vol.56, No.1, p.188-203, 2008.

Scarf, H., A min-max solution of an inventory problem in *Studies in the Mathematical Theory of Inventory and Production*, K. Arrow et al., Eds. Redwood City, CA: Stanford Univ. Press, 1958, p.201-209.

초 록

본 연구는 불확실성이 존재하는 단기 뉴스벤더 모델에서의 주문량 결정 방법에 대해 다룬다. 특히, 최대 엔트로피접근법에 기반을 둔 주문량 결정법에 대해 다각적인 분석을 시도하였다. 이는 다른 형태의 정의역에서 추정된 최대 엔트로피 분포의 확률밀도함수의 형태에 대한 분석과 근사적 풀이에 대한 시도를 포함한다. 최대 엔트로피 접근법과 분포자유접근법을 이용하여 도출된 해와 몇 가지 지수족 분포를 가정하였을 때의 최적해를 비교하고, 나아가 고차 모멘트 정보가 존재하는 상황으로의 확장을 시도하여 예측 정확성을 높일 수 있는 방안을 제시한다.

주요어: 수요의 불확실성, 분포 자유 접근법, 최대 엔트로피 방법론, 뉴스벤더 모델

Student Number: 2016-21126