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Checking the power of the Econometric method presented in A Random Attention Model with a special case

랜덤 주의 모형에서 제시된 계량 검정법의 검정력을 특별한 사례로 재고

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Checking the power of the Econometric method presented in A Random Attention Model with a special case

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Abstract

I test the power of the econometric method presented in Masatlioglu et al. (2018) using simulation. The choice dataset is generated by the model presented in Aguiar (2017), RCG model, which is a special case of the model presented in Masatlioglu et al. (2018), RAM model. The observable choice dataset satisfying the RCG model reveals no preference relation according to RAM model. I question if the econometric method can still verify the underlying true preference order as compatible and correctly reject the false preference orders. I show that the econometric method presented in RAM has trivial power in the special case since the construction of the dataset admits the test statistics to exist on the boundary of the moment inequality. However, the method relatively performs well when there exist few alternatives in the choice problem, narrowing down the candidates for the underlying preference order that are compatible with the given dataset. On the other hand, when many alternatives exist, the method fails to give any information about what the true underlying preference might be. When this boundary issue is anticipated, I suggest a one-sided hypothesis test to circumvent this problem and provide simulation evidence to support it. The evidence shows that the one-sided hypothesis test has higher power and gives information about what the true preference order might be in this special case.

Keywords: Decision theory, revealed preference, limited attention, bounded rationality, nonparametric identification, moment inequality.

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1. Introduction

In general, standard Economics models had assumed that agents who participate in economic activity have perfect rationality. Before the choice is made, the agents know their true preference with certainty and can consider all possible outcomes and probabilities associated with the choice. However, numerous literatures in marketing and experimental economics document that the agents seem to not behave congruently with the assumption.[1] One example is that people make different choices even though the situation seems identical. This phenomenon is called random/stochastic choice and is extensively demonstrated in experiments.[2] One explanation is provided through the Random Utility Model that asserts there exists a probability distribution over the preference ranking. (Block and Marschak (1960)). Another is that our attention is limited and we fail to consider all possible alternatives presented.

This explanation is enumerated as limited attention and a concept called 'consideration sets' emerges based on the assumption, which is the set of subsets

[1] See for example, Kahneman, Daniel,. Teversky, Amos. (1979) "Prospect Theory: An Analysis of Decision under Risk". *Econometrica.* 47 (2): 263.

[2] See for example Agranov and Ortoleva. (2017). "Stochastic Choice and Preferences for randomization". *Journal of Political Economy.* Vol .125, 40-68.

of the grand set of alternatives that the agent actually considers when making decision. The impact of limited attention is non-trivial and allocating attention or gathering information can lead to a different prediction with Standard Economics.[3] (Stigler (1961)). A natural extension is the formation of the consideration sets. Sims (2003) theoretically demonstrated that "rational inattentive" agents choose information optimally in a given problem based on the Shannon mutual information which measures the amount of information that one random variable contains about another random variable. Caplin and Dean (2015) establishes axiomatic conditions that characterize the observed dataset which is a consequence of a costly information acquisition representation such as Sims (2003) model.

Meanwhile, Masatlioglu, Nakajima and Ozbay (2012) (MNO) shows that when limited attention exists, classical revealed preference is inapplicable since we do not know whether a choice is from preference or limited attention. They provide axioms to reveal preference and attentions from choice data in limited attention setting. This analysis can be important if a researcher can only observe the choice of the people and needs to estimate the demand of a product or the welfare effect of a given policy when the classical revealed preference might

[3] The definition of Standard Economics is unclear and misleading but I use it following the convention to distinguish behavioral economics with non-behavioral economics.

fail because of limited attention. Thus, the researcher might need a method to accurately derive the revealed preference.

While MNO is a deterministic consideration set model, Manzini and Mariotti (2014) analysis a random consideration set model introducing the concept of attention parameter which is a (unobservable) probability that the agent will consider a given alternative.

Masatlioglu et al. (2018) is an extension of random consideration set model and tries to incorporate randomness in the consideration set formation. It introduces μ which is the probability of a certain subset of all feasible alternatives to be considered as a consideration set. They employ an intuitive assumption, 'monotonic attention', which states that the probability of considering a certain subset should not decrease when an alternative not considered is removed. They provide a theory of revealed preference based on the assumption and develop econometric method to test whether a preference order is compatible with the data generating process that generates the observable choice data set. The paper uses two assumptions, monotonic attention and triangular attention rule, to make inequality constraints for identification.

I test this nonparametric identification method and see the performance of it. I used the Random categorization rule (RCG) presented in Aguiar (2017), which is a special case of RAM, since it has an advantage in recovering preference and attention probabilities. If the dataset satisfies acyclicity and Weakly Decreasing Marginal Propensity of Choice with totality, we can recover the underlying unobservable preference up to a monotonic transformation and the attention probability using mobius inversion formula. I used this theorem and generated datasets following the RCG model for simulation.

When using RCG model to generate choice dataset, the dataset reveals no preference order according to the RAM while the true preference order and attention probabilities is retrieved following the RCG model. My object is to see whether the econometric method provided in the RAM paper successfully not reject the true preference order and give information about the underlying true preference when we can infer nothing by following the RAM.

The simulation shows that even though the power is trivial, the econometric method performs relatively well when there exist only 3 alternatives with an additional default option. The true preference has an empirical rejection probability close to zero while 20 out of 24 preferences have slightly higher empirical rejection probability than the pre-selected level. Therefore, the candidate for the underlying true preference can be narrowed down.

However, the power is trivial and the test fails to reveal any information when there exist 4 alternatives and an additional default option. This is caused by the fact that the false preference orders admit the test statistics to be on the boundary of the moment inequality in this special case, making the test to be asymptotically non-similar and have trivial power.

When the test statistic is anticipated to exhibit the boundary issue, I suggest to do a one-sided hypothesis test to circumvent the problem. Since the test statistic is expected to be close to zero under the false preference order, I propose to change the null hypothesis to $\mathrm{H}_0\colon \max(\mathbf{R}_{\succ}\boldsymbol{\pi}) = 0$ and the alternative to $\mathrm{H}_1\colon \max(\mathbf{R}_{\succ}\boldsymbol{\pi}) < 0$. Under the false preference, the ERP should be similar with or below the preselected nominal level while the ERP should be higher than the nominal level under true preference. Simulation evidence shows that this one-sided test can help to narrow down the candidates for the underlying true preference order.

2. Existing Models and Methods

In this section, I introduce and compare the concepts and methods from Masatlioglu et al. (2018) and Aguair (2017). I clearly note that all of the theorems, lemmas, definitions in this section are from the two literatures and are not mine.

2.1 Random Attention Model

RAM builds up a model based on a situation where agents pay attention to a subset of X - the universal set of all mutually exclusive alternatives- with a given probability μ. For example, an agent who is facing a problem to choose what to drink might consider {apple juice, beer, soda, Coke} with probability $\frac{1}{2}$ and consider {orange juice, vodka, soda, water} with a probability $\frac{1}{2}$. The agent will select the best alternative according to her preference order after the consideration set is realized. I follow the notation of Masatlioglu et al. (2018) and directly use the theorems and definition documented there.

Let $\mathcal{X}$ denote the set of all nonempty subsets of X and $\pi(a|S)$ represent the probability that an agent chooses a from the choice problem S. $\mu(T|S)$ denotes the probability that an agent pays attention to T, which is a subset of $S \in \mathcal{X}$, when the choice problem is S. $\sum_{a \in T \subset S} \mu(T|S)$ denotes the probability of an agent to consider alternative $a \in S$. RAM reckons that decision makers choose the best alternative among the alternatives in the consideration set according to a fixed preference order. The authors suggest an intuitive assumption that conveys the logic that attention is scarce and alternatives compete to gain attention.

Assumption 1 (Monotonic Attention in Masatlioglu et al. (2018)). For any $a \in S - T$, $\mu(T|S) \leq \mu(T|S - a)$.

The probability of considering set T should not decrease when an alternative which is not considered is removed from the problem. It can be understood as the regularity condition on attention probabilities.

Using the notations and assumption 1, Masatlioglu et al. (2018) introduces RAM representation.

Definition 3 (in Masatlioglu et al. (2018)). A choice rule π has a random attention representation if there exist a preference ordering $\succ$ over X and a monotonic attention rule μ such that

$$\pi(a|S) = \sum_{T \subset S} 1(a \; is \; best \; in \; T) \cdot \mu(T|S)$$

For all $a \in S$ and $S \in \mathcal{X}$ where $1(\cdot)$ is an indicator function. Define π to be a RAM if it satisfies Definition 3.

Now the RAM model gives the definition of revealed preference. a is revealed to be preferred to b if a is preferred to b for all possible RAM representations of π. Then the model can find revealed preference relation if the following lemma is satisfied.

Lemma 1 (in Masatlioglu et al. (2018)). Let π be a RAM. If $\pi(a|S) > \pi(a|S-b)$, then a is revealed to be preferred to b.

The Theorem 1 in Masatlioglu et al. (2018) states that if π is a RAM, checking whether distinct $a, b \in S$ satisfies Lemma 1 is sufficient to find the revealed preference relation between a and b from the given dataset.

The authors also provide how to characterize whether a dataset is consistent with RAM in Theorem 2. When choosing not to choose is included in the model, the characterization becomes as following:

Remark 4 (in Masatlioglu et al. (2018)). A choice rule π has a RAM representation with a default option {o} if and only if it satisfies acyclicity of P and regularity on default.

Regularity on the default states that $\pi(o, S) \leq \pi(o, S-a)$ for all $a \in S$, $S \in \mathcal{X}$.

I need to introduce one more concept in Masatlioglu et al. (2018) to use the econometric method provided in RAM which is Triangular Attention Rule. I explain what it means in the Appendix.

TA rule makes $\pi\left(a_{k,\succ}|S\right) = \sum_{T \subset S} 1\left(a_{k,\succ} \text{ is best in } T\right) \cdot \mu(T|S)$ be represented in a simpler form as $\pi\left(a_{k,\succ}|S\right) = \mu(T|S)$ where $T = S \cap L_{k,\succ}$. It is shown in the literature that if a choice rule can be represented by a monotonic attention rule, it can also be represented by a monotonic triangular attention rule.

Masatlioglu et al. (2018) provides a nonparametric method to identify and test what preference order is compatible with the observable choice dataset. Θ_π is the set of preferences that are compatible with π. Compatible means that a certain preference order can have a RAM representation for a given choice dataset.[4]

[4] See Appendix for more detail.

Let $\boldsymbol{\pi}$ be a long vector that has elements as $\pi(a|S)$ for all alternatives $a \in S$.

Theorem 4 (Nonparametric Identification in Masatlioglu et al. (2018)). Given any preference $\succ$, there exists a unique matrix $\mathbf{R}_{\succ}$ such that $\succ \in \Theta_{\pi}$ if and only if $\mathbf{R}_{\succ}\boldsymbol{\pi} \leq 0$.

Theorem 4 in Masatlioglu et al(2018) gives a sufficient inequality condition to check whether a preference is $\succ \in \Theta_{\pi}$ with the observable choice dataset. $\mathbf{R}_{\succ}$ is a product of two matrices $\mathbf{R}$ and $\mathbf{C}_{\succ}$. By combining $\mathbf{R}$ and $\mathbf{C}_{\succ}$, monotonicity assumption becomes $\pi\left(a_{\ell,\succ}|S\right) - \pi\left(a_{\ell,\succ}|S - a_{k}\right) \leq 0$, for all $a_{k} \succ a_{\ell,\succ}$ in S.

The estimated choice rule is $\hat{\pi}$:

$$\hat{\pi}(a|S) = \frac{\sum_{1\leq i\leq N} 1(y_i = a, Y_i = S)}{\sum_{1\leq i\leq N} 1(Y_i = S)}, a \in S, S \in \mathcal{X}.$$

Let $\hat{\boldsymbol{\pi}}_S$ be the vector of $\hat{\pi}(\cdot\,|S)$ with each alternative $a \in S$ being an element of $\hat{\boldsymbol{\pi}}_S$ as $\hat{\pi}(a|S)$. $\hat{\boldsymbol{\pi}}$ is a long vector that is stacked with choice rules of $S \in \mathcal{X}$ and $\boldsymbol{\pi}$ be the population counterpart.

Also, let $\sigma_{\pi,\succ}$ be the standard deviation of $\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}}$, and $\hat{\sigma}_{\succ}$ be its plug-in estimate.[5]

The null hypothesis is $\mathrm{H}_0 = \succ \in \Theta_{\pi}$ and is rejected if $\mathbf{R}_{\succ}\boldsymbol{\pi} \leq 0$ is violated. Using this fact, the test statistic is:

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[5] $\sigma_{\pi,\succ} = \sqrt{\mathrm{diag}(\mathbf{R}_{\succ}\Omega\mathbf{R}'_{\succ})}$ and $\hat{\sigma}_{\succ} = \sqrt{\mathrm{diag}\left(\mathbf{R}_{\succ}\hat{\Omega}\mathbf{R}'_{\succ}\right)}$,

$$\mathcal{T}(\succ) = \sqrt{N} \cdot \max\{(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ}), 0\},$$

Where $\oslash$ denotes the Hadamard division. The null hypothesis is rejected if the test statistic exceeds the critical value which is obtained from simulation. We can rewrite the test statistics as,

$$\mathcal{T}(\succ) = \max\{(\mathbf{R}_{\succ}\sqrt{N}(\hat{\boldsymbol{\pi}} - \boldsymbol{\pi}) + \sqrt{N}\mathbf{R}_{\succ}\boldsymbol{\pi} \oslash \hat{\sigma}_{\succ}), 0\}.$$

By central limit theorem, the first component $\sqrt{N}(\hat{\boldsymbol{\pi}} - \boldsymbol{\pi})$ has an asymptotic normal distribution $\mathcal{N}(0, \Omega_{\pi})$ and the second component $\sqrt{N}\mathbf{R}_{\succ}\boldsymbol{\pi}$ is bounded above if $\succ \in \Theta_{\pi}$ by monotonicity assumption. Simulated test statistics can be written as

$$\mathcal{T}^*(\succ) = \max\{(\mathbf{R}_{\succ}\mathbf{z}^* \oslash \hat{\sigma}_{\succ} + \psi_N(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ}), 0\}.$$

Where $\mathbf{z}^*$ is a random vector simulated from the asymptotic normal distribution $\mathcal{N}(0, \Omega_{\pi})$. There are several choices for ψ_N according to Andrews and Soares (2010), but Masatlioglu et al. (2018) employs

$$\psi_N(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ}), = \frac{1}{\sqrt{\ln N}}(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ})_{-},$$

Where $(\mathbf{a})\_ = \mathbf{a} \odot 1(\mathbf{a} \leq 0)$, with $\odot$ denoting the Hadamard product and $\mathbf{a}$ is a vector. The literature picks the BIC value $\kappa_N = \sqrt{\ln N}$ following the recommendation in Andrews and Soares (2010). From 2000 simulations, we obtain a simulated distribution of the test statistics. The critical value is defined as

$$c_\alpha(\succ) = \inf\left\{t : \frac{1}{M}\sum_{m=1}^{M} 1(\mathcal{T}^*(\succ) \le t) \ge 1-\alpha\right\}$$

Where α is the preselected level of the test. The critical value is the smallest value of t such that the proportion of $\mathcal{T}^*(\succ)$ being larger than t, $\mathcal{T}^*(\succ) > t$, out of M simulations is smaller than α. The null hypothesis is rejected if and only if $\mathcal{T}(\succ) > c_\alpha(\succ)$.

Masatlioglu et al. (2018) shows that uniformity issues is well controlled with theorem 5 in the same literature by proving that $\mathcal{T}^*(\succ)$ stochastically dominates $\mathcal{T}(\succ)$.

To see how the simulation evidence is obtained and is interpreted, we need to look back at Masatlioglu et al (2018). First, we need to generate sample data from the stochastic choice dataset using multinomial distribution. After the sample data is generated, we need to estimate

$$\hat{\pi}(a|S) = \frac{\sum_{1\le i\le N} 1(y_i = a, Y_i = S)}{\sum_{1\le i\le N} 1(Y_i = S)}, a \in S, S \in \mathcal{X}.$$

Which is the moment estimator. We then calculate the test statistics

$$\mathcal{T}(\succ) = \sqrt{N} \cdot \max\{(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ}), 0\}.$$

By the central limit theorem, we simulate 2000 test statistics following a normal distribution $\mathcal{N}(0, \Omega_\pi)$ to make a hypothetical distribution for $\mathcal{T}^*(\succ)$. We then obtain $c_\alpha(\succ)$. If the estimated test statistics $\mathcal{T}(\succ)$ exceeds the critical value $c_\alpha(\succ)$, the null hypothesis, which is $H_o = \succ \in \Theta_\pi$ is rejected.

Monte Carlo simulation is executed to this process for a given number, M. Then we obtain the result of each execution whether the test statistic is rejected or not rejected. The literature define empirical rejection probability as the total number of rejection in M simulations divided by M:

$$empirical\ \ rejection\ \ probability \equiv \frac{\text{total number of rejection in } M \text{ simulations}}{M}.$$

A specified preference order that is considered as the null hypothesis may not be compatible if the ERP substantially exceeds the pre-selected level, $\alpha,$ and may not have a RAM representation.

If the underlying true preference order is known and the test is done on a false preference order, the ERP can be interpreted as the power of the test since it is the probability to reject the null hypothesis when the alternative is true. Conversely, if the test is done on the true preference order, the ERP can be interpreted as the size of the test since it is the probability to reject the null hypothesis when the null is true.

2.2 Random Categorization Rule

I use the model proposed by Aguiar (2017) to generate the dataset that is used in simulation to test the proposed econometric method for partially identifying the underlying preference.

The reason I have brought the model is because I can generate the choice dataset not arbitrarily but following the model and can test the econometric method presented in Masatlioglu et al. (2018) with all information needed. To elaborate, we can recover the underlying true preference order and the probabilities to consider the consideration set if the dataset satisfies WDMP and acyclicity. Therefore, there is an advantage since we can create a choice dataset satisfying the two conditions and then recover all the information needed. Also, RCG is a special case of RAM. Thus, the econometric method which is used to test whether a given preference order can have a RAM representation should work well under a dataset that is generated by RCG.

Random Categorization (Aguiar (2017)) models a situation where a decision maker is faced with a set of alternatives and considers the subsets with categories. For an example, a DM facing a problem to select a drink might mentally categorize beers and choose to consider only beers among the alternatives that are in display. I directly follow the notation in Aguiar (2017).

Let X denote the finite choice set with the default option {o}. A stochastic choice dataset is a pair of menus, which is called as choice problem in RAM , and choice probability $p(a, A)$, which is denoted as $\pi(a|A)$ in RAM: $\{A, p(a, A)\}_{A \in \mathcal{M}, a \in X \cup \{o\}}$. The literature sets $p(o, \emptyset) = 1$ and $\sum_{a \in A} p(a, A) + p(o, A) = 1$. Also, RCG model assumes that the dataset is complete, meaning $p(a, A)$ is observed for all $a \in A, A \in \mathcal{M} : \mathcal{M} \equiv 2^X$.

A category is a subset of X, denoted as D, and the collection of D is $\mathcal{D} \subseteq 2^X$. $m(D)$ represents the probability that the DM pays attention to category D. Moreover, RCG assumes that the DMs have only injective utility functions, u, excluding indifferent preferences.

The consideration set is the intersection of the given menu, A, and the category D: $\Gamma(D, A) = D \cap A$. The DM chooses the alternative that gives the most utility among the consideration set. Define the set $\mathrm{B}_A(a) = \{b \in A; u(b) > u(a)\}$ which is the set of all alternatives in A that gives higher utility than alternative a. Then, the choice rule following the RCG model becomes:

$$P_{RCG}(a, A) = \sum_{\{a\} \cap D \neq \emptyset; B_A(a) \cap D = \emptyset : D \in \mathcal{D}} m(D).$$

Where the probability of choosing the default is $P_{RCG}(o, A) = \sum_{A \cap D = \emptyset : D \in \mathcal{D}} m(D)$ and $\sum_{a \in A} P_{RCG} + P_{RCG}(o, A) = 1$. $P_{RCG}(a, A)$ means that the probability of choosing $a \in A$ is the sum of $m(D)$ such that D contains a but no other alternatives that give higher utility.

Definition 1 in Aguiar (2017) states that if there is a triple u, m and $\mathcal{D}$ satisfying $P_{RCG}(a, A)$, a stochastic choice dataset has a Random Categorization rule representation.

Definition 2 in Aguiar (2017) states that $p(b, A \cup \{a\}) \neq p(b, A)$ if and only if a is stochastically revealed preferred to b. The condition that the choice rule should satisfy to know the revealed preference relation is different for RAM and RCG. If a is revealed to be preferred to b, it should satisfy $\pi(a|S) > \pi(a|S-b)$ according to theorem 1 (RAM) while it should satisfy $\pi(b, S \cup \{a\}) \neq \pi(b, S)$ by definition 2 (RCG). In RCG model, the revealed preference is set as the definition, not a lemma derived from assumptions.

To characterize the RCG model, I need to introduce one more concept in Aguiar (2017) which is WDMP. I explain it briefly in the Appendix.

Now RCG model can be characterized.

Theorem 1 (in Aguiar (2017)). A complete stochastic dataset admits a Random Categorization rule (RCG) representation if and only if it satisfies acyclicity and the WDMP.

When a stochastic choice data set satisfies acyclicity and WDMP with preference satisfying totality[6], the underlying unobservable preference order and m can be recovered according to Corollary 1 and 2 in Aguiar (2017).

By checking $p(b, A \cup \{a\}) \neq p(b, A)$, it is straightforward how to find the revealed preference order.

m is recovered using the Mobius inversion formula on posets:

[6] Condition 3(RCG). $\succ$-Totality. $\succ$ is total if and only if for any $a, b \in X, a \succ b$ or $b \succ a$

$m(D) = \sum_{A \subset D : D \in \mathcal{D}} (-1)^{|D \setminus A|} (1 - \varphi(X/A))$ where φ is a hitting function: $\varphi: 2^X \longmapsto [0,1]$, $\varphi(A) = \sum_{A \cap D \neq \emptyset : D \in \mathcal{D}} m(D)$. And using the fact that $1 - \varphi(X/A)) = p(o, X/A)$, we can calculate every $m(D)$ if $p(o, A)$ for all $A \in \mathcal{M}$ is observable and given as satisfying WDMP.

2.3 RCG is a special case of RAM

Looking at Theorem 1 (Aguiar (2017)), A complete stochastic dataset admits a Random Categorization rule (Aguiar (2017)) representation if and only if it satisfies acyclicity and WDMP. Since Remark 4 (Masatlioglu et al. (2018)) states that a choice rule π has a RAM representation with a default option if and only if it satisfies acyclicity of **P** and regularity on the default, RCG is included in the group of models that satisfies RAM because WDMP is a stronger condition than regularity.

If the collection of categories is same as the power set of X, $\mathcal{D} \equiv 2^{\mathrm{X}}$, the probability mapping μ can be expressed in terms of $m(D)$:

$$\mu(T|S) = \sum_{D \cap S = T : D \in \mathcal{D}} m(D)$$

and $\mu(\emptyset|S) = m(\emptyset)$.

The probability of the set T to be the consideration set from choice problem S is the sum of $m(D)$s that satisfies $D \cap S = T$. Following this representation of μ, the

RCG model satisfies the monotonicity assumption of RAM.[7]

Finally, the RCG models do not generally satisfy the triangular attention rule. However, since RCG model is a special case of RAM, by Theorem 2 (Masatlioglu et al. (2018)) and remark 2 (Masatlioglu et al. (2018)), the stochastic dataset can also be represented by a monotonic triangular attention rule.

3. Simulation evidence

I have used the econometric package provided by the authors of Masatlioglu et al. (2018), which is called "ramchoice" used in "R". However, I have modified the data generating process. First I specified the choice probabilities of the default to satisfy WDMP and gave an underlying preference order as $a(1) \succ b(2) \succ c(3) \succ \cdots$, making $a(1)$ the most preferred alternative with the others following the order. I only consider alternatives that are better than the default option. Thus, the alternative ranked at the last is always the default option. Then, I recovered the whole choice dataset and $m(D)$s following the corollary in Aguiar (2017). I matched the choice probabilities to each choice problem and generated sample data using multinomial distribution. Then I used the "rAtte" function in "ramchoice" to test if a preference order of interest represents the generated choice dataset.

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[7] See Appendix.

3.1 Simulation evidence in Masatlioglu et al. (2018).

First I show the exact replication of one simulation evidence presented in Masatlioglu et al. (2018). In Masatlioglu et al. (2018), it tests a model which follows the logit attention model:

$$\mu_{\varsigma}(T|S) = \frac{|T|^2}{\sum_{T' \subset S} |T'|^2}$$

Where $\omega_{T,\varsigma} = |T|^{\varsigma}$ and ς=2.

Five hypotheses of interest are as following.

Table 1 Logit attention model example, H1~H4.

H1	1	2	3	4	5
H2	2	3	4	5	1
H3	3	4	5	2	1
H4	4	5	3	2	1
H5	5	4	3	2	1

H1 is the true preference order and others are not. The order in H2 is marginally changed since only 1 is placed at the end of the order but the others maintain their true order. However, H5 is the totally inverse order of the true preference. The table below is the result of the test from sample size 10000 and 2000 Monte Carlo simulation.

Table 2 Logit attention model results.

	H1	H2	H3	H4	H5
N=10000	0.0000	0.0000	0.7650	0.9450	0.9655

The ERP is close to zero under H1 and H2 as expected. Meanwhile, H3, H4, H5 shows ERP close to 1, meaning that all three preference orders are mostly rejected in M simulations. Also, as the order becomes further from the true preference order, the ERP rises, meaning that the statistical power increases as the order changes more. The test seems to work well under the logit attention model. However, I found a special case where it fails.

3.2 Example 1 (Three alternatives and default)

The following table shows the choice dataset that I have generated to satisfy acyclicity and WDMP. The underlying true preference order is $a(1) \succ b(2) \succ c(3) \succ \mathrm{o}(4)$. Therefore, the choice dataset should have a RCG representation, which is a special case of RAM representation.

Table 3 Example 1, underlying preference $\boldsymbol{a \succ b \succ c \succ \mathrm{o}}$.

$\boldsymbol{P(\cdot\,\vert S)}$	$\{\boldsymbol{a,b,c}\}$	$\{\boldsymbol{a,b}\}$	$\{\boldsymbol{a,c}\}$	$\{\boldsymbol{b,c}\}$	$\{\boldsymbol{a}\}$	$\{\boldsymbol{b}\}$	$\{\boldsymbol{c}\}$	$\emptyset$
o(4)	1/16	2/16	3/16	3/16	5/16	5/16	6/16	1
$a(1)$	11/16	11/16	11/16	·	11/16	·	·	·
$b(2)$	3/16	3/16	·	11/16	·	11/16	·	·
$c(3)$	1/16	·	2/16	2/16	·	·	10/16	·

Table 4 Example 1, $\boldsymbol{m(D)}$.

$m(\{a,b,c\})$	$m(\{a,b\})$	$m(\{b,c\})$	$m(\{a,c\})$
7/16	1/16	1/16	1/16
$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\emptyset)$
2/16	2/16	1/16	1/16

Recall that in RAM, a is revealed to be preferred to b if and only if $\pi(a|S) > \pi(a|S-b)$. However, when following RCG, $\pi_{RCG}(a|S) = \sum_{\{a\}\cap D\neq\emptyset;B_A(a)\cap D=\emptyset:D\in\mathcal{D}} m(D)$ where $\mathrm{B}_A(a) = \{b \in S; u(b) > u(a)\}$. Therefore, if $a \succ b$, $\pi_{RCG}(a|S) = \pi_{RCG}(a|S-b)$ because the categories that satisfy $\{a\}\cap D \neq \emptyset$ and $B_A(a) \cap D = \emptyset$ do not change when an alternative that is less preferred is removed . In conclusion, there is no partial revealed preference relation according to RAM when we can only observe the choice dataset generated by the RCG model. I question whether the economic method presented in Masatlioglu et al. (2018) can correctly not reject the true preference order while successfully rejecting preference orders that are not the true one in this special case.

In the first row of table 5, 10,000 samples were generated based on the dataset presented in table 3 and 2,000 Monte Carlo simulation was executed to see the empirical rejection probability when the level of the test was predetermined to 0.05. Moreover, the data was generated via complete choice problem/menu which means that all choice problems $S \subset \{o, a, b, c\}$ are observed. The test was executed on all 24 preference order that 4 alternatives can have.

As shown below, the true preference H1: $a(1) \succ b(2) \succ c(3) \succ \mathrm{o}(4) \in \Theta_{\pi}$ has an empirical rejection probability close to zero as expected. The size of the test is relatively well controlled since the ERP is much lower than 0.05 under H1. There are 3 other preference orders that have ERP close to zero. This is because the test statistics $\mathbf{R}_{\succ}\pi$ contains zero under a false preference order which makes the test statistic be on the boundary of $\mathbf{R}_{\succ}\pi \leq 0$. According to Andrews and Soares (2010), there exist alternatives that have lower rejection probability than α as the test is not asymptotically similar on the boundary of the null hypothesis. They state that this is a common feature of test of multivariate one-sided hypothesis.

The totally inverse order H24: $\mathrm{o}(4) \succ c(3) \succ b(2) \succ a(1) \in \Theta_{\pi}$ is included in the set of hypotheses that have higher rejection probability than the predetermined level. There are 20 out of 24 preference orders showing ERP slightly above 0.05. Therefore, the test relatively performs well in a sense that it can narrow down the candidates for the true preference to 4 hypotheses, H1, H2, H4, H19.

However, the power of the test has trivial power, being only slightly above the preselected nominal level. Moreover, the power does not increase as the preference order moves further from the true preference order. H13, H14, H15, H17, H23 even have a higher rejection probability than H24. This problem is also a result of the test statistics being on the boundary of the null. This is exacerbated when one more alternative is added to the choice problem. I will show this in the next section.

I have tested if the sample size would be important to obtain the result in the first row of table 5 and also checked if the power of the test increases as the sample size increases. However, when the sample size is set to 2000, 4000, 6000 and 8000, the result does not deviate significantly and the power also does not seem to increase as the number of sample increases. However, when the sample size is changed to 2000, the power of H5 and H6 falls below the preselected nominal level 0.05, which makes it hard to interpret whether this is a result from the proper size control under the true underlying preference or a false order having trivial power.

Table 5 . Empirical rejection probabilities for example 1.

	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12
N=10000	0.0000	0.0000	0.0600	0.0000	0.0600	0.0600	0.0530	0.0530	0.0620	0.0620	0.0530	0.0620
N=8000	0.0000	0.0000	0.0605	0.0000	0.0605	0.0605	0.0535	0.0540	0.0600	0.0600	0.0540	0.0600
N=6000	0.0000	0.0000	0.0630	0.0000	0.0625	00625	0.0530	0.0530	0.0725	0.0725	0.0530	0.0725
N=4000	0.0000	0.0000	0.0650	0.0000	0.0640	0.0640	0.0505	0.0515	0.0610	0.0610	0.0515	0.0625
N=2000	0.0000	0.0000	0.0540	0.0000	0.0495	0.0495	0.0525	0.0575	0.0660	0.0600	0.0575	0.0620

	H13	H14	H15	H16	H17	H18	H19	H20	H21	H22	H23	H24
N=10000	0.0640	0.0640	0.0653	0.0635	0.0640	0.0635	0.0000	0.0600	0.0530	0.0620	0.0640	0.0635
N=8000	0.0735	0.0735	0.0705	0.0705	0.0735	0.0705	0.0000	0.0605	0.0540	0.0600	0.0735	0.0705
N=6000	0.0765	0.0765	0.0765	0.0765	0.0765	0.0765	0.0000	0.0630	0.0530	0.0725	0.0765	0.0770
N=4000	0.0685	0.0685	0.0655	0.0655	0.0685	0.0655	0.0000	0.0650	0.0515	0.0625	0.0685	0.0660
N=2000	0.0585	0.0590	0.0645	0.0645	0.0595	0.0645	0.0000	0.0540	0.0575	0.0620	0.0615	0.0655

N=10000, 8000, 6000, 4000, 2000 samples for each row respectively. 2000 Monte Carlo simulation, complete menu, 24 preference set.

Table 6 H1~H24.

H1	1	2	3	4
H2	1	2	4	3
H3	1	4	3	2
H4	1	4	2	3
H5	1	3	2	4
H6	1	3	4	2
H7	2	1	3	4
H8	2	1	4	3
H9	2	3	1	4
H10	2	3	4	1
H11	2	4	1	3
H12	2	4	3	1
H13	3	1	2	4
H14	3	1	4	2
H15	3	2	1	4
H16	3	2	4	1
H17	3	4	1	2
H18	3	4	2	1
H19	4	1	2	3
H20	4	1	3	2
H21	4	2	1	3
H22	4	2	3	1
H23	4	3	1	2
H24	4	3	2	1

Also, I tested if the econometric method performs well under limited data setting, not under complete dataset. Limited data states that the observable choice problem is not the full power set of $\mathcal{X}$. For example, only menu $\{a, b, c, o\}$, $\{a, b, o\}$, $\{b, c, o\}$, $\{a, c, o\}$ might be able to be observed in the choice problem. Then, to increase the statistical power, we need to slightly modify Assumption 1 (Mastalioglu et al. (2018)) and Definition 6 (Mastalioglu et al. (2018)) which are documented in the appendix of Masatlioglu et al. (2018). The econometric program provided by the authors, 'rAtte', has an input to consider this limited data setting. I have generated data where only menu size of 4 and 3 exist and then tested with the econometric method with limited data setting. As looking at table 7, the result is very similar to the complete data setting as the true underlying preference has ERP close to zero with 3 other alternatives having a similar ERP. Although the number of null hypothesis which has size below the nominal level has increased to 12, we can still narrow down the true preference with the econometric method, which are H1, H2, H4, H19.

Table 7 10000 sample, 2000 Monte Carlo, Limited data setting.

H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12
0.0000	0.0000	0.0490	0.0000	0.0485	0.0485	0.0440	0.0440	0.0530	0.0530	0.0440	0.0530
H13	H14	H15	H16	H17	H18	H19	H20	H21	H22	H23	H24
0.0760	0.0760	0.0670	0.0670	0.0760	0.0670	0.000	0.0490	0.0440	0.0530	0.0760	0.0670

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3.3 Example 2 (four alternatives and default)

I have generated a larger example with 4 alternatives and a default option based on the RCG model. Table 8 and 9 shows the result. The underlying true preference is $a(1) \succ b(2) \succ c(3) \succ d(4) \succ \mathrm{o}(5)$.

Table 8 Example 2, four alternatives and default.

$\mathrm{P}(\cdot\,\|S)$	$\{a,b,c,d\}$	$\{a,b,c\}$	$\{a,b,d\}$	$\{a,c,d\}$	$\{b,c,d\}$	$\{a,b\}$	$\{a,c\}$	$\{a,d\}$
o(5)	1/32	2/32	2/32	2/32	3/32	4/32	3/32	4/32
$a(1)$	26/32	26/32	26/32	26/32	·	26/32	26/32	26/32
b(2)	2/32	2/32	2/32	·	26/32	2/32	·	·
$c(3)$	2/32	2/32	·	3/32	2/32	·	3/32	·
$d(4)$	1/32	·	2/32	1/32	1/32	·	·	2/32
$\mathrm{P}(\cdot\,\|S)$	$\{b,c\}$	$\{b,d\}$	$\{c,d\}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{d\}$	$\emptyset$
0(4)	4/32	4/32	5/32	6/32	6/32	10/32	10/32	1
$a(1)$	·	·	·	26/32	·	·	·	·
$b(2)$	26/32	26/32	·	·	26/32	·	·	·
$c(3)$	2/32	·	22/32	·	·	22/32	·	·
$d(4)$	·	2/32	5/32	·	·	·	22/32	·

Table 9 Example 2, $\boldsymbol{m}(\boldsymbol{D})$.

$m(\{a,b,c,d\})$	$m(\{a,b,c\})$	$m(\{a,b,d\})$	$m(\{a,c,d\})$	$m(\{b,c,d\})$	$m(\{a,b\})$	$m(\{a,c\})$	$m(a,d)$
16/32	3/32	4/32	0	0	1/32	0	0
$m(\{b,c\})$	$m(\{b,d\})$	$m(\{c,d\})$	$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\{d\})$	$m(\emptyset)$
1/32	0	1/32	2/32	1/32	1/32	1/32	1/32

Table 10 shows the result of the test after generating 8000, 10000, 12000 samples from the dataset in each 2000 Monte Carlo simulation. The choice problem is complete. For simplicity, I have tested 5 preferences where H1: $a(1) \succ b(2) \succ c(3) \succ d(4) \succ \mathrm{o}(5) \in \Theta_\pi$ is the true preference while H5: $\mathrm{o}(5) \succ d(4) \succ c(3) \succ b(2) \succ a(1)$ is the totally inverse order of the true preference. As shown in table 10, the results show that all preference orders have a slightly higher empirical rejection probability than the preselected level of the test which is 0.05, which means that the size is relatively controlled well under H1 but have trivial power.

Overall, the test fails to give any information about the underlying true preference from the dataset generated by the RCG model in all three sample sizes. This problem can be explained by the fact that the false preference order makes $\max(\mathbf{R}_\succ \hat{\boldsymbol{\pi}})$ close to zero when the dataset follows the RCG model.

Table 10 Example 2 results.

	H1	H2	H3	H4	H5
N=8000	0.0625	0.0660	0.0725	0.0520	0.0690
	H1	H2	H3	H4	H5
N=10000	0.0700	0.0740	0.0785	0.0670	0.0720
	H1	H2	H3	H4	H5
N=12000	0.0755	0.0780	0.0825	0.0635	0.0705

N=8000, 10000, 12000 sample for each row respectively, 2000 Monte Carlo simulation, complete choice problem, 5 preferences.

Table 11 Example 2, H1~H5

H1	1	2	3	4	5
H2	1	2	3	5	4
H3	5	1	2	3	4
H4	2	3	4	1	5
H5	5	4	3	2	1

I have increased the simulation to obtain the critical value from 2000 to 5000, while the Monte Carlo for creating ERP is 2000 to see if the number of simulation was insufficient. However, the result of the test has not improved. The ERPs are all similar to each other. Therefore, the test seems not to perform well when the number of alternative increases.

Table 12 10000 sample, 2000 Monte Carlo simulation, complete choice problem, 5 preferences, 5000 bootstrap simulations for obtaining critical value.

H1	H2	H3	H4	H5
0.0675	0.0715	0.0755	0.0655	0.0725

Recall the construction of the $\mathbf{R}_{\succ}\boldsymbol{\pi}$ which became $\pi(a_{\ell,\succ}|S) - \pi(a_{\ell,\succ}|S - a_{\Bbbk}) \leq 0$, for all $a_{\Bbbk} \succ a_{\ell,\succ}$ in S using monotonic attention rule and triangular attention rule. In the RCG model, when the preference order is not true, $\pi(a_{\ell,\succ}|S) - \pi(a_{\ell,\succ}|S - a_{\Bbbk}) = 0$ for $a_{\ell,\succ} \succ a_{\Bbbk}$. This is the result of simply using the definition of $\pi(a_{\ell,\succ}|S) = \sum_{\{a_{\ell,\succ}\}\cap D\neq\emptyset; B_A(a_{\ell,\succ})\cap D=\emptyset: D\in\mathcal{D}} m(D)$. To specify, recall that in the previous section, if $a \succ b$, $\pi_{RCG}(a|S) = \pi_{RCG}(a|S - b)$, because the categories that satisfy $\{a\} \cap D \neq \emptyset$ and $B_A(a) \cap D = \emptyset$ do not change, the probability do not change. Thus, when following the order that is not true, there is at least one element in $\mathbf{R}_{\succ}\boldsymbol{\pi}$ that has the value $\pi(a_{\ell,\succ}|S) - \pi(a_{\ell,\succ}|S - a_{\Bbbk}) = 0$, for $a_{\ell,\succ} \succ a_{\Bbbk}$. Since the elements of $\mathbf{R}_{\succ}\boldsymbol{\pi}$ under the false preference order will contain values that are below or equal to zero, the maximum element will be zero, being $\max(\mathbf{R}_{\succ}\boldsymbol{\pi}) = 0$. If the order is totally inverse from the true order, then $\mathbf{R}_{\succ}\boldsymbol{\pi}=\mathbf{0}$, where $\mathbf{0}$ is the zero vector. Therefore, the estimated test statistics $\max(\mathbf{R}_{\succ}\widehat{\boldsymbol{\pi}})$ will be close to zero under false preference order.

Lemma 1. If the choice dataset is generated by following the RCG model, the maximum element of $\mathbf{R}_{\succ}\boldsymbol{\pi}$ will be zero under the false preference order and $\mathbf{R}_{\succ}\boldsymbol{\pi} = \mathbf{0}$ under the totally inverse order. Thus, under false preference orders, $\max(\mathbf{R}_{\succ}\widehat{\boldsymbol{\pi}}) \approx 0$.

If the parameter is on the boundary of the inequality constraint, then the estimator has a truncated distribution. In the case of $\mathbf{R}_{\succ}\pi = 0$ and $\mathbf{R}_{\succ}\sqrt{N}(\widehat{\boldsymbol{\pi}} - \boldsymbol{\pi}) \rightarrow N(0, \Omega_{\pi})$, our moment estimator $\sqrt{N}\mathbf{R}_{\succ}\widehat{\boldsymbol{\pi}}$ will have a "half truncated-normal" distribution[8], causing problems if using just a normal distribution for inference. According to Masatlioglu et al. (2018), the test statistics, $\mathcal{T}(\succ) = \sqrt{N} \cdot \max\{(\mathbf{R}_{\succ}\widehat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ}), 0\}$, is appropriate when only a few moment inequalities are violated. Therefore, the test statistics performed relatively well when there were only a few alternatives even though there exists a boundary issue. However, if the size of the alternatives is relatively large and $\max(\mathbf{R}_{\succ}\pi) = 0$, many inequalities can be violated and using a truncated normal distribution for inference will perform better than our test statistics.[9]

When the test statistics $\mathbf{R}_{\succ}\pi$ under false preference is not on the boundary of the null hypothesis but violates the inequality constraint, then the test statistics gives high power with appropriate size control. The logit attention model presented as simulation evidence in Masatlioglu et al. (2018) shows violation of the inequality

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[8] See Hansen. *"Econometrics"*, CHAPTER 7. RESTRICTED ESTIMATION, P. 181.

[9] See Masatlioglu et al. (2018), section 7.2.

constraint under false preference order. For a clearer example, see example 6 which is introduced in Masatlioglu et al. (2018). The example violates the regularity condition but satisfies the RAM model.

Example 3. (regularity violation presented in Masatlioglu et al. (2018))

Table 13 Example 3. μ.

$\mu(T\|S)$	$T=\{a,b,c\}$	$\{a,b\}$	$\{a,c\}$	$\{b,c\}$	$\{a\}$	$\{b\}$	$\{c\}$
$S=\{a,b,c\}$	2/3	0	0	1/6	0	0	1/6
$\{a,b\}$	·	1/2	·	·	0	1/2	·
$\{a,c\}$	·	·	1/2	·	0	·	1/2
$\{b,c\}$	·	·	·	1/2	·	0	1/2

Table 14 Example 3. $P(\cdot\,|S)$.

$\boldsymbol{P(\cdot\,\|S)}$	$\{\boldsymbol{a,b,c}\}$	$\{\boldsymbol{a,b}\}$	$\{\boldsymbol{a,c}\}$	$\{\boldsymbol{b,c}\}$
$a(1)$	2/3	1/2	1/2	·
$b(2)$	1/16	1/2	·	1/2
$c(3)$	1/16	·	1/2	1/2

When 10000 sample data is generated by the choice dataset as table 13, the test rejects the false preference orders with ERP close to 1 while the underlying true preference order, H1: $a(1) \succ b(2) \succ c(3) \in \Theta_\pi$ is not rejected with ERP close to 0. Therefore, when the test statistics becomes $\mathrm{R}_{\succ}\pi > 0$ under the false preference order and the extent of the violation is sufficiently large, the test works well, almost always rejecting the false preference order and almost always accepting the true order.

Table 15 Regularity violation case. 10000 sample, 2000 Monte Carlo simulations, complete choice problem, 4 preferences

H1	H2	H3	H4
0	1	1	1

Table 16 example 4. H1~H4.

H1	1	2	3
H2	2	1	3
H3	3	1	2
H4	3	2	1

Back to my example, I have also tested the econometric method under limited data settings and present an example when only choice problem of size 5, 4, 3 are observed. As shown in table 17, even in the limited data setting with slightly modified monotonicity assumption and compatibility definition, the test gives no information about the underlying true preference order.

Table 17 10000 samples, 2000 Monte Carlo simulation, Limited data setting, 5 preferences.

H1	H2	H3	H4	H5
0.0640	0.0670	0.0710	0.0550	0.0795

4. Suggesting an auxiliary method: one-sided hypothesis test.

So far, under the dataset generated with the RCG model, we have seen that it admits $\max(\mathbf{R}_{\succ}\boldsymbol{\pi}) = 0$ under the preference orders that differ with the underlying true preference order while the moment inequality will be satisfied in a large extent only when the preference order is identical with the true one. I suggest to use this information and do another test when this boundary problem is expected to arise under false preference orders.

By setting the null hypothesis to $\mathrm{H}_0\colon \max(\mathbf{R}_{\succ}\boldsymbol{\pi}) = 0$ and the alternative hypothesis $\mathrm{H}_1\colon \max(\mathbf{R}_{\succ}\boldsymbol{\pi}) < 0$, the inference method becomes a simple one-sided hypothesis test. Now the false preference order is at the interest to be tested and the test should fail to reject the null hypothesis under false preference order while it should reject the null under the true preference order.

I still use the same test statistic proposed in Andrews and Soares (2010), GMS, but slightly modify the method from Masatlioglu et al. (2018) since the null have changed. Originally, the test statistics was $\mathcal{T}(\succ) = \sqrt{N} \cdot \max\{(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ}),\ 0\}$ considering the fact that the test statistic should only have a positive value if the inequality constraint $\mathbf{R}_{\succ}\boldsymbol{\pi} \leq 0$ is violated. I modified the test statistics to $\mathcal{T}(\succ)' = \sqrt{N} \cdot \max\{(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ})\}$ as we are now interested if the value of $\max(\mathbf{R}_{\succ}\boldsymbol{\pi})$ is below zero. This is plausible since the null is now $\max(\mathbf{R}_{\succ}\boldsymbol{\pi}) = 0$ and the violation occurs if $\max(\mathbf{R}_{\succ}\boldsymbol{\pi}) < 0$ in our situation. Also, the approximate distribution of $\mathcal{T}(\succ)'$ is now $\mathcal{T}^{*\prime}(\succ) = \sqrt{N} \cdot \max\{(\boldsymbol{R}_{\succ}\boldsymbol{z}^{*} \oslash \hat{\sigma}_{\succ} + \psi_N(\boldsymbol{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ})\}$ where $\psi_N(\boldsymbol{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ})$ is the same as in Masatlioglu et al.(2018):

$$\psi_N(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ}), = \frac{1}{\sqrt{lnN}}(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ})_{-}\ ,$$

Where $(\mathbf{a})\_ = \mathbf{a} \odot 1(\mathbf{a} \leq 0)$, with $\odot$ denoting the Hadamard product. This part could be set to zero as the null hypothesis is $\mathbf{R}_{\succ}\boldsymbol{\pi} = 0$. However, this makes the critical value too high by ignoring the extent of how far $\max(\mathbf{R}_{\succ}\boldsymbol{\pi})$ is from zero.[10] In this case, only the totally inverse preference order has the ERP close to the preselected nominal level but all other preferences have much higher ERP. This occurs because the test statistic, $\max(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}})$, has little difference under the true and false preference orders in example 2 and 3, making them hard to distinguish. However, if the slackness of the inequality constraint is considered in $\mathcal{T}^{*\prime}(\succ)$, the test statistics can be constructed to have a smaller critical value while the asymptotic size of it is correct.(Andrews and Soares (2010)). Therefore, I keep the

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[10] See appendix for results.

term in Masatlioglu et al. (2018) to make more difference between the preferences that are far from the true one with ones that are close. The critical value is obtained from the simulated distribution as before but with modification with the direction of the inequality.

$$\mathrm{c}'_\alpha(\succ) = \inf\left\{t : \frac{1}{M}\sum_{m=1}^{M} 1(\mathcal{T}^*(\succ) \geq t) \geq 1-\alpha\right\}$$

The null hypothesis is rejected if $\mathcal{T}(\succ)' < \mathrm{c}'_\alpha(\succ)$.

$\mathcal{T}^{*\prime}(\succ)$ stochastically dominates $\mathcal{T}(\succ)'$ according to the proof of Theorem 5 in the appendix of Masatlioglu et al. (2018).[11] Therefore, $\mathbb{p}(\mathcal{T}^{*\prime}(\succ) \geq \mathrm{c}'_\alpha(\succ)) \geq \mathbb{p}(\mathcal{T}(\succ)' \geq \mathrm{c}'_\alpha(\succ))$ or equivalently, $\mathbb{p}(\mathcal{T}^{*\prime}(\succ) \leq \mathrm{c}'_\alpha(\succ)) \leq \mathbb{p}(\mathcal{T}(\succ)' \leq \mathrm{c}'_\alpha(\succ))$. Since we have found the smallest t that satisfies,

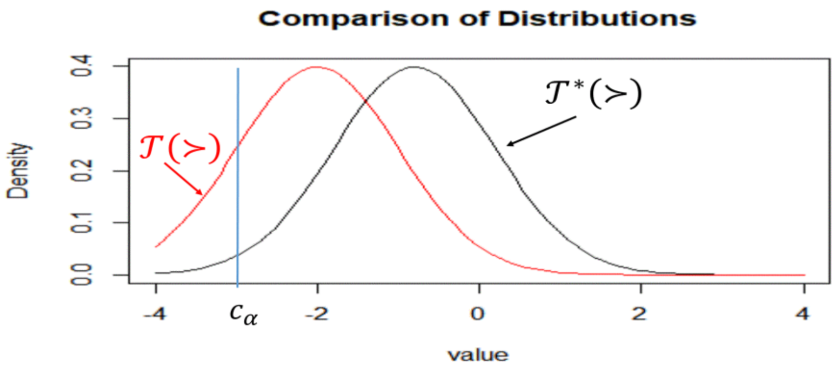
$$\frac{1}{M}\sum_{m=1}^{M} 1(\mathcal{T}^*(\succ) \geq t) \geq 1-\alpha,$$

as the critical value, $\mathbb{p}(\mathcal{T}^{*\prime}(\succ) \leq \mathrm{c}'_\alpha(\succ)) \approx \alpha$. Thus, under the underlying true preference order, we expect that $\mathbb{p}\left(\mathcal{T}^{*\prime}(\succ) \leq \mathrm{c}'_\alpha(\succ)\right) \approx \alpha \leq \mathbb{p}(\mathcal{T}(\succ)' \leq \mathrm{c}'_\alpha(\succ))$. This means that the ERP should be higher than the preselected nominal

[11] The proof presented in Appendix B of Masatlioglu et al. (2018) shows the result where $\mathcal{T}^*(\succ) = \sqrt{N}\cdot\max\{(\boldsymbol{R}_\succ \boldsymbol{z}^* \oslash \hat{\sigma}_\succ + \psi_N(\boldsymbol{R}_\succ \hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_\succ), 0\}$ stochastically dominates $\mathcal{T}(\succ) = \sqrt{N}\cdot\max\{(\mathbf{R}_\succ \hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_\succ),\ 0\}$. However, the result holds even when the positivity constraint is dropped. See appendix.

level under the true preference order. The following graph shows the intuitive reason why this works.

Figure 1 The reason why ERP is higher than the preselected level under the true preference order.



On the other hand, the distribution of $\mathcal{T}(\succ)'$ and $\mathcal{T}^{*\prime}(\succ)$ will be close to each other under false preference orders since we expect a boundary issue, meaning that the ERP should be a similar value with the preselected nominal level.

I conducted a simulation to see the performance of the suggestion on the same dataset presented in example 2. I generated 10000 samples and conducted 2000 Monte Carlo simulation.

Table 18 One-sided hypothesis test result for example 2, sample size 10000, 2000 Monte Carlo simulation.

	H1	H2	H3	H4	H5
original	0.0700	0.0740	0.0785	0.0670	0.0720
One-sided test	0.1100	0.0920	0.0445	0.0520	0.0240

As shown in the table, the size of the test is relatively controlled well as the ERP of H3 and H5 is below the preselected level, 0.05. Note that H5 is the order that is the complete inverse of the true underlying preference order enumerated as H1. The ERP under H5 is the smallest, meaning that the test fails to reject the null hypothesis the least among the tested preference orders as expected. Also, recall that H1 was the true preference order and H2 was a preference order that was only slightly different with H1. In both cases, the ERP is higher than the level. Therefore, we can narrow down the compatible preference orders to H1 and H2 since they have high ERPs than the preselected level. H3 also have higher ERP but the amount is significantly smaller than H1 and H2.

Compared to the original econometric method presented in Masatlioglu et al. (2018), my suggestion makes a distinction of what can be the true underlying preference order while the original could not. Therefore, the suggested one-sided hypothesis test can help to make inference about the preferences that are compatible with the choice dataset.

I have conducted the same test with the dataset presented in example 1. The following table shows the result of sample size 10000 and 2000 Monte Carlo simulation. The table shows that 20 out of 24 preference orders show ERP less than 0.05 which coincide with the 20 preference that originally had higher ERP than 0.05, suggesting these preferences might be the false preference orders. We can also see that the size of the test is well controlled since 20 preference orders that originally had higher ERP than 0.05 now have smaller ERP than 0.05.

Moreover, the true preference H1 shows ERP almost close to 1, meaning that the power of the test is high. Compared with the original econometric method presented in Masatlioglu et al. (2018), there is no additional information about the underlying true preference order by using the suggested one sided test since H1, H2, H4 and H19 have ERP close to one which is the same result with the original test. However, the power of the test has increased significantly. We could not affirmatively claim that 20 out of 24 preferences orders are not compatible with the choice dataset with the original test statistics since the ERP was similar to the nominal level. However, with my suggestion, we can affirmatively claim that H1, H2, H4 and H19 are the candidates as they show high ERP.

Table 19 One-sided hypothesis test result for example 1, sample size 10000, 2000 Monte Carlo simulation.

	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12
N=10000	0.0000	0.0000	0.0600	0.0000	0.0600	0.0600	0.0530	0.0530	0.0620	0.0620	0.0530	0.0620
One-side	1.0000	1.0000	0.0145	1.0000	0.0365	0.0365	0.0330	0.0300	0.0200	0.0200	0.0300	0.0190
	H13	H14	H15	H16	H17	H18	H19	H20	H21	H22	H23	H24
N=10000	0.0640	0.0640	0.0653	0.0635	0.0640	0.0635	0.0000	0.0600	0.0530	0.0620	0.0640	0.0635
One-sided	0.0315	0.0315	0.0270	0.0270	0.0315	0.0270	1.0000	0.0145	0.0295	0.0190	0.0290	0.0265

The one-sided test I suggest is not a solution to the boundary issue or an alternative method for the one presented in Masatlioglu et al. (2018) but simply a method to circumvent the anticipated problem when we expect that the boundary issue may arise in a special case. When the result is given as table 5 and 10, we can observe that the power is trivial across all the preference orders and suspect that there is a boundary issue. If we have pre-knowledge or expectation that the dataset is generated by following the RCG model (Aguiar (2017)) or the MM model (Manzini and Marioti (2014)), then the boundary issue is anticipated and the suggested one-sided test might be preferable. Therefore, I suggest to do a one-sided test after testing the preference orders with the econometric method provided in Masatlioglu et al. (2018)

if the results show trivial power across all preference orders.

5. Conclusion

I have tested the econometric method presented in Masatlioglu et al. (2018) with simulation data generated via RCG rule introduced in Aguiar (2017). The RCG rule is a special case of RAM model. When the data is generated through the RCG rule, no preference relation is revealed through the lemma presented in RAM while the RCG model can recover the underlying true preference order and the unobservable probability m over categories if the dataset satisfies WDMP and acyclicity.

The simulation evidence suggest that the econometric method performs relatively well when there is only small size of alternatives. If there exist three alternatives and an additional default option, the true preference gives empirical rejection probability close to zero as expected when the sample size is 10000 with 2000 Monte Carlo simulation under complete choice data setting. There are only 3 other hypotheses that also give ERP close to zero while 20 out of 24 preference orders give ERP slightly over the preselected nominal level, 0.05. Therefore, the method can narrow down the candidates for the true underlying preference. However, the power of the test is trivial since the ERPs for the false preference orders are only marginally higher than the nominal level of the test. Also, the power does not increase as the preference order moves further from the true order nor as the size of the sample increases. One problem is that the RCG model makes $\max(\mathbf{R}_{\succ}\boldsymbol{\pi})$ be on the boundary of the moment inequality $\mathbf{R}_{\succ}\boldsymbol{\pi} \leq 0$, which makes the asymptotic size of the test non-similar and the test have trivial power.

The problem is exacerbated when the size of the alternatives is large. When one more alternative is added and there exist all 4 alternatives with an additional default option, the moment inequality test gives no information about the true underlying preference. The simulation test fails under each tasks that I took which were increasing the sample size to 12000, increasing the Monte Carlo simulations for estimating ERP to 2000, increasing the Monte Carlo simulations for obtaining the critical value to 5000.

I suggest a one-sided hypothesis test to circumvent the boundary issue when it is expected that $\max(\mathbf{R}_{\succ}\boldsymbol{\pi}) = 0$ under false preference orders. Set H_0: $\max(\mathbf{R}_{\succ}\boldsymbol{\pi}) = 0$ and the alternative hypothesis $\mathrm{H}_1: \max(\mathbf{R}_{\succ}\boldsymbol{\pi}) < 0$. Modify the test statistics and the simulated test statistics to $\mathcal{T}'(\succ), \mathcal{T}^{*\prime}(\succ)$ respectively. Find the critical value that satisfies

$$\mathrm{c}_{\alpha}'(\succ) = \inf\left\{t : \frac{1}{M}\sum_{m=1}^{M} 1(\mathcal{T}^{*}(\succ) \geq t) \geq 1 - \alpha\right\}$$

And reject the null hypothesis if $\mathcal{T}(\succ)' \leq \mathrm{c}_{\alpha}'(\succ)$. The size of the test is well controlled and the ERP should be higher than the preselected level under the true preference order since $\mathcal{T}^{*\prime}(\succ)$ stochastically dominates $\mathcal{T}'(\succ)$. The simulation evidence shows that the suggested methods helps to narrow down the candidate for the underlying true preference order with non-trivial power and proper size control.

In conclusion, the econometric model presented in Masatlioglu et al. (2018) has limitation if the data set is generated by the special case of RAM, which is the RCG rule. The problem emerges because the population constraint matrix $\mathbf{R}_{\succ}\boldsymbol{\pi}$ contains values that are zero or below zero under the false preference order if the data is generated by the RCG rule, making the test statistic $\max(\mathbf{R}_{\succ}\boldsymbol{\pi})$ exist on the boundary of the moment inequality. When this problem is expected to occur, I suggest a one-sided hypothesis test to help. This one-sided test is not a solution to the boundary issues of the moment inequality tests but a helpful method when we suspect the specific problem in the paper to happen.

6. Related literature

My paper is closely related to Masatlioglu Y., Xinew Ma, Cattaneo M. D. and Elchin Suleymanov (2018) and Aguiar (2017) as I have tested the econometric method presented in the Masatlioglu et al. (2018) by using the model introduced in Aguiar (2017). The model presented in Aguiar (2017) is a special case of the model presented in Masatlioglu et al. (2018).

The paper is also related to Manzini and Mariotti (2014) since the model presented in MM (2014) is a special case of both Aguiar (2017) and Mastalioglu (2018). The Manzini and Mariotti (2014) can be viewed as a paper that linked limited attention to stochastic choice. The other two paper are generalizing the result of MM (2014) in a different way.

Andrews and Soares (2010) is the base of developing and interpreting the econometric method in RAM. RAM uses one of the proposed test statistics in Andrews and Soares (2010) which is GMS. The GMS test has an advantage that the power of the test is better than other tests presented in Andrews and Soares (2010) such as PA or LF tests.

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Appendix.

A. 1. Triangular Attention Rule and Definition 6 in Masatlioglu et al. (2018)

Definition 5 (Triangular Attention Rule in Masatlioglu et al. (2018)). An attention rule μ is triangular with respect to $\succ$ if for any $S \in \mathcal{X}$ and $T \subset S,\ \mu(T|S) > 0$ only if $T = S \cap L_{k,\succ}$ for some $k \in \{1, \dots,\ K\}$ where $L_{k,\succ}$ is the lower counter set of $a_{k,\succ}$.

When the alternatives in the grand set X is reordered such that $a_{1,\succ} \succ \cdots \succ a_{k,\succ} \succ \cdots\ \ a_{K,\succ},\ L_{k,\succ}$ is a set such that $\{a_{k,\succ} \succ \cdots\ \ a_{K,\succ}\}$.

Definition 6 in Masatlioglu et al. (2018) states that preference $\succ$ is compatible with $\pi,$ denoted by $\succ \in \Theta_{\pi}$, if there exists some monotonic attention rule μ such that $(\pi, \succ, \mu)$ is a RAM.

A. 2. The construction of $\mathbf{R}_{\succ}$.

$\mathbf{R}$ is a matrix that contains constraints on the attention rules following the monotonic attention rule. Each element is the form of $\mu(T|S) - \mu(T|S - a) \leq 0$ for all $a \in S - T$ based on the preference order. Using the definition of triangular attention rule, $\mathbf{C}_{\succ}$ transforms the choice rule π back to a unique triangular attention rule.

A. 3. Showing that $\boldsymbol{\mu(T|S) = \sum_{D\cap S=T:D\in\mathcal{D}} m(D)}$ is monotonic.

If the choice dataset has the Random Categorization rule representation such that there exists a triple $u, m, \mathcal{D}$ satisfying $P_{RCG}(a, A) = \sum_{\{a\}\cap D\neq\emptyset; B_A(a)\cap D=\emptyset:D\in\mathcal{D}} m(D)$, then u can be expressed with $m(D)$ where $D \in \mathcal{D}$. Then u satisfies the monotonic attention assumption.

Proof) The part that u can be expressed with $m(D)$ is shown above.

Suppose the choice dataset has the RCG representation. Then $\mu(T|S) = \sum_{D\cap S=T:D\in\mathcal{D}} m(D)$. Let every $D \in \mathcal{D}$ satisfying $D \cap S = T$ be denoted as $D_1, D_2, \dots, D_n$ where $n \in \mathbb{N}$. Let $a \in S - T$. Since $a \in S - T$, $D_j \cap S - a = T$ for all $1 \leq j \leq n$. Therefore, since $\mu(T|S - a) = \sum_{D\cap S-a=T:D\in\mathcal{D}} m(D)$, $\mu(T|S) \leq \mu(T|S - a)$

A. 4. RAM$\not\subset$RCG

A stochastic choice dataset that satisfies RAM is not included in the models that can have a RCG representation since regularity on the default option is not a sufficient condition for WDMP.

Counter example 1.

Table 20 counter example, satisfy regularity but not WDMP.

$P(o,\{a,b,c\})$	$P(o,\{a,b\})$	$P(o,\{a,c\})$	$P(o,\{b,c\})$	$P(o,\{a\})$	$P(o,\{b\})$	$P(o,\{c\})$	$P(o,\emptyset)$
$1/16$	$4/16$	$3/16$	$2/16$	$5/16$	$4/16$	$4/16$	1

The example satisfies regularity on the default option but not WDMP since

$$[\![p(o,\{a\}) - p(o,\{a,b\})]\!] - [\![p(o,\{a,c\}) - p(o,\{a,b,c\})]\!] < 0$$

The following example shows that there is an RAM representation but no RCG representation for a given dataset.

Table 21 Example (from the RAM literature example 2).

$\mathbf{P}(\cdot\,\|\mathbf{S})$	$\boldsymbol{S = \{a,b,c\}}$	$\boldsymbol{\{a,b\}}$	$\boldsymbol{\{a,c\}}$	$\boldsymbol{\{b,c\}}$
a	$1/2$	1	0	$\cdot$
b	$1/2$	0	$\cdot$	1
c	0	$\cdot$	1	$\cdot$

Then by Theorem 1(RAM) and Lemma 1(RAM), the revealed preference is $a \succ b \succ c$ and the probability mapping of the consideration set, μ, as listed below can generate the given dataset.

Table 22 Attention probability from the RAM paper example 2.

μ	$\{a,b,c\}$	$\{a,b\}$	$\{a,c\}$	$\{b,c\}$
$\{a,b,c\}$	$1/2$	$\cdot$	$\cdot$	$\cdot$
$\{a,b\}$	0	1	$\cdot$	$\cdot$
$\{a,c\}$	0	$\cdot$	0	$\cdot$

$\{b,c\}$	$1/2$	$\cdot$	$\cdot$	1
$\{a\}$	0	0	0	$\cdot$
$\{b\}$	0	0	$\cdot$	0
$\{c\}$	0	$\cdot$	1	0
$\{\emptyset\}$	0	0	0	0

If there exists a set of categories that satisfy the upper example, since $\mu(\{b,c\}|\{a,b,c\}) = 1/2$, there exists D_1 such that $D_1 \cap \{a,b,c\} = \{b,c\}$ and $m(D_1) > 0$. Now D_1 also satisfies $D_1 \cap \{a,b\} = \{b\}$. However, $\mu(\{b\}|\{a,b\}) = 0$, which is a contradiction.

A. 5. WDMP

Definition 3(Successive Difference of the Probability p in Aguiar(2017)). For the probability $p(a,A)$ for any (fixed) $a \in A \cup \{o\}$ and $A \in \mathcal{M}$, for all integers $n \geq 1$, and any given arbitrary sequence of menus $\{A_i\}_{i=1}^{\mathrm{n}}$ with $A_i \in \mathcal{M}$; define recursively its successive difference: $\Delta_{\mathrm{A_n}} \dots \Delta_{\mathrm{A_1}} p(a,A) = \Delta_{\mathrm{A_{n-1}}} \dots \Delta_{\mathrm{A_1}} p(a,A) - \Delta_{\mathrm{A_{n-1}}} \dots \Delta_{\mathrm{A_1}} p(a, A \cup A_n)$ for $n \geq 2$.

Condition 2(Weakly Decreasing Marginal Propensity of Choice in Aguiar (2017)). For all $A \in \mathcal{M}$, for all integers $n \geq 1$, and any given arbitrary sequence of menus $\{A_i\}_{i=1}^{n} \in \mathcal{M}^n$ the successive difference for the outside option o and any menu $A \in \mathcal{M}$ are non-negative $\triangle_{A_n} \dots \triangle_{A_1} p(o,A) \geq 0$.

It is similar with the regularity condition for the default option but also adds a condition that states the amount of difference caused by regularity should not increase as the menu gets larger.

For $n = 1{,}2$ the condition is as following.

$\Delta_{A_1} p(a, A) = p(a, A) - p(a, A \cup A_1) \geq 0$ for n=1 and,

$\Delta_{A_2}\Delta_{A_1} p(a, A) = \Delta_{A_1} p(a, A)$ - $\Delta_{A_1} p(a, A \cup A_2) = p(a, A) - p(a, A \cup A_1)$ - $[p(a, A \cup A_2)$-$p(a, A \cup A_2 \cup A_1)] \geq 0$ for n=2.

A. 6. Attention probability in example 1.

Table 23 Attention probability in example 1.

	$S = \{a, b, c, \mathrm{o}\}$	$\{a, b, \mathrm{o}\}$	$\{a, c, \mathrm{o}\}$	$\{b, c, \mathrm{o}\}$
$T = \{a, b, c, \mathrm{o}\}$	7/16	·	·	·
$\{a, b, \mathrm{o}\}$	1/16	8/16	·	·
$\{a, c, \mathrm{o}\}$	1/16	·	8/16	·
$\{b, c, \mathrm{o}\}$	1/16	·	·	8/16
$\{a, \mathrm{o}\}$	2/16	3/16	3/16	·
$\{b, \mathrm{o}\}$	2/16	3/16	·	3/16
$\{c, \mathrm{o}\}$	1/16	·	2/16	2/16
$\{\mathrm{o}\}$	1/16	2/16	3/16	3/16

A. 7. when $\boldsymbol{\psi_N(\mathrm{R}_{>}\hat{\pi} \oslash \hat{\sigma}_{>}) = 0}$.

I show the result when $\psi_N(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ})$ is set to zero for example 1 and example 2.

Table 24 ERP for example 1 when $\psi_N(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ}) = 0$. Sample size 10000, 2000 Monte Carlo simulation.

	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12
N=10000	1.0000	1.0000	0.5765	1.0000	0.6810	0.6455	0.6935	0.6625	0.4830	0.4290	0.5930	0.3505
	H13	H14	H15	H16	H17	H18	H19	H20	H21	H22	H23	H24
N=10000	0.4480	0.3940	0.3255	0.2730	0.2950	0.1864	1.0000	0.3860	0.3860	0.1435	0.1110	0.0480

Table 25 ERP for example 2 when $\psi_N(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ}) = 0$. Sample size 10000, 2000 Monte Carlo simulation.

	H1	H2	H3	H4	H5
N=10000	0.6520	0.6285	0.4175	0.4210	0.0445

We can see that only the totally inverse order has ERP less than the preselected nominal level, 0.05. This is because when we set $\psi_N(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \hat{\sigma}_{\succ})$ to zero, we do not consider the slackness when creating a simulated distribution for $\mathcal{T}(\succ)$, making the critical value too high to give any distinguishable results.

A. 8. $\mathcal{T}^{*\prime}(\succ)$ stochastically dominates $\mathcal{T}'(\succ)$.

The proof in Appendix B in Masatlioglu et al. (2018) holds even if the positivity constraint is dropped. Proposition A.2 in B.3 in Masatlioglu et al. (2018) holds since the coupling argument in proposition A.1 does not require positivity:

$$\mathbb{E}\left[\left|\sqrt{N}(\bar{\pi}-\pi)-\sqrt{N}\tilde{\mathrm{z}}_{\pi}\right|\right] \leq \text{Const.} \sqrt{\frac{1}{\mathrm{Min}_{s\in\mathcal{X}} N_s}}$$

Thus, the last line can be extended to:

$$\leq \left|\mathbb{E}_{\mathrm{f}}\left[\sqrt{N}\mathrm{Max}(\mathbf{R}_{\succ}\hat{\boldsymbol{\pi}} \oslash \sigma_{\pi,\succ})\right] - \mathbb{E}_{\mathrm{f}}\left[\sqrt{N}\mathrm{Max}(\mathbf{R}_{\succ}(\tilde{\mathbf{z}}_{\boldsymbol{\pi}}+\boldsymbol{\pi}) \oslash \sigma_{\pi,\succ})\right]\right| \leq \sqrt{\frac{1}{\mathrm{Min}_{s\in\mathcal{X}} N_s}}$$

Also, Proposition A.1 uses the fact that $|\sqrt{N}(\hat{\pi}-\pi)-\sqrt{N}\tilde{\mathrm{z}}_{\boldsymbol{\pi}}| = O_{\mathbb{P}}(1/\sqrt{N})$ by the central limit theorem in each step, which is not dependent on positivity. Therefore, Proposition A.1 holds even when the positivity constraint is dropped.

In conclusion, the proof can be done similarly as in Appendix B.3 in Masatlioglu et al. (2018) to show that $\mathcal{T}^{*\prime}(\succ)$ stochastically dominates $\mathcal{T}'(\succ)$.

A. 9. R codes.

I upload all the codes in my personal blog:

https://blog.naver.com/alexkmc/221327446707

초록

본 연구는 Masatlioglu et al. (2018)에서 제시한 계량 검정법의 검정력을 시뮬레이션을 통해 점검하고자 한다. 선택에 관한 데이터 집합은 Aguiar (2017)에서 제시한 RCG 모델을 따라 형성했으며 이 모델은 Masatiloglu et al. (2018)에서 제시한 RAM 모델의 특수한 사례이다. RCG 모델을 따르는 관측 가능한 선택 데이터에서는 RAM 모델을 통해 어떠한 현시 선호 관계도 밝힐 수 없다. 본 연구는 이러한 상황에서도 이 검정법이 선택의 기저에 있는 실제 선호 관계는 데이터와 부합한 것으로 보여주는 한편 실제와 다른 선호 관계는 기각하는지 점검한다. 연구 결과 RAM 논문에서 제시한 검정법은 이 특수한 사례에서 낮은 검정력을 지닌 것으로 밝혀졌다. 이는 RCG 모델을 따르는 경우 통계량이 적률 부등식의 경계에 존재하기 때문이다. 하지만 적은 대안이 제시된 경우에서는 이 검정법을 통해 실제 선호 관계가 무엇인지 구분할 수 있어 상대적으로 잘 작동한다. 반면에 대안의 수가 큰 경우 이 검정법은 실제 선호 관계가 무엇인지에 대한 정보를 주지 못하게 된다. 이러한 경계 문제가 발생할 것이 예측 되는 경우, 연구자는 단측 검정을 문제의 해결책으로 제안하며 시뮬레이션 증거를 제시한다. 시뮬레이션 증거에 따르면 이 특수한 사례 안에서 단측 검정이 더 높은 검정력을 주며 기존의 검정법이 주지 못한 실제의 선호 관계에 대한 정보를 제공한다.

주요어: 결정 이론, 현시 선호, 부주의, 제한된 이성, 비모수적 검정, 적률 부등식.

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