교육학석사학위논문

A Case Study on Design and Application of Creative Tasks in School Mathematics

: Focused on Percentage Bars and Pie Charts

학교 수학에서의 창의적 과제의 설계와 적용

- 비율그래프를 중심으로 -

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송 밝 음

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위 원 장 김 서 령 (인)

부위원장 유 연 주 (인)

위 원 이 경 화 (인)

ABSTRACT

A Case Study on Design and Application of Creative Tasks in School Mathematics

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Balgeum Song

The Graduate School of Mathematics Education

Seoul National University

Mathematical creativity plays crucial role in mathematical discovery, invention, understanding and construction of meaning. In school mathematics, students' conceptual understanding and mathematical thinking develops when they learn contents by using their creativity. In this sense, the creative tasks have to be implemented with appropriate instructional strategies in regular classes, which foster students' mathematical creativity, support conceptual learning of content knowledge of curriculum and elicit the development of mathematical thinking.

This study investigated whether the instruction of percentage bars and pie charts with the creative tasks and instructional strategies support the development of students' conceptual understanding and statistical thinking, and examined the role of creativity played in enhancement of conceptual understanding and statistical thinking. To this end, the literature on mathematical creativity, percentage bars, pie charts and statistical graph were analyzed, and then drew the strategies for creative task design and creative instructional strategies. Based on these strategies, the creative tasks were designed and implemented with the instructional strategies in regular classes. The responses of the students during the teaching sessions were collected and analyzed according to the phases of learning percentage bars and pie charts.

The findings showed that creative tasks and instructional strategies supported students' conceptual understanding and their construction of global view from local view related to percentage bars and pie charts, by fostering students' fluency, flexibility, originality and elaboration. Furthermore, the analysis of the results indicated that the creative tasks provide various learning opportunities for students who differ in their levels. This implies the possibility of teaching mathematical content of curriculum and stimulating students' mathematical creativity in school mathematics through creative tasks and instructional strategies.

Keyword : Mathematical creativity, creative task, percentage bar, pie chart, local view, global view

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CHAPTER 1

INTRODUCTION

1. Purpose of Research

Great attention has been shown to the creative education in school mathematics. The creative education in school mathematics implies that students' creativity should be cultivated in their regular classes. This means that students' creative thinking should be fostered when they are learning the mathematical content knowledge of the curriculum. In this regard, a number of researchers have reported that creative tasks and creative instructional strategies effectively support conceptual learning and mathematical thinking (Lee, 2015; 2016; Luria, Sriraman, Kaufman, 2017; Mann, 2006; Nadjafikhah, Yaftian, & Bakhshalizadeh, 2012). However, still many teachers have difficulties in teaching curriculum content and fostering students' creativity in the regular classes (Baer & Garrett, 2010). These difficulties come from the lack of clear and concrete guidelines related to design of tasks and lessons which support learning mathematical content knowledge along with students' creativity. While there has been a great deal of research on mathematical creativity, there are a few empirical studies about the cultivation of mathematical creativity which enhance mathematical learning in school mathematics (Nadjafikhah et al., 2012). In this overall perspective, there arises the need for empirical studies about creative tasks and instructional strategies which support mathematical learning in school mathematics.

Among diverse statistical graphs, percentage bars and pie charts are the most commonly used in daily life and in professional work (Goldstone, 1982; Hunt, & Mashhoudy, 2008; Reys, Lindquist, Lambdin, & Smith, 2012; Spence, 2005). Although percentage bars and pie charts have high utilization, misleading graphs are often used

(Hunt, & Mashhoudy, 2008; Watson, 2011). In this sense, the ability to spot a distorted graph is required, however, students show difficulties in construction and interpretation of percentage bars and pie charts (Friel, Curcio & Bright, 2001; Hunt, & Mashhoudy, 2008; Li, & Shen, 1992; Watson, 2011). With this mind, there is a need for investigation on the teaching and learning method in percentage bars and pie charts.

In this study, the literature on mathematical creativity, percentage bars, pie charts and statistical graph will be reviewed in order to draw strategies for creative task design and creative instructional strategies with the purpose of enhancement of learning percentage bars and pie charts. Furthermore, the creative tasks for teaching and learning percentage bars and pie charts will be developed and implemented in teaching sessions. The results of the teaching sessions will be analyzed focused on students' development of conceptual understanding and statistical thinking and on the mathematical creativity. This research aims to obtain implications for teaching and learning mathematical content knowledge through creative tasks and creative instructions in regular classes.

2. Research Questions

As stated above, this study will investigate whether the creative tasks and instructional strategies support the development of students' conceptual understanding and statistical thinking in teaching and learning percentage bars and pie charts. Additionally, this research will examine the role of creativity played in enhancement of conceptual understanding and statistical thinking. This research seeks to address the following research questions:

2.1. How does students' conceptual understanding develop during the instruction of percentage bars and pie charts in which the creative tasks and instructional strategies implemented?

2.1.1. How does students' understanding about the concept of percentages develop?

2.1.2. How does students' understanding about the relationship between percentage and percentage bars develop?

2.1.3. How does students' understanding about the relationship between percentage and pie charts develop?

2.1.4. How do students integrate the data and the contextual knowledge in interpretation of percentage bars and pie charts?

2.2. How does students' statistical thinking develop during the instruction of percentage bars and pie charts in which the creative tasks and instructional strategies implemented?

2.2.1. How does students' view of the data, percentage bars and pie charts develop from local view to global view?

2.3. How does mathematical creativity affect students' learning of percentage bars and pie charts during the instruction in which the creative tasks and instructional strategies implemented?

2.3.1. What are the aspects and the features of creative tasks which contribute to the development of conceptual learning and statistical thinking related to percentage bars and pie charts?

2.3.2. What are the aspects and the features of instructional strategies which contribute to the development of conceptual learning and statistical thinking related to percentage bars and pie charts?

The aim of this study is to shine new light on the debates through the examination of the role of mathematical creativity which contributed to the development of conceptual understanding and statistical thinking in the instruction of percentage bars and pie charts in which creative tasks and instructional strategies implemented. Especially, this research provides an opportunity to investigate the learning aspects of average students in regular classroom in which their mathematical creativity is stimulated. Based on the investigation, this study aims to identify the implication of the design creative tasks and instructional strategies for enhancement of conceptual understanding and statistical thinking related to percentage bars and pie charts.

CHAPTER 2

LITERATURE REVIEW

1. Literature on Percentage Bars and Pie Charts

1.1. Conceptual Understanding of Percentage bars and Pie charts

A percentage bar and a pie chart are a straight bar and a circle divided into sections respectively. These graphs show how the parts comprise the whole and relate to each other. Thus, they are useful to represent relative size of data (Reys et al., 2012; Spence, 2005). The visualization of proportional relationship of each part to another or to the whole is the core of percentage bars and pie charts.

Learning percentage bars and pie charts require students to use, understand and combine various mathematical concepts, such as fractions, ratios, proportions and percentages. Especially, percentages are used in percentage bars and pie charts to denote the relative value of each data. A percentage is a value of a part compared to a whole of 100. In the same data set, the percentage of each data has to be calculated to the same whole amount, and the total sum of these percentages has to be 100 (Reys et al., 2012). Hunt and Mashhoudy (2008) found that when students were given a data set, they created incorrect pie charts in which the total sum of the percentages was greater or lower than 100. Li and Shen (1992) reported that students often calculated the percentage of each part considering different whole amount. These errors are due to a lack of understanding of the meaning and the features of percentage. Therefore, students' conceptual understanding of percentage is essential for learning percentage bars and pie charts.

Percentage is a ratio of two numbers: the part amount and the whole amount. Therefore, ratio is also related to learning percentage bars and pie charts. As a ratio is a relationship between two amounts, the ratio differs depending on two parts under consideration even in the same data set. For example, considering a data set in which the percentage of the category A is 20%, B is 40% and C is 40%, the ratio of B to A is 2:1, but the ratio of B to C is 1:1. In like fashion, teachers should guide carefully if students try to construct percentage models and to interpret percentage bars and pie charts using ratio (Reys et al., 2012).

Learning percentage bars and pie charts also requires understanding of knowledge of geometry since they involve lengths of rectangles, central angles of sectors, areas of rectangles and of sectors. A percentage bar is comprised of the rectangles with the same width. This bar represents the whole amount of the data set and each rectangle corresponds to each part amount of the data set. Thus, the length and the area of rectangles should be proportional to the percentages. In case of pie chart, since each sector of a circle shows each portion amount and the circle displays the whole amount, the area and the central angle of sectors should be proportional to the respective frequencies (Li & Shen, 1992; Hunt, & Mashhoudy, 2008; Reys et al., 2012).

These features allow different ways to construct a percentage bar and a pie chart. The whole size of a bar or a circle can be defined and drawn at first. Then the parts can be depicted by calculating the lengths of the rectangle or the central angles of the sectors proportional to the respective percentages. Another method is that after defining the whole size, the bar or the circle can be divided into equal size pieces which can be used as unit. In case of percentage bars, the size of small rectangle which serves as unit can be defined first. Then the parts can be depicted proportional to their percentage by using this piece. Students, however, often overlook conceptual features of percentage bars and pie charts, and create the one in which each part is not corresponding with the percentage (Li, & Shen, 1992). These errors occur when students just follow the construction procedures and create meaningless graphs without reflection on the mathematical concepts embedded in the graphs. Thus, it is important to support students to construct meaningful graphs. One way to help students create personally meaningful graphs is giving opportunities to construct graphs in their own way and to refine them. This kind of activity can help not only develop graphing skills but also understand the meaning and the necessity of graph components (Lee & Ji, 2008; Watson, 2011)

Besides these syntactic meanings, semantic understanding is needed as well (Friel, Curcio, & Bright, 2001). Since "data are just not numbers, they are numbers with a context" (Cobb & Moore, 1997, p. 801), the context situation of the graph must be considered in graph comprehension. Statistical thinking involves this integration of mathematical, statistical and context knowledge. However, the work of Li and Shen (1992) and of Watson (2011) showed that students interpreted graphs and data without taking account of context at all, or focusing on the context without consideration on data. It happens because, in general, mathematical classes lead students to be independent from context and to focus on the mathematical structure inherent in the task (del Mas, 2004). Accordingly, it is necessary to deliberate on which kind of tasks and teaching strategies can support students to interpret graphs connecting the data and the context. An answer to that can be realistic situations. Curcio (1987) argued that the familiar context assists graph interpretation by recalling relevant information in mind. Therefore, the instructional tasks whose context is related to students' everyday life would help students to integrate the data and the context when interpreting data and graphs.

If instructional tasks ask students to construct graphs following conventional procedures and simply to obtain numerical information from the data and the graphs, it would be difficult to understand for what the percentage bars and the pie charts serve and the meaning of graph components (Friel, Curcio & Bright, 2001; Watson, 2011). Exploring the concepts inherent in percentage bars and pie charts would help understand the features of the graphs, the meaning of their components and their convention. In addition, students would be able to construct the graphs and interpret them by reflecting on the mathematical and statistical meaning, concepts and relation inherent in the graphs and data.

2. Literature on Statistical Thinking and Statistical Graph

2.1. Global View and Local View

Statistics and data analysis facilitate to describe and predict the general feature of data set which is not evident with individual data (Lee & Ji, 2008; Ciancetta, 2007). The general feature of the data set means the trends and the patterns of the data. These trends and patterns are used for statistical comparison, explanation and decision, and for analysis on the data set. This is the basic statistical methods, and it is called as statistical investigation process (Lee & Ji, 2008). However, in the beginning of learning statistics, most students have difficulties in seeing the entire data set as one cluster, then tend to focus on individual data (Lee & Ji, 2008; Ben-Zvi & Arcavi, 2001). Focusing only on each individual data or considering that individual data has its own characteristic is called local view, whereas comparing between individual data, recognizing the entire data as a whole entity and exploring the shape, spread, patterns of the data set is called global view (Lee & Ji, 2008; Bakker & Gravemeijer, 2004; Ben-Zvi & Arcavi, 2001; Ciancetta, 2007). Looking globally at data and at graph is indispensable for statistics and data analysis. The development from local view to global view of data and graph allows to gain insight into the entire data set and to discern the relationship between the insight and the real-world phenomena represented in the data set. Ultimately, it allows statistics to become an important tool of understanding the real world (Kim et al., 2017). This aggregated-based reasoning is one of the fundamental type to statistical thinking (Wild & Pfannkuch, 1999). Thus, it is necessary to support the construction of global views in teaching and learning of percentage bars and pie charts.

One way to facilitate looking globally at the data is observation using different representations of the data (Lee & Ji, 2008). Tukey introduced an approach of exploring and analyzing the data, called

Exploratory Data Analysis (EDA), with the purpose of revealing the underlying structure of the data. It is usually performed through graphical representation of the data (Woo, 2000). Beyond the visualization of data, graph facilitate discovery and speculation of the attributes which cannot be easily detected with numerical form of data, such as the relationships between individual data, and the structures and features of the whole data set. Graphical representation helps to understand and analyze globally the data set, and by doing so, helps to derive a variety of meaning inherent in the data (Kim et al., 2017; Woo, 2000; Lee & Ji, 2008). In other words, the construction and interpretation of the graphical representation of the data assist in the development from local view to global view.

2.2. Level of Graph Construction and Comprehension

Teaching and learning statistical graphs can largely be divided into graph construction and graph interpretation (Reys et al., 2012). Researchers investigated and suggested students' level of graph construction and graph interpretation. With regard to graph construction, Ben-Zvi and Arcavi (2001) classified six phases of construction of global view from local view analyzing two seventh-grade students' activities of data representation in graph as follows: (a) recognizing relevant information; (b) reading and understanding local data in tables and in graphs; (c) observing the differences between adjacent local data; (d) learning basic concepts of global view by understanding trends and patterns, and learning the language to describe them; (e) constructing global view by handling special local points; (f) transferring flexibly between local and global view by rescaling graphs.

Graph interpretation refers to the abilities to derive meaning from graphs (Friel, Curcio & Bright, 2001). It includes both reading the data from the graph and discerning their meaning by connecting the data with the context. In relation to graph comprehension, Curcio (1987) suggested three levels of graph comprehension. Simply obtaining information from the title, legend, axes and data value, such as "A accounts for 30%" is classified as the elementary level, named as "reading the data". The intermediate level, called "reading between the data", refers to comparing data by using other mathematical concepts and abilities. The example expressions are "the largest part is C", "the least is A", and "A is half of B". The advanced level, "reading beyond the data", relates to the abilities to make inferences and predictions by connecting the data and the context. Discerning the trends of the data, generalizing it and finding implications belong to this level. Students can easily perform "reading the data" task, but they have difficulties in "reading between data" and "reading beyond the data" tasks. The errors that students make while performing the "reading between data" task can be originated from misreading or lacking in mathematical knowledge. "Reading beyond the data" task is more challenging as it requires not only finding facts explicit in the graphs but also drawing students' opinion, verifying and evaluating their ideas. In order to overcome the difficulties in the latter level tasks, students need to engage in the questions which allow students to practice reasoning, synthesizing and evaluating based on data (Friel, Curcio & Bright, 2001). This Curcio's (1987) classification can be related to development of view from local view to global view. If a student's level of graph comprehension progress from the elementary to the advanced level, it can be considered that their view of data and graph has developed from local view to global view.

Regarding percentage bars and pie charts, it can be considered that local view of data and graphs involves focusing on individual values and global view entails comparing proportional relationship between a part and another or between the part and the whole. As mentioned in the previous section, the common students' errors in construction of percentage bars and pie charts are considering different whole amount when calculating the percentage of each part and creating graphs in which the parts do not proportional to the percentage (Li, & Shen, 1992; Watson, 2011). These errors occur when students focus only on individual values and do not take account of the comparative relation between a part and another or between the parts and the whole. That is, those errors occur when students are fixed to local view of data, and percentage bars and pie charts. Local view is a starting point of data and graph exploration and a part of the process of global view. Students' view can develop to global view or stay in local view depending on the instruction (Lee & Ji, 2008). Thus, in teaching and learning percentage bars and pie charts, teachers need to lead students to develop the view of data and graph from local view to global view by helping them to see relative values and to discern the comparative relationship among the data.

3. Literature on Mathematical Creativity

3.1. Various Definitions of Creativity

Although the creativity has been widely investigated, in the literature there is no consensus about the definition of creativity (Mann, 2006; Sriraman, 2005). Hence researchers have discussed about the creativity from various perspectives.

Creative thinking was distinguished by Guilford (1967) between divergent and convergent thinking. While divergent thinking involves exploring various aspects of a problem situation to generate multiple alternative solution from available information, convergent thinking involves exploring deeply and improving an aspect to produce a single correct solution to a problem (Cropley, 2006; Lee, 2015; Leikin, 2009; Tabach & Friedlander, 2017). On the one hand, the typical process of divergent thinking is "making un expected combinations, recognizing links among remote associates, transforming information into unexpected forms. (Cropley, 2006, p. 391)" On the other hand, the typical process of convergent thinking is "emphasizing speed, accuracy, logic (Cropley, 2006, p. 391)" and "recognizing the familiar, reapplying set techniques, and accumulating information. (Cropley, 2006, p. 391)"

The Torrance's model (1974) was developed to measure and evaluate individual creativity. In order to evaluate the creative products from the test, Torrance (1974) defined fluency, flexibility, originality and elaboration as main component of creativity. Fluency refers to producing multiple ideas, and flexibility relates to varying ideas, using various approaches to a problem, producing qualitatively different solutions. Originality is characterized by novel or unusual idea, and elaboration involves ability to organize, generalize and express clearly ideas. Comparing the work of Torrance (1974) and Guilford (1967), fluency, flexibility and originality can be associated with divergent thinking, whereas elaboration can be associated with convergent thinking.

There have been investigations which suggested criteria for judgement of creative products. Vygotsky (1967, 2004) differentiated between absolute and relative creativity. While absolute creativity stands for historical accomplishments at a global level, while relative creativity refers to novel and meaningful ideas or products in a specific reference group (Lev−Zamir & Leikin, 2011). Kaufman and Beghetto (2009) suggested Four C Model of Creativity, which classified creativity into four levels building on this distinction of Vygotsky (1967, 2004): mini−c, little−c, Pro−C, Big−C. All these four levels indicate novel and meaningful behaviors, ideas or products, however, they differ in concerning the reference group. The mini−c level refers to novel and meaningful experience in light of each individual. Novel and task appropriate behaviors, ideas or products are considered as little−c. Pro−C involves novel and significant discoveries, ideas or products to a group of experts in a specific field, and Big−C refers to legendary achievements, ideas or products from a historical perspective, which can affect an entire field of study or domain. As mini−c, little−c and Pro−C discuss creativity in light of a specific reference group, they correspond to relative creativity, and Big−C corresponds to absolute creativity.

3.2. Mathematical Creativity in School Mathematics Level

The studies on mathematical creativity have been conducted focusing on different subjects. There are studies about creative people, which focused on their characteristics, habits and working environment (Sriraman, 2009), studies about creative thinking process (Silver, 1997; Haylock, 1987, 1997), studies on evaluating creative products (Leikin, 2009; Leikin, & Lev, 2007), and studies about tasks, instructional strategies or setting to foster creative thinking (Lee, 2017; Luria et al., 2017; Nadjafikhah et al., 2012; Sriraman, 2005).

The reason for these various subjects of research stems from the absence of an agreed definition of mathematical creativity, like as general creativity (Lee, 2015; Leikin & Pitta-Pantazi, 2013; Singer, Sheffield & Leikin, 2017; Sriraman, 2017; Tabach & Friedlander, 2017). However, the mathematical creativity is generally understood to refer the ability to produce novel and useful mathematical concept, knowledge or perspective based on prior knowledge and experience, or to reorganize existing mathematical concepts or knowledge in a unique way (Lee, 2015; Nadjafikhah et al., 2012). The determination whether the new work is novel and useful can differ depending on the reference group and on the point of time. In this regard, mathematics education researchers have concurred with considering mathematical creativity in school level as relative creativity (Ching, 1997; Lev-Zamir & Leikin, 2011; Sriraman, 2005; Zazkis, 2017).

Sriraman (2005) proposed mathematical creativity at the professional and K-12 levels in line with the relative creativity perspective with the object of discussing on mathematical creativity in school level, as follows:

> At the professional level, mathematical creativity can be defined as (a) the ability to produce original work that significantly extends the body of knowledge, and/or (b) the ability to open avenues of new questions for other mathematicians. (…) On the other hand, mathematical creativity in grades K-12 can be defined as (a) the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or (b) the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle requiring imagination. (Sriraman, 2005, p.23-24)

Since mathematical creativity at the professional level refers to accomplishment which mathematical society recognizes and approves, it corresponds to Pro−C of the Four C Model. In case accomplishment is significant historically and globally in a specific field, or beyond, it can be classified as Big−C. In contrast, as mathematical creativity at K−12 level relates to mathematical idea or product which is novel and useful to individual student, it can be considered as mini−c. However, if it is task appropriate, or/and novel and meaningful to the class, it corresponds to little−c. If students' mathematical discovery or invention is novel and meaningful in their view, it can be considered creative even though it is already known in mathematicians' community. Thus, task appropriateness, newness, usefulness and meaningfulness in school level are verified with reference to students' previous experience as well as the performance of peers (Hershkowitz, Tabach, & Dreyfus, 2017; Leikin, 2009; Lev−Zamir & Leikin, 2011; Leikin & Pitta−Pantazi, 2013). In case, therefore, a student generates new idea using previous knowledge and experience, solves a problem which has not yet been dealt with, the student can be considered to be creative. The same is when a student's idea and solution is novel and meaningful in comparison to other students' who have common educational history, or a student contributes to extend mathematical knowledge of the class.

3.3. Education of Mathematical Creativity in School Mathematics

Up to now, the majority of education programs and studies on mathematical creativity in school mathematics have been conducted performed on gifted students (Lee, 2015; Sriraman, 2005). Gifted students can be evaluated as more creative comparing with peers, however, considering the Four C Model (Kaufman & Beghetto, 2017), every student is able to manifest creativity, beginning in mini−c. In recent years, inasmuch as, there have been an increasing number of researchers who concurred with the claim that every student has creative potential, and that creativity can be developed through education, the need for nurturing creativity in education has steadily been arisen (Lee, 2015, 2016; Luria et al., 2017; Sriraman, 2017; Sternberg, 2017; Zazkis, 2017).

Although there has been sustained effort to educate average students by promoting creativity in school mathematics (Beghetto, 2017; Hoth, Kaiser, Busse, Dohrmann, Konig, Blomeke, 2017; Lee, 2017; Zazkis, 2017), many regular classroom teachers still feel burdened with teaching curriculum content while fostering students' creative thinking (Baer & Garrett, 2010). Because they find that this needs extra time on top of regular classes and instructional materials. In this context, Sternberg (2017) pointed out that teaching for creativity separately from regular classes runs a risk of leading students to differentiate creative mathematics from "normal" mathematics. He argued, therefore, that it should be actualized in regular classes.

In this regard, there has been discussion about compatibility between teaching content knowledge and promoting creative thinking in daily practice. Following to Baer and Garrett (2010), it is possible because emphasizing creative thinking effectively helps in acquiring and understanding content knowledge. Beghetto and Plucker (2006) argued that students' understanding will deepen engaging in the creative process which involves pursuing efficient solution approach, considering how and why a certain solution works and exploring the meaning of that solution. In the same vein, Lev-Zamir and Leikin (2011) also argued that new mathematical concepts can be constructed by organizing and combining new and previous knowledge, and by developing abstract ideas They stated that this process requires creativity, and that teaching with and for creativity can reinforce students' learning.

The studies presented thus far support the notion that teaching through creativity helps not only to develop new content knowledge but also to deepen conceptual understanding of it. After all, it is necessary to research and develop creative tasks and instructional strategies which facilitate achievement content standards goals, support for students’ conceptual understanding of the content to be learned and enhancement of mathematical thinking in regular class with average students.

3.4. Creative Tasks

In order to teach students by promoting creative thinking, appropriate tasks for this should be used in lessons. Lee (2017) argued that the use of tasks which gives learners the opportunity to pose various questions, to approach from diverse perspectives, and to find multiple answers permit creative learning. In general, open tasks are widely used for creativity education.

Open task is a task whose starting situation or goal situation is open. One kind of open-start tasks is an ill-structured task which consists of incomplete elements and provides incomplete information to find the solution (Lee, 2015; Beghetto, 2017). Hence it requires students to make assumptions in order to structure the problem situation by resolving the uncertainty. Such desire to resolve that uncertainty leads to think and act in new way (Lee, 2015; Beghetto, 2017). Students can approach the task in various and novel ways depending on the viewpoint and the structuring method. This feature of the task can foster students’ creative thinking.

The task in which the goal situation allows many possible approaches, solutions and answers (Luria et al., 2017). Leikin and Lev (2007) stated that performing the task by various means is a trait of creative mathematical thinking and argued that open-ended task stimulates creative thinking. Leikin’s (2009) findings supported this argument through the measurement of the level of the students’ solutions to the open-ended tasks.

Open−ended task can also promote mathematical development of the classroom community. The feature that it allows multiple solutions stimulates mathematical discussion among students and gives them opportunities to compare the solutions, to check the validity of the solutions, and to find mathematical relationship and common mathematical feature existing in their solutions. As a result, open−ended task facilitates the exploration of mathematical properties and principles embedded in the task by connecting and exploring the mathematical ideas involved in problem solving (Leikin, 2009; Zaslavsky, 1995).

Considering diverse task situations and exploring a variety of approaches, solutions and answers are beneficial for conceptual understanding along with practicing divergent thinking (Luria et al, 2017). Students need more than drill and practice, and rote memorization of mathematical facts to learn mathematics. Learning mathematics involves incorporating knowledge, experience and conceptual understanding of (Mann, 2006). The open task requires and enables learners to find mathematical meaning from the task, and to focus on mathematical concepts embedded in the task rather than simply to use certain algorithms, rules or procedures (Luria, Sriraman, & Kaufman, 2017; Mann, 2006; Nadjafikhah et al., 2012). Thus, the open task is appropriate for fostering students' creativity and conceptual learning.

Although numerous researchers equated creative thinking with divergent thinking (Cropley, 2006), together with divergent thinking, convergent thinking is also essential for creative thinking (Baer & Garett, 2010; Cropley, 2006; Lee, 2017; Runco, 2003; Sriraman, 2017). Cropley (2006) emphasized the importance of convergent thinking in creativity by explaining the cases of acceptance or rejection of divergent thinking and convergent thinking (see *Figure 1*). As shown in *Figure 1*, an original product can be generated by divergent thinking, however, it can be meaningless if consideration of accuracy, significance and validity is not accompanied. Convergent thinking critically evaluates and distinguishes the results of the divergent thinking (Runco, 2003). Thus effective and meaningful creativity requires the combination of the production of ideas through divergent thinking and the consideration through convergent thinking (Cropley, 2006; Runco, 2003).

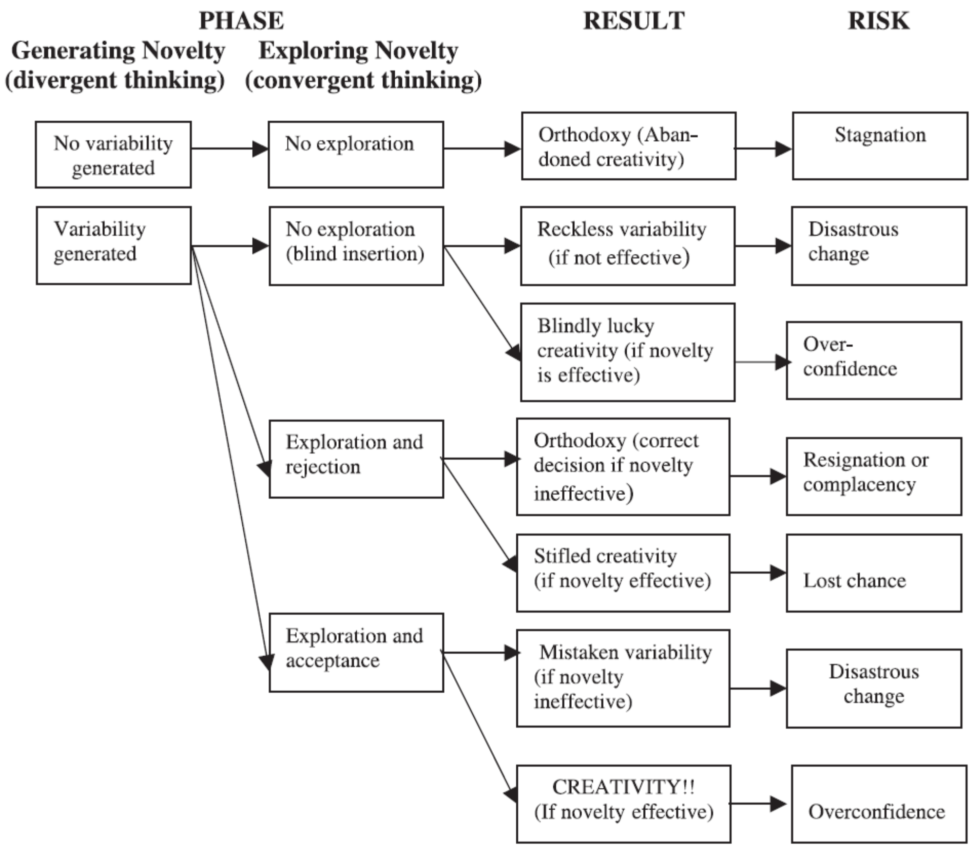

Figure 1. Results and risk of differing combination of divergent thinking and convergent thinking (Cropley, 2006, p.400)

Convergent thinking is also crucial for the knowledge acquisition and for the development of conceptual understanding and mathematical thinking because on the one hand, it manipulates existing knowledge, and on the other hand, it results increased knowledge (Cropley, 2006). Having roots in the work of Cropley (2006), Lee (2017) suggested the four-quadrant model, which categorized the types of tasks for mathematical creativity education in accordance with the degree of requirement of divergent thinking or/and convergent thinking when performing the task (see *Figure 2*).

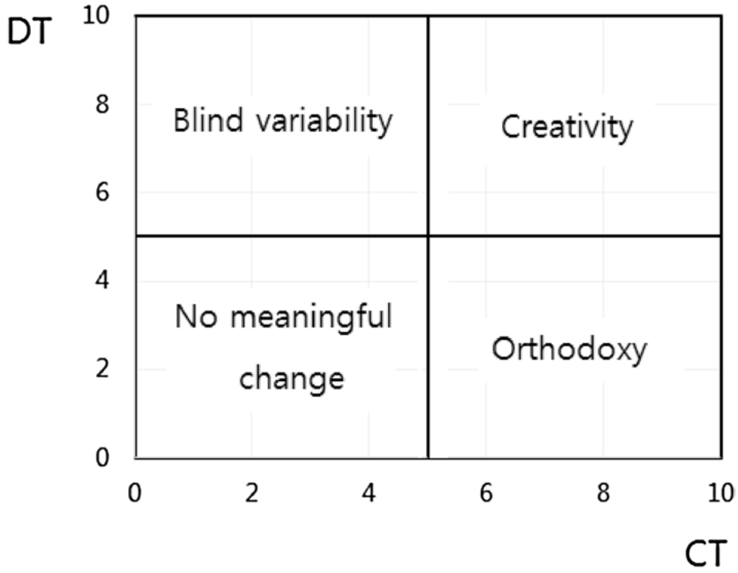

Figure 2. The four−quadrant model (Lee, 2017, p. 998)

As Lee (2017) pointed out, the task classified as "blind variability", which requires high divergent thinking but low convergent thinking, allows free−thinking, considering different possibilities, varying viewpoint and producing not only one answer or solution, however, it provides less opportunities for connecting relevant mathematical concepts or exploring inherent mathematical logic. Therefore, it is necessary to design tasks which requires high both divergent and convergent thinking for mathematical creativity education as well.

Besides the indispensable role of convergent thinking along with divergent thinking in creativity education, pre−existing knowledge is also important. Cropley (2006) asserted that divergent thinking builds on prior knowledge, and it leads to extension of knowledge passing through critical evaluation of convergent thinking as "intuition may well derive from convergent thinking at least as much as from divergent thinking" (p. 393) and "the basis of intuition (…) is knowledge, and knowledge is acquired via convergent thinking" (p. 394). This implies that it is necessary to design task in which students can engage with previous learned knowledge and personal experience in order to enable learning mathematical content through creativity. Because it allows learners to reinterpret and to reconstruct pre-existing knowledge and their experience from different perspective, and lead them to produce new mathematical representation or solution (Lee, 2015; Luria, Sriraman, & Kaufman, 2017). In addition, this kind of task encourages students to explore the essence of mathematics, that is, mathematical pattern, relation, and structure, by using informal reasoning and representation. Teaching and learning mathematics creatively can be realized through these activities (Lee, 2015; Mann, 2006).

Based on literature review, here in this study creative task is defined as the task which has not only one answer, solution or approach; requires high both divergent and convergent thinking, being classified as "creativity" in the four-quadrant model (Lee, 2017); and students can engage in by using previous learned knowledge, personal experience, informal reasoning, and informal representation, or mathematical pattern, relation and structure. This study aims to develop creative tasks for four instructional sessions on percentage bars and pie charts following this definition.

3.5. Instructional Strategies for Implementation of Creative Tasks

The implementation of tasks in classroom necessitates appropriate instructional strategies to achieve instructional goals and to maximize students' learning. The same is true for mathematical creativity. The emergence of creativity in class depends greatly on the teaching strategies and the educational environment (Luria et al., 2017; Mann, 2009; Nadjafikhah et al., 2012; Sriraman, 2005). Although the creative tasks are implemented in the class, its effect will be minimal if the appropriate instructional strategies do not support students' creativity. Thus, there is a need to examine the instructional strategies and the educational environment for learning mathematical concept through creative thinking.

The literature on mathematical creativity commonly considered the following four instructional strategies to enhance mathematical creativity in classroom. First, teachers should support students for construction of mathematical knowledge by themselves (Nadjafikhah et al., 2012; Sriraman, 2005). To put it concretely, teachers should help students to understand the task situation and to engage in the task on their own by asking indirect questions or/and by providing minimal hints. The questions and hints should be those give students opportunities to "search, explore, make conjectures, hypothesize, examine, refute, adapt strategies, devise plans, conclude reason and justify their conclusions and reflect on them, monitor, and experience the processes that mathematicians have gone through" (Nadjafikhah, Yaftian, & Bakhshalizadeh, 2012, p. 289). In this way, students can construct personally meaningful mathematical concepts and ideas.

Another strategy is to give students enough time to engage in tasks (Sriraman, 2005; Mann, 2006). To experience authentic mathematics, students should confront mathematical tasks like mathematicians. As mathematicians devote themselves to problem for considerable time before discovery, students need to be provided time to struggle with tasks (Mann, 2006). This time allows learners to examine various aspects of the given tasks, and it may lead them to think creatively. Moreover, students can experience the moment of insights and feel satisfaction and delight of that moment (Sriraman, 2005).

The social interaction between students is significant to promote students' creative thinking in classroom (Mann, 2006; Zaslavsky, 1995). They can reflect on their ideas, refine, defend and justify them, and also examine and validates peers' ideas through discussion on diverse ideas, solutions and answers (Sriraman, 2005). Additionally, the share and the exchange of ideas among students influence each other. Then, it helps to learn conceptually (Mhlolo, 2017) and gain a new insight to the task (Nadjafikhah et al., 2012; Sriraman, 2005).

The classroom environment which is open to express opinions, comment, question, and try new things has a positive effect on the emergence of students' creative thinking (Luria et al., 2017). This environment also permits failure and error, thereby it leads students to search for alternatives and to open new perspectives (Mann, 2006). In cultivation of this kind of environment, teachers play a crucial role (Nadjafikhah et al., 2012). They also have to be flexible and open to alternative approaches to the tasks (Sriraman, 2005). If teachers' thought is fixed in certain approaches, solutions or answers to the tasks, they can respond negatively on diverse students' ideas (Beghetto, 2017), and this response will hinder students' creativity. Thus, as long as teachers acknowledge the possibility of learning mathematical content with implementation of creative tasks, the whole classroom would be provided opportunities to think creatively.

3.6. Components of Mathematical Creativity and Percentage bars and Pie charts

Researchers have developed creative tasks and drawn instructional strategies on the basis of the components of mathematical creativity which varied depending on researchers. Following the work of Torrance (1974), this study considers fluency, flexibility, originality and elaboration as the main components of mathematical creativity, as much of the literature on mathematical creativity (Park, 2017; Lee & Lee, 2010). In terms of mathematics education, fluency relates to the ability to suggest numerous ideas, approaches, solutions or mathematical products, such as graph, sign, model; flexibility to generate qualitatively different ideas, approaches, solutions or products, and to vary standpoint or thinking strategy when dealing with a mathematical problem or object; originality to generate unique or insightful ideas, solutions or products and to approach from unusual and new angle; elaboration to generalize, organize, and improve ideas, thought processes, or expressions (Leikin & Pitta-Pantazi, 2013; Lev-Zamir & Leikin, 2011, 2013; Singer, Sheffield & Leikin, 2017; Park, 2017; Lee & Lee, 2010).

These four components can be related to construction of percentage models and interpretation of data, percentage bars and pie charts. In teaching and learning percentage bars and pie charts, fluency refers to the construction of numerous percentage models, the use of numerous ways of constructing a percentage model, the recognition of a plenty of mathematical relationships related to data, areas, lengths of rectangles, and central angles of sectors, and a variety of interpretations of data, percentage bars or pie charts. Flexibility relates to the construction or interpretation of percentage models with variation of approaches by considering which has not discerned before; and the qualitatively different comparison of areas or amounts between one part and another or between one part and the whole when dealing with percentage models. In case of originality, it involves the construction of unusual, unique and insightful percentage models; the use of novel methods to construct percentage models; the unusual and meaningful interpretation of data, percentage bars, pie charts and mathematical concepts or relationship inherent in them; the insight into data, percentage models, areas, lengths of rectangles or central angles of sectors. The ability to verify or refine the constructed percentage models and their methods of construction, to verify calculation process related to percentage or proportion in construction or interpretation of data, percentage bars and pie charts; to organize or generalize ideas and thinking process; and to express ideas related to the construction or interpretation of data or percentage models are characteristic of elaboration.

The mathematical situations in which fluency, flexibility, originality and elaboration can be used encourage the application of prior knowledge to new and unfamiliar problem situations. Consequently, such application of knowledge allows not only acquisition of new knowledge, but also conceptual understanding of new knowledge (Mann, 2006). In this study, we intended to foster these four components by creative tasks and teaching strategies to enhance students' conceptual understanding of percentage bars and statistical thinking.

CHAPTER 3

METHODOLOGY[1]

1. Case Study

This study employs the case study methodology. According to Yin (2016), the case study is used in empirical research in order to verify and extend theories, and to derive implications by analyzing phenomenon in depth through specific cases. In this way, it enables researchers to investigate complex social phenomenon and to answer "how" and "why" questions regarding the phenomenon.

Despite the mathematical creativity graphs have been widely investigated, there have been few empirical studies that examined in details of the implementation of creative tasks and instructional strategies for teaching curriculum content in regular classroom (Luria et al., 2017). Hence this study aimed to take a closer look at teaching and learning case of percentage bars and pie charts in which creative tasks and instructional strategies implemented, and, doing so, to add new meaning to previous research. Since the purpose of this study is to investigate how students' conceptual understanding and statistical thinking develop during the instruction of percentage bars and pie charts in which the creative tasks and instructional strategies implemented and how mathematical creativity influences the learning, as stated in the Introduction, it was deemed appropriate to use case study as a research methodology.

1. This study is aligned with Lee, Moon & Song (2018).

2. Research Design

2.1. Participants

The two teachers participated voluntarily in this study. They were working in the school where the teaching experiments were conducted. They joined for creative task design willing to improve their lessons. The teacher A was in charge of the sixth-grade class which participated in this study. She was in her first year of teaching as this project began. The teacher B had taught for eight years in the elementary grades and she had experience in teaching percentage bars and pie charts.

A total of 28 sixth-grade students learning in a class of one urban middle-class school participated in the study. These students had average achievement. The teaching experiments were conducted right after they learned about the percentage, within regular classes. Four lessons were designed and implemented using the creative tasks and instructional strategies developed in this study. The pupils were accustomed to engage in critical debates as the teacher A had held discussion since the beginning of the semester by encouraging students to exchange their ideas freely.

2.2. Research Procedures

This research was carried out by a team of university-based researchers in mathematics education working in concert with two elementary teachers in order to design and develop the creative tasks and the lessons for percentage bars and pie charts. The research team had one-hour bi-weekly meeting during six months.

Figure 3 illustrates the research procedures. The research started with literature review of learning percentage bars and pie charts, development of mathematical creativity, design of creative tasks and their implementation. The team had examined the mathematical concepts which compose percentage bars and pie charts and statistical thinking related in construction and interpretation of statistical graphs. The team then had drawn teaching and learning content of percentage bars and pie charts, and its phases of learning percentage bars and pie charts. Based on analysis of literature of mathematical creativity, the strategies to design creative tasks and teaching method had been drawn.

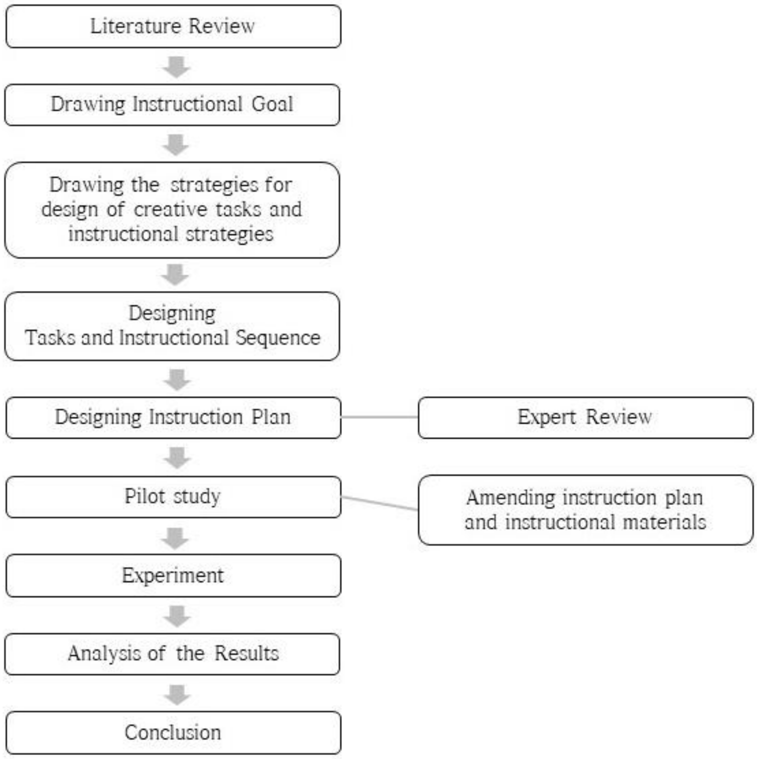

Figure 3. The research procedures.

Standing on the analysis of previous investigation, a series of tasks were developed and an instructional sequence was designed for four instructional sessions of percentage bars and pie charts. For each instructional session, a lesson plan was also designed including hypothetical learning trajectories. The creative tasks and lesson plans which were developed following the drawn strategies were revised by one researcher from mathematics education. The pilot study was then carried out in different five sixth-grade classes in the same school as the participants in the main study. Five teachers from the same school who conducted or observed the pilot study provided feedback on the developed tasks and the lessons. Taking account of the feedback, there were time adjustment to give more enough time for discussion, modification of difficult terms for students in worksheets, and addition of PowerPoint slides to support students' understanding of the tasks. The revised version of tasks was implemented in the main study by the teacher A who had participated in development of the tasks and the lessons.

3. Overview of the Tasks

The first task was adapted from the task "Making Per Sense of It" (Romberg, 1997, p.1−4) which requires expression of opinion in regard to given percentage information and the task "The School Theater" (Romberg, 1997, p.5−6) of the Mathematics in Context(MiC) unit "Per Sense" (Romberg, 1997). The task situation related to the concert ticket sales, which devised to be familiar to students. The task was intended to focus on the relationship among the whole, the part and the percentage. For this, the students were asked to discuss about the given claim which includes certain percentage value and to visualize given percentage value. To do so, ill−structured task was used, which provided only percentage value, in order that students suppose freely to complete the task situation for appropriate judgement based on the given information. The task was designed to make students able to visualize the given percentage value using their prior experience and knowledge related to percentage, ratio, fraction, and statistical graphs.

The second and the third task was about construction of percentage model from given data table which showed the number of each zone seating of concert. The students were asked to construct models in their own way using their mathematical knowledge. The only difference between the second and the third was the shape of the model. In the second task, students were asked to construct models in shape of a bar, but in the third, in circle shape. Especially in the third task, they were provided worksheets with circles divided into four, eight, ten or twenty sections.

The last task was about interpretation of percentage bar and pie chart which illustrate the proportion of the means of transportation workers used to commute. This was aimed to focus on the fact that the length of each part of percentage bar or the central angle of pie chart is proportional to its percentage. In addition, it was aimed to facilitate interpretation of the graph connecting data with context as the task involves students' everyday life experience. To this end, the ill-posed task was used again. Each group of students was given a percentage bar or a pie chart without label (see *Figure 4*). There was no information about the part amount, the whole amount, percentage value and the means of transportation in the task. They were asked to infer the percentage and the item of each sector and then interpret the given graph by using prior mathematical knowledge and personal experience. At the end of the task, the students were required to discuss how to relieve traffic congestion.

The developed tasks are presented in *Figure 4, Figure 5*, and *Figure 6*. Table 1 presents the topic of each session

Table 1. The topics of each session

Session	Topic
1	Visualization of percentage
2	Construction of percentage bars
3	Construction of pie charts
4	Interpretation of percentage bars and pie charts

Lesson 1: Proportional thinking and visualization of percentage

1. Consider the text below. Do you agree with Sujin? Use specific reasons and details to support your opinion.

> The singer A held a concert in Korea. 75% of the tickets for the concert were sold. "The singer A is so popular.", said Sujin after hearing the news.

2. Make drawings or graphs of the given situation to support effectively your opinion about Sujin's comment.

Figure 4. The creative tasks for the first instructional session

Lesson 2,3: Visualization of the given data table in a bar-shaped model

	Zone S	Zone R	Zone A	Total
No. Seat	60	40	100	200

Let's create a graph to represent percentage considering the data table below. Create a model individually then, discuss with your group members.

Figure 5. The creative tasks for the second and third instructional session

Lesson 4: Interpretation of Percentage Bars and Pie Charts

The chart below shows which means of transportation workers used to commute. Work with your group members to answer the following questions:

(a) What is the percent of each sector of the graph? Explain how you did it.

(b) Guess which transportation can be matched to each sector of the graph. Explain your answer.

(c) Based on your answers to (a) and (b), share your idea with your group members about the way to reduce traffic congestion during rush hours.

Figure 6. The creative tasks for the fourth instructional session

4. Data Collection

Data for the research was collected by audio-recording of the research team meetings, audio- and video-recording of the instructional sessions, and writing field notes during classroom observation. The documents related to design and development of the tasks, instructional strategies, and instructional sequence and the students' worksheets were also collected.

The four instructional sessions in which the developed tasks were implemented were conducted in April, 2017 by the teacher A who is in charge of the class and participated in development of the tasks. As the purpose of this study is to see the aspects of teaching and learning percentage bars and pie charts during regular classroom where the creative tasks and instructional strategies were implemented, each session was conducted during 40 minutes. In the case of the first session. The block lesson was conducted to provide students rich opportunities of sharing various opinions amongst them

The participating students had learned about ratio and proportion. Hence they encouraged to perform the tasks using mathematical knowledge related to fraction, ratio, proportion, and statistical graphs which they had learned before, such as pictograph, bar graph, line graph. Moreover, the lessons were conducted focusing on development of conceptual understanding and of statistical thinking with regard to percentage bars and pie charts.

In all teaching-experiment sessions the participating students were provided individual worksheets to record their thoughts, their reason, their own percentage models and the process of the construction of their own models. All students' work recorded on the worksheets were digitized. In order to confirm the evidence of students' development of conceptual learning and statistical thinking related to percentage bars and pie charts during task performance, group discussion and whole-class discussion which could not be confirmed through worksheets, All the lessons included cameras and recorders, one for each group, respectively. One camera was used to videotape the whole classroom. These data were transcribed later for analysis.

The researchers attended all classes. The researchers recorded field notes and videotaped noteworthy students' responses and teacher's pedagogical action by moving camera. The observation was centered on the moment of that students' mathematical creativity was fostered by creative tasks and instructional strategies, and whether students' conceptual understanding and statistical thinking were developed through it, in order to answer the research questions.

These collected data served as base data for analysis. The analysis was conducted selecting the episodes which met the purpose of this study.

5. Data Analysis

In this study, the collected data was analyzed to examine whether students succeeded in conceptual learning of percentage bars and pie charts and in development of statistical thinking related to the percentage bars and pie charts during the lessons in which creative tasks and instructional strategies were implemented.

The phases of learning percentage bars and pie charts are devised to determine whether students' conceptual understanding and statistical thinking related to the percentage bars and pie charts. It built on Ben−Zvi and Arcavi (2001)'s research about the process of the construction of global view from local view and Curcio (1987)'s three levels of graph comprehension These researchers investigated students' development of global view by focusing separately on graphic representation of data or on situation of obtaining information from data table or graph. Since teaching and learning statistical graphs involve both construction and interpretation, this study focused on the progress from local view to global view in both construction and interpretation activities. Interpretation of statistical graphs requires numerical interpretation of data and semantic interpretation of data connecting with data context. Thus, in this study, students' interpretation of data, percentage bars and pie charts were analyzed in two ways considering numerical aspect and semantic aspect, respectively.

As seen in the literature reviews in Chapter 2, that the most important in construction of percentage bars and pie charts is the visualization of the proportional relationship applying the concept of ratio with consideration of the characteristic of percentage is the most important. Therefore, in this study, regarding percentage bars and pie charts, it was considered that local view of data and graphs involves focusing on individual data and global view involves focusing on proportional relationship between the part and another part, or the part and the whole. According to this, the learning phase of percentage bars and pie charts is divided into four phases from zero to three. First, with regard to construction and numerical interpretation of graphs, the phase 0 is characterized by focusing on the context, because it is not relevant to construction or numerical interpretation of graphs and data. The case of focusing on pointwise information is classified as the phase 1, like as the first level of graph comprehension of Curcio (1987). Focusing on the difference between individual data and focusing on the proportional relationship among data were considered as the phase 2 and 3, respectively. When interpreting semantically the graphs, it is required to consider the task requirement and the context situation. If only personal experience or anecdote is considered in interpretation, it does not involve task requirement and the context situation. Therefore, this case is classified into the phase 0. The phase 1 is characterized by consideration of appropriate context situation or of the qualitative features of percentage. If the data or the graph is interpreted comparing the meaning of each part, it was considered as the phase 2. The suggestion of alternative, prediction, implication is considered as the phase 3 (see Table III-2).

Table 2. The phases of learning percentage bars and pie charts

Phase	Construction	Interpretation	
		Data	Context
0	Focused on context		Considering personal experience
1	Focused on pointwise information		Considering appropriate context situation
2	Focused on the difference between individual data		Comparison of the meaning of each part
3	Focused on the proportional relationship among data		Suggestion of alternative, prediction, implication

Students' responses and models were classified according to these four phases and were analyzed with respect to the development of conceptual understanding and statistical thinking, and the emergence of mathematical thinking. When cases of disagreement occurred, a consensus was reached by discussion. In Chapter 4, the work of S2, S3, S7, S8, S9, S10, S11 who showed clearly the conceptual learning and development of statistical thinking related to percentage bars and pie charts using mathematical creativity through their words, action and worksheets during the task performance and discussion.

CHAPTER 4

RESULTS [2]

1. Visualization of Percentage and Mathematical Creativity

The most important function of percentage bars and pie charts is visualization of the proportion between the parts and the whole. Thus, it is required the concept of proportion. Having little understanding of the concept of proportion results in visualization of each individual data without considering the comparative relation among them.

The first item of the task used in the first session of the teaching gave opportunities of critical exploration of validity of the given claim which includes certain percentage value. This task led students to recall the concept of percentage which they had learned before by stimulating mathematical creativity. For example, most of the students started to engage in the task with the discussion about the assumption of the whole and the whole amount. They supposed diverse situations and made various assumptions of the subject to be considered as the whole and of the whole amount by using fraction and ratio, as shown in the excerpt below:

> 35 S3: It means 75 out of 100 people. so it is 75 over 100. Representing it as an irreducible fraction, it is 3 over 4. As it is the same as 3 out of 4 people, many people bought the tickets. So it’s right that the singer A is popular in Korea.

The students’ action of making assumptions of the whole amount shows their understating of that a percentage is a value

2. The results of this study are from the same experiment of Lee, Moon & Song (2018).

compared to some amount considered as a whole. This means that students considered the given percentage as a relative value instead of absolute one. This results show that they successfully recalled the concept of percentage.

Furthermore, the task allowed the students who had little acquaintance with percentage to engage actively in the task, and they could evaluate the validity of the given claim at their own level. This feature of the task also allowed students to examine various aspects of percentage. One of them is the qualitative information of the percentage. The students debated on it when sharing their opinions about the given claim and the contribution of the group 1 to whole-class discussion triggered it. In group discussion, the group 1 concluded that the singer A was not popular since there was a gap in concert ticket sales comparing with the popular singers in reality. During whole-class discussion the teacher wrote students' opinions on the board to be shared with other students. Furthermore, she asked group 1 to explain about the ticket sales of the popular singers in reality and how it is different from the singer A of the given task. Teacher's question also gave other students from different groups opportunities to engage in the discussion and they explained that the concerts of famous singers were sold out within a very short time. This led students to examine the data acquisition time. As can be seen from the excerpt below, the conversation between S1 and S2 led other students to make different assumptions of data acquisition time and to think what the given percentage represents depending on the assumption.

77 S1 : The singer A
78 T : Uhum.
79 S1 : Well, what's the word for it? For a few days? Can I say for a few days?
80 S2 : There is no such thing.
81 S1 : Ah yeah... It wasn't...

(…)

86 S3 : Yeah, right! It doesn't tell about the time.

87 S4 : Then, 75% within a week.

88 S3 : If so, the singer is not popular.

89 S4 : That's it.

(…)

91 S3 : Perhaps tickets were sold that much within 1 second.

92 S4 : Perhaps tickets were sold that much within 3 years.

(…)

95 S5 : 75% within a year.

96 S4 : Yeah, we also need to know about the time.

As the interaction among students in whole-class discussion had stimulated their fluency and flexibility, they could change their perspective and pay attention to this qualitative information of percentage which they had not considered before. If the task required rote application of percentage calculation procedure, the students might not have chance to change their point of view and look into the data collection process, which is one of fundamental statistical thinking types. Moreover, it was possible because the teacher stimulated students' elaboration, in other words, she assisted students to organize their thought, to clarify their claims and to give grounds for their opinion.

The second item of the task asked students to represent the given percentage on their own and they made a variety of models. Most of the students used the background knowledge to perform the task. S6 tried to visualize it remembering the statistical graphs that he had learned, such as pictographs, bar graphs and line graphs. Initially, he intended to use line graph, but immediately changed his mind by recognizing that percentage is not used for the data which changes over time. In the end, he used pictograph due to a lack of understanding of effective visualization of percentage (see *Figure 7*). His model showed the individual data, which is 75, however, as it could not show the relationship between the data, it can be considered that he was in the phase 1 of learning percentage bars and pie charts.

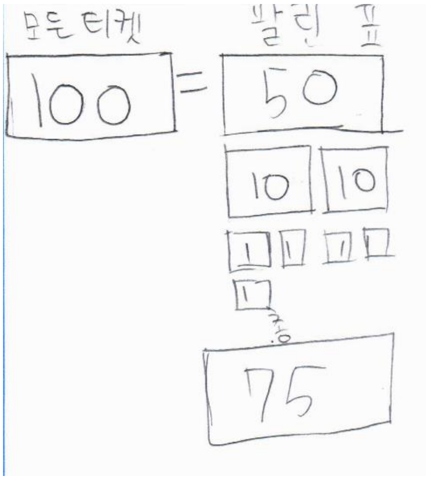

Figure 7. Visualization in the phase 1

Some students succeeded in visualizing the percentage by considering the relative size among data. S7 explained about her group's percentage model (see *Figure 8*) as follows and showed attainment of the phase 2 and 3.

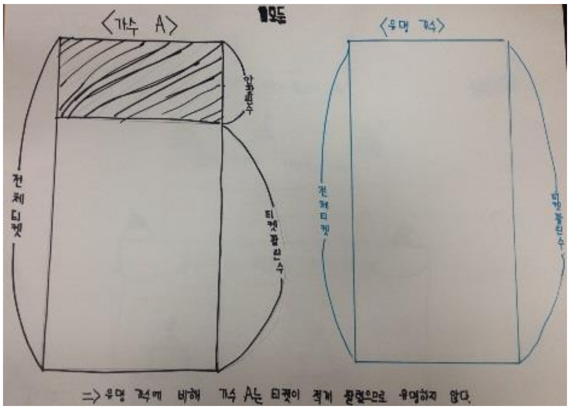

Figure 8. Visualization in the phase 2

The left model in the *Figure 8* was constructed by understanding that 75% is equivalent to $\frac{3}{4}$, and the right model is the visualization of 100% as a whole. Her group could explain how much popular the singer A is by comparing 75% with 25%, 50% or 100% based on such visualization. These additive comparisons serve as the evidence for the phase 2 and the visualization of 75% considering the proportional relationship between 75% and 100% serves as the evidence for the phase 3.

208 T : S7, how can we that this part shows 75%?
209 S7 : Yes.
210 T : How?
211 S7 : Expression of this $\frac{3}{4}$ of the whole (rectangle) is 75%.
212 T : How did you draw $\frac{3}{4}$?
213 S7 : With ruler.
214 T : Oh, yeah. But how can we know that this part shows $\frac{3}{4}$?
215 S7 : Cut it in half, and then cut it in half again.

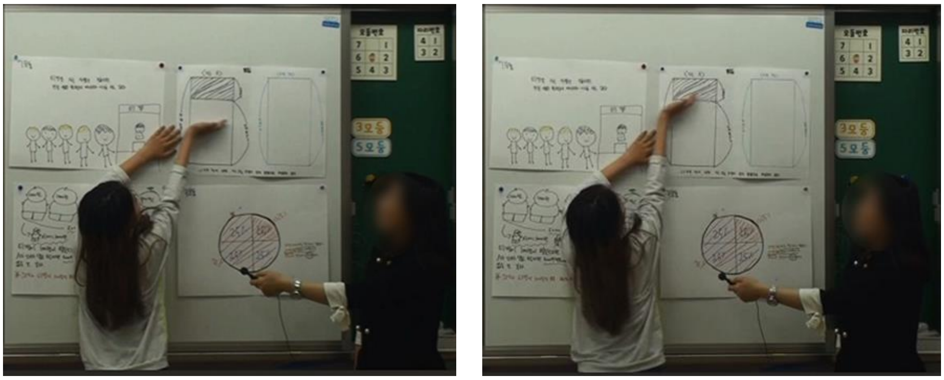

Figure 9. The moment of the attainment of the phase 3

In *Figure 10*, it can be seen that students created a variety of percentage models. There were students who represented each individual data, or who tried to represent the difference between individual data. Some recognized that the percentage could be

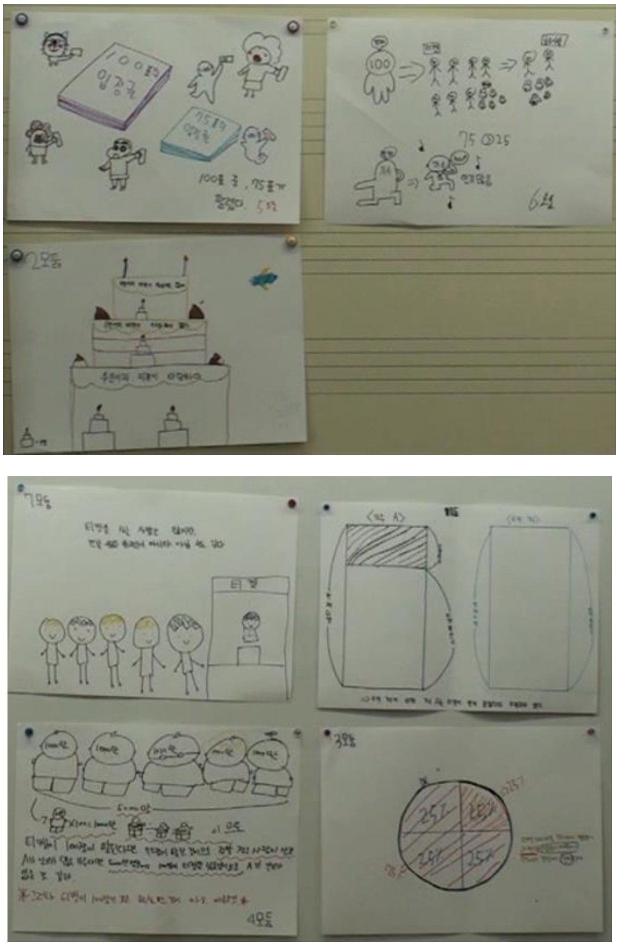

Figure 10. A variety of percentage models

converted to equivalent fraction and they used this fact in comparison of percentages. The opportunity to create various models in different ways fostered students' fluency. By doing so, it allowed students to examine and to explore deeply the concept of percentage from various angles. It also enabled to apply previously learned fraction to the construction of percentage model which is new for them. Furthermore, sharing their own models and responding to teacher's questions about the construction methods and their ideas helped students to understand the meaning of percentage and percentage models. These activities also fostered students' elaboration, resulting in students' development of the learning phases.

2. Construction of Percentage bars and Creativity

The second task was designed aiming at understanding of the proportion among the data given in table and visualizing them in shape of a bar. The teacher gave examples of battery power indicator and mobile application to monitor data usage to help students use their everyday experience to construct a percentage bar model. Most of the students, including those reached the phase 3, returned to the phase 1 or 2 when constructing percentage bar models. For example, as *Figure 11* shows, S3 visualized the data incompletely by representing the whole amount separately.

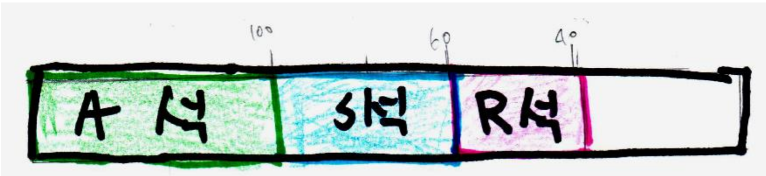

Figure 11. S3' s percentage bar model in the phase 1

S3' s model displayed each individual data, but as she did not understand that the parts constituted the whole, S3 was considered as the phase 1. In group discussion, S8 examined whether S3' s model is appropriate to the given data and gave his opinion about her model. This discussion related to her model stimulated her statistical thinking and led S3 to refine it.

110 S8 : (Looking at S3' s percentage bar model (*Figure 11*)) Why is there nothing next to the R seats?

111 S3 : Because there is nothing next to the R seats.

112 S8 : (Showing his own model and pointing to A, S and R on his model) In total, there are 200 seats. If you have 60, 40 and 100, you have 200 in total.

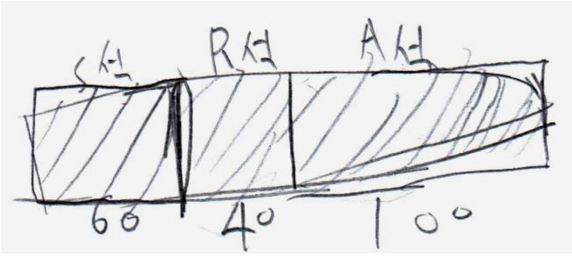

Figure 12. S8’s percentage bar model in the phase 2

S3 could not understand what S8 meant by "Why is there nothing next to the R seats?" right away. However, S8’s comment promoted her elaboration and made S3 to check her model out. After looking at S8’s bar model (see *Figure 12*), S3 took note of the relationship between each part and the whole, which is the key element in the concept of proportion. Then, S3 modified her percentage bar model, as shown in *Figure 13*. The modified model was classified as the phase 2 since it did not show the relative size among the data.

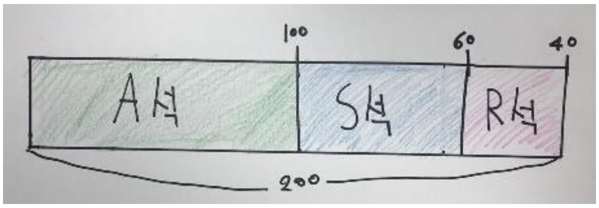

Figure 13. S3’s modified model in the phase 2

While paying attention to the relationship between the part and the whole, the proportional relationship between one part and another or between the part and the whole should be visualized using the relative position. Being able to do so means a complete understanding of the meaning and the structure of the percentage bar. S10 accomplished it. As shown in *Figure 14*, S10 simply displayed the data without considering the proportional relationship between the data. S10’s early model corresponded to the phase 1. In group discussion, her group member S9 pointed out the error in S10’s model, as shown the excerpt below.

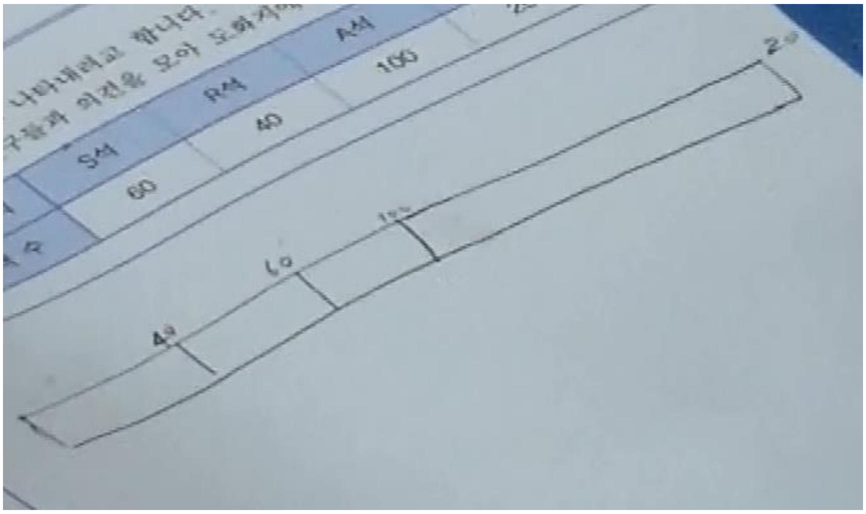

Figure 14. S10’s Percentage model in the phase 1

85 S9 : It doesn’t make sense. (Pointing to the first left section of S10’s model) 40 is this big, but why.... I can understand if it (pointing to the second left section of S10’s model) is 80, but this is 60. (pointing between the first and the second left sections) If it is 20, it has to be half of this (pointing to the first left section).

S9 explained to S10 that the length of each rectangle had to be proportional to the data, by using the rectangle which represented 40. This S9’s explanation helped S10 to understand the proportional relationships between the data and led her to modify the model according to the relative position (see *Figure 15*).

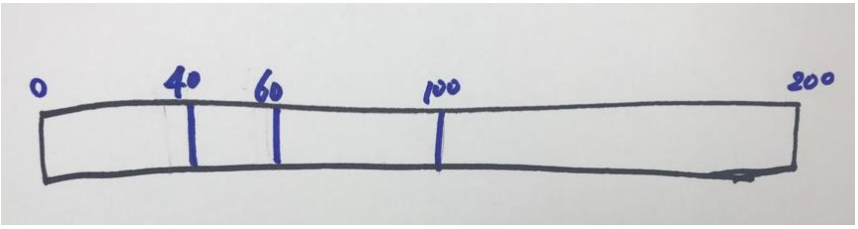

Figure 15. S10’s Percentage model in the phase 3

During the class, some students created unique percentage bar models. S11’s model (see *Figure 16*) was the case. While most of the students constructed a bar to visualize the data, S11 represented it creating three percentage bar models. It was unusual, but as it showed the proportional relationship among the data, it can be considered meaningful and original.

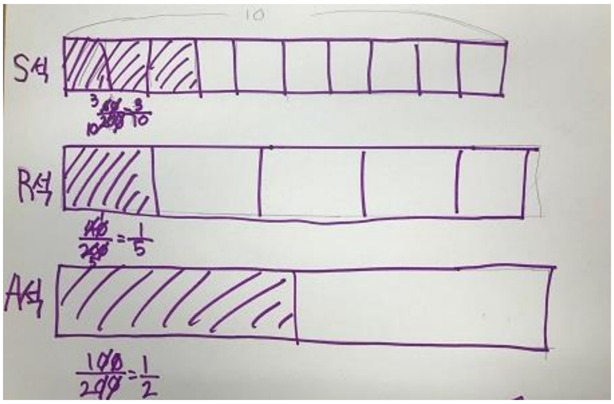

Figure 16. S11’s unique percentage bar model

The defect of S11’s model was that three bars had different entire lengths. The teacher showed her model to the class during whole-class discussion and asked her why the entire lengths of bars were different. The teacher did it to bring students’ attention to the whole amount. S11 replied that they had to be the same, but she could not explain why they should be the same size. At that moment, other students gave their opinions.

107 T : Why do you think those should have the same size?
108 S2 : Because the whole is the same, but only the whole amounts are different.
109 T : The whole,
110 S2 : Is the same, but the part amounts are different.
(…)
117 S3 : Because there are 100 seats, 40 seats and 60 seats

within 200. They are within the same 200 seats. So they should have the same size. Don’ t you think?

As the percentage of each data from the same data set needs to be calculated on the same base, these percentage models also have to be the same size. This mathematical fact could be checked and shared with the whole class through S11’ s novel model, the teacher’ s questions and whole−class discussion.

The task aspect that encouraged students to create percentage bar models in their own way stimulated students’ fluency and originality. The gradual improvement of the learning phases from the phase 1 to 2, and from the phase 1 to 3 could be made due to the promotion of flexibility and elaboration. Critical discussion among students played a crucial role in understanding the meaning of the components of percentage bar and the proportional relationships among data. Meanwhile, it can be seen the development of students’ conceptual understanding of percentage bars and of their statistical thinking related to percentage bars.

3. Construction of Pie charts and Creativity

The third session was conducted as like the second. The data was given in table and students were asked to visualize the proportional relationship among the data in circles and to refine the created models by critical examination. The worksheets with circles equally divided into four, eight, ten or twenty sections were used as instructional material. Representing data in a circle would be difficult task for them as they had not enough understood the properties of circles. Hence initially most of the students were in the phase 0, however they showed improvement through the exploration of the equally divided sections.

At the beginning of the class, the teacher drew students' attention to the different number of sections of the circles in order that they choose convenient one for the representation of the data. At that moment, S9 said that the number of sections was irrelevant to represent data. This implied that he was in the phase 0, as he could not relate the number of sections and the given data. However, the following discussion after S9's comment led him to pay attention to the relation between the data and the number of the sections of the circles.

47 S12: Well, the sections have the same size. That (pointing the data table) has digits in the tens place. If there are digits in the units place, it would be difficult (to represent). But there are digits only in the tens and the hundreds place, we can make it more easily.

48 S9: Ah~ I see! I think I understand. Look. Imagine there are digits in the units place. Like 21. Then we have to mark out here (pointing between the two marks). But there are digits in the tens and the hundreds place, so we can mark out using these

marks. But, if we have 23, we have to mark out between two marks. Like this.

(…)

52 S9: I mean. Hey, there are 60, 40, and 100. Right? But if there is a number with a digit different from zero in the units place, like 63 or 66. Let' s consider this mark is for 60. Then, if we have 66, we have to mark here between two marks, right? But this number (pointing the data table) has zero in the units place, right? So we can mark out on the mark.

Figure 17. S9' s exploration of the number of sections

As shown in *Figure 17*, S9 claimed that it was able to construct more elaborate pie chart by creating more marks between the given marks on the circle. This S9' s change in perspective could occur because S12' s comment promoted S9' s flexibility in whole-class discussion. The teacher noticed that S9 was paying attention to the number of the marks and asked him to share what he had found related to the number of the marks in whole-class discussion. The teacher led him to think about a circle with 5000 marks and another one with 100 marks, in order to foster his fluency and flexibility through examples. Then, S9 responded to this question saying that the marks the circle had, the easier it was to represent the data if the data have digits different from zero in the units places. This implied that S9 did not fully understand the proportional relationship between the data and the number of marks. Thus, S9 was in the phase 2 at that moment. The teacher kept discussing about the number of the marks with S9 (see the excerpt below).

259 T : Look, S7. Which circle do you think is better to represent this data between the circle with 10 marks and 20 marks?
260 S9 : 10 marks.
261 T : Why? You have said before that the more marks the circle has, the better it is to display (the data), haven' t you?
262 S9 : I mean, when the data have digit (different from zero) in the units places.
263 T : Then, in this data?
264 S9 : 10 marks.
265 T : Why?
266 S9 : Because it' s enough with digit in the tens places.
267 T : Because it has digits in the tens places?
268 S9 : 6 sections for S, 4 sections for R, 10 sections for A.
269 T : Huh? It sounds a little strange. Can you collect your thoughts? Once again?
270 S9 : Oh wait, well, it' s percentage. Ah! This is a pie chart. Ah, there are 10 marks. Since the percentage of S seats is 30%, the percentage of R seats is 20 % and the percentage of A seats is 50%, for S seats, out of 3, no, 3 sections out of 10; for R seats, 2 sections; for A, 5 sections.

S9 noticed that the circle with many marks was not always a good one to represent data. He realized that depending on given data changed the number of the marks which allowed to represent the data more easily. After this discussion, S9 focused on the proportional relationship between the parts and the whole; converted the proportions between the parts and the whole to percent; and represented the data in a pie chart. This implied that S9 succeeded in development to the phase 3.

The teacher supported S9 to reflect on himself by repeating his words, to change his perspective by stimulating fluency and elaboration and by doing so to construct systematic and structural concepts. Other students could also construct pie charts successfully following the same trail as S9. At that time, the fact that the task allowed students to choose one from different kinds of circles, and that group and whole discussion promoted interaction between peers contributed to the development of the learning phase.

4. Interpretation of Percentage bars and Pie charts and Creativity

The task used in the fourth teaching session required integrated understanding of percentage and contextual knowledge. Each group of students was given a percentage bar or a pie chart without label. They were asked to infer the percentage of each transportation and to deduce plans for reduction in traffic congestion. This task is to verify whether students understood the need to consider proportional size between the parts and the whole when visualizing percentage information, and whether students were able to connect the data and the context in interpretation of graphs.

At first, the students found ways to speculate the percentage of each section of the given graph. The group S2 belonged to was given a percentage bar. S2 started to measure the length of each section and the entire length of the graph with a ruler which had centimeters on one side and inches on the other side. When measuring the lengths, he used the side of the ruler with the inch markings, but he did not realize it. As S2 thought that he was measuring in centimeter, one of his group members, S13, pointed it out.

1 S2 : 5cm. Here, 5cm is 100%. Okay? Well...
2 S13: 5cm? It's 5 inches.
3 S2 : It's 5cm. Why is it not 5cm? Ah~ Okay. Okay. It's 5 using this side (pointing the side of inches of the ruler). Then, we can measure using this side (pointing the side of inches of the ruler).
4 S2 : No. No. No. This is 3.7. No. No. No. This is 2.6%
5 S13: What are you doing now? In inch again?
6 S2 : As we measured in inch there, we have to measure this in inch, too.

The excerpt above showed that S2 fully understood that percentage is used to indicate relative size, not absolute quantity.

The aspect of the task that allowed students to perform in their own way without following any procedure stimulated S2' s originality and gave chance to confirm the principal concept of the percentage. After that, S2 calculated the percentage by comparing the length of each part with 5.

59 S2 : As this is 5 inches, 20% per 1 inch.

60 S2 : 1.5. This is 30%. 1.1. This is 22%. One point.. 1.15. Thirty.. Hmm.. How much would it be.. 2.3. 23%. 0.85. 17%.

As shown the excerpt above, S2 multiplied the length of each part by 20 to calculated the percentage by using the fact that 1 inch represents 20% in the given percentage bar. This implied that S2 was in the phase 3 since he was aware of the proportional relationship between the length of each rectangle and the entire length of the percentage bar. S2 proceeded to guess which transportation would correspond to each part and completed the percentage bar, as shown in *Figure 18*. *Figure 19* shows the original percentage bar with labels. These figures indicate that the percentages that S2 inferred basing on the lengths are approximate to the original percentage.

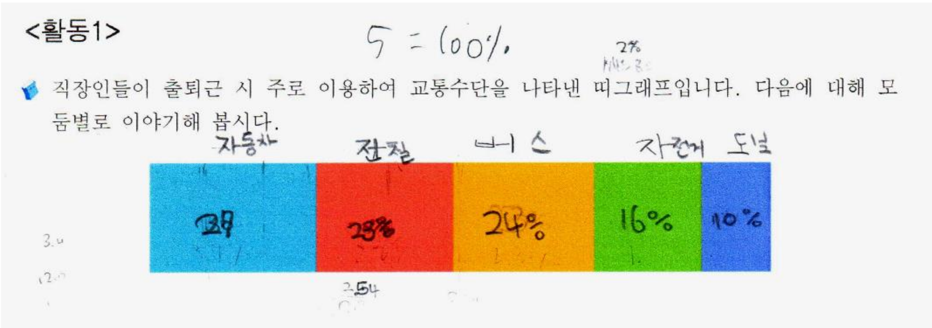

Figure 18. S2' s inference

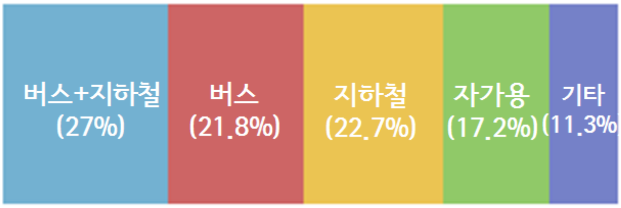

Figure 19. The original percentage bar labeled with percentages

The students started the subtask of matching each part with transportation by listing various means of transportation and looking back upon their personal experiences without regard to the problem situation and the percentages that they calculated. But soon, students took account of the task requirement and managed to integrate contextual knowledge. For example, S7 tried to infer from facts as the excerpt below.

4 S7 : The biggest part is bus.
5 S17: Why?
6 S7 : Basically, there are cities without a subway. So it seems that bus is the most common transportation.

S7' s argument on the basis of transportation infrastructure which other students could not take into account can be considered as unique. A possible explanation for this might be that the task allowed various possible answers and by doing so students could consider various cases related to the topic.

This speculation about correspondent means of transportation to each part followed the group discussion about the way to relieve traffic congestion on the basis of the percentage bar or pie chart completed by themselves. The group S8 belonged to supposed that car was the most used mean of transport by commuter (see *Figure 20*). As S8 in the excerpt below, gradually a large number of students engaged in the group discussion by integrating the percentages and context knowledge.

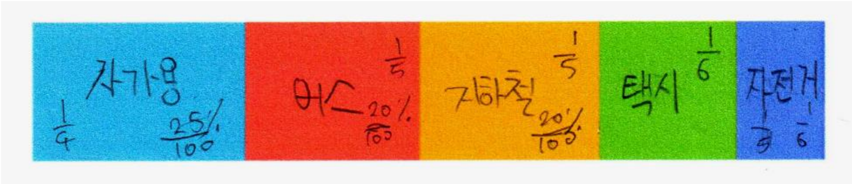

Figure 20. The percentage bar completed by the group 2

52　　S8 : As one vehicle holds many people

(…)

58　　S8 : Since there is no car on the road. Since there are just buses on the road, the traffic congestion will reduce.

After that, this group suggested the extension of bus route and the widening of the road as a method to improve traffic flow (see *Figure 21*). As these alternatives were based on the data of the graph and understanding of the traffic situation of real life, they could be considered as an evidence of the phase 3.

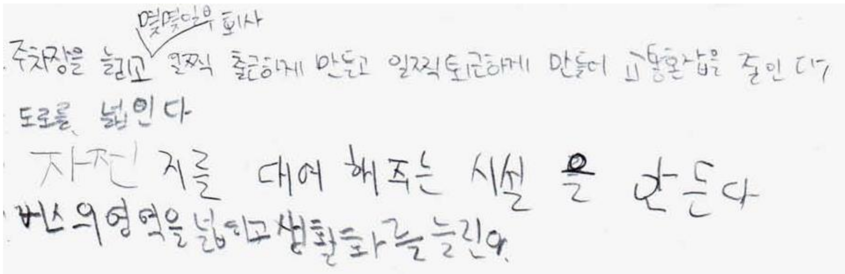

Figure 21. The suggestion of the group 2

The aspects of the task that dealt with familiar topic to students and allowed diverse answers stimulated students' fluency, and by doing so, facilitated appropriate interpretation and suggestion of alternatives by integrating the percentage values and the real context. This kind of task was able to give opportunities to understand the meaning of percentage bars and pie charts and to realize how useful they were in interpretation of real context and decision making.

CHAPTER 5

CONCLUSION

This study investigated whether creative tasks and instructional strategies designed for lessons on percentage bars and pie charts elicit students' growth in conceptual understanding and statistical thinking. The research questions related to students' understanding about the concepts embedded in percentage bars and pie charts, the phases of learning percentage bars and pie charts, and the role of mathematical creativity played in students' learning development. The conclusion which can be drawn from this study are as follows.

With respect to the first research question, it was found that the creative tasks and the instructional strategies helped to develop students' conceptual understanding of percentage bars and pie charts by fostering their mathematical creativity. Performing the creative tasks, students examined the claim which included a percentage value and constructed the percentage models in various and ingenious ways by utilizing the fraction, ratio, proportion, percentage and statistical graphs which they had learned before. They also interpreted the percentage bars and pie charts from various perspectives in light of the data and the context. In the meantime, students could explore deeply the following concepts embedded in percentages, percentage bars and pie charts: the comparative relation between the whole amount and the part amount; the qualitative attributes of the percentage related to time; the proportional; the proportional relationship among percentage, lengths of rectangles which represent the parts in percentage bar, the central angles of sectors which represent the parts in pie chart, areas of the rectangles and areas of the sectors; the integration of the data and the contextual knowledge in interpretation of graphs. Students revealed their fluency, flexibility, originality and elaboration throughout the instructional sessions in order to perform the tasks.

These results are in line with the research of Luria, Sriraman and Kaufman (2017), who suggested that encouraging students to approach problems with various perspectives and to involve coming up with many different solutions helps them to build conceptual understanding and to construct meaning, as well as to nurture creative thinking. If the tasks did not allow diverse approaches and representations using students' own method to perform the tasks, we believe that it would be hard to expect the exploration of the concepts involved in percentage bars and pie charts, as well as the promotion of fluency, flexibility and originality. In addition, the findings showed that students' conceptual understanding enhanced when students organized, generalized, justified and refined their ideas and their percentage models using their elaboration. This implies that convergent thinking was one of the important factor which enabled conceptual learning of percentage bars and pie charts, as claimed by Cropley (2006) and Lee (2017). Therefore, it is necessary to consider the graph construction and interpretation activities using creative tasks for the sake of conceptual learning of percentage bars and pie charts.

Secondly, the findings indicate that the creative tasks and the instructional strategies helped to develop students' statistical thinking related to percentage bars and pie charts by fostering their mathematical creativity. Initially, the students focused on individual data, but performing the tasks they could detect and represent the difference and the proportional relationship between a part and another or between the parts and the whole during the construction and the interpretation of percentage bars and pie charts. Students' improvement of the phase of learning percentage bars and pie charts indicated that students' point of view developed from the local view to global view. With regard to interpretation of percentage bars and pie charts, students showed the development of their statistical thinking. Initially, the students looked back on their personal experiences when interpreting the graphs, however, they gradually considered the data and appropriate contextual knowledge. With this, they could integrate them and consequently suggest alternatives. The creative tasks in which the starting situation and goal situation was open led students to see the given problem situation from various perspectives. Sharing their opinions with peers in discussion also helped them to approach from various angles. This attribute of the creative tasks and the aspect of the instructional settings fostered students' fluency and flexibility. Consequentially, students could develop their view from local view to global view. A possible explanation for these results is that the task was able to be solved with its underlying mathematical relation and structure, by using informal reasoning and representation. These findings corroborate the ideas of previous work in this field (e.g. Cropley, 2006; Lee, 2015; Luria, Sriraman, & Kaufman, 2017; Mann, 2006). In contrast to Friel, Curcio & Bright (2001) and Gal (1998), who stated that students have difficulties in reading between data and reading beyond data, the students could suggest alternatives without difficulty, which correspond to the level of reading beyond data. This result may be explained by the fact that the familiar topic to students facilitated the suggestion of alternatives. This is in good agreement with Curcio (1987), who argued that familiarity allows readers' graph interpretation.

The most interesting finding was that the creative tasks and instructional strategies provided various learning opportunities for students who differed in their levels. To a greater or lesser extent, every student engaged in the task and performed the task at their level. Most of students could create their own percentage models and interpret the data and the graphs by utilizing their background knowledge. Regardless of achievement level, students voiced a variety of ideas. These ideas promoted other students' creative thinking and resulted in novel and unique ideas. It can be explained that the creative tasks which allowed to use students' previous mathematical knowledge and personal experiences made this possible. Furthermore, the fact that there were various possible approaches, solutions and answers to the tasks and the permissive classroom environment for failure and discovery led the students, even who were with low achievement, to feel free to perform the tasks and participate in discussion actively, as claimed various researchers, for example, Luria et al. (2017), Nadjafikhah et al. (2012) and Sriraman (2005). As Shayshon et al. (2014) pointed out, the classes of the present day consist of more heterogeneous students comparing with the past. It is not easy to satisfy the needs of all the different levels of students in the same classroom. However, the finding of this study has implications for using creative tasks and instructional strategies in the classroom to meet the demand of students who vary in achievement level. Nonetheless, special care is needed when implementing creative tasks in classroom. In this study, the aspects of the creative tasks which were designed based on the familiar context to students and the permissive environment encouraged students' participation in discussion, whereas distracted students from the instructional goal. This confirms those of Li and Shen (1992) and of Watson (2011) who pointed out that students feel difficult to consider the contextual knowledge adequately when interpreting data. However, it could be overcome giving students enough time and guiding teacher's questions. Thus, it can be assumed that this attempt to instruct students in percentage bars and pie charts using context-based creative tasks with understanding of their limits and controlling that gave students good learning opportunities. These results empirically support the arguments of previous studies (e.g. Lee, 2015, 2016; Kaufman & Beghetto, 2009; Sriraman, 2017; Sternberg, 2017; Zazkis, 2017) which claimed that it is possible to educate average students using creative tasks in their regular classroom.

The current study aimed to determine that conceptual learning and statistical thinking related to percentage bars and pie charts can be promoted by implementation of creative tasks with appropriate instructional strategies through a case study. To do this, the four instructional sessions were administered to sixth-grade average students in their regular classroom. The results of this investigation show that the creative tasks and the creative instructional strategies support in enhancement of conceptual understanding and statistical thinking with regard to percentage bars and pie charts, and give diverse learning opportunities to students who vary in achievement level. Taken together, these findings implicate and provide empirical evidence that it is possible to teach average students new mathematical content knowledge, which is included in national curriculum, through design of creative tasks and their implementation with creative teaching strategies within regular classroom as daily practice. The current study is limited to the domain of percentage bars and pie charts with small number of students. Further research is needed to investigate its generalizability.

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국문 초록

학교 수학에서의 창의적 과제의 설계와 적용
– 비율그래프를 중심으로 –

수학적 창의성은 수학적 발견과 창조, 이해 및 의미 구성에 있어 중요한 역할을 한다. 학생들은 학교 수업에서 수학적 창의성을 이용하여 수학을 학습할 때 수학적 개념에 대한 이해가 풍부해지고 수학적 사고가 향상된다. 이를 위해 학교 수업에서는 학생의 수학적 창의성을 촉진하는 동시에 교육과정의 내용 지식을 개념적으로 학습하고 관련된 수학적 사고를 향상시킬 수 있는 창의적 과제가 적절한 교수 전략과 함께 사용되어야 한다.

본 논문에서는 창의적 과제 설계와 적용을 통해 비율 그래프 개념 학습과 비율 그래프와 관련되는 통계적 사고가 신장되는지, 그 때에 수학적 창의성의 역할은 무엇인지 살펴보았다. 이를 위해 비율 그래프, 통계적 사고와 통계 그래프 그리고 수학적 창의성에 대한 선행연구들을 분석하였다. 그런 다음 비율 그래프 개념적 이해와 사고력 신장을 촉진하기 위한 창의적 과제 개발 원리와 교수 전략을 도출하였다. 이를 토대로 창의적 과제와 수업을 설계 및 개발하여 실제 수업에 적용하였다. 수업 중 학생들의 반응을 수집하여 이를 비율 그래프 학습 단계에 따라 분석하였다.

연구 결과, 창의적 과제와 교수 전략을 적용한 수업은 학생들의 유창성, 유연성, 독창성, 정교성을 자극하여 비율그래프에 대한 학생들의 개념적 이해를 돕고, 국소적 조망에서 전체적 조망으로의 관점 전환을 촉진하는 것으로 확인되었다. 또한 이러한 수업이 다양한 수준의 학생들에게 서로 다른 수준의 학습 기회를 제공함을 확인할 수 있었다. 이는 학교 수학에서 창의적 과제와 교수 전략을 이용하여 일반 학생을 대상으로 교육과정의 내용을 지도하는 동시에 수학적 창의성 교육이

가능함을 시사한다.

주요어 : 수학적 창의성, 창의적 과제, 비율 그래프, 국소적 조망, 전체적 조망
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