교 육 학 석 사 학 위 논 문

A diffractive experiment on the use of digital technology

Moving bodies and the density of rational numbers

디지털 테크놀로지 이용에 대한 회절 실험: 움직이는 물체들과 유리수의 조밀성의 관계

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Abstract

A diffractive experiment on the use of digital technology

Moving bodies and the density of rational numbers

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Digital technologies define human life and thinking in contemporary society, yet the research on mathematics learning has not granted adequate power to them. This thesis aims to show how digital technologies configure a mathematical concept through material and discursive intra-action in a mathematics classroom. A group of elementary school students manipulate a number line, points and decimals in the screen with touchpad keyboards in a teaching experiment using GeoGebra. Drawing upon the inclusive materialism as a framework and through a diffractive experiment, this study highlight that the emergent meaning of the concept is entangled with the digital artifact. The results reveal that the students' discourse of rules and the hand gesture on and off touchpads co-constitute the meaning of density by evoking movement across the bodies in the activity. This finding corroborates the agency that the non-human acts upon the human and the significance of the physical mode of engagement in mathematical activities.

Keywords: Touchpad; GeoGebra; Density of rational numbers; Body; Inclusive materialism; Diffractive apparatus

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1 Introduction

Contemporary digital technologies have become more 'body-friendly' (Sinclair, 2014) with the widespread adoption of haptic input devices. In a haptic technological environment - such as that with touchpad or touchscreen - users interact with technologies through more direct body movements compared to the keyboard-and-mouse environment, and hence a more powerful status is granted to the user's body. This change in the interaction asks the studies concerning the technologies in mathematics classrooms to include our body as one of the central interests (Ferrara, Faggiano, & Montone, 2017).

On this basis, this thesis aims to examine the dynamic relationship between the body and the meaning-making process in mathematics classroom with digital technologies. We study a small group of grade 5 students exploring the meaning of density of rational numbers using multi-touch dynamic digital apparatus. We take the inclusive materialist perspective (de Freitas & Sinclair, 2014) to focus on the body movement in mathematical activity with digital technologies. Through a microethnographic analysis, we pay attention to the bodily movement toward the given technology and the development of meaning during the activity.

Our specific research question in this thesis is as follows: *In a multi-touch dynamic digital technology environment, how does the meaning of the density of rational numbers evolve in relation to the students' body movements toward the technology?*

2 Theoretical Background

2.1 Existing theoretical approaches to digital technologies

When theorizing the relation between the meaning-making process and technological environment in the mathematics classroom, two theoretical perspectives have been dominant in recent studies, namely the instrumental approach and the semiotic perspective (Drijvers, Kieran, & Mariotti, 2010).

At the heart of the instrumental approach lies the conceptual distinction between an artifact and an instrument. While an artifact is a given material object, an *instrument* is a psychologically constructed entity in which the artifact and the utilization scheme of it are entangled (Trouche, 2014). Thus, the studies based on the instrumental approach focuses on the specific technique of learner in order to investigate the scheme. Also, it is not only that the learner manipulates the artifact, but the artifact shapes learner's mathematical thinking as well. The meaning that the learner makes is considered to be closely related to the specific utilization scheme and evolves as he/she accomplish a given task.

The semiotic perspective, on the other hand, focuses on the semiotic potential of the artifact. A *Semiotic potential* is generated when there is a discrepancy between the personal meaning the learner makes from its use and the mathematical meaning of the experts. Then, an artifact functions as a tool of *semiotic mediation* when it is intentionally set up to mediate the mathematical meaning by the teacher (Bartolini Bussi & Mariotti, 2008). As the teacher orchestrates this mediating process, guiding the evolution of the learners' meaning, new signs - words, diagrams, gestures, and so on - emerge to signify the new meaning. An instrumental approach is fundamental to both identifying the semiotic potential of an artifact and designing appropriate tasks (Drijvers, Kieran, & Mariotti, 2010).

While these two perspectives together seize and integrate the material, social and psychological nature of mathematical activities with tools to some extent, they do not offer much theoretical room to integrate the role which the body plays (Drijvers & Ferrara, 2018). The inclusive materialism, based on a new materialist ontology in which the physical and the mathematical reside on the same ontological realm, reevaluate the role the material elements - including the body - play in mathematical activity. In the context of research concerning the multi-touch digital technologies, this approach started to get adopted in recent years to investigate the relationship between the hand movement and mathematical thinking (Chorney & Sinclair, 2018; de Freitas & Sinclair, 2017; Sinclair & de Freitas, 2014).

2.2 Embodied cognition and mathematics learning

2.2.1 What is cognition to be embodied?

Cognition lies at the heart of learning, and thus any educational enterprise. It is not just about knowing something, according to Shapiro (2011), but about “who we are as a human and how we think”. Therefore, it is indispensable procedure to contemplate about which perspective to cognition is of value, before engaging oneself into any sort of discussion of mathematics education. To this end, this review will begin by briefly examining two predominant perspectives to cognition: The *Standard cognitive science* and the *Embodied cognition*.

At the early stage of its emergence, cognitive science concerns with the human mentality to which its predecessor, Behavioral psychology, paid little heed. Cognitive scientists in 1960`s and 70`s conceptualize our mind as a symbol system. Later in 1980`s along with the swift development of computer system, the conception of the mind as computational system began to obtain wide consensus among cognitive scientists (이정모, 2010). These two are the main characteristic of the *Standard cognitive science*: *Representationalism* and *Computationalism*. The mind was considered to be an abstract information processor (Wilson, 2002), which processes(computes), internal symbols(representations) associated to external stimuli.

In early 90’s, however, there emerged a movement which lays a strong emphasis on the role of interaction between the human body and the environment in mental operation as an alternative to the standard cognitive science. Here, the mind is conceptualized as a phenomenon which is a connected continuum of the brain, the body, and the environment (이정모, 2010). It poses that “the cognitive processes are deeply rooted in the body’s interactions with the world” (Wilson, 2002). This movement is called the *Embodied cognition* or *embodied perspective to cognition*.

It seems natural at this point to ask whether the embodied cognition is indeed a better approach to human cognition than the standard cognitive science. Shapiro (2011) and many other researchers who are going to appear in this review believe so. One of the major shortcomings of the investigation in the standard cognitive science is that they are not concerned with the understanding the cognizer's environment; and they do not mind examining the interactions between the cognizer and the environment. However, ample amount of evidence, which are about to be presented soon, tells that the environment plays an essential role in cognition of a person. Instead, the embodied cognition research, in general, claims either that (a) organism's conception is necessarily determined by its physical body; (b) Cognitive processes occur without representation; or (c) the body is a constituent of cognition, rather than a mere cause.

The three modes of arguments mentioned at the end of the last section, are indeed what Shapiro (2011) called the 'three themes of embodiment'. The concept 'embodiment' is often interpreted in multiple different ways (Núñez, Edwards, & Matos, 1999). For example, Wilson (2002) said there are 6 different views to of embodied cognition; There is a 419-page-handbook entirely dedicated to show how diverse this interpretations could be (See Shapiro, 2014). There is no single uniform thread of theoretical discussion. To this end, Shapiro distinguished three general, most salient themes among the conducted research of embodied cognition, without which, he believes, the research of embodied cognition would be described only in incomplete manner. The themes also referred as hypotheses

The first theme is called *Conceptualization* theme. A study that support this theme would argue that the properties of an organism will necessarily determine, or rather constrain in a sense, the conception it acquires of the world. It sees “a connection between the kind of body an organism possesses and the concepts it is capable of acquiring” (Shapiro, 2011, p. 112). Thus, if two organisms were heteromorphic, their understanding of the world would be different since their given frame of understanding is not the same. Lakoff and Johnson (1980) were the leading advocates among the others. They argued that metaphors, while grounded on the everyday physical experience constrained by properties of the human body, form the basic abstract concepts which later would form complex abstract concepts.

The second theme is *Replacement*. Replacement studies are explicit in manifesting their anti-representational, anti-computational stand point. They envision cognition as “emerging from continuous interactions between a body, a brain, and a word” (Shapiro, 2011, p. 156). In favor of the hypothesis, dynamical systems theory is preferred, to computational theory, to account for cognitive processes. By such theory, supporters of Replacement believe that they can get by without representations or computation of them.

Constitution is the third theme of embodiment. It maintains the environment, including the body, plays a constitutive role rather than a causal role in cognitive processing. In the standard cognitive science, body had to be satisfied with its role as a primer of cognition, since it was only regarded as a cause of physical stimuli, or perceptions whilst cognition occurs ‘inside’ the body. However, Constitution hypothesis argues that the body is a constituent of cognition, i.e. we cannot explain cognitive phenomena without discussing the role the body plays.

This distinction is neither exhaustive nor exclusive. However, since this is well elaborated meta-framework for research in embodied cognition. I will adopt this as the framework for the analysis of this review.

2.2.2 Research trend in mathematics education

This review selected articles which meet the following two conditions. One, articles have to be published in SSCI (Social Science Citation Index)-indexed journals, whose primary concern is mathematics education. In other words, I investigated: Educational Studies in Mathematics (Springer), Journal for Research in Mathematics Education (NCTM), International Journal of Science and Mathematics Education (Springer), Mathematical Thinking and Learning

(MTL). Secondly, articles must focus on the three things simultaneously: (a) Mathematics education, (b) mathematics learner, and (c) the role of the body in cognition. For example, if a study had concerned mathematics teaching in terms of embodied cognition (e.g., Abrahamson, 2009; Font, Bolite, & Acevedo, 2010; Yoon, Thomas, & Dreyfus, 2011), it was not included in the analysis.

As a result, 36 articles were selected and analyzed. Several articles form non-SSCI journals and published books were included if necessary. Among them, only 6 are Conceptualization research and the rest belongs to Constitution category. Surprisingly, and unfortunately, none of the selected article could have been classified as Replacement research even in the most generous sense. Therefore, only the analyses on Conceptualization and Constitution research will be presented below. Under each theme of research, the most prominent trend will be presented along with a brief commentary.

Embodiment as conceptualization

Conceptualization studies in mathematics education appear to find their theoretical ground in *Conceptual metaphor theory*, with no exception. Conceptual metaphor theory is first proposed by linguist-philosophers, George Lakoff and Mark Johnson (1980). It illustrates how our concepts are grounded onto our basic physical experience shaped by our body. This theory was later imported into the field of mathematics by Rafael Nunez (Lakoff & Núñez, 1997, 2000; Núñez et al., 1999), arguing that even the complex, abstract mathematical knowledge – which seems to be the opposite of the human, bodily entity – is indeed grounded in the daily, bodily experience. For example, Lakoff and Nunez (2000) show that image schema such as ‘Container schema’, which is related to the sense of in and out, structures the set theory of mathematics using ‘Metaphorical mapping’. By continuously linking and building up metaphors, they contend that even Euler’s equation is, in basic level, grounded in human body. Bazzini (2002) built further up on these works, claiming that embodied activities in mathematics classroom should be socially approved by peer students and teachers, i.e. ‘legitimatized’, with the help of technological instruments.

No matter what kind of success had conceptual metaphor theory enjoyed in linguistics, however, that in mathematics faced multiple different criticisms from many researchers. Among them, Schiralli and Sinclair (2003) criticized that the conceptual metaphors fail in multiple was to be logically sufficient to account for or even simply associate with the entire mathematical knowledge. Thom and Roth (2011) pointed out that it still remains unexplained how linguistic representation emerges from embodiment, and attempted to provide an alternative theory which allegedly bridges the gap.

In sum, Conceptualization research in mathematics education is outdated and seems to be sterile. The Lakoff & Nunez’s book in 2000 was admittedly so sensational that many researchers were intrigued and provided his or her own comments on that. However, since then there has been no succeeding research published in SSCI-journals in mathematics education, except for Bazzini (2002), for over 15 years. Only related research other than Bazzini's, was David Tall's research (e.g., de Lima & Tall, 2008; Kidron & Tall, 2014; Tall & Katz, 2014). However, it turned out that these studies, which on the surface appears that they are based on conceptual metaphor theory, misinterpreted the concept of 'embodiment', ending up failing to provide no theoretical contribution to the body of Conceptualization research. This implies either researchers are no more intrigued by this theory; or the quality research based on this theory is not superb enough to be featured in these, so called, 'premium' journals.

Embodiment as constitution

Most of the Constitution research in mathematics education is dedicated to study human gestures. Since Constitution research dominates the field of embodied cognition research in mathematics education, it is safe to say that most of them are focused on the study of gestures. Gesture is considered as an essential part of semiotic resource, communication and thus of cognition (Sfard, 2009). Radford (2009), devising a concept of 'sensuous cognition', tried to present an alternative to standard cognitive science. In this framework, cognition is irreducible to intangible concepts, instead comprises speech, gestures, and artifacts. Radford is not alone in conceptualizing gestures with the other semiotic modalities. Gesture is, in many case, discussed along with speech (or prosodies), inscriptions, artifacts (or tools) and these collectively referred as 'semiotic bundle' (Arzarello, Paola, Robutti, & Sabena, 2009; Maschietto & Bartolini Bussi, 2009). The body of research on gesture continues to grow.

On the other hand, building on the effort based on orthodox conception of embodied cognition, there exists a few, yet remarkable theoretical attempts to better conceptualize the cognitive phenomena through materialistic approach.

Wolff-Michael Roth is one of the leading researchers of this movement. Relying on geometrical concepts and shapes, he theorizes mathematics cognition in light of philosophical phenomenology. Drawing on philosophers such as Henry, Merleau-ponty and Marion, he poses the concept 'mathematics in flesh' (Roth & Thom, 2009; Thom & Roth, 2011). Here, increased amount of attention is imposed upon the body itself, more specifically on its immanent properties and the invisibles (Roth & Maheux, 2015). Ultimately, in materialistic perspective, the distinction between individual knowing and mathematical knowledge loses its power, giving way to let learners and the curricular material grow together (Roth, 2016).

Meanwhile, Elizabeth de Freitas and Nathalie Sinclair lead another line of the movement with different philosophical stance. They primarily draw on French mathematical philosopher Gilles Chatelet for the concept 'virtuality' and take more materialistic stance which assumes learner's body as a (becoming of) dynamic mathematical material (de Freitas & Sinclair, 2012, 2013, 2014).

Recently, while Roth agree on the usefulness of Chatlet's 'virtuality' (See, for example, Roth & Maheux, 2015) and the view that agency is distributed not only to mind or body, but also to other worldly materials (See, for example, Roth, 2016), each of their work contains explicit criticism to the other, and the two squads never cooperate with each other.

2.2.3 Prospects and challenges

As I have presented in this review, especially in covering the research with Constitution theme, mathematics education research based on embodied cognition not only overcomes the weaknesses of standard cognitive science but promises an open possibility for further theoretical development. Among them, I insist, we should take careful heed of, and put more effort to the recent materialistic approach. This is not a mere philosophical discussion remote from the practice of mathematics education. First and foremost, it not only breaks down the distinction between the inside and the outside, but it actually connects the two historically-long-divided realms. In addition, it has explanatory power to account for inner 'construction' without appealing to representations by illuminating source of out intentionality, thus is expected to resolve learning paradox that constructivism has long fought against.

However, research of embodied cognition in mathematics education also faces immediate challenges. First, like the other embodied cognition research in cognitive science, it must embrace empirical result from cognitive neuroscience. Neural evidences are accumulating on mathematics cognition, and some researchers endorse its educational values (e.g., Campbell, 2006, 2010; Lee & Ng, 2011; Van Nes, 2011). Second, current rather theoretical discussion must proceed to the practical realm of mathematics teaching. Only a few studies has been considered as concerning with teaching (e.g., Alibali & Nathan, 2007; Yoon et al., 2011). In order to harvest fruitful results in real education practices, we have to resolve these challenges.

2.3 Theoretical framework: Inclusive materialism

This study takes 'inclusive materialism' (de Freitas & Sinclair, 2014) as the primary theoretical lens to address the phenomenon of mathematics learning. Criticizing the existing approaches of embodiment in mathematics education, de Freitas and Sinclair (2014) attempted to theorize the body and mathematics learning in the new materialist light, specifically that of Karen Barad (2007, 2011). As a result, they have proposed the inclusive materialism, an onto-epistemology for mathematics classrooms based on the new materialist thinking.

Inclusive materialism is acknowledged for successfully appreciating the various human and nonhuman material aspects of learning (Ferrara & Ferrari, 2017; Roth, 2016), distancing from the anthropo-centric views on technology use in the classroom. For this reason, it is specifically adopted in some studies to illuminate the power of dynamic multi-touch digital technologies in mathematics learning (e.g., Chorney & Sinclair, 2018; Sinclair & de Freitas, 2014).

This section first reviews the relevant post-humanist background of Barad's new materialism. Then, from the inclusive materialist point of view, we discuss how the body is reconceptualized; how meaning of mathematical concept is bodily and thus material; and thereby how mathematics learning is reimagined.

2.3.1 New materialism and post-humanist ideas

What is 'new' in the new materialism? As Coole and Frost (2010) write, the key characteristic of new materialism is decentering from the anthropo-centric ontology, or in other words, post-humanist approach. Previously, in the humanist social science, there existed a clear ontological distinction between the human and the nonhuman. Based on the Cartesian dualist thinking, matter, including the body is distinguished from human mind for that matter has a physical, tangible dimension and (human) mind consists of subjectivity and free-will (Coole & Frost, 2010). The human was naturally considered as superior or located at the center of various social inquiries, whereas the nonhuman was marginalized.

New materialism, however, like the other post-humanist ideas in general, criticizes this dichotomy and accounts for how these two realms are ontologically entangled and equivalent, instead of being distinct. Fox and Alldred (2015) concisely summarize this flat and relational ontology which the new materialism heavily owes to Deleuze and Guattari (1987). First, new materialism considers both human and nonhuman entities as relational, "having no ontological status or integrity other than produced through their relationship to other similarly contingent and ephemeral bodies, thins and ideas" (Fox & Alldred, 2015, p. 401). 'Assemblage' is a useful concept in the new-materialist discussion in that regards. In contrast to set of individual elements, which only conceptualizes the inert and immovable aspects, assemblage "captures(s) the structural arrangements of human and non-human that sustain a given social-material configuration or a kind of interaction" (Ferrara & Ferrari, 2017, p. 23), which "develop[s] in unpredictable ways around actions and events" (Fox & Alldred, 2015, p. 401). Assemblages, just like territories, are constantly territorializing, deterritorializing, and reterritorializing by the relational flows. Due to its ever-changing nature, assemblage is always indeterminate and in the process of becoming, rather than being something definite.

Second, new materialism attends to the agency of nonhumans and the consequential differences, instead of who or what caused the differences. Unlike the humanist social science, agency, the capacity to affect does not belong to only humans any more, also to nonhumans.

Then, what is nonhuman? What does it mean for nonhumans to have agency in the post-humanist idea? This may sound counter-intuitive at first. Do material objects have the consciousness and intentionality as we humans do? Do they autonomously cause effects? The answers to both questions are no. Sayes (2014) surveyed a wide range of publications positioned in Actor-Network Theory, a post-humanist social theory which shares the same vein with new materialism, to examine such questions. 'Nonhuman' in this context could refer to (a) a condition for the possibility of human society, such as machines, computers or palaces; (b) mediators which not only convey the actions and forces among actors, but also continually modify the relations between actors; (c) members of moral and political associations like sensors and ignition control technology in the case of seat-belt enforcement; or (d) the gathering of actors. Considering that, the claim that nonhumans have agency does not stipulate that they have the consciousness or intentionality by which it wields its agency as in the case of humans. Rather it is:

> ... a complicated but nonetheless minimal conception of agency. It is minimal because it catches every entity that makes or promotes a difference in another entity or in a network…[O]ne need only ask of an entity '[d]oes it make a difference in the course of some other agent's action or not? Is there some trial that allows someone to detect this difference?' If we can answer yes to these two questions, then we have an actor that is exercising agency – whether this actor is nonhuman or otherwise (Sayes, 2014, p. 141).

We will later discuss the methodological implications of this ontology in the methodology section.

2.3.2 Inclusive materialism and mathematics learning

The primary aim of Inclusive materialism is to theorize the relationship between mathematics and the material world(de Freitas & Sinclair, 2014). de Freitas and Sinclair (2014) critically reviewed existing works on the embodiment in mathematics learning, which purportedly encompass the bodily, hence material dimension of mathematics learning. However, the existing studies either reduce mathematics learning onto the sensory-motor plane; or they depict learner as a subjectivity who is capable of constructing knowledge only with the autonomy and intentionality. They were problematic in that the former does not account for the social and environmental factors and the latter is too anthropo-centric.

Therefore, instead of taking body as purely physiological matter or learner as isolated willful subjectivity, the inclusive materialism reconceptualize body as an assemblage (*body-assemblage*). Following the new materialism in feminist studies, it proposes the body be "an assemblage of human and non-human components, always in a process of becoming that belies any centralizing control"(de Freitas & Sinclair, 2014, p. 25). By doing so, the boundaries are rather flexible and porous, and agencies are distributed across the environment of the mathematical activity. By proposing such extensive definition of the body, they suggest that we pay delicate attention to the matters and their material configuration - the physical bodies, utterance, artifacts, signs, representations, and their dynamic relationship - in mathematical activity. The concept of body is extended beyond the physical skin, and could include learner, the physical body, technological artifacts, language and even concepts. However, concept and matter seem to be far apart from each other if not ontologically disjoint. In fact, the incorporation of (mathematics) concept in the body-assemblage is what distinguishes inclusive materialism from the other post-humanist assemblage theories (Ferrara & Ferrari, 2017). to understand what a mathematical concept is and how it is material, we must understand Karen Barad's quantum ontology, upon which its materiality is drawn.

Barad (2007, 2011) takes a step further here and take an epistemological turn. Building upon this relational ontology and the Copenhagen interpretation of Niels Bohr to the famous Double-slit experiment, she argues the fundamental indeterminacy of knowing and its entanglement with matter.

> The objective referent for concepts like ‘wave’ and ‘particle’ is not a determinately bounded object with inherent characteristics [. . .] but rather what is called a phenomenon – the entanglement/inseparability of ‘objects’ and ‘apparatus’ (which do not preexist the experiment but rather emerge from it) (Barad, 2011, p. 142).

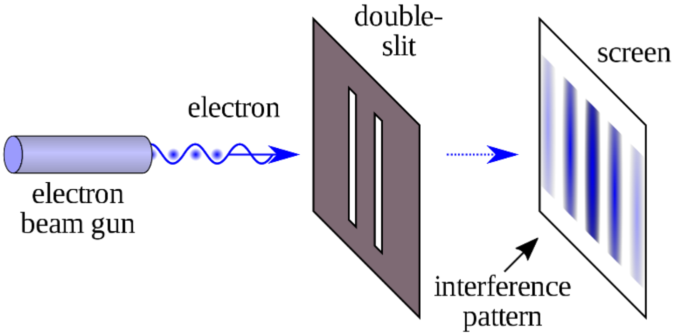

Figure 1. Double-slit experiment

What does the material configuration have to do with the meaning of a mathematical concept? The inclusive materialism takes an ontological turn by which the meaning is not abstract or disembodied, but rather material and concrete. Thus, the meaning of a concept is ontologically entangled with the specific physical arrangements. In a mathematical activity constructing a circle with a compass, for instance, it is not the case that the compass is a disposable medium for some transcendental, determinate a priori meaning of the concept 'circle'; the meaning of circle is performed through the process where the circle and the compass are assembled. In other words, meaning emerges through mathematical activity and entails all the material specificity implicated in the activity.

In terms of research, the culmination of the inclusive materialism lies in capturing the emergent and creative aspect of the meaning-making process in mathematical activity. Drawing on the historical insights of a French mathematical philosopher Châtelet (2000) regarding the mathematical inventions, de Freitas and Sinclair (2014) deem the mobility critical for such inventiveness. Moving the given (mathematical) objects in an activity or attending how they are materially determined in terms of the movement, - rather than logically - configures a new physical arrangement latent in the material environment. Therefore, the movement actualizes the potential mobility of mathematical objects is a major source for creating a new material assemblage, and hence a new meaning emerges. In that, the design of mathematical activity should encourage the learners to attend to the mobility, instead of the rules of the given logical structure.

3 Research Context and Method

3.1 Methodology

3.1.1 New materialist insights for research methodology

Since the inclusive materialism calls for an ontological turn, not just an epistemological one, we must rethink research methodology as well. What does it mean to conduct research on a phenomenon based on a new materialist onto-epistemology and how are we supposed to do it? In this section, I first review Fox and Alldred (2015) in details to get an insight for new materialist research methodology. Then I specifically chose 'diffractive experiment' as the methodology for this study and explain how it produces research-assemblage.

Fox and Alldred (2015) dwell on the questions of the new materialist social inquiry and discuss a wide range of implications. Placing emphases on the concept *assemblage* and *affect* and *territorialization* as the key characteristics of new materialism, as briefly discussed in Section 2.1, they suggest the five implications of new materialist ontology for research.

First, new materialist research should be concerned with affect in general, not only with human agency. Also, the unit of analysis must be the assemblage, not human actors. Smythe and her colleagues (2017) addressed the similar point and argue that the question of a new materialist educational research is no longer "who is learning what", but "what is becoming", and "what is new" (Smythe et al., 2017, p. 116).

Second, new materialist research should focus on the difference and indeterministic nature of assemblage. We must pay our attention to singular affects as well as we did to aggregative affects in the humanist approach. This point aligns with de Freitas and Palmer's (2016) approach where they call for the shift of focus from merely being engaged in (mis)recognizing the pre-determined concepts toward the process in which children and concepts are entangled.

Third, new materialist social inquiry should be sensitive to the possibility of assemblages comprising the elements from the different levels - micro, meso, or macro - and that the affective flows among them are dynamic rather than unidirectional like 'top-down' or 'bottom-up'. The multi-layered nature of an assemblage is primarily due to the dissolution of the dichotomy between the material(individual) and the cultural(social) in new materialism. To elaborate on the dynamic, multidirectional nature of the affective flows in assemblage, de Freitas and Sinclair liken an assemblage to a knot of many different threads in which there is "no inside or outside, no beginning or end - one is always in the middle of the knot, always moving along its various threads" (2014, p. 34).

Fourth, new materialist investigation should seek its utility at a micro(political) level. Although we should be open to the possibility of inclusion of larger macro element in an assemblage, a new materialism requires us to track the molecular flows rather than molar flows. It is because the study of bodies and assemblages is "the study of movements of desire, power, resistance and becoming other" and those molecular flows "signal moments of deterritorializations, becomings and lines of flight" (Fox & Alldred, 2015, p. 403) with which we must be concerned with as discussed in the second point. de Freitas and Palmer (2016) also suggest classroom data be studied with focuses on children's micro-perception and bodily movement with regard to the evolution of concepts

Fifth, and most importantly, new materialist social inquiry must see researcher and data as a knowledge-producing assemblage, instead of independent entities. From the new materialist view, research is "a territorialization that shapes the knowledge it produces according to the particular flows of affect produced by its methodology and methods" (Fox & Alldred, 2015, p. 403). Assemblage is a machine to produce something because of its affects (Deleuze & Guattari, 1987). In that regard, research as an assemblage, or simply *research-assemblage* is a collection of machines where:

> [T]he affects in a 'data collection machine' apprehend aspects of an event, and act on these to produce an output called 'data'. An 'analysis machine' processes this data according to rules of logic, deduction or inference to produce 'findings' in the form of generalities or summaries(Jackson & Mazzei, 2013). A 'reporting machines takes these outputs of data analysis and creates knowledge products for dissemination: theory, policy and practice implications and so forth (Fox & Alldred, 2015, p. 403 emphasis added).

Based on this discussion, Fox and Alldred (2015) suggest the following framework for new materialist methods.

Research design	• Attend not to individual bodies, subjects, experiences or sensations, but to assemblages of human and non-human, animate and inanimate, material and abstract, and the affective flows within these assemblages. • Explore how affects draw the material and the cultural, and the 'micro', 'meso' and 'macro' into assembly together. • Explore the movements of territorialisation and de-territorialisation, aggregation and disaggregation within the assemblages studied, and the consequent affect economies and micropolitics these movements reveal.
Data collection	• Identify assemblages of human and non-human, animate and inanimate, material and abstract, cutting across what are traditionally considered 'micro' and 'macro' levels. • Explore how elements in assemblage affect and are affected and assess what bodies and other things do: the capacities these affective flows produce. • Identify territorialization and de-territorialization, and aggregating and singular flows within assemblages.
Data Analysis	• Take the assemblage as the primary focus for analysis, incorporating both nonhuman elements and human relations. • Explore affect economies and the territorializing and de-territorializing capacities produced in bodies, collectivities and other relations in assemblages. • Examine how flows of affect within assemblage's link matter and meaning, and 'micro' and 'macro' levels. • Acknowledge the affective relations within the research-assemblage itself.
Research reporting	• Be reflexive, recursive and rhizomic • Offer deterritorializations and lines of flight to event assemblages and affects • Draw research audiences into the research-assemblage, to contribute their own affects and capacities to its affective economy and micropolitics

3.1.2 Diffractive experiment

Having adopted the insights and suggestion for a new materialist social inquiry, therefore, I deem 'diffractive method' to be a suitable research methodology to examine a mathematics classroom. Barad (2007) provides the foundation of this methodology as she proposes the entanglement between matter and meaning, inspired by Haraway (1997).

> Neither discursive practices nor material phenomena are ontologically or epistemologically prior. Neither can be explained in terms of the other. Neither is reducible to the other. Neither has privileged status in determining the other. Neither is articulated or articulable in the absence of the other; matter and meaning are mutually articulated (Barad, 2007, p. 152).

This perspective calls us for a new way of understanding what knowing is. Here, knowing is redefined to be a material-discursive intra-activity. As Barad writes, knowing is “a matter of part of the world making itself intelligible another part of the world” (Barad, 2007, p. 185). It is not identifying differences from of between bodies to produce codes and categories. Rather, it is a process of interference and overlapping.

Diffraction is a methodological means to oppose the humanist methodology of 'reflection' by which researchers look for similarities among independent bodies. Instead diffraction suggest to study “how differences get mad in such process of interference and the effects that differences make”; and “what is excluded and how these differences and exclusions matter” (Barad, 2007,

p. 30). Diffractive methods directly address the question of doing research where discourse and matter are mutually constituted in producing knowledge has its upper hand especially in considering the agency of the material in the production of knowledge (Lenz Taguchi, 2012).

According to Hill's (2017) review, there are two types of diffractive methods in education: diffractive reading and in situ experimentation with a diffractive apparatus. The former aims to read data through various theoretical lenses to generate new insights and questions (e.g., Chorney, 2014). The latter, as de Freitas (2017) calls *diffractive experiment*, concerns the new phenomenon produced as a result of the interference between the apparatus and environment. This is based on Barad's (2007) understanding of experiment, as we discussed in Chapter 2, where the experimental apparatus (the double slit) and object (the photon) are intrinsically entangled in producing meaning of concept (light, or what light is). Designing such diffractive apparatus in educational setting help us better understand the intra-action between matter and meaning (de Freitas, 2017).

Therefore, the diffractive experiment is appropriate especially in examining mathematical activities with tools. This is because any apparatus produces an effect where meanings of concepts are entangled with the physical apparatus and intrinsically indeterminate, instead of merely being a mediator of learning or a tool that learners use to acquire particular concepts (Smythe et al., 2017). Smythe and her colleagues (2017) also made clear that it is not to study the effect of the apparatus on learning that separate the students from the apparatus and study students' acquisition of concepts. Through diffractive experiment, we can observe how a concept is created and re-created through the children's gesture and touch in this experiment, and we explore the non-human power and performativity that traverses and sustains the learning assemblage in this context.

3.2 Participants and activity

Participants involve a group of grade 5 students in a suburban elementary school. Through a preliminary oral interview, four students were selected (A, B, C, D) who had exhibited mid-level achievement in mathematics previous semester and, at the same time, displayed a low-level understanding of density in response to a series of questions based on Vamvakoussi and Vosniadou's series of cognitive studies (Vamvakoussi & Vosniadou, 2004, 2007, 2010).

Figure 2. Overview of the classroom setting

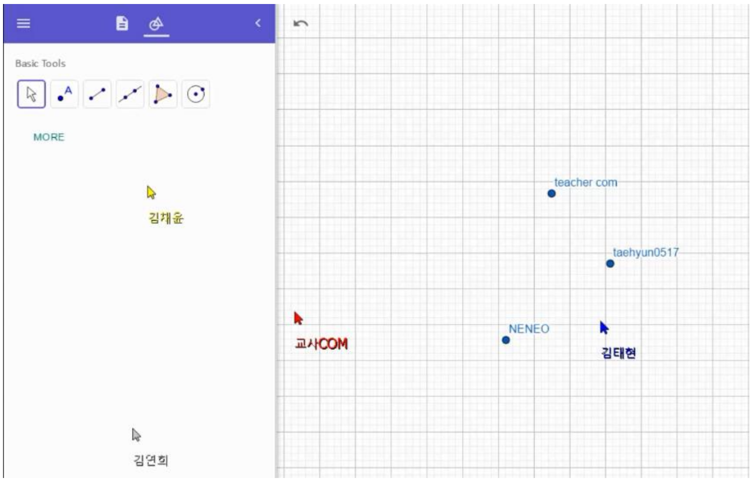

Figure 3. Simultaneously connected collaborative GeoGebra environment

The activity is a game called 'point-and-name,' a part of a teaching experiment designed by the author to explore the number line with regard to the density using GeoGebra, which spans over three lessons. Students sat facing each other and were given touchpad keyboards that were connected to one GeoGebra environment on a shared monitor screen simultaneously [Figure 3 and Figure 4].

Figure 4. Students with touchpad keyboards

In the beginning, on the screen was shown a line with three round-shaped points on it and each named 0, 0.5, 1 respectively. On the background lied grid lines: In between two adjacent thick grid lines, four equidistant thin lines were placed and divided the space into five parts (Figure 5).

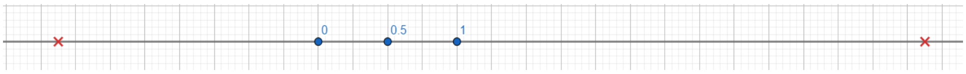

Figure 5. The initial setup of the activity

As students zoom in, the gaps across the grid lines would appear to widen, and new grid lines come froth filling the gap. Split into two teams (AB vs. CD) and taking turns, each team was asked to place a point anywhere on the line in between the two Xs on left and right; name it with a rational number in reference to the previously placed points and their names. The game ends when a team fails to place a point or give an appropriate name to their points. One of the researchers assumed the teacher's role without offering any help but the technical ones during the activity.

3.3 Data collection

For fine-grained qualitative analysis, the researcher collected three types of data. First, video data of the students' body movement. As we have discussed in Section 2.3, from the inclusive materialist perspective, it is of the utmost importance to observe the virtual mobility and material effect in a mathematical activity which includes the body movements and pattern. To capture every single moment, the research has installed five different high definition cameras (See Figure 2 for the camera configuration). Two cameras were looking at each team, capturing individual or team level movements. The other two cameras looked down upon the touchpads, recording the students' hand movement on the touchpads (Figure 4 was captured by one of these two cameras). The remaining one camera recorded the overview of the classroom in case the rest of the cameras miss any macro scale details (Figure 2 was captured from this camera's video).

The second type of data is an audio data of the students' discourse. The researcher placed voice recorder on the desks to record students' discourse in a close distance. In addition, to not miss the teacher's voice who were relatively far away from the recorder, the researcher recorded teacher's speech primarily using the laptop's recording application. Those two audio files were later enhanced and merged so that the voices of all participants were clearly intelligible.

Third, the video data of screen during the activity was also collected. Not only the movement of the human bodies, but also that of non-human bodies are as equally as important. In this teaching experiment, the screen is major playfield for most of the non-human bodies, in other words, the mathematical objects. Therefore, the researcher screen-recorded the GeoGebra environment and captured all the movements of cursors and screen itself, such as panning and zooming. Later this date was calibrated with the other two data and provided a holistic view of the mathematical activity.

3.4 Analytical framework

In mathematics education research, those who wish to focus on the body movement usually had turned to anthropological methods. Miroethnographic analysis(Streeck & Mehus, 2005) is one of them. This method has been preferred by the researchers who studies mathematics learning from the interactionist perspective focused on the body movement occurring in micro scales or a relatively short period of time. Specifically, those who studies gestures and mathematical conceptualization reckon this as a suitable research methodology. Nemirovsky, Kelton and Rhodehamel (2013), in there rational for the microethnography, wrote it is " the study of multimodal strands of activity over short periods of time" which traces include "talk, gesture, facial expression, body posture, drawing of symbols, manipulations of tools, pointing, pace and gaze." Also, it attends "the cultural and lived circumstances of the studied events, striving to achieve the thick descriptions" (p. 385). In short, microethnographic analysis has been effective for in-depth examination of the interplay among speech, gesture and artifacts in a mathematical activity.

Thereby, we pay special attention to the following elements to understand the interaction between the learners' physical body and the digital apparatus: (a)the hand motion on touchpads, (b)the cursor movement on the screen, (c)the movement of screen itself (e.g. zooming in/out, panning), (d)the speech of students, (e)the hand gesture in the air. We must watch the hand motions on the touchpad since they are the primary means of manipulating the apparatus for students. The cursor and screen movements are also important for they embody the learner's attention and its shift. Furthermore, in a multi-touch digital technology environment, the gestures in the air are specifically important not only because they communicate meanings coupled with the speech but also they are part of a gestural continuum together with the gestures toward the haptic input device, preserving senses from one to the other (de Freitas & Sinclair, 2017).

On the cognitive side, a series of studies had identified various aspects of understanding on the density of rational numbers(Vamvakoussi & Vosniadou, 2004, 2007, 2010). Vamvakoussi and Vosinadiou's result suggests that a key cognitive action in developing the meaning of density is to find a smaller unit than the least common unit of numbers in an interval in order to conceive a number with the smaller unit which would locate in somewhere 'in-between' the existing numbers (e.g., ¾ in the interval (1/2, 1)). Understanding that this action could be implemented in any interval of two rational numbers is considered as indicating a sophisticated development of the meaning of density.

4 Result and Discussion

Our major findings are drawn from three notable sequences of the episode. As students carried out the activity through these sequences, the meaning of density had become more and more sophisticated. Each sequence is thematized by a significant development of the meaning of density that had come to the surface. We present each sequence through transcription or a brief depiction that highlights key events, then follow up by discussion.

4.1 Episode

4.1.1 Sequence 1: “You must follow the rules.”

Shortly after the beginning, students started to collectively establish a set of implicit rules of engagement (*First-rules*) and eventually all began to abide by them to play the game:

First-rule 1. *A new point must be 0.5-away from an existing point and named accordingly*

First-rule 2. *A new point must be placed on a thick grid line. The highlight of the first sequence is presented below.*

1 C: (*C drags his index finger to the left and then gently taps*.) [C’s cursor moves to the left side of '0', and a point appears on the left side of the line.] [Figure 6]

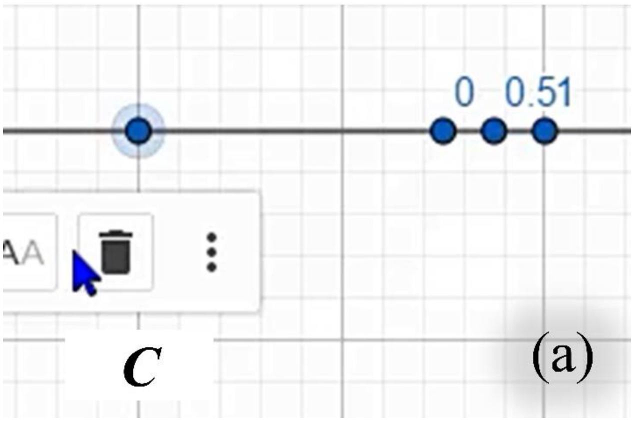

Figure 6. C pointing on the left side of 0

2 D: Why are you putting it there?

3 C: You don’t like it?

4 D: No. You can’t just do this in any way you want.

5 C: Then, how?

6 D: You must follow the rules. See, it goes from 0, to 0.5, then to 1. What do you think comes next?

7 C: 1.5, it is. Let me throw this away. (*C right-clicks with his middle finger*) [C opens the menu of the point and deletes it.]

8 D: (*D spread her index and middle fingers away on the touchpad, then double-taps the pad, and drags with her index finger to the right.*) [The screen zooms in. The

point appears on the line and is dragged right to the place where a thick grid line adjacent to '1' meets the line.] [Figure 7]

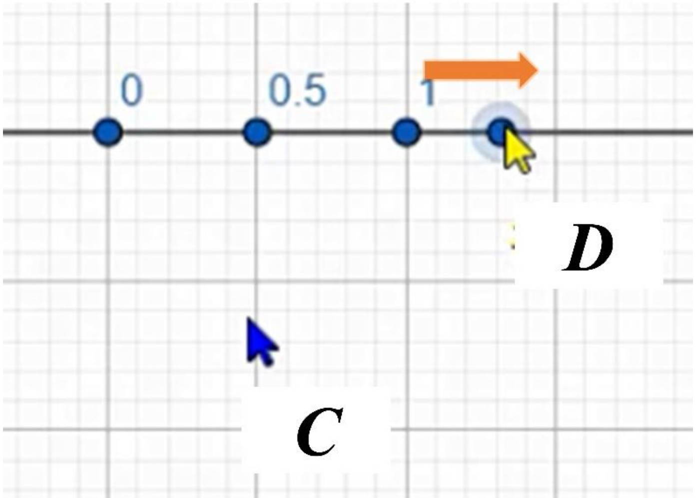

Figure 7. D sliding the point to right

9 C: Now you name the point.

10 D: (*D tabs and then types '1.5'*)[The name '1.5' appears above the point.][Figure 8]

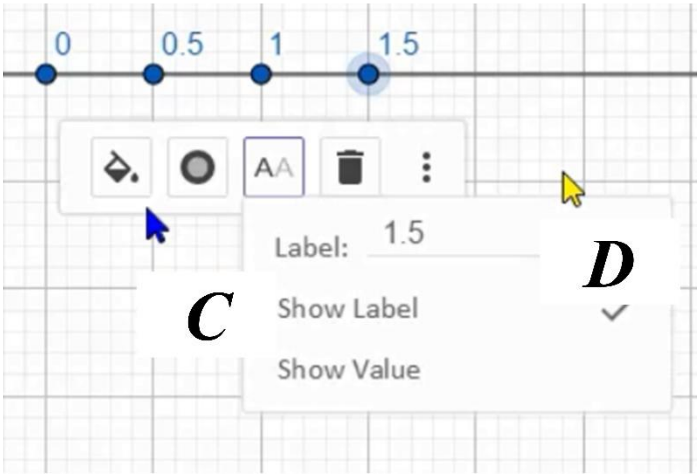

Figure 8. D naming the new point '1.5'

After Team CD pointed ‘1.5’, A began to led Team AB’ turn. A zoomed in, and ‘0.5’ and ‘1.5’ got off the thick grid line because of that. A started counting the number of thick grid lines - including newly appeared ones - between '1' and '1.5' with his cursor. After expressing confusion about how far they should zoom in, A zoomed out, coming back to the scale at which they were. Then A placed '2' on a thick grid line as far away as the distance between '1' and '1.5'. From then on, every new point ended up on a thick grid line, 0.5-away from the point either at far-right or far-left, subsequently producing '2.5', '3', '3.5' and so on [Figure 9].

Figure 9. The Screen Capture at the end of Sequence 1

Although the students did not articulate the rules explicitly, we could infer the First-rules from their discourse and actions in this sequence. Line 6 and Line 10 are good examples where they defined the way the game should be played and complied. These rules suggest that 0.5, the least common unit of numbers given in the initial setup, remained indivisible to students and they did not conceive any smaller unit by which they could play the game. The multiples of 0.5 (e.g., 0, 0.5, 1.5, …) are treated as if they were successive.

We must note that these rules are neither pregiven by the teacher nor randomly established by the students. Rather, they resulted from the structure consisted of the salient perceptional features of the initial technological terrain: the three points on the line with names '0', '0.5', '1'; evenly distanced from each other; all aligned with the thick grid lines [Figure 5]. This regulation was not included in the instructions from the teacher. Students could have laid their point anywhere in between the Xs and claim its legitimacy by naming it with a proper rational-number. Instead, their attention was captivated by the given structure and, as a result, their subsequent practice began to adapt to it. Although Team AB had zoomed in and thus had a better chance to place their point in between the existing points, they were only *looking closely* at the line to measure the distance between 1 and 1.5. Instead, having their attention fixed only at the thick grid lines, they failed to obtain the measure they wanted since 1.5 was not on one of the thick grid lines. Eventually, they zoomed out so that they could place '2' one thick-grid-line away to the right.

4.1.2 Sequence 2: “We can’t go beyond X.”

In the following sequence, to resolve the problem on hand, students devised new rules (*Second-rules*):

> ***Second-rule 1.*** *A new point must be 0.1-away from an existing point and named accordingly;*
>
> ***Second-rule 2.*** *A new point must be placed on a grid line, either thick or thin.*

Once Team CD placed '4', the last point in Figure 3, students encountered an issue: There is no available place for new points. According to the First-rules,

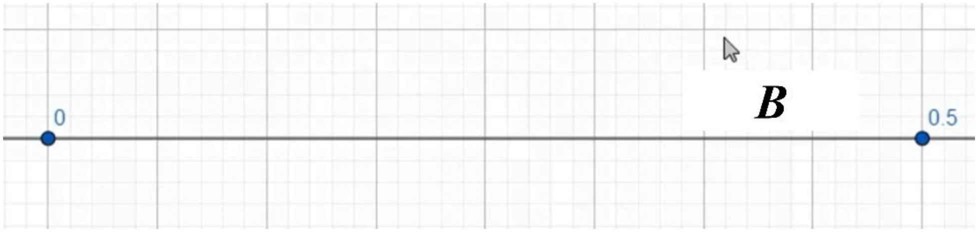

Figure 10. B zooming in and the eight thick grid lines between '0' and '0.5'

they were only supposed to point on the thick grid lines between the two Xs. Zooming in at the interval (3.5, 4), frustrated, A commented, "We can't go beyond X." Instead of giving up, however, Team AB then decided to seek a new place for '0.1'. B zoomed in and started counting thick grid lines between '0' and '0.5', of which she counted eight [Figure 10].

However, students therein encountered the second issue: No thick grid line corresponded to the proper location of '0.1'. While B was having a hard time finding a thick grid line corresponding to '0.1', A started to notice the use of thin grid lines and suggested B to make use of them. Not long after, B agreed on A.

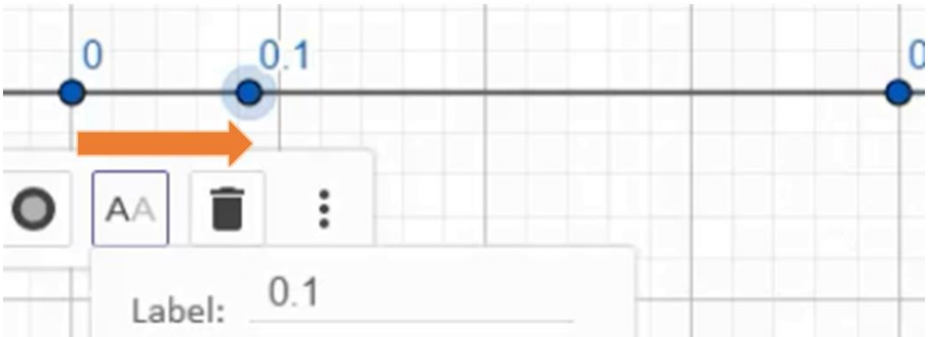

Figure 11. B pointing and naming '0.1' on a thin grid line

There, they began to count, finding out the interval (0, 0.5) comprised 20 thin grid lines and thus 0.1 corresponded to four of them. At last, a new point appeared near 0 and got dragged slowly to the fourth thin grid line and so did the name '0.1', shortly [Figure 11].

From then on, students unanimously followed Team AB's method with little verbal exchange, counting four thin grid lines to the right and pointed '0.2', then '0.3' [Figure 12]. The activity had to stop there since the lesson time was over.

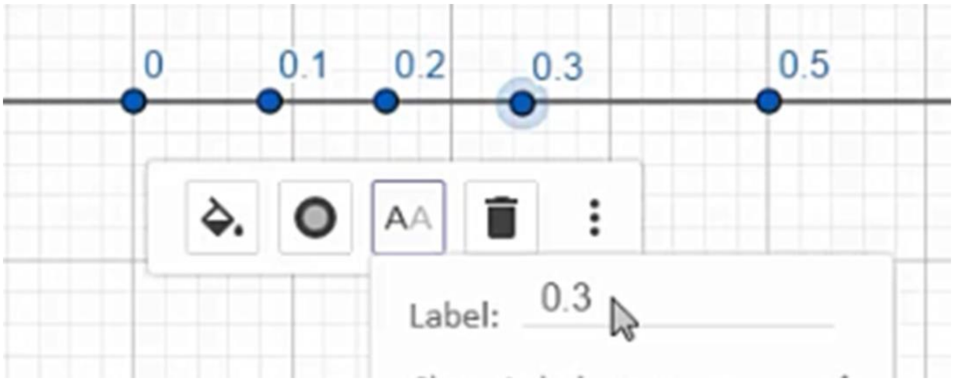

Figure 12. Subsequent pointing and naming of ‘0.2’ and ‘0.3’ on thin grid lines

After Team AB had devised a solution to place '0.1' in violation of the First-rules, Team CD simply followed their solution for '0.2' with no dispute at all, and Team AB repeated the action for '0.3' in the same way. Even though we were not able to observe further after '0.3', it is reasonable to assume they would have continued to play in the same manner judging based on their previous performance.

This new set of rules suggests the students became able to conceive a smaller unit (e.g., 0.1). Moreover, it shows that they could come up with the numbers which can be placed in between two once-seemingly-successive numbers. At this point, 0.5 was not the indivisible least common unit, and the multiples of 0.5 were not successive for students any longer.

Throughout this sequence, the body movements were at the center. Especially, the act of zooming in and out, rendered by spreading and pinching fingers on the touchpad, eventually evoked the change in students' perceptual habit and made them recognize the potential of thin grid lines – which was imperceptible earlier.

Another noteworthy thing is that the same perceptual change did not occur in the first sequence even though they zoomed in/out in precisely the same manner. The fundamental difference here lies in the degree of X's agency-in-play in the two situations. Once the students reached the barricades, unlike the first sequence, X became a major player in this student-technology-concept assemblage, which dismissed the students' propensity to go further outward according to the First-rules. Instead, it turned their attention to the intervals in between the points. It was not until the formation of this particular configuration that spreading fingers catalyzed such a dramatic shift in practice. Only then, students were *stretching* the number line and space rather than merely *looking closely* at them.

4.1.3 Sequence 3: "We keep stretching."

After the activity was over, the teacher conducted an ill-structured group interview with students on-site. Through students' discourse and gestures, it became apparent that the meaning of density had evolved further, and it was indeed inseparable from the technological environment of the activity.

11	T:	Let's say we already have placed like, so many points and, say, y'all are really smart people. Then, can we place another point at this time?
12	All:	Yes.
13	B:	By stretching more and more *(palm facing forward and all fingers open, B spreads her thumb and index and close them in repeatedly)* [Figure 13].

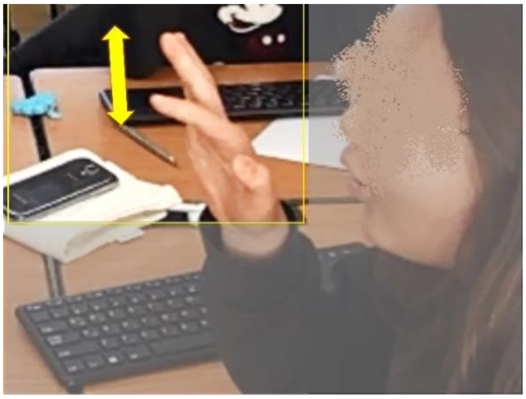

Figure 13. B's hand gesture: "By stretching more and more"

14	T:	Then, until when?
15	A, B:	Forever.

16 B: We do **<u>zero point zero, zero, zero, zero, zero, zero, one</u>** [**<u>0.0000001</u>**]

17 A: **<u>Zero point zero, zero, zero, zero, zero, zero,</u>** … zero, one [**<u>0.000000</u>**01][1] (…)

18 C: The further you stretch *(palm facing downward and only index and middle finger open, C spreads them away and close in quickly and repeatedly)* [Figure 14], the more lines you will have *(palm facing forward, C*

Figure 14. C' hand gesture: "The further you stretch"

repeats to spread and squeeze all of his fingers swiftly) [Figure 15].

[1] This bolded and underlined part in the transcript indicates the moment when speeches of A and B were overlapping. Note that B deliberately continued to add a zero even after A stopped.

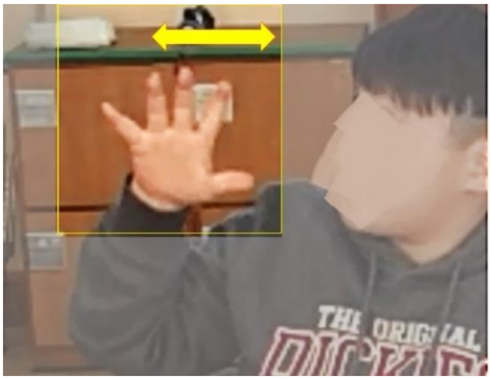

Figure 15. C' hand gesture: "the more lines you will have."

19 T: Then, let's say, I would place my points at all the empty place on the line there, and now we don't see any empty place. Can we still lay another point?

20 A: We can.

21 B: Yes. We keep stretching *(palm facing downward, B spreads her index and middle finger away and bring them in repeatedly)* [Figure 16].

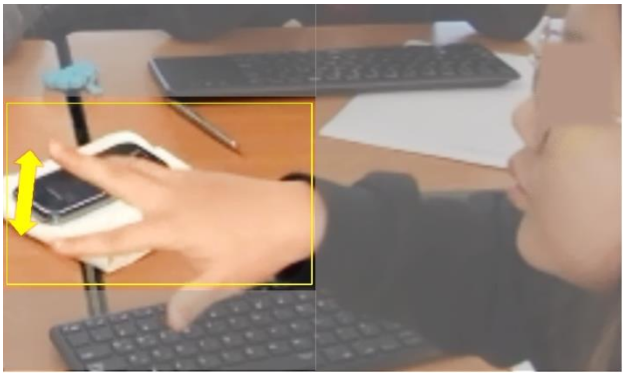

Figure 16. B' hand gesture: "We keep stretching"

The students realized that no matter how many points are on the line and how dense they appear to be, they could always find a place for another point in between them. B was enumerating zeros in 0.0000001 out loud (Line 16) to imply that the game could go on by finding a number with a sufficiently small unit in response to the teacher's question (Line 11). Moreover, while enumerating zeros out loud simultaneously with B, A noticed that he could find an even tinier unit if he continued to add zeros before finishing with one. This made A take a brief halt when B finished and then quickly added one more zero and one (Line 17). Also, Line 13, 18 and 21 indicate that students became able to conceive a room to put such numbers by virtually stretching the line and space.

4.2 Discussion

As we have observed from the episode, the body movement is a key factor in the invention of new meanings. Students' hand gestures on and off the touchpads demonstrate the catalyzing role of the mobility in the meaning-making or, in other words, becoming the assemblage of hands-touchpads-number line-space-'density'. No one possessed the new meaning from the beginning. No one was able to produce it by repeating what seemed possible. The meaning did not exist until the finger movements actualized the latent material configurations and allowed new senses in unanticipated ways. It is this emergent nature what makes the meaning genuinely new and inventive.

Using the GeoGebra environment as a diffractive analysis, the analysis provided in the previous section sought to trace the concept of the density of rational numbers as it emerged from the students' activities on the touchpad keyboards with the monitor. To clarify, the goal was not to address epistemological issues – what concepts were learned – which frame the concept as an abstract a priori. To the extent that new concepts of density emerged, the digital apparatus acted as a useful tool for experimentation: it enabled us to delve into the indeterminacy of matter. This indeterminacy arises in part from the accidental nature of students' use of the digital apparatus, for example, accidental pointing of a point on the far-left side of zero, accidental encounter of the X as barricade, uncontrolled zooming-in and subsequent appearance of unanticipated number of grid lines. Yet within this indeterminacy, there is also the strict adherence of the number names and symbols to a determine order: the names of points are always in decimal expression and they tend to stick to their rules until they face a challenge.

5 Conclusion

This thesis has scrutinized a case of mathematical activity to examine the relationship between the body movement and the meaning development in a multi-touch digital technology environment. The researcher designed a digital apparatus for students to explore the meaning of the density using GeoGebra and touchpad keyboards. A diffractive analysis on the case of four elementary school students were subsequently provided, focused on the movements of bodies – e.g. fingers, cursors, points and screen space - and change of their perceptual habits. The result suggests such body movements played an essential role and catalyzed the emergence of the developed meaning of density. Through the inclusive materialist lens, we could observe how the finger movements toward the digital apparatus played a crucial role in developing the meaning of the density of rational numbers.

The educational implications of this study are two-folds. First, multitouch dynamic digital technologies have a potential to promote new bodily mode of mathematics learning. The 'new' mathematical meaning in the case of this study emerged around the body movements interacting with the technological environment instead of a problem-solving with pencil and paper or a pure linguistic discourse. In other words, the movements are legitimate constituents of such meaning. This mode of learning was unprecedented, at least unobservable, especially in case of the conventional classroom settings with paper textbook or even keyboard-and-mouse. For this reason, the researcher reckon this type of technologies bear the potential to evoke and promote new mode of mathematical learning.

Second, mathematical activities to promote the develop of new meanings should be designed in the way that breaks the existing perceptual habits of students. The result of this study suggests that it is not helpful to obeying to the given structure – whether it is physical or perceptual – and maintain the original habit in developing more sophisticated meanings. Although such obedience do yield a product in the end, there is nothing extraordinary in the product than a repetition of logical inferences – i.e. the realization of the possible – and failure to metamorphose into a new meaning. Therefore, this thesis propose that it would be helpful to design mathematical activities that keep students from following the old habits by, for instance, setting up obstacles in their way.

The researcher envisions the direction of future research as follows. First, future research may examine the effect of the dynamic number line on concepts from the area *Numbers and Operations* by modifying the activity presented in this thesis. The presented mathematical activity help learners attend to smaller units by manipulating objects and screen, and express various numbers on the line. This feature may be suitable for the concepts like reduction to a common denominator, cancel of a fraction, infinite decimal or irrational number in the process of which the attention to smaller units is integral.

Second, future research is strongly encouraged to develop the assessment tool aligned with the inclusive materialist perspective. The study documented the emerging process of a mathematical meaning. Thus, it is reasonable to claim an acquisition of an intuition on the density instead of a solid and robust learning. As Sinclair, de Freitas, and Ferrara (2013) noted, this type of knowing is rather 'murky' and 'furtive' and this draws this study's boundary. The existing assessment system, which is focused on the determinate concepts and results of learning, is not appropriate to evaluate this 'murky and furtive' change. One may criticize this inclusive materialist approach for its failure to capture 'learning'. However, considering our learning has always been an aggregation of tiny bits and pieces of – sometimes tacit – changes, the type of change reported in this thesis bears significant educational implication. Therefore, the researcher of this study urges future research to develop a tool to capture and evaluate this emergent process.

In conclusion, I hope this study will contribute both to the embodied cognition theory and the studies on the integration of digital technologies in mathematics education by encouraging to incorporate the body and the material as central foci of investigation.

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국문 초록

현대 디지털 테크놀로지는 현대 사회인의 생활방식과 사고를 규정하는 요소다. 최근의 디지털 테크놀로지는 인간의 신체를 더 직접적으로 이용하는 입력방식을 택함으로써 그 어느 때 보다도 신체 친화적으로 변하고 있다. 나아가 상호작용 방식의 이와 같은 변화는 인간이 디지털 테크놀로지를 이용해 사고하는 방식에 큰 영향을 줄 것으로 예상된다.

그러나 수학 학습에 관한 기존 연구에서는 아직 디지털 테크놀로지와 이와 같은 신체친화적 변화가 주는 영향에 대한 조명이 충분히 이루어지지 않았다. 이는 신체적인 측면을 포착할 수 있는 이론적 틀이 부재했기 때문이다. 테크놀로지와 도구를 다루는 기존의 관점들은 테크놀로지 조작의 절차적 측면이나 그로 인해 발생하는 도식에 집중하여 수학 학습의 심적, 사회적 측면을 포착하는 데에는 성공했으나, 신체적이고 물질적인 상호작용 방식이 수학적 의미 형성에 주는 영향을 담을 수는 없었다.

이에 본 연구는 한 교수실험의 사례를 통해 역동적 멀티터치 디지털 테크놀로지와의 신체적 상호작용이 수학적 의미 형성에 주는 영향을 탐구하고자 한다. 교수실험에서는 한 무리의 초등학생들이 지오지브라를 이용하여 점, 유리수, 그리고 수직선을 탐구하였으며, 이를 통해 유리수 조밀성의 의미를 형성하는 활동을 했다. 사례 분석을 위해 신체와 비인간 물질의 상호작용을 묘사하는 데 특화된 포괄적 유물론을 이론적 배경으로 채택하였으며, 동일한 존재론 및 인식론적 배경에 기반한 회절실험을 방법론으로 채택했다.

연구 결과, 신체적 상호작용이 학생들에게 새로운 수학적 의미를 형성하는 데에 결정적인 역할을 한 것으로 드러났다. 학생들의 터치패드 상에서의 손가락 움직임, 그리고 그로 인한 점, 직선, 공간의 움직임이 조밀성 의미 형성을 유발하는데 촉매와 같은 역할을 했으며, 새로운 조밀성의 의미는 학생들의 게임 실행 규칙에 대한 담화, 터치패드를 조작하는 제스처, 의사소통에 사용하는 제스처 등과 함께 구성되어 있는 것으로 드러났다.

이와 같은 결과는 수학 학습이 갖는 신체성과 물질성을 강조함과 더불어, 신체 및 수학적 대상의 움직임이 새로운 수학적 의미를 창발할 수 있음을 조명한다. 이에 비추어 볼 때 본 연구는 향후 역동적 멀티터치 디지털 테크놀로지를 이용한 과제 설계에 시사점을 줄 수 있을 것이다.

주요어: 터치패드; 지오지브라; 유리수의 조밀성; 신체; 포괄적 유물론; 회절 장치

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