

The effect of restoration on the resilience of transportation infrastructure

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ABSTRACT: Coping with the resilience of infrastructure systems is important in order to be prepared for disruptive events such as natural hazards. It is essential to understand how and in what order damaged infrastructure objects should be restored so that they can provide adequate service again. Infrastructure managers making such decisions must also take into account other constraints such as available funds, personnel, available resources, and any external constraints, e.g. which objects on which roads should have priority. In this work, a mathematical optimization model was used to determine such a restoration program, by minimizing the direct and indirect costs of the event, considering constraints such as budget, resource and traffic flow. With this approach, a restoration programme for a real road network in Switzerland after the occurrence of an extreme flood event is investigated.

INTRODUCTION

The primary objective of infrastructure managers is to ensure that their transportation infrastructure provides adequate service. This means that they have the continuous task of executing interventions to help prevent the loss of service and to restore service after it is lost, which can happen, for example, due to natural hazards such as floods, landslides, and earthquakes. In other words, they have the continuous task of making the infrastructure resilient. As natural hazards can result in substantial negative economic and societal consequences, a significant portion of the resilience of infrastructure can be attributed to how infrastructure managers restore service once it is lost.

Finding optimal restoration strategies requires consideration of both direct costs, which are associated with the execution of interventions, such as cleaning-up, reparation, rehabilitation or reconstruction, and indirect costs, which include additional travel time costs, additional vehicle operating costs and additional accident costs caused by changes in the traffic flow while the restoration in-

terventions are being executed. It also requires investigating the effect on the restoration of service of constraints, such as money available, personnel available, and resources available, as well as any external restrictions as to which objects on which roads should have priority.

An example of the effect of these constraints on the ability to restore service following natural hazards is presented in this work. The investigation is focused on the restoration of the real world network in the Chur region of Switzerland following the occurrence of heavy rainfalls of different intensities that lead to flooding. The restoration model used is based on the one presented in Hackl et al. (2018a). To give a complete picture of the effect of the constraints, simulations of network performance were done assuming different states of the infrastructure objects when the natural hazards occur, and different traffic configurations used during the restoration of the network. The effects on the restoration of service of each of the constraints are shown, as well as indications as to how to better develop risk-reducing intervention programs.

METHODOLOGY

Considering that severe disruption to infrastructure services result in potentially high costs, the quick restoration of damaged infrastructure following a natural hazard event is critical. Hence infrastructure managers have the challenging task to determine optimal restoration programs, i.e. the optimal plans of how, and the order in which, the damaged infrastructure objects will be restored so that they provide adequate levels of service, taking into consideration the possible improvements in service, the costs, and the limited available budget and resources. In mathematical terms, such a problem can be described as the following optimization problem (Hackl et al., 2018a):

$$\min Z^R = \sum_{t \in T} \left[\sum_{n \in \mathcal{N}^s} \sum_{i \in \mathcal{I}(n|s)} \delta_{n,i,t} \cdot C_{n,i} + \gamma \sum_{\substack{P \in \mathcal{P}_{od}^{s \setminus g} \\ e \in P}} \Pi(t|x_{e,t}, \delta_{n,i,t}) - \Pi^0(t|x_{e,0}) + \sum_{P \in \mathcal{P}_{od}^g} \Lambda(t|\delta_{n,i,t}) \right] \quad (1a)$$

subject to

$$\sum_{i \in \mathcal{I}(n|s)} \sum_{t \in T} \delta_{n,i,t} \leq 1 \quad \forall n \in \mathcal{N} \quad (1b)$$

$$\sum_{n \in \mathcal{N}^s} \sum_{i \in \mathcal{I}(n|s)} \sum_{t \in T} \delta_{n,i,t} \cdot C_{n,i} \leq \Omega_t \quad \forall t \in \mathcal{T} \quad (1c)$$

$$\sum_{n \in \mathcal{N}^s} \sum_{i \in \mathcal{I}(n|s)} \sum_{j \in \mathcal{I}(n|s)} \delta_{n,i,j} \cdot r_{n,i,k} \leq \Psi_{k,t} \quad \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \quad (1d)$$

$$x_{e,t} \in \min Z^T = \sum_{e \in \mathcal{E}^s} \int_0^{x_{e,t}} C_e^T(\omega) d\omega \quad (1e)$$

subject to

$$\sum_{P \in \mathcal{P}_{od}^{s \setminus g}} f_{od}(P) = d_{od} \quad \forall od \in V^H \quad (1f)$$

$$f_{od}(P) \geq 0 \quad P \in \mathcal{P}_{od}^{s \setminus g}, \forall od \in V^H \quad (1g)$$

The objective of the mathematical model is to find a restoration program that minimizes the sum of costs (Eq.(1a)), considering the direct costs (C) and the indirect costs due to temporal prolongation of travel (Π) and loss in connectivity (Λ).

Here, direct costs are intervention costs, i.e., the costs of restoring objects to states where they function again as intended. For each object (e.g. road section, bridge) with reduced functionality $n \in \mathcal{N}^s$, a (finite) set of possible interventions $\mathcal{I}(n|s)$ is assigned. Only one of these can be selected at a time $t \in T$ to restore functionality of n . This is indicated by the binary decision variable $\delta_{n,i,t}$, which has a value of 1 if intervention $i \in \mathcal{I}(n|s)$ is executed on object n in state s , initiated at period t and 0 otherwise. In the case of prolongation of travel, indirect costs are principally caused by such things as additional travel time, additional vehicle operating costs and additional accidents. While in the case of a loss in connectivity, indirect costs are principally due to the loss of economic activity that occurs while travel is not possible. The magnitude of each depends on the network design, the traffic flow, and the considered interventions. A complete list of both types of indirect costs can be found in Adey et al. (2012). The indirect costs due to temporal prolongation of travel are measured as the difference between indirect costs at t and the indirect costs at $t = 0$ (i.e., Π^0) when the network was fully functional. The cost function Π dependent on the link traffic flow $x_{e,t}$ on edge e in period t , which is influenced by the restoration decisions made ($\delta_{n,i,t}$). Since temporal prolongation only occurs when traffic flow is possible, $\mathcal{P}_{od}^{s \setminus g} \subseteq \mathcal{P}_{od}$ refers to the set of od -paths where at least some flow is still possible, while $\mathcal{P}_{od}^g \subseteq \mathcal{P}_{uv}$ refers to the set of od -paths where no flow is possible. If now the flow is only observed when the subnetwork is disconnected from the main road network. In this case, it is assumed that travelers cannot make their trips, causing costs of Λ as long not a minimum of service is restored.

Since the quantification of indirect costs is a non-trivial task, and there are high levels of uncertainty associated with the estimated values, a weighting factor γ is used to allow a relative weighting be-

tween both costs; i.e., decision makers can decide to which extent indirect costs will affect the determination of the optimal restoration program.

The accompanying constraints of Eq. (1a) are continuity constraints, budget constraints, and resource constraints. The continuity constraints Eq. (1b) force the model to select only one intervention per object, which is executed in a one-time interval, throughout the investigated time period. The budget constraint Eq. (1c) forces the model to select no more interventions than for which funding is available, i.e., the total cost of all interventions cannot exceed Ω for the investigated time period t . The resource constraint Eq. (1d) forces the model to select no more resources than available in a time interval t , i.e., the number of resources k in time interval t , needed for the interventions, cannot exceed $\Psi_{k,t}$. Other constraints for the direct costs could be added if desired; for example, time constraints, accessibility constraints, or maximum or minimum work-zone constraints (Hajdin and Adey, 2006; Lethanh et al., 2018).

As the indirect costs depend on the traffic flows and they change according to the restoration strategy, the traffic assignment has to be taken into account in the optimization problem Adey et al. (2014). Using simple approaches such as a user equilibrium assignment, the traffic problem can be treated as a convex optimization problem (Beckmann et al., 1959) leading to a bilevel optimization problem. Thereby, the link traffic flow $x_{e,t}$ is estimated, by solving the user equilibrium assignment, Eq. (1e) subjected to Eqs. (1f) and (1g). The costs of travel on each edge change with the flow and, therefore, the costs of travel on several of the network paths change as the edge flow changes. A stable state is reached only when no traveler can reduce his costs of travel by unilaterally changing routes Sheffi (1985).

The lower-level optimisation Eq. (1e) for the user equilibrium assignment, corresponds to the sum of the integrals of the travel cost function C_e^T for all edges e in the network. The demand constraint Eq. (1f), stating that the flow on all od -pairs has to equal the demand $d_{od} \geq 0$ for all $od \in \mathcal{V}^H$. The non-negativity constraints Eq. (1g) are re-

quired to ensure that the solution of the program will be physically meaningful. Eq. (1e) is formulated in terms of edge flows, whereas the constraints are formulated in terms of path flows. $x_e = \sum_{od \in \mathcal{V}^H} \sum_{\substack{P \in \mathcal{P}^{s \setminus g} \\ e \in P}} f_{od}(P)$ expresses the relationship between edge and path flow, and incorporates the network design into the optimisation problem.

NOTE: Substantial research has been carried out on this traffic assignment problem and its extension to more practical approaches, including the representation of dynamic traffic phenomena such as queues, spillbacks, wave propagation, capacity drops and so on. In this work, this simplified approach was used due to the universality of the problem and its simple mathematical handling. In addition, the simple and commonly used Frank-Wolfe algorithm was used to solve the optimization problem (Chen et al., 2002). However, this does not affect the applicability of the proposed method, as it can be applied independently of the problem. An overview of more detailed traffic assignment methods can be found in Nagurney (2007); de Dios Ortuzar and Willumsen (2011); Hoogendoorn and Knoop (2012); Patriksson (2015). Furthermore, A similar approach can be used for other transportation networks such as power grids, utility networks, or communication networks.

While there are solution strategies for solving the upper-level and lower-level optimization separately, classical methods fail to solve this multilevel optimization model exactly. This is due to the computational complexity, including non-linearity, non-convexity, and non-differentiability of the combined problem. Furthermore, the upper-level optimization can be classified as a combinatorial optimization, where the optimal restoration program comes from a finite set of restoration program, i.e., the combinations of different interventions in time are finite. In many such problems, an exhaustive search is not feasible, but heuristic procedures can provide a way of computing near-optimal solutions. In this work, a simulated annealing (SA) based metaheuristic procedure is used to approximate an optimal solution of the upper-level optimization. The lower-level optimization is embedded in the SA but solved with classical methods.

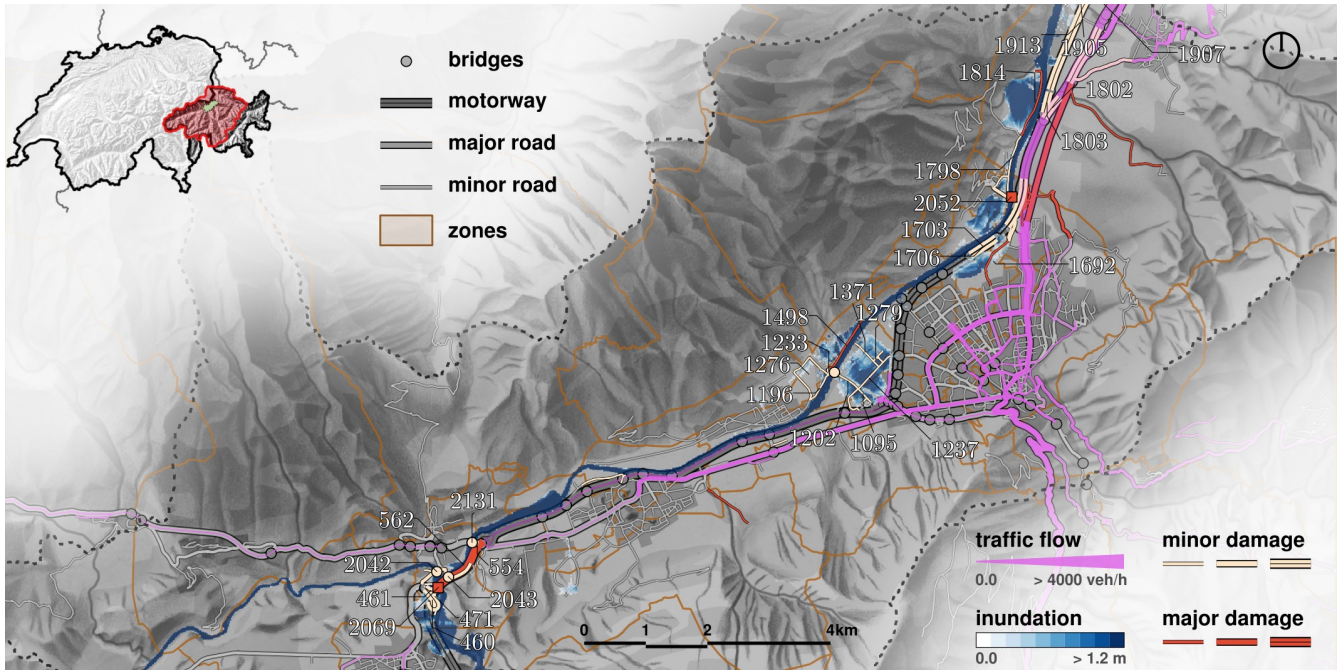


Figure 1: Area of interest

APPLICATION

The application presented in this section is used to demonstrate the usefulness of the methodology considering a specific problem. The application shows the design and implementation of a restoration program focused on estimating the resilience related to a road network in the Canton of Grisons in Switzerland. This area was suspected to have an unacceptable level of road network-related risk due to riverine floods. Historical records and previous studies suggested that the heavy rainfalls in this area have the potential to result in severe flood events. The road network in the area of study, which is considered to play an essential role in the economy of the eastern part of Switzerland, consists of circa 51 km national roads, 165 km main roads, and 395 km minor roads, with many of these objects exposed to the hazards of interest. The example network consists of 2153 objects which include 2037 road sections and 116 bridges, as shown in Figure 1. A detailed description of the area in the context of the example can be found in Hackl et al. (2016).

The trips in the region were considered to start and end in the 37 zones based on judicial districts as shown in Figure 1. All trips made from an origin

to a destination during a particular time period are stored in a so-called *OD* matrix. Since not enough trip distribution information was available for the area of interest, a gravity distribution model (de Dios Ortuzar and Willumsen, 2011) was used to estimate the trips based on the population of zones. The obtained gravity model was calibrated based on the Swiss national traffic model (FOSD, 2015), which provides data for the motorway and main roads.

Functional losses due to local scour at bridge piers, and inundation of road sections caused by an extreme flood event was considered. The states of the objects were derived by fragility and functional loss functions, which were used in a flood simulation with a 500 year return period. The detailed quantification and computer-supported model used to determine the damage state and functional losses are described in Hackl et al. (2018b). For illustration purposes only three damage states were considered: s_0 *no damage* - object is in a normal state; s_1 *minor damage* - some service can be provided; and s_3 *major damage* - no service can be provided;

The total costs $C_{n,i}$ for intervention $i \in \mathcal{I}(n|s)$ executed on object n in state s , were considered to be the summation of the fixed costs (e.g. build-

Table 1: Intervention types and associated recovery rate, resources and costs for bridge local scour.

State	Intervention	LOS recovery	Duration	Resources	Fixed costs	Variable costs	Resource costs
for bridges	type	[%]	[h/el]	[#rwc]	[mu]	[mu/el]	[mu/(rwc · h)]
s_1 (minor)	level 1	100	20	2	16,000	24,000	900
	level 2	100	40	1	10,000	15,000	900
	level 3	20	35	1	10,000	13,000	900
s_2 (major)	level 1	100	90	2	48,000	64,000	1200
	level 2	100	160	1	30,000	40,000	1200
	level 3	10	145	1	30,000	37,000	1200
for roads	type	[%]	[h/m ²]	[#rwc]	[mu]	[mu/m ²]	[mu/(rwc · h)]
s_1 (minor)	level 1	100	0.001	2	5250	22.00	500
	level 2	100	0.003	1	3500	16.50	500
	level 3	30	0.003	1	3500	14.50	500
s_2 (major)	level 1	100	0.006	2	14,400	110.00	700
	level 2	100	0.012	1	9600	82.50	700
	level 3	10	0.010	1	9600	78.50	700

el ... affected elements of the bridge (e.g. pier(s), and/or abutment(s))

rwc ... restoration work crews

mu ... monetary unit (e.g. USD, EUR, CHF)

ing site facilities), the variable costs (e.g. mu/m² pavement, mu/m³ concrete), and resource-related costs (e.g. labor costs) to execute the intervention. The terms fixed, and variable costs are used for non-resource related material costs, where resource costs are used to describe labor and construction machinery cost of the restoration work crews.

These costs vary depending on the type of object and the type of intervention. For each type of object, three different intervention types are considered: *level 1 interventions*, *level 2 interventions*, and *level 3 interventions*. Level 1 interventions require less time than level 2 interventions, but more resources and additional costs. Level 2 interventions restore the capabilities in a default way. Level 3 interventions require less time and costs than a level 2 intervention. Both level 1 and level 2 interventions restore objects so that they once again provide the expected LOS, level 3 interventions do not. The LOS recovery, durations, resources and costs related to interventions of each type for road sections and bridges are given in Table 1.

Travel time costs are estimated as the increased amount of time that people spend traveling. They are linked directly to the flow on an edge, due to the cost-flow relationship C_e^T where cost is handled in terms of travel time per unit distance de Dios Ortuzar and Willumsen (2011). The cost-

flow relationship used is the function proposed by the Bureau of Public Roads Bureau of Public Roads (1964):

$$C_e^T := t_{e,t}(x_{e,t}) = t_e^0 \left(1 + \alpha_e \left(\frac{x_{e,t}}{y_{e,t}} \right)^{\beta_e} \right) \quad (2)$$

where $t_{e,t}$ is the travel time at edge e in period t given the traffic flow $x_{e,t}$, t_e^0 is the free flow travel time, $y_{e,t}$ the edge capacity, and α and β are parameters for calibration, typical chosen as $\alpha = 0.15$ and $\beta = 4$. Based on the work of the Swiss Association of Road and Transport Experts VSS (2009), it was assumed that one hour of travel time can be estimated with 23.02 mu/hour for cars and 130.96 mu/hour for trucks. The costs due to a loss in connectivity are estimated based on the unsatisfied demand per time period t and the resulting costs due to a loss of labour productivity. Based on the data from the Federal Statistical Office Reutter and Bläuer Herrmann (2016), the labor productivity per hour worked was assumed to be 83.27 mu/hour.

In order to estimate the optimal restoration program, different assumptions were made, including the number of restoration work crews available (#rwc = 3), the considered time intervals ($\Delta t = 4$ hour), the working hours per day (8 hour), and the weighting factor for indirect costs ($\gamma = 1$).

The considered scenario had a budget constraint of 10 million mu, and it was assumed, that the second restoration work crew (resource B) was not available in the first four days, while restoration work crew three (resource C) was not available between the 10th and the 15th day of the restoration program.

Due to the complexity of the problem and the extensive solution space, it is not possible to solve this mathematical model exactly by analytics or an exhaustive search. In order to evaluate the obtained results, a comparison between the proposed model and a (standard practice) benchmark model was made.

The traditional methods to develop restoration programs are mostly heuristic and based on subjective ranking and priority rules developed by domain experts. These prioritization rules can be based either on economic or engineering criteria, such as structure vulnerability, road class, traffic volume, and various socioeconomic factors. For example Buckle et al. (2006) prioritize road object based on expected damage ranks, where the object with the highest damage or functional loss are given the highest priority for restoration. Because this ranking does not account for the importance of the object in the network, Miller (2014) proposed prioritization based on the average daily traffic volume for each object under normal conditions, as a benchmark model. Since this model does not account for disconnected parts of the network, a modified version was implemented as the benchmark:

1. sorting the objects based on their average daily traffic volume under normal conditions,
2. restoring objects that cause a loss of connectivity,
3. restoring the remaining objects;

For illustration purposes, Figure 2 presents only the results of one scenario (which corresponds to the scenario shown in Figure 1). Figures 2a and 2b show performance-based resilience curves of the system, with the ordinate representing the loss of service and the abscissa the time after the hazard event. In this example, it was assumed that the LOS for the system was determined by (a) the temporal prolongation of travel and (b) the loss in connectivity, so two resistance curves were obtained.

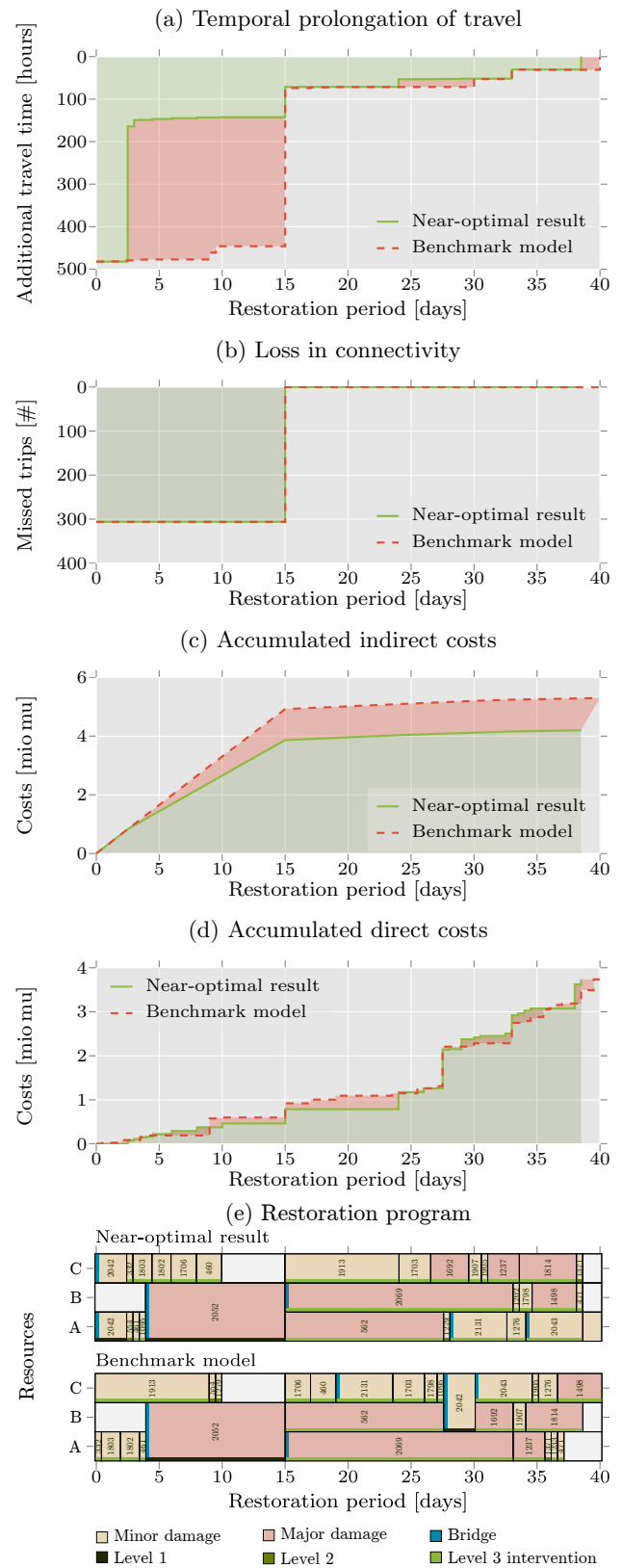


Figure 2: Results for one scenario. Comparing the near optimal results with the benchmark model.

The areas above the curves are directly proportional to the indirect costs of the optimization problem, but the shape of the curves depends on the chosen restoration program and thus affects the direct costs. Therefore, minimizing the area above the curve (i.e., increasing the resilience of the system) may not always be the best solution as the direct costs may be disproportionately high.

Figure 2 shows the results of the optimized restoration program in green, while the results of the benchmark model are shown in red. Since the benchmark model prioritizes objects that cause a loss of connectivity, in this scenario objects were first fixed that connect the village Haldenstein with the rest of the network. Without explicitly specifying this constraint in the optimization model, the proposed restoration program also fixes these objects first, so that both models restore the connection loss as quickly as possible (here after 15 days), as shown in Figure 2b. However, since the benchmark model does not take into account the traffic flow and the associated complexity (e.g., capacity overruns), the resilience of the network is worse than it could be (with the same investment in restoration), as shown in Figure 2a. In other words, the resilience of the system depends not only on the money and resources invested but also on the decisions as to how and when the objects should be restored.

Figures 2c and 2d show the costs associated with the hazard event. Due to the high cost of missing a trip, the accumulated indirect costs initially increase faster until the objects causing the loss of connection are repaired. After that, only the costs for the temporal prolongation are added. Thereby, the slope of the benchmark model is greater than that of the optimized restoration program. At the end of the restoration period (here 40 days), the cumulative indirect costs for the benchmark model amount to 5,296,760 mu, while the estimated indirect costs for the optimized program amount to 4,199,330 mu, i.e., 1,097,430 mu could be saved through intelligent planning of the restoration sequence. In order to allow a direct comparison between the different resilience curves and the associated indirect costs, it was assumed that the same

strategies were used in the benchmark model as in the optimized results, as there is generally no description of the type of interventions of such a benchmark model. I.e., the total direct costs for the scenario were 3,735,850 mu in both cases, only the order of the interventions was different, as shown in Figure 2d.

Finally, Figure 2e shows the detailed restoration programs for the scenario. Above is the program that provides the near-optimal solution, while below is the result of the benchmark model. As mentioned above, the same intervention types were used for both approaches, so that only the position of the interventions and not the length or type is different.

CONCLUSIONS

Coping with the resilience of infrastructure systems is important in order to be prepared for disruptive events such as natural hazards. It is essential to understand how and in what order damaged infrastructure objects should be restored so that they can provide adequate service again. Infrastructure managers making such decisions must also take into account other constraints such as available funds, personnel, available resources, and any external constraints, e.g. which objects on which roads should have priority.

In this work, a mathematical optimization model was used to determine a near-optimal restoration program that considers different objects in different states of damage, different intervention strategies per object, budget and resource constraints, as well as direct and indirect costs, last caused by diverging traffic flows through the restoration process. The approach was demonstrated by using it to determine the near-optimal restoration program for a real-world road network in Switzerland following the occurrence of an extreme flood event.

In comparison to a classical restoration model, it was shown that minimizing the area above the resilience curve (i.e., increasing the resilience of the system) is not always the best solution, since the direct costs can be disproportionately high. In addition, it was shown that the resilience of the system depends not only on the money and resources invested but also on the decisions on how and when to restore the objects.

REFERENCES

- Adey, B. T., Lethanh, N., Hartmann, A., and Viti, F. (2014). "Evaluation of intervention strategies for a road link in the Netherlands." *Built Environment Project and Asset Management*, 4(2), 180–198.
- Adey, B. T., Lethanh, N., and Lepert, P. (2012). "An impact hierarchy for the evaluation of intervention strategies for public roads." *Proceedings of the 4th European Pavement and Asset Management Conference (EPAM)*, Malmö, Sweden, EPAM.
- Beckmann, M. J., McGuire, C. B., and Winsten, C. B. (1959). *Studies in the economics of transportation*. Yale University Press, New Haven, CT.
- Buckle, I. G., Friedland, I., Mander, J. B., Martin, G., Nutt, R., and Power, M. (2006). "Seismic Retrofitting Manual for Highway Structures : Part 1 - Bridges." *Report No. FHWA-HRT-0*, Federal Highway Administration (FHWA), McLean, VA.
- Bureau of Public Roads (1964). "Traffic Assignment Manual." *Manual*, Urban Planning Division, US Department of Commerce, Washington, DC.
- Chen, A., Jayakrishnan, R., and Tsai, W. K. (2002). "Faster Frank-Wolfe Traffic Assignment with New Flow Update Scheme." *Journal of Transportation Engineering*, 128(1), 31.
- de Dios Ortuzar, J. and Willumsen, L. G. (2011). *Modelling transport*. John Wiley & Sons, West Sussex, UK, 4 edition.
- FOSD (2015). "Nationales Personenverkehrsmodell des UVEK, Aktualisierung auf den Basiszustand 2010." *Report*, Federal Office for Spatial Development (FOSD), Bern, Switzerland.
- Hackl, J., Adey, B. T., and Lethanh, N. (2018a). "Determination of Near-Optimal Restoration Programs for Transportation Networks Following Natural Hazard Events Using Simulated Annealing." *Computer-Aided Civil and Infrastructure Engineering*, 33(8), 618–637.
- Hackl, J., Heitzler, M., Lam, J. C., Adey, B. T., and Hurni, L. (2016). "Final Model, Methodology and Information Exchange." *Deliverable 4.2*, Novel indicators for identifying critical, INFRAstructure at RISK from Natural Hazards (INFRARISK), Zurich, Switzerland.
- Hackl, J., Lam, J. C., Heitzler, M., Adey, B. T., and Hurni, L. (2018b). "Estimating network related risks: A methodology and an application in the transport sector." *Natural Hazards and Earth System Sciences*, 18(8), 2273–2293.
- Hajdin, R. and Adey, B. T. (2006). "Optimal worksites on highway networks subject to constraints." *Proceeding of the International Forum on Engineering Decision Making, Lake Louise, Canada*, Vol. 2, Lake Louise, Canada, IFED, 1–11.
- Hoogendoorn, S. P. and Knoop, V. (2012). "Traffic flow theory and modelling." *The Transport System and Transport Policy: An Introduction*, B. vanWee, J. A. Annema, and D. Banister, eds., Edward Elgar Publishing Limited, Cheltenham, UK, Chapter 7, 125–159.
- Lethanh, N., Adey, B. T., and Burkhalter, M. (2018). "Determining an Optimal Set of Work Zones on Large Infrastructure Networks in a GIS Framework." *Journal of Infrastructure Systems*, 24(1), 04017048.
- Miller, M. K. (2014). "Seismic Risk Assessment of Complex Transportation Networks." Ph.D. thesis, Stanford University, Stanford, CA.
- Nagurney, A. (2007). "Mathematical Models of Transportation and Networks." *Mathematical Models in Economics*, Zhang, ed., Vol. 2, Eolss Publishers, Paris, France, 346–384.
- Patriksson, M. (2015). *The Traffic Assignment Problem: Models and Methods*. Dover Publications, Mineola, NY.
- Reutter, R. and Bläuer Herrmann, A. (2016). "Arbeitsvolumenstatistik (AVOL)." *Report do-d-03-svolta-02.01*, Federal Statistical Office (FOS), Neuchâtel, Switzerland.
- Sheffi, Y. (1985). *Urban transportation networks: equilibrium analysis with mathematical programming methods*. Prentice-Hall, Englewood Cliffs, NJ.
- VSS (2009). "Kosten-Nutzen-Analysen im Strassenverkehr: Zeitkosten im Personenverkehr." *Swiss Standard SN 641 822a*, Swiss Association of Road and Transport Experts (VSS), Zurich, Switzerland.