Probabilistic fatigue design of reinforced-concrete wind turbine foundations

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ABSTRACT: Probabilistic fatigue design of wind turbines is a new approach to optimize the design by reducing in a reliability- and cost-optimal way the amount of materials used for the construction, ultimately reducing the cost of energy. This paper presents such a probabilistic framework for reliability assessment of onshore wind turbine foundations with aim to optimize the design. This framework includes stochastic modelling of fatigue strength based on a large database of test results, stochastic modelling of the fatigue load (wind), modelling of the related epistemic and aleatory uncertainties, along with a case study showing how optimization could be exercised using the reliability-based framework.

1. INTRODUCTION

It is important to design wind turbine structures to a specific (target) reliability level, in order to avoid conservative designs and excessive use of materials with the overall aim to minimize the cost of such installations and ultimately the cost of energy (Sørensen & Toft, 2006).

Wind turbines are exposed to cyclic load from wind. This causes fatigue of all components of the wind turbine including the reinforced concrete foundation. In most of the cases, fatigue design governs the structural dimensions or the amount of reinforcement in the foundation of an onshore wind turbine, (Görasson & Nordenmark, 2011). However, estimating the level of damage in the foundation is complex and therefore prediction of the actual fatigue life is difficult. The current international codes use models for damage accumulation with respect to fatigue of concrete, which are generally conservative; they are not able to predict the real behavior accurately but can only predict the remaining useful life with uncertainty.

This paper presents a probabilistic framework for reliability assessment with respect to fatigue failure of an onshore reinforced concrete wind turbine foundation. This includes stochastic modelling of the fatigue strength, stochastic modelling of fatigue loads, uncertainties associated with strength and load modelling, and reliability-based calibration of material partial safety factors (γ_C) for design with respect to fatigue failure of the concrete. Examples of reliability assessment and optimization of design parameters will be presented.

2. DETAILS OF THE ONSHORE WIND TURBINE FOUNDATION

A gravity spread foundation is the most commonly used foundation for onshore wind turbines. This is due to ease of construction with little excavation and refill work. This foundation consists of a large slab, which could be square, octagonal or circular in plan, with or without thickness variation. Typically the reinforcement is placed at top and bottom layers of the slab with orthogonal grids and radial pattern through an embedded ring. This radial pattern can also be used for circular slabs. Typically, this kind of foundation transmits forces from wind turbine tower to soil to a larger area by a spreading action. The foundation slab transmits the forces from the tower to the soil by a cantilever bending action,

and thus the cross section of the slab experiences bending moment and bending stresses. These variations in the fatigue forces / stresses are modelled by force / stress ranges and mean values in the form of Markov Matrices (Sandia National Laboratories, 1999).

This paper deals with fatigue reliability assessment of a similar reinforced concrete foundation as described in (Svensson, 2010) along with a Markov-matrix based on 20 years of wind data for a Siemens wind turbine described in (Görasson & Nordenmark, 2011). The basic features of the wind turbine foundation is shown in Figure 1 and Table 1.

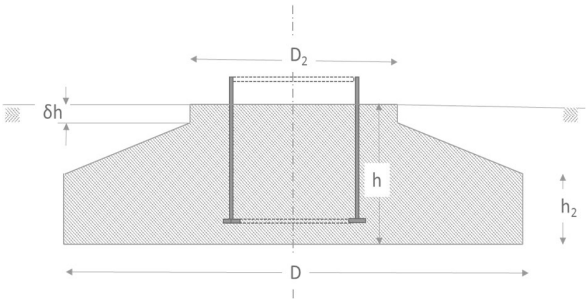

Figure 1: Principal geometry of the foundation

Table 1: Main Design characteristic of the wind turbine foundation.

Hub-height	99.5 m
Design wind speed at hub-height	8.5 m/s
Shape	Circular
Concrete class	C30/37
Diameter (D & D_2)	15.66 m, 6.0 m
Thickness (h & h_2)	3.0 m, 1.73 m
Pedestal height (δh)	0.27 m
Reinforcement type	B500B
Reinforcement	1540 mm^2/m

Part of the Markov matrix is shown in Table 2 and Figure 2. Heading in Table 2 shows the mean values of overturning moment. The values in Table 2 show amplitudes to be added or subtracted from mean values in the heading to obtain maximum and minimum overturning moments. The first column indicates the number of cycles observed during one year.

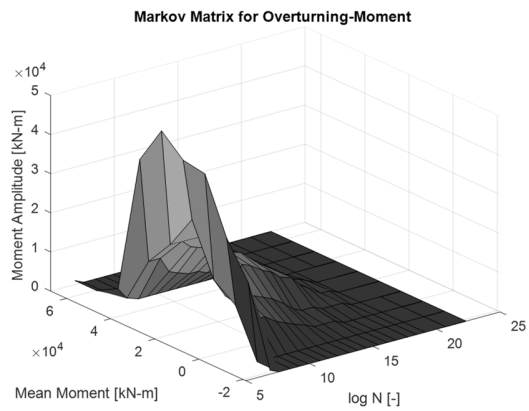

Figure 2: Markov matrix for overturning moment

3. MODELLING OF UNCERTAINTIES

The following uncertainties are considered in the current paper: concrete strength, fatigue strength model for linear damage accumulation (S-N) curves, wind load effects and Miner's rule. This includes modelling of all epistemic and aleatory uncertainties.

3.1. Concrete strength

The reference property of concrete is the compressive strength f_c of standard test specimens (cylinder of 300 mm height and 150 mm diameter), tested according to standard conditions at a standard age of 28 days (ISO 1920, 2004) and (ISO 3893, 1977). All other properties (e.g. tensile strength, modulus of elasticity, and compressive strain) are related to the reference strength of concrete.

However, the reference strength of concrete is subject to both aleatory and epistemic uncertainties. Stochastic modelling of f_c is explained in various international standards, guidelines and background documents e.g. (JCSS, 2000), (DS-INF 172, 2009), (EN 1992-1, 2004) and (MC2010, 2013).

The stochastic modelling applied in (DS-INF 172, 2009) is adopted here. Accordingly, the concrete compression strength is assumed to be lognormal distributed with a mean value of

f_{cm} and a standard deviation σ_{fc}. The associated Coefficient of Variation (CoV) becomes $V_{fc} = 0.14$.

Table 2: Fatigue overturning moment spectrum (Görasson & Nordenmark, 2011).

	Peak-to-peak fatigue load bins [kNm]									
	mean values of moment [kNm]									
(ni)	-21300	-11450	-1600	8250	18100	27950	37800	47650	57500	67300
1.0E+9	0	0	0	0	0	0	0	0	0	0
5.0E+8	0	0	0	0	0	0	0	0	0	0
2.0E+8	0	0	0	5	5	5	0	0	0	0
1.0E+8	0	0	0	5	1000	1000	5	0	0	0
5.0E+7	0	0	5	1000	1000	2000	5	0	0	0
2.0E+7	0	0	5	1000	3000	3000	1000	0	0	0
1.0E+7	0	0	5	1000	4950	4950	2000	0	0	0
5.0E+6	0	0	1000	2000	6950	6950	3000	0	0	0
2.0E+6	0	0	1000	2000	7950	8900	4000	0	0	0
1.0E+6	0	0	1000	3000	9900	10900	4950	0	0	0
5.0E+5	0	0	2000	4950	10900	12900	5950	0	0	0
2.0E+5	0	0	3000	6950	12900	13850	5950	0	0	0
1.0E+5	0	0	4000	7950	13850	14850	6950	0	0	0
5.0E+4	0	0	4950	8900	6850	15850	6950	5	0	0
2.0E+4	0	0	6950	9900	17800	17800	7950	5	0	0
1.0E+4	0	0	6950	9900	19800	17800	8900	5	0	0
5.0E+3	0	0	9900	10900	23750	19800	12900	5	0	0
2.0E+3	0	5	11900	15850	28700	19800	12900	1000	0	0
1.0E+3	1000	20800	29700	43550	44500	49450	39600	4000	1000	1000

Number of cycles

3.2. Fatigue strength model

The FIB Model Code 2010, (MC2010, 2013), presents a model to estimate the number of fatigue cycles to failure by equation (2), see below, which then can be used to estimate the level of fatigue damage in concrete. For reliability analysis equation (2) is modified to equation (4) removing the safety factors and characteristic values applied in (MC2010, 2013). A few stochastic variables are introduced X_1, X_2 & ϵ. Here, X_1 corresponds to condition of $S_{max} = 1$ at $\log N = 0$, X_2 corresponds to limit of linearity of these curves presented, see Figure 3, and ϵ models the model uncertainty associated with the fatigue strength model. Using a large database complied from literature (Lantsought, 2014), (Lohaus, Oneschkow, & Wefer, 2012) & (Thiele, 2016) of fatigue laboratory tests, estimates of X_1, X_2 and σ_ε are obtained using the Maximum Likelihood Method (MLM), and corresponding parameter uncertainties (standard deviations and correlation coefficients) are estimated, see (Mankar, Bayane, Sørensen, & Brühwiler, 2018) & (Mankar, Rastayesh, & Sørensen, 2018). To obtain a stochastic model for fatigue of concrete for the wind turbine foundation, fatigue test data within the range of compressive strength varying from 20 MPa to 60 MPa from database (described above) are used. This range represents the variability of the compressive strength in the foundation concrete, which is class C 30/37. See Table 3 for stochastic parameters described and obtained by MLM for this wind turbine. Figure 3 shows the mean failure curves obtained using equation (4) for the chosen dataset within range of 20 MPa to 60 MPa.

3.3. Wind load

A model uncertainty related to fatigue load assessment X_W is introduced in order to capture the uncertainty related to the assessment of the fatigue load from wind turbine structure, controller, turbulence simulation and aero-elastic model, see (Toft, Svenningsen, Sørensen, Moser, & Thøgersen, 2016).

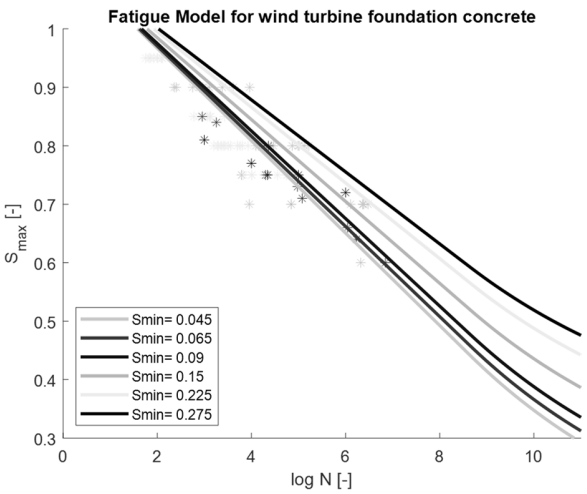

Figure 3: Fatigue strength model of concrete for onshore wind turbine foundation

4. DESIGN-EQUATION

A design-equation is formulated for fatigue failure of concrete based on Miner's rule of linear damage accumulation, as shown in equation (1).

$$G(T_L, Z) = 1 - Damage = 1 - \sum \frac{n}{N} = 1 - \sum_{i=1}^{N_{smax}} \sum_{j=1}^{N_{smin}} \frac{n_{ij} \cdot \mathrm{T_L}}{N_{Dij}} \quad (1)$$

where

T_L service life of the structure

N_{Smax} and N_{Smin} are the number of bins for $S_{Cd,max}$ and $S_{Cd,min}$ respectively

n_{ij} experienced / observed number of stress cycles of $S_{Cd,max,i}$ and $S_{Cd,min,j}$ in each bin (i,j) per year

$N_{D,ij}$ required number of stress cycles of $S_{Cd,max,i}$ and $S_{Cd,min,j}$ in each bin (i,j) per year for failure calculated deterministically based on equation (2).

Z design parameter, for current case it is tensile reinforcement area.

$$\log N_{D,ij} = \frac{8}{(Y-1)} \cdot \left(S_{cd,max,i} - 1\right), \text{if } \log N_{D,ij} \leq 8$$

$$\log N_{D,ij} = 8 + \frac{8 \cdot \ln(10)}{(Y-1)} \cdot \left(Y - S_{cd,min,j}\right) \cdot \log\left(\frac{S_{cd,max,i} - S_{cd,min,j}}{Y - S_{cd,min,j}}\right), \text{if } \log N_{D,ij} > 8 \quad (2)$$

where

$Y = \frac{0.45 + 1.8 \cdot S_{cd,min}}{1 + 1.8 S_{cd,min} - 0.3 \cdot S_{cd,min}^2}$

$S_{cd,max,i} = \frac{\gamma_{ED} \cdot |\sigma_{c,maxi}| \cdot \eta_c}{f_{cd,fat}}$

$S_{cd,min,j} = \frac{\gamma_{ED} \cdot |\sigma_{c,min,j}| \cdot \eta_c}{f_{cd,fat}}$

γ_{ED} the partial safety factor for fatigue load. For this case, γ_{ED} is considered as 1.0 as direct strain measurements are available, (MC2010, 2013).

η_c the averaging factor for concrete stresses in the compression zone considering stress gradient. The recommended value 1.0 for uniform stress i.e. for the case of no stress gradient.

$f_{cdfat} = \beta_{c,sus(t,t0)} \cdot \beta_{cc(t)} \cdot f_{cd} \cdot (1 - f_{cd}/400)$, is the design reference fatigue strength.

$f_{cd} = f_{ck}/\gamma_c$

γ_c the partial safety factor for material; 1.5 is recommended in (MC2010, 2013).

$\beta_{cc(t)}$ factor considered for strength gain over time due to continued hydration.

$\beta_{c,sus(t,t_0)}$ is a coefficient, for fatigue loading it may be taken as 0.85.

$\sigma_{c,max,i}$ & $\sigma_{c,max,i}$ are max. and min. stresses used to obtain $S_{cd,max,i}$ and $S_{cd,min,j}$

$\sigma_{c,max,i} = \sigma_{DL}(Z) + \sigma_{WL,max}(Z)$

$\sigma_{c,min,j} = \sigma_{DL}(Z) + \sigma_{WL,min}(Z)$

σ_{DL} stress due to dead load

σ_{WL} stress due to wind load on the turbine

E_c modulus of elasticity of concrete in MPa, $E_c = 4700 \cdot \sqrt{f_{ck}}$

f_{ck} characteristic static compressive strength of concrete in MPa.

5. LIMIT-STATE-EQUATION

A limit-state-equation corresponding to design-equation (1) is formulated, see equation (3).

$$g(t,Z) = \Delta - \sum_{i=1}^{N_{Smax}} \sum_{j=1}^{N_{Smin}} \frac{n_{ij} \cdot t}{N_{S,ij}} \tag{3}$$

where

Δ model uncertainty associated with PM rule

t time in years $0 < t < T_L$

$N_{S,ij}$ required number of stress cycles of $S_{Cd,max,i}$ and $S_{Cd,min,j}$ in each bin (i,j) per year for failure calculated using stochastic variables described in equation (4).

$$\log N_{S,ij} = \frac{X_2}{(Y - X_1)} \cdot \left(S_{c,max,i} - X_1\right) + \epsilon \,, \quad \text{if } \log N_{S,ij} \le X_2$$

$$\log N_{S,ij} = X_2 + \frac{X_2 \cdot \ln(10)}{(Y - X_1)} \cdot \left(Y - S_{c,min,j}\right) \cdot \log\left(\frac{S_{c,max,i} - S_{c,min,j}}{Y - S_{c,min,j}}\right) + \epsilon \quad \text{if } \log N_{S,ij} > X_2 \tag{4}$$

where,

$S_{c,max,i} = |\sigma_{c,max,i}| / f_{cfat}$

$S_{c,min,j} = |\sigma_{c,min,j}| / f_{cfat}$

$\sigma_{c,max,i}$ and $\sigma_{c,min,j}$ are maximum and minimum stresses used to obtain $S_{c,max,i}$ and $S_{c,min,j}$

$\sigma_{c,max,i} = \sigma_{DL}(Z) + X_W \cdot \sigma_{WL,max}(Z)$

$\sigma_{c,min,j} = \sigma_{DL}(Z) + X_W \cdot \sigma_{WL,min}(Z)$

$f_{cfat} = \beta_{c,sus(t,t0)} \cdot \beta_{cc(t)} \cdot f_c \cdot (1 - f_c/400)$

$f_c = X_{f_c} \cdot f_{cm}$

Table 3: Stochastic parameters in limit-state-equation

Parameter	Dist* Type	Parameters Mean	Parameters Std. Dev.	Ref**
Δ	LN	1.0	0.30	SJ
X_{fc}	LN	1.00	0.14	Sec. 3.1
X_1	N	1.13	0.03	Sec 3.2
X_2	N	8.66	0.37	
ε	N	0.00	σ_ε	
σ_ε	N	0.88	0.07	
$\rho_{X_1,\sigma_\varepsilon}$	-	0.01		
$\rho_{X_2,\sigma_\varepsilon}$	-	-0.01		
ρ_{X_1,X_2}	-	-0.84		
X_W	LN	1.00	0.10	Sec 3.3

**: N-Normal, LN-Log-Normal*

*** SJ:* (Márquez-Domínguez & Sørensen, 2013)

6. STRESS CALCULATION AND MODELLING OF DESIGN PARAMETER

The stresses ($\sigma_{c,max}$ *and* $\sigma_{c,min}$) in the reinforced concrete section are obtained at the face of the embedded ring (at ground level, see red dot) considering the cantilever section of foundation as shown Figure 4.

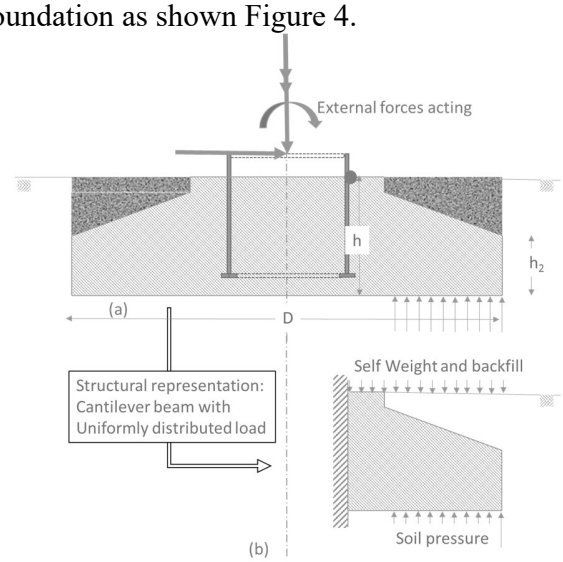

Figure 4: (a) geometry of foundation with external forces, (b) cantilever beam representative structure for (a).

The compressive stresses ($\sigma_{c,max}$ *and* $\sigma_{c,min}$) in the foundation concrete at the face of embedded

ring (at ground level where stresses in concrete in most cases are compressive) are obtained for various magnitudes of the reinforcement area (which is the design-parameter, Z).

The tensile reinforcement is chosen as the design-parameter, Z. since reinforcement area is a more costly material compared to concrete and optimizing amount of reinforcement is therefore more relevant than optimizing the quantity of concrete. The diameter (D) of the foundation is mostly governed by soil parameters or geotechnical aspects. The thickness of the foundation near the face of the embedded ring (h) and outer-thickness (h_2) is modelled as proportional to each other to keep the same slope of the foundation. The design value of thickness is obtained from the deterministic design equation (1).

Understanding and modelling the relationship between the design-parameter and the stresses in concrete is an important aspect as the design-parameter is the only connection between the design-equation and the limit-state-equation. The change of design-parameter may have different effects on mean level of stresses and amplitude of stresses. Such a relation is plotted in Figure 5.

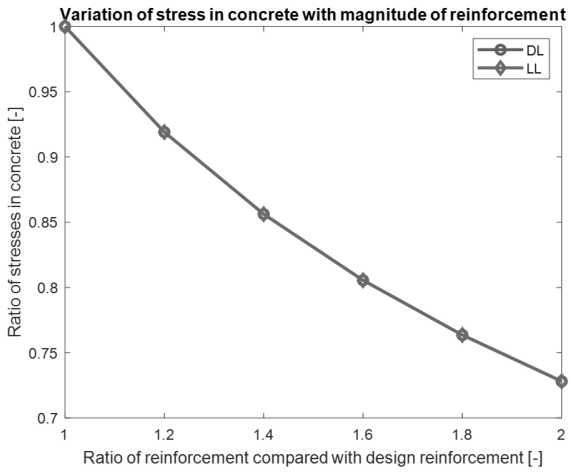

Figure 5: Stress variation as a funtion of variation in the design parameter.

The effect shown in Figure 5 reflects that increasing the magnitude of reinforcement increases bending stiffness that again reduces the dead-load-stress as well as live-load-stress, and thus the total-stress. The change in self-weight of the foundation when reinforcement area is changed is negligible, and hence not considered.

7. RELIABILITY ANALYSIS

A reliability analysis is performed using the limit-state-equation (3). The cumulative (accumulated) probability of failure in time interval $[0, t]$ is obtained by equation (5).

$$P_F(t) = P(g(t) \leq 0) \qquad (5)$$

The probability of failure is estimated by FORM, see (Sørensen J. D., 2011). The corresponding reliability index $\beta\ (t)$ is obtained by equation (6).

$$\beta(t) = -\phi^{-1}(P_F(t)\ ) \qquad (6)$$

where, $\Phi(\ )$ is standardized normal distribution function.

The annual probability of failure conditional on survival up to year $t - \Delta t$ is obtained from:

$$\Delta P_F(t) = \frac{P_F(t) - P_F(t - \Delta t)}{1 - P_F(t - \Delta t)}, \quad t > 1\ year \qquad (7)$$

where Δt =1 year.

8. RESULTS

Results include comparison the annual-reliability-index ($\Delta\beta$) values obtained from analysis with international code requirements. Further an example of optimization of the design parameter is presented.

8.1. Code requirements of reliability

When designing the wind turbine foundation according to IEC 61400-1 ed. 4 a recommended target annual probability of failure throughout planned fatigue life should be 5.10^{-4} and corresponding annual reliability index is 3.3, see background document for safety factors in IEC 61400-1, (Sørensen & Toft, 2014). This reliability level corresponds to minor /moderate consequences of failure and moderate / high cost of safety measure. It is noted that this reliability level corresponds approximately to the reliability level for offshore structures that are unmanned or evacuated in severe storms and where other

consequences of failure are not very significant. The same conditions could also be assumed for an onshore wind turbine where these wind turbines are installed sufficiently away from inhabitants and consequence in terms of loss of human life due to failure of wind turbine are negligible.

When designing the wind turbine foundation according to DNVGL an annual target probability of failure should not exceed 10^{-4}, which corresponds to annual reliability index of 3.7, see DNV-OS-J101. However, this standard is superseded by (DNVGL-ST-0126, 2016) and this new standard recommends using target reliabilities for similar existing design solutions or internationally recognised codes and standards.

8.2. Optimal design parameter

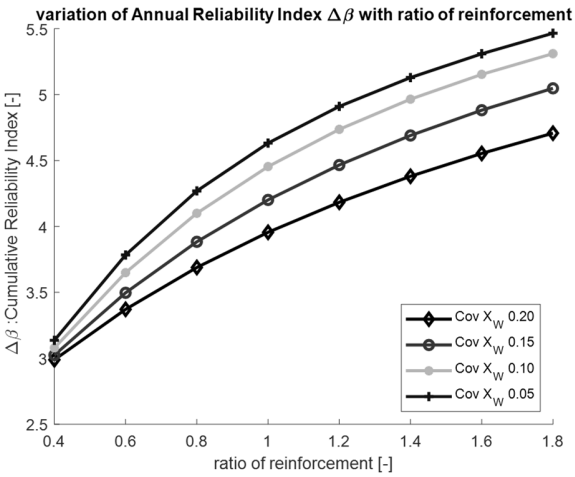

Figure 6: Annual reliability (Δβ) as function of design parameter

Figure 6 shows the annual reliability index $\Delta\beta$ as function of design parameter. To meet the target annual reliability indices $\Delta\beta$ set forth by codes in range of 3.3 to 3.7, the design-parameter (ratio of the reinforcement) to achieve this range of $\Delta\beta$ is 0.50-0.75. This shows that, only 50-75% (of 1540 mm^2/m) reinforcement is required to satisfy the reliability requirements with respect to fatigue failure of the concrete as compared to design requirements. Which means, in case of new structures these design requirements provide margin for life extension of the structures. It is noted that other design requirements may be governing for the design.

Figure 7 shows the annual reliability index ($\Delta\beta$) as function of material partial safety factor (γ_C) for T_L of 25years. The partial safety factor for load is assumed equal to 1.0. It can be seen that for the case with CoV_X_W = 0.15, a material partial safety factor (γ_C) of around 1.3 together with a partial safety factor on the load γ_{ED} = 1.0 is sufficient to meet the target annual reliability index Δβ > 3.3. Next, for the case with CoV_X_W = 0.20 a partial safety factor on load (γ_{ED}) can be calibrated to 1.45/1.3 = 1.1, to be used together with γ_C = 1.3.

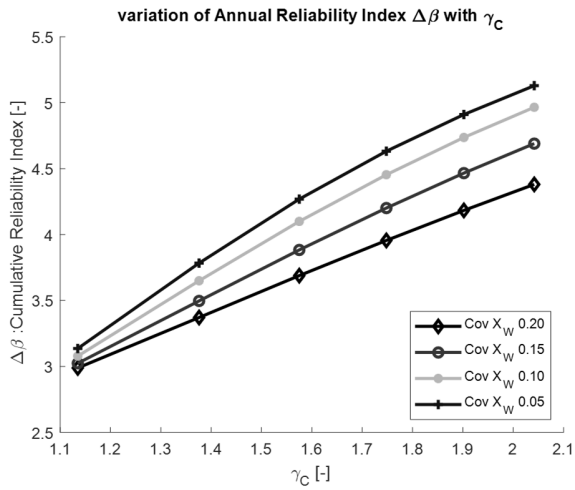

Figure 7: Annual reliability (Δβ) as function of γ_C

9. CONCLUSIONS AND FUTURE WORK

This paper presents a general approach for reliability assessment of wind turbine concrete foundations with respect to fatigue failure of the concrete. The approach can be used as basis for probabilistic design. The approach is illustrated in an example showing that the reliability level using present recommendations in standards is acceptable, and also that material savings could be possible. For future work an extension from component level reliability to system level reliability analysis is recommended. Further, combination with other limit states related to fatigue of reinforcement and ultimate failure due to extreme loads should be included.

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