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Reliability Assessment of Reinforce Concrete Structures Subjected to Progressive Collapse

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ABSTRACT: Progressive collapse has attracted more and more attention due to the happened extreme events. The static pushdown method is widely used to analyze the progressive collapse capacity of RC structures. However, the previous research hardly takes the random variables (e.g., geometric dimension and material properties) into consideration. This paper develops a reliability analysis framework for RC structures subjected to different column removal scenarios. In the framework, an efficient deterministic model is firstly developed based on the fiber element in software OpenSEES. Then a reliability analysis method is proposed based on the probability density evolution method (PEDM). Two typical RC frames are designed and tested by the reliability analysis method. The results show that the reliability analysis framework works well on the two designed RC frames.

1. INTRODUCTION

Progressive collapse of reinforced concrete (RC) structure has attracted more and more concerns around the world, since the frequently happened extreme event, such as terrorist attacks, explosion, vehicle impact and so on, have caused a lot of death and financial loss. It usually begins with the failure of local elements caused by accidental load, which then causes continuous failure of the elements connected with the failure elements, and finally leads to the global failure of the structure.[1,2] Disproportion is one of the biggest characters of progressive collapse. Although the initial loss may be small at the beginning, but the consequence will be a disaster. Therefore, it is necessary to study the progressive collapse behavior of the structures and develop an effective method to mitigate progressive collapse.

Because the extreme events are hard to identify and model, the treat-independent method is used more widely [2]. Treat-independent method pre-defines the position and extent of the local damage, and it doesn't consider the effect of extreme events. Among various treat-independent method, the alternative load path (ALP) method is used most. A lot of relative experiments have been done [3,4,5,6]. However, experiments cost too much on both money and time, and therefore numerical analysis also plays a significant role in progressive collapse research. There're two major families in numerical analysis, the high fidelity 3D finite element[7,8,9,10] and the efficient macro-level model [11,12,13,14]. Although the first approach is able to obtain the detailed local performance of a structure, the modeling procedure may be complicated and the computational burden is heavy. On the other hand, the macro-level model use fiber beam elements and/or macro joint elements to simplify the structures. In this way, the computational burden is greatly reduced, and it will be more convenient to get the global-level structural response, thus the second approach is more popular.

Additionally, the structural capacity of resisting progressive collapse is affected by various parameters, e.g., the load actions, the material properties and the geometric dimensions. These factors have great randomness. For example, the compressive strength of concrete is highly influenced by the curing condition, and therefore it's really an uncertainty. However, the considerations of randomness are rarely found in previous research, and thus a stochastic analysis method is greatly needed to take the uncertainties into considerations.

This paper presents a reliability analysis method for RC structure subjected to progressive collapse based on the well-known PDEM. The static pushdown method is used to calculate the structural capacity of resisting progressive collapse, and the efficient force-based element in OpenSEES is adopted to develop the macro-level model. Meanwhile, the PDEM [15,16] is used to consider the uncertainties, and thus the reliability of the structures under different column-removal scenario can be obtained. Additionally, two RC frames are designed according to the Chinese design code, and the reliability of the two is analyzed based on the above method.

2. DETERMINISTIC MODELING OF RC STRUCTURES UNDER PROGRESSIVE COLLAPSE

2.1. FE model for RC structures

Before considering the uncertainties of RC structures, a finite element model should be proposed firstly to simulate the deterministic behaviors of the structures. To achieve this goal, a micro-level model is set up based on software OpenSEES.

As shown in Fig.1, the efficient force-based fiber element (FBE) is selected to simulate the behaviors of beams and columns. The widely-used fiber section is employed here also. By dividing the section into several fibers with different stress-strain relationships, it will be convenient to consider the confined effect of concrete caused by transverse reinforcements [17]. Meanwhile, the co-rotational formulation is used to take the geometric nonlinear effect into account. In addition, the Krylov Newton method is selected as the nonlinear solution algorithm since it is proved to be effective.

As for material models, the bilinear model, which is named steel01 in OpenSEES, is adopted for steel rebars. The concreteD material is adopted to simulate the behavior of concrete, as it is developed based on the plastic-damage mechanics and can accurately reflect the plastic and damage behavior of concrete.

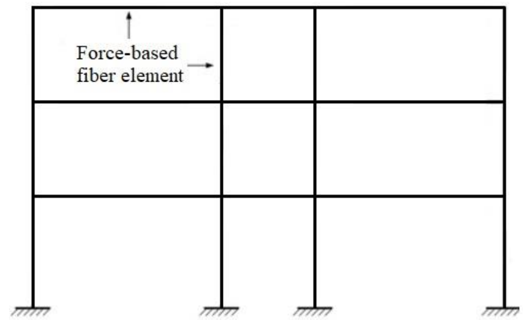


Figure 1: Finite element model for static pushdown analysis of RC structures

2.2. Pushdown analysis method

Static pushdown method is one of the wildly-used approach while analyzing the capacity of resisting progressive collapse. In the method, the critical elements are removed at first, and then the vertical load is applied to the remaining structure to test whether it still has the capacity of bearing the load. According to DoD (2013) and GSA (2013), the vertical load is usually calculated as 1.2×Dead Load (DL) + 0.5×Live Load (LL) . [18] Additionally, it should be paid attention to that only the vertical load in the damage bays is applied progressively, while the load in the other bays keeps as a constant, as shown in the Fig.2. By applying the load progressively, the relation between the vertical load and structure deformation can be obtained, from which we can analyze the bearing capacity of resisting progressive collapse.

The displacement-controlled procedure is the most common way to apply load, because it can still simulate the behavior of structures even in the later period of progressive collapse. When the vertical displacement Δ is applied to the damage element, the corresponding reaction force can be recorded, which usually written as $\alpha\times(1.2\times DL + 0.5\times LL)$. α is called the load factor, and there will always be a peak value α_{max} in the pushdown curve (α - Δ curve). α_{max} represents the maximum bearing capacity, and a larger α_{max}

means that the structure has a better capacity of resisting progressive collapse for a known column removal condition.

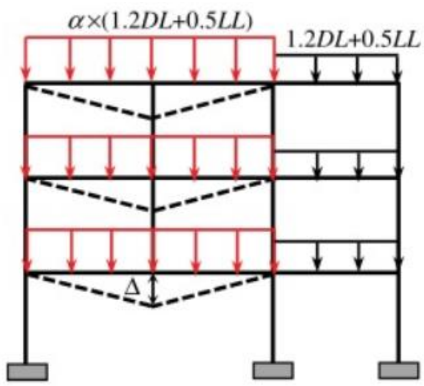


Figure 2: Static pushdown analysis under a interior column removal scenario

3. PROBABILITY DENSITY EVOLUTION METHOD FOR RELIABILITY ASSESSMENT

3.1. Fundamentals

Using the finite element method, the deterministic behavior of RC structures can be simulated and analyzed. However, uncertain variables always exist in RC structures, and therefore a stochastic analysis method is needed to perform the probabilistic analysis to reflect the influence of these variables. The PDEM, which is based on the principle of preservation of probability, is used in this paper. Through PDEM, not only the statistical information but also PDFs of the response can be obtained. It has been applied in several fields, and proves to be effective with small computational cost.

In general, the response of a random system can be expressed as

$$X(t)=H(\Theta,t),\quad \dot{X}(t)=h(\Theta,t) \qquad (1)$$

where t is the generalized time variable; X is the response of system (e.g., stress/strain); Θ is a vector that represents the random parameters in the system (e.g., material properties); $H(\cdot)$ is a transfer function that connects the response X with random parameters, and obviously $h=\partial H/\partial t$.

Eq. (1) holds for arbitrary structural systems, and during the system evolution, vector Θ has represented the same random parameters, thus it is a probability preserved system. In other words, we can set up the following formula,

$$\frac{D}{Dt}\int_{\Omega_t\times\Omega_\Theta} p_{x\Theta}(X,\Theta,t)dXd\Theta=0 \qquad (2)$$

where Ω_t is the distribution domains of the generalized time; Ω_Θ is the distribution domains of the random parameters; $p_{x\Theta}(X,\Theta,t)$ is the is the joint probability density of X(t) and Θ.Expand the formula, the following equation, which is called the generalized probability density evolution equation, can be obtained [15]

$$\frac{\partial p_{x\Theta}(X,\Theta,t)}{\partial t}+\frac{\partial p_{x\Theta}(X,\Theta,t)}{\partial X}h(\Theta,t)=0 \qquad (3)$$

To solve this formula, the initial condition as follow should be considered

$$p_{x\Theta}(X,\Theta,t)\big|_{t=0}=\delta(X-X_0)p_\Theta(\Theta) \qquad (4)$$

where X_0 is the initial value of X and $\delta()$ is the Dirac function. Once the $p_{x\Theta}(X,\Theta,t)$ is gotten, the PDF of X can also be obtained through integration

$$p_X(X,t)=\int_{\Omega_\Theta} p_{X\Theta}(X,\Theta,t)d\Theta \qquad (5)$$

3.2. Extreme-value event for reliability

Generally, the reliability of a structure can be expressed as the following formulation,

$$R=\Pr\left\{X(\Theta,\tau)\in\Omega_s,\tau\in[0,T]\right\} \qquad (6)$$

where $\Pr(\cdot)$ represent the probability of the given event; $X(\Theta,\tau)$ is the response of a structure; Ω_s is the security domain. In fact, the formulation is usually written in another way,

$$R=\Pr\left\{g(\Theta,\tau)>0,\tau\in[0,T]\right\} \qquad (7)$$

where $g(\Theta,\tau)$ is called the limit state function. Notice that for each step, the $g(\Theta,\tau)$ can be regarded as a random, thus the Eq.(8) is re-written as follows,

$$R=\Pr\left\{\bigcap_{0\le\tau\le T}(g(\Theta,\tau)>0)\right\} \qquad (8)$$

To solve the above formulation to obtain the reliability, the theorem proofed by Li and Chen will be helpful. According to the theorem, if we assume that $Z_{\min} = \min_{0\le\tau\le T}(g(\Theta,\tau))$, in which $Z_{\min}$ is so-called the extreme value event,[19] then the Eq.(8) can be changed as,

$$R = \Pr\left\{\bigcap_{0\le\tau\le T}(g(\Theta,\tau)>0)\right\} = \Pr\{Z_{\min}>0\} \quad (9)$$

Once we know the distribution of $Z_{\min}$, the reliability of the structure can be calculated as

$$R = \Pr\{Z_{\min}>0\} = \int_0^{\infty} pz_{\min}(z)dz \qquad (10)$$

where $pz_{\min}$ is the distribution of the extreme value $Z_{\min}$, and it is obvious that $pz_{\min}$ can be conveniently obtained by the above-mentioned PDEM.

In summary, the implementation of the proposed method is indicated in Fig.3.

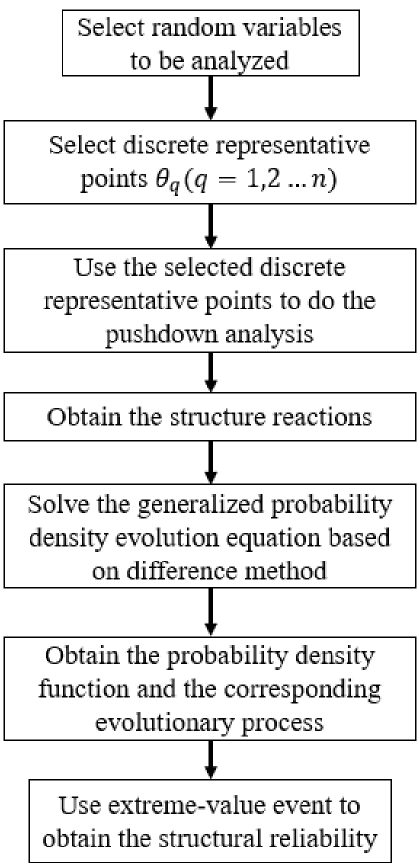


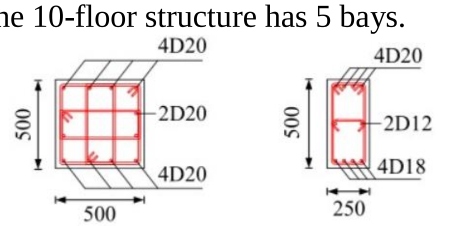
Figure 3: The process of the proposed method

4. NUMERICAL EXAMPLE

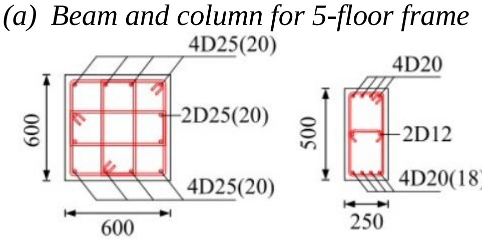
4.1. Design of structure

In this paper, two RC frames are designed to verify the proposed reliability-based robustness quantification method. Both of them are designed based on the Chinese design code of concrete structure [20], and one of them is 5-floor high while another is 10-floor high.

The two frames share something in common. For example, the first floor of the two are both 4500mm high, while the other floors are 3600mm high; the span for each bay is 6000mm. However, due to the difference of height, the two are different in element dimension and reinforcement area. More details can be seen in Fig.4. In addition, it should be noticed that for 10-floor frame, the reinforcement areas are different in 1-4 floors and 5-10 floors, the rebar diameters outside the bracket are used for 1-4 floors while the diameters inside the bracket are for 5-10 floors, as shown in Fig.4. In addition, the 5-floor structure has 4 bays while the 10-floor structure has 5 bays.



(a) Beam and column for 5-floor frame



(b) Beam and column for 10-floor frame
Figure 4:Design of RC structures

The two frames are designed based on the same load parameters. According to Chinese design code, the dead load is set as 5 kN/m^2 for floors and 7kN/m^2 for the roof, while the live load is set as 2kN/m^2 for both the floor and the roof. As for seismic load, the two structures are assumed to be built in Nanjing, China, where the seismic intensity is set to be 7.0.

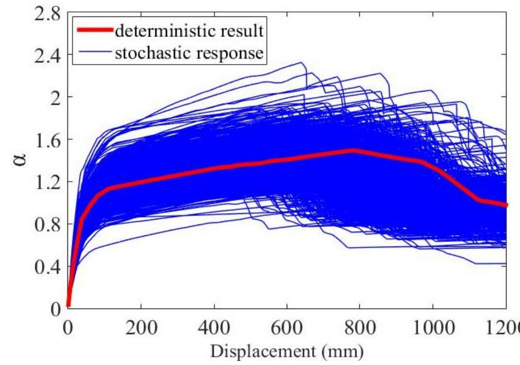
4.2. Reliability analysis results

Using the above-mentioned deterministic modeling method and the PDEM-based reliability analysis method, the stochastic responses of the two RC frames can be obtained and the reliability of them can be greatly estimated. The geometric dimensions, material properties, and the dead/live loads are all set as random variables, just as listed in Table.1. Based on PDEM, 800 representative points are selected for the stochastic/reliability analysis.

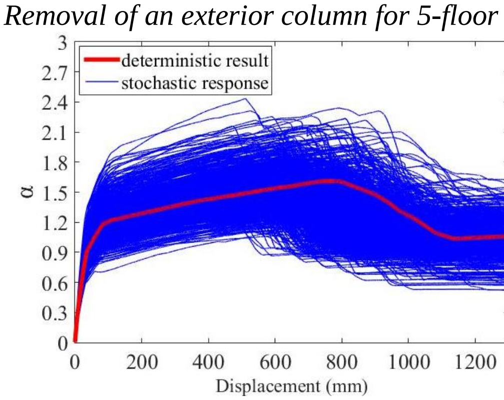
Table 1: Probability information of random variables

Variables	*Mean value*	*COV*	*Probability distribution*
Span	*6000mm*	*0.003*	*Normal*
Storey height	*4500/ 3600mm*	*0.003*	*Normal*
Column width	*600mm*	*0.01*	*Normal*
Beam height	*500mm*	*0.01*	*Normal*
Beam width	*250mm*	*0.01*	*Normal*
Cover depth	*30mm*	*0.01*	*Normal*
Rebar diameter	*12/18/20/25mm*	*0.04*	*Normal*
Concrete compressive strength	*20/26MPa*	*0.18*	*Normal*
Concrete elasticity modulus	*30000MPa*	*0.15*	*Lognormal*
Concrete tensile strength	*2.0/2.6MPa*	*0.18*	*Normal*
Steel elasticity modulus	*200000MPa*	*0.033*	*Normal*
Steel yield strength	*400MPa*	*0.093*	*Beta*
Steel ultimate strength	*650 MPa*	*0.08*	*Beta*
Steel ultimate strain	*0.12*	*0.15*	*Lognormal*
Floor dead load	*5.0kN/m2*	*0.1*	*Normal*
Roof dead load	*7.0kN/m2*	*0.1*	*Normal*
Live load	*2.0kN/m2*	*0.4*	*Beta*

Fig.5 shows part of the stochastic responses under different column-removal scenarios. As we can see, uncertainties do have a significant influence on the structural capacity of resisting progressive collapse, and the capacity may even be reduced by half under some extreme conditions. It can also be seen that when it comes into the nonlinear phase, the uncertainties show a stronger influence on structural responses. It means that the uncertainty and the nonlinearity have a coupling amplification effect, and it is hard to consider this effect in theoretical calculation, but a reliability-based design method may be helpful to it.



(a) Removal of an exterior column for 5-floor frame



(b)Removal of an interior column for 10-floor frame

Figure 5: Part of the stochastic results for the two designed RC frame

Based on the PDEM, the distribution of the extreme values of the given event can be obtained, and then the reliability of the structures can be analyzed. Table 2 gives the reliability analysis results for 6 different column-removal scenarios. The columns are recorded as A, B and so on from left to right. In the figure, it can be seen that different failure modes will greatly affect the structural reliability. When the exterior column is removed, it show a less reliability than the interior column-removal scenario. The possible reason is that there will be a tensile catenary action in the interior column-removal scenario, while the exterior column-removal scenario does not.

Table 2: Reliability under different column-removal scenario.

The 5-floor frame		*The 10-floor frame*	
Column removal	*Reliability*	*Column removal*	*Reliability*
A	*94.62%*	*A*	*50.86%*
B	*96.12%*	*B*	*58.38%*
C	*96.25%*	*C*	*59.86*

5. CONCLUSIONS

The present paper develops a reliability analysis framework for RC structures under different column-removal scenario. The framework can be divided into two parts, a deterministic modeling method based on fiber elements and a reliability analysis method based on the PDEM. Meanwhile, two typical RC frames are designed as the numerical examples, and they are analyzed by the proposed reliability analysis method. Based on the results, the following conclusions can be drawn:

- The stochastic analysis method can obtain the stochastic responses of RC structures effectively, and it lays a foundation for the reliability analysis.
- The reliability analysis framework works well on the two designed RC frames.
- In the progressive collapse caused by column failure, the exterior column removal shows a less reliability than the interior column-removal scenario.

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