A Multistage Stochastic Model for the Management of Flexibility in Infrastructure Systems

Samuel Torres Rincón
Graduate Student, Dept. of Civil and Environmental Engineering, Univ. de los Andes, Bogotá, Colombia

Mauricio Sánchez Silva
Professor, Dept. of Civil and Environmental Engineering, Univ. de los Andes, Bogotá, Colombia

Daniel F. Villarraga
Graduate Student, Dept. of Civil and Environmental Engineering, Univ. de los Andes, Bogotá, Colombia

ABSTRACT: Flexibility can be an attractive property to add the infrastructure systems during the design phase to reduce the impact of uncertainty. The management of this flexibility, however, is a complicated sequential decision problem that must be solved to extract all the benefits that flexibility may offer. In this paper, a multistage stochastic model is proposed to simulate the flexibility management process. The model combines a novel mathematical representation of flexibility with a set of policies and a scenario-based representation of uncertainty to determine the optimal initial configuration and the optimal flexibility for the system. The results showed that, under specific conditions, flexibility can be a desirable property to add to the system. In addition, the model provides valuable insight about the interaction between system design properties and management policies.

1. INTRODUCTION

Infrastructure systems are susceptible to the effects of uncertainty due to their long life-cycles, constant exposition to external agents, and fixed nature. When the external conditions change the system state may be affected, decreasing the benefits the system provides. In other cases, the external changes may offer new opportunities that the system is incapable to exploit. To face the negative effects of uncertainty, designers can add special characteristics to the system such as robustness, redundancy, or flexibility.

Flexibility, understood as the ability of a system to easily modify some of its operational or design variables, can be a desirable characteristic to add during the design phase. According to some authors, flexibility has the potential to decrease the undesired effects of uncertainty while enabling the system to benefit from unexpected opportunities (Ross et al., 2008; Cardin et al., 2015). Other authors suggest that flexibility may enable close to optimal performance over a range of diverse conditions (Fitzgerald, 2012).

Even though flexibility has been studied adequately to understand the benefits it may offer, the concept is still far from being completely understood. Specifically, the previously provided definition is not standard, and it was derived from the works of a segment of researchers. Although the core elements form the definition seem to be common in the most recent works, there are still many interpretations due to the multidisciplinary nature of the concept (Saleh et al., 2009). In addition, the lack of consensus in the conceptual definition has, expectedly, transferred to the mathematical representation. Nearly every author that has written about the topic has proposed their own measurement of flexibility, with few similarities between works. For instance, Swaney and Grossmann (1985), and later Pistikopoulos and Mazzuchi (1990), proposed to measure flexibility in chemical processes as the maximum fluctuation allowed in the system's operational variables that maintain the operation inside the feasible range. In the context of transportation, Morlok and Chang (2004) defined flexibility as the maximum traffic that a network can mobilize. In a completely different approach, Olewnik and Lewis (2006) coupled flexibility with the concept of adaptation effectiveness. The relationship was established by describing flexibility as the distance between extreme Pareto points inside the system's performance space. These examples show that even if the word flexibility is ubiquitous, the interpretation of the concept is far from being unique.

Despite these limitations, other areas of flexibility research have advanced independently with promising results. For instance, the problem of determining the value of adding flexibility has prompted the development of new methodologies such as real options analysis (DeNeufville, 2003). The flexibility valuation problem is closely related with the flexibility management problem where the system manager must determine the system's elements to change, the timing of the change, and the magnitude of the change, given a vector of uncertain parameters. This problem is often analyzed as a sequential decision problem and solved using techniques such as dynamic programming, control theory, and stochastic programming.

Specifically, stochastic programming(SP) is a modeling framework used to provide an approximate initial solution to the decision variables. Because it focuses on finding the initial decision instead of the sequence of future decisions, the SP approach has been successfully used to determine the set of policies that control future changes. Cardin et al. (2017) developed a method to manage flexibility that uses different types of decision rules (linear, constant, conditional) to help decide the system manager when to execute the adaptation options. The authors proposed a multistage stochastic model to determine the optimal set of decision rules. Zhao et al. (2018) used a similar approach to solve a multiple-facility capacity expansion Problem. Other approaches to determine the optimal decision rules include the use of techniques such as Differential evolution (Hu et al., 2018). The previous approaches are focused on determining the optimal set of policies to manage the system; however, the design flexibility is not considered explicitly.

In this paper, a novel approach that combines a flexibility index and a multistage stochastic program is proposed. By taking into account the flexibility designed into the system, the future decisions will be limited by the initial design characteristics,i.e., the initial design value and the available flexibility. Under these conditions, the stochastic program can determine the optimal initial design characteristics.

2. MANAGING FLEXIBILITY THROUGH POLICIES

The value provided by the addition of flexibility to the design highly depends on how the flexibility is managed across the system's life-cycle. If the available flexibility is executed too early or too late, the system may not receive all the potential benefits. Similarly, this potential loss may happen if the magnitude of the changes is higher or lower than needed. These decisions are affected by the uncertainty surrounding the system. The addition of flexibility only provides the potential to adapt; the decisions are the real value driver behind flexibility.

The decisions in flexibility management depend on a set of policies that inform the system manager the subsystems that should be modified and the timing and magnitude of the modifications. The policies can take different forms such as exact numerical values, conditional thresholds, qualitative descriptors, and functions. Some of the policies may be arbitrary values while others may be determined as optimal solutions of a mathematical program. In a more strict sense, a policy works as a mapping between states of the world or the system, and decisions made on the system characteristics at every time instant.

Determining the optimal decision path given a policy set is not a trivial problem. It can be formulated as a sequential decision problem and the methods to solve it are severely limited by the curse of dimensionality. A typical example of a solution method is the dynamic programming (DP) approach, which rapidly explodes when the number of variables or the number of analysis periods increase. A different method which relies in approximations is the multistage stochastic programming (MSP) approach. In this case, the problem may also become intractable when the model grows in complexity, but the real value of the method lies in the focus of the solution. Compared with the DP approach, the MSP approach focuses on finding a robust initial decision. Even if a MSP model is capable of finding a decision at every stage for every possible outcome of the uncertain parameter, its real strength lies in being capable of finding optimal stage zero decisions (policies, initial design, flexibility) by taking into account all the possible decisions that can be made in the future.

In the next section a brief discussion on the Multistage Stochastic Programming theory is presented.

3. MULTISTAGE STOCHASTIC PROGRAMMING

Multistage stochastic programming (MSP), specifically linear multistage stochastic programming with recourse, is a modeling framework used to solve sequential decision problems. The multistage approach is a generalization of the two stage problem where a decision is made at the initial stage -stage zero- without complete knowledge of an uncertain parameter. At the second stage, the realization of the uncertain parameter becomes known and the decision maker has the option to make a second decision -the recourse- to adjust the initial decision. When the approach is extended to the multistage case every recourse decision will affect the state of the system at all the subsequent stages. For this reason, the stage zero decisions and every recourse decision up to stage $T-1$ are calculated based on the expected value of the system future state, as a function of the uncertain parameter at each stage. Formally, a MS program can be formulated as follows (Birge and Louveaux, 2011):

$$
\begin{aligned}
\min_{x} \quad & c^0x^0 + E_{\xi}[\min c^1x^1 + \dots \\
& + E_{\xi}[\min c^T x^T]\dots] \\
\text{s.t.} \quad & W^0x^0 = b^0, \\
& H^0x^0 + W^1x^1 = b^1, \\
& \dots \\
& H^{T-1}x^{T-1} + W^Tx^T = b^T, \\
& x^t \geq 0 \quad t = 2,\dots,T
\end{aligned}
\tag{1}
$$

Where x^t are the decision variables, c^0 are the known costs, W^0 and b^0 are the known matrix and vector, respectively; ξ represents the random parameters in W^t, H^t, b^t and c^t, for $t = 2,\dots,T$. The general formulation is divided in two main elements: the known costs for the stage zero, and the expected costs for all the future stages. The second element is usually called the value function.

Before solving the problem described by Eq. 1, an approximation of the distribution of the uncertain parameters ξ is needed. The approximation is usually carried out through methods that discretize the original continuous distribution (e.g. Gauss-Hermite) (Brandimarte, 2006). The resulting discretization defines a finite number of future states at every stage, which can be represented by a scenario tree structure (Figure 1). In this structure every node represents a discretization point of the distribution, while every preceding branch has associated the probability of that particular value. At the final stage, there will be as many nodes as scenarios, each one with a specific probability. As expected, the number of scenario can grow exponentially and the problem may become intractable if there are many uncertain parameters or if the discretization is too detailed.

To solve a MS program, different methods can be used depending on the particular problem structure. If the multistage program is linear, a deterministic equivalent linear program can be constructed from the scenario tree. However, the resulting structure may be too complex to solve by simply using linear solvers. In some cases, an approximation to the value function in the form of outer or inner linearizations may be required. In other cases, con-

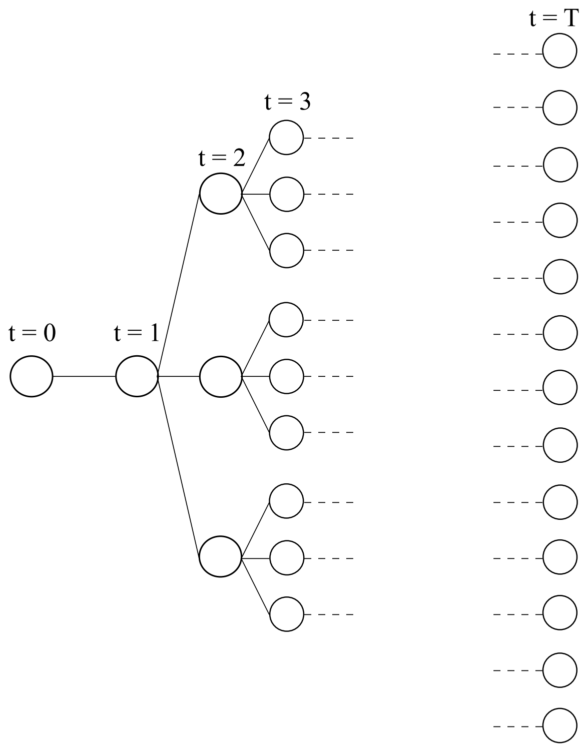

Figure 1: Scenario tree representation for the uncertain parameter discretization

straint relaxation or solving the dual problem can increase the efficiency of the solution process. Finally, Monte-Carlo methods are available to obtain a representative sample of the uncertain parameter, reducing the problem size (Birge and Louveaux, 2011).

4. MEASURING FLEXIBILITY

The development of a multistage stochastic model that returns the optimal flexibility that should be designed into the system requires a numerical representation of the attribute. Torres Rincón et al. (2018) developed a straightforward measure of flexibility based on four components, as shown in Eq. 2:

$$F_i(t) = k\frac{s_i(t)}{c_i} \tag{2}$$

Eq. 2 represents the flexibility of a design or operational variable i that was designed to be able to change inside the available flexible range $s_i(t)$. The unitary cost of executing an adaptation inside the available flexible range is c_i. These two elements characterize the change process when the system is designed for flexibility. The parameter k is defined as the ratio $c_{out}/x_i(0)$, where the additional elements c_{out} and $x_i(0)$ represent the cost of modifying the system's variable X_i *outside* the design range $s_i(t)$, and the initial value of the variable, respectively. The inclusion of these two elements increases or reduces the impact of the flexible properties $s_i(t)$ and c_i on the overall perception of the variable flexibility. For instance, a system whose design variable can be changed 4 units will be considered more flexible if the initial value of the variable was 10 units, compared with a initial design of 20 units. In the first case, the option to adapt is more impactful in relation with the initial configuration.

The index presented in this section provides a straightforward approach to include flexibility into a multistage stochastic model, which is presented in the following section.

5. OPTIMAL FLEXIBILITY

The flexibility management problem can be modeled as a sequential decision process under uncertainty, and can be solved using a multistage stochastic program. Because MS programs focus on obtaining a robust stage zero solution rather than finding the optimal evolution process, the proposed model focuses on finding the optimal initial system capacity and the optimal flexibility range, by minimizing the expected costs for all future stages. The model considers as stage zero costs the cost of building the initial capacity and the cost of introducing a certain level of flexibility, which is introduced in the model as the available flexible range. In addition, the model assigns a decision variable to the magnitude of the change required for each stage; this change can happen inside the available flexible range, or outside if the range is exhausted. These additional variables are not as important as the stage zero variables because they are scenario dependent; i.e., for each possible value of the uncertain parameter a solution for these variables will be computed. In contrast, the stage zero variables solution will be unique for all possible scenarios.

The costs at any future stages will be the cost of any executed adaptation inside or outside the flexible range, the costs of operating the current capacity and the revenue captured due to the current demand. The restrictions of the model establish that the current capacity must exceed the current demand by a factor determined by the flexibility policy; additionally, the changes made inside the flexible range must be equal or lower than the available flexible range. The last set of restrictions are the non-anticipativity constraints required in any MS program. The following equation describes the proposed model:

$$\begin{aligned}
\min_{\mathbf{X},f} \quad & C_0X^0 + C_f(f) + E_\xi[\min\gamma(c_iX^1(\xi^1) \\
& c_{out}Y^1(\xi^1) + c_{op}W^1 - b\xi^1) + \ldots \\
& + E_\xi[\min\gamma(c_iX^T(\xi^T) + c_{out}Y^T(\xi^T) \\
& + c_{op}W^T - b\xi^T)]\ldots] \\
\text{s.t.} \quad & W^t \geq R\xi^t \quad t = 1,\ldots,T \\
& \sum_{i=1}^{t} X^i \leq S_i \quad t = 1,\ldots,T \\
& X_t^s = X_t^{s*}, \quad \forall t,s,s* \in \{s\}_t
\end{aligned} \tag{3}$$

In Eq. 3, C_0 represents the unitary cost of building the initial system configuration, C_f represents the unitary cost of introducing a f amount of flexibility into the design, γ is the discount factor, c_i is the unitary cost of deploying an adaptation at every stage t, c_{out} is the unitary cost of modifying the existing design *outside* the available flexible range, c_{op} is the unitary cost of operating the current design, and b represents the unitary revenue. X^t,Y^t, W^t and S_i are the decision variables that represent the initial configuration and the adaptations inside the flexible range, the adaptations outside the flexible range, the total capacity, and the available flexible range, respectively. R is an element of the flexibility policy that represents a safety ratio between the available capacity and the current demand. The set $\{s\}_t$ is the set of all the scenarios that are equal to scenario s up to stage t.

The novelty of the proposed model is the inclusion of an indicator of flexibility as a decision variable. While the flexibility index proposed in the previous section is not explicitly used, two components of the index, the available flexible range and the initial capacity, are used as decision variables. The other two components, namely the unitary cost of a flexible change ci and the unitary cost of change outside the flexible range c_0, become parameters of the model. In this way, the proposed measure of flexibility is included in the MS program implicitly.

6. NUMERICAL EXAMPLE

6.1. Description

In this section a straightforward example is presented to illustrate the capabilities of the model. The MS program was formulated and solved using the commercial software AIMMS and the commercial solver CPLEX, both under academic licenses. The system simulated only has one variable of interest (capacity) affected by the uncertainty in one external parameter (demand). The system initial design value and the available flexible range for that design variable are the decisions to optimize. The costs to minimize depend on the unitary cost and unitary revenue parameters, listed in Table 1.

One of the most difficult problems when solving a MS program is the approximation of the uncertain parameter. If a finely discretization of the distribution is used, the number of scenarios may render the problem intractable. Conversely, if the discretization is too sparse, the solution may be useless. For this example, however, a strong approximation had to be used to reduce the computational times necessary to find a solution. This decision to approximate is justified because at this stage of the research analyzing the general behavior of the model provides more insight than a particular result.

For this example the uncertain parameter was discretized into three cases: low growth, medium growth, and high growth. For each of the cases there is a probability associated and a range of values that the demand may take at each stage, as shown in Table 2. At the final stage there are 3^n possible scenarios, with n equal to the number of stages, for the uncertain parameter that must be evaluated by the program. For this reason, in an attempt to reduce the number of scenarios, this numerical example considers 8 stages where 1 stage covers a period of 5 years, modeling a typical system's life-cycle of 40 years. The decision to group many periods into 1 stage is justified because a typical infrastructure system is, usually, not expected to be subject to changes very frequently.

The model computes the expected costs for each of the demand scenarios at each stage. As a result, the optimal initial capacity and the optimal available flexible range are determined, while a solution for the adaptation magnitude inside and outside the flexible range for each demand scenario are also calculated. The adaptation magnitude variables are not as important because they are scenario dependent; the main results are the stage zero decisions. The next section summarizes the results of the numerical computations.

Table 1: Model costs

Unitary cost	Value
C_i	2.50
C_0	3.00
Op	0.10
$Revenue$	0.20
$FlexCost$	1.50

Table 2: Uncertain parameter discretization

Case	Lower limit	Upper limit	Probability
Low growth	0.40	0.80	0.30
Medium growth	0.80	1.20	0.40
High growth	1.20	1.60	0.30

6.2. Results and Discussion

Figure 2 shows two instances of demand scenarios and the response suggested by the model to the system design variable. This response is conditioned by the initial design characteristics (initial value and available flexible range), the uncertain parameter distribution, and the flexibility policy. Specifically, the policy is determined by two elements, the inspection frequency (each stage comprises 5 years) and the conditional threshold R, discussed in Section 5. While the inspection frequency is an intrinsic part of the model, the conditional threshold is introduced as a constraint. In both cases, however, the values for these elements are defined by the user.

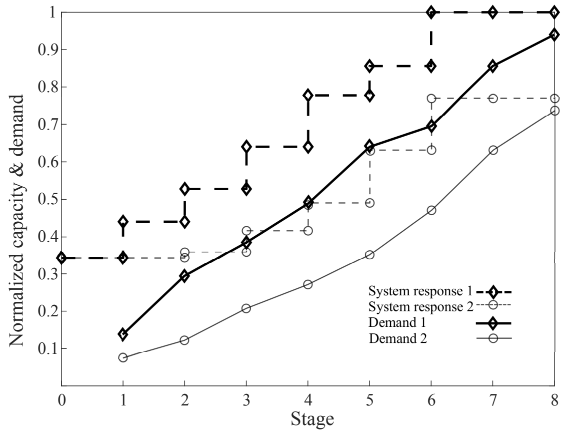

Figure 2: Two different realizations for the uncertain parameter and the system adaptive response

Figure 3 presents the effect of the conditional threshold on the initial decision variables, X^0 and S_i. Increasing the threshold increases the overall magnitude required for the design variable. This increment is shared uniformly between X^0 and S_i without preference. If the unitary cost c_f of introducing flexibility behaved exponentially, or any non-linear trend, this behavior would not have been observed.

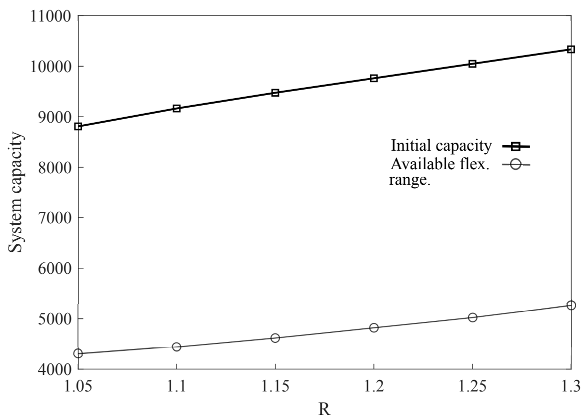

Figure 3: Effect of the threshold ratio, defined by the policy, on X^0 and S_i

Nonetheless, c_f still has a remarkable impact on the results of the model, as shown in Figure 4. This figure illustrates how the decision variables X^0 and S_i are affected by the ratio between the unitary cost of introducing flexibility c_f and the unitary cost of building the initial design c_0. When the ratio is small, the model will allocate more space for future adaptations. Conversely, when the ratio is high the model will prefer to invest in a larger initial design. While these results are expected, it is valuable that the model is capable of replicating this behavior organically.

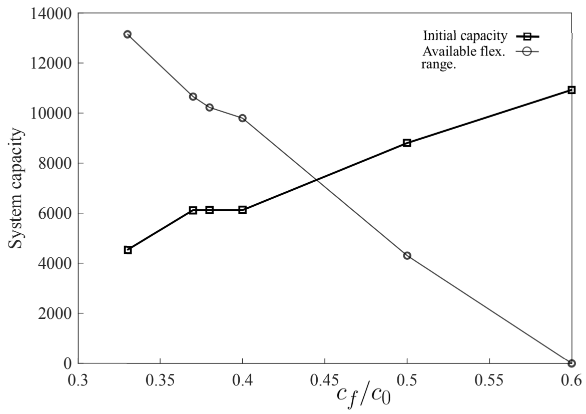

Figure 4: Effect of the cost c_f of introducing flexibility on X^0 and S_i

Finally, Figure 5 shows the effect that the ratio between the costs c_{out}/c_i has on the desirability of flexibility represented by the available flexible range S_i. If the system, due to its characteristics, is easy to modify without being designed for flexibility, or if the added flexibility is not enough to reduce the cost of flexible adaptations c_i, the ratio c_{out}/c_i is low and the model recommends not to introduce flexibility. In contrast, if the cost of modifying the system without flexibility is high, or if the added flexibility manages to reduce considerably the adaptation costs, the ratio c_{out}/c_i increases, raising the desirability of flexibility.

7. CONCLUSIONS

Flexibility is property that has the potential to increase infrastructure systems' performance in the face of uncertainty. The realization of this potential highly depends on the management of flexibility. The management process is a complicated sequential decision problem where many elements converge. The initial system configuration, the flexibility design measures, the adaptation policy, and the perception of uncertainty determine the system

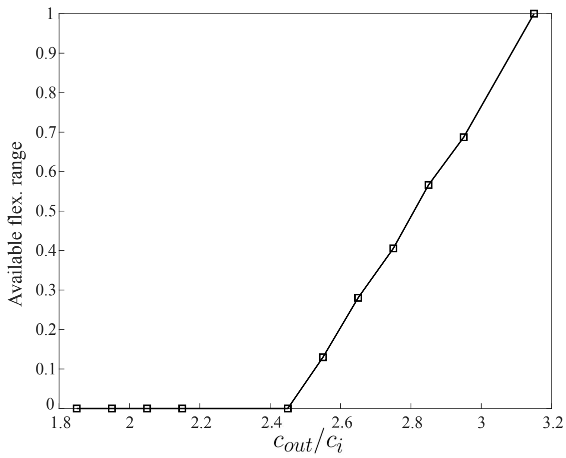

Figure 5: Effect of the costs of adaptation inside and outside the available flexible range, c_i and c_{out}, on the desired flexible range S_i

adaptation trajectory, and the value generated by flexibility.

As a contribution to improve the understanding of this problem, a multistage stochastic program was formulated and solved. The model combined a novel mathematical representation of flexibility with a specific flexibility policy that determined some of the model parameters. While previous works have focused on determining the optimal set of policies (Cardin et al., 2017), the proposed model focused on finding the optimal initial configuration and the optimal flexibility by considering multiple evolution trajectories. The model was successful in simulating the many interactions that exist between system properties, costs, and policies.

Additional research is necessary to formulate a more robust model that can consider multiple design variables. Furthermore, the development of a model that can determine the optimal sequence of actions given a flexibility policy would also be of value to the field.

8. ACKNOWLEDGMENTS

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