Risk-Informed Hazard Loss of Bridges in a Life-Cycle Context

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ABSTRACT: Coastal bridges are vulnerable to typhoon hazards due to increasing exposure to climate change over the life-cycle. The consequent hazard-induced loss is affected by uncertainties arising from the typhoon hazard model in terms of increasing occurrence of typhoon frequency and intensity. In the literature, researchers highlighted hazard model of the stochastic homogeneous Poisson process in an infinite time interval and neglected quantification of uncertainty caused by climate change. This paper estimates the expectation and variance of lifetime typhoon-induced loss of bridges considering both with and without climate change effects using renewal theory and moment generating function methods. This paper aims to aid the design of coastal bridges by considering the changing climate effects on hazard management process.

1. INTRODUCTION

Global climate change is expected to enlarge the vulnerability of infrastructure worldwide due to increasing exposure to warming temperature, sea level rise and coastal typhoons (IPCC 2014). The resulting impacts considerably increase the probability of occurrence of typhoon and the consequent monetary loss. Therefore, consequence evaluation of civil infrastructures considering climate change is cumulatively important. The hazard-induced loss can be one of the most uncertain components in life-cycle analysis, because of the inherent uncertainties in terms of occurrence and intensity of hazard events (Frangopol *et al* 2017). To quantify the hazard uncertainties, researchers have highlighted the homogeneous Poisson process to model typhoon arrivals (Elsner and Bossak 2001; Vanem 2011; Dong and Frangopol 2016). Based on the stochastic process, hazard-induced loss within structural life-cycle has been widely studied. Wen and Kang (2001) investigated the minimization of expected life-cycle cost of an office building against earthquakes and wind hazards based on homogeneous Poisson Process. Takahashi *et al* (2004) proposed a decision methodology to manage seismic risk of a building, in which the life-cycle cost analysis according to a renewal model is proposed. Unlike the stationary hazard model of earthquakes, typhoon model can be non-stationary because it has been suggested that occurrence of typhoon activities can be affected by climate change (Bender *et al* 2010). Recent studies considered a non-stationary stochastic process to model the typhoon activities. Lin and Shullman (2017) used non-homogeneous Poisson process to model the expected monetary loss of New York city induced by hurricane surge flooding with a changing climatic environment. The non-stationary lifetime loss is obtained from a time-varying loss annually.

Although the lifetime hazard-induced loss has been partially investigated in these literatures, loss models based on homogeneous Poisson process has limited to the expectation of lifetime loss neglecting the variance. The loss estimation of non-stationary typhoon process is simplified by the sum of annual loss, which may result in underestimate of the uncertainty of the non-stationary stochastic model. Therefore, this paper proposed renewal theory and moment generating method to address these issues. Expectation and variance of lifetime typhoon-induced loss are investigated. The proposed analytical methods are validated by Monte Carlo simulations numerically. The renewal theory and moment generating function approaches are proposed to estimate typhoon-induced loss for coastal bridges in a life-cycle context considering the effects of climate change to aid climate adaptation of civil infrastructures. A framework of the life-cycle risk assessment of bridges under typhoon hazard considering climate change is shown in Fig. 1. The risk is the product of probability of structural failure from vulnerability model and the consequence from loss model. The hazard model and loss model are emphasized in this paper.

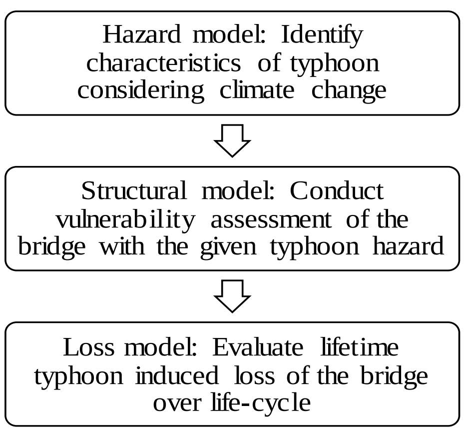

Fig. 1. Framework of the risk assessment of bridges involving hazard, consequence, and loss models.

2. STOCHASTIC HAZARD MODELS

The uncertainty of typhoon occurrence is widely quantified by a stochastic process model. In probability theory, a counting process denoted as $\{N(t), t \geq 0\}$ can be used to model event arrivals, in which $N(t)$ represents the number of events within time interval t. In this process, the arriving times of each typhoon can be denoted as $0 < T_1 < T_2 < \dots$ until T_k is the arrival epoch at the k_{th} occurrence with $T_k \leq t$. The inter-arrival time between each two adjacent typhoons can be expressed by X_1, X_2, … and the inter-arrival time X_k represents inter-arrival interval between the k_{th} and $(k-1)$th event for $k > 1$. The definition gives $X_k = T_k - T_{k-1}$ and $T_k = X_1 + X_2 + \dots + X_k$.

The Poisson process has been widely used to model the random occurrence of hazard arrivals, such as in the typhoon risk analysis. A counting process becomes a homogeneous Poisson process (HPP) when the inter-arrival times are independently and identically distributed (IID) and follows an exponential distribution. Therefore, the inter-arrival time of an HPP typhoon model can be expressed as $X \sim \text{EXP}(\lambda)$ with a mean value of $1/\lambda$, where λ is the annual occurrence rate denoting the number of typhoon events per year. Therefore, the mean value of the number of typhoons over the life-cycle $(0, t_{\text{int}}]$ is $E[N(t_{\text{int}})] = \lambda t_{\text{int}}$. The probability of k number of events $N(t_{\text{int}}) = k$ can be written as

$$\text{P}[N(t_{\text{int}}) = k] = \frac{(\lambda t_{\text{int}})^k e^{-\lambda t_{\text{int}}}}{k!}, k = 1, 2, \cdots \tag{1}$$

However, a stationary incremental arrival rate is less likely applicable to a changing environment. The probability of typhoon occurrence is expected to increase due to exposure to warming climate. Hence, a non-homogeneous Poisson process (NHPP) is suggested with respect to the model considering climate change. A NHPP typhoon model has a time-dependent occurrence rate $\lambda(t)$ and has the mean number of typhoon events over the life-cycle t_{int}

$$E[N(t_{\text{int}})] = \int_0^{t_{\text{int}}} \lambda(t)dt \tag{2}$$

The probability of k number of events over period t_{int} becomes

$$\text{P}[N(t_{\text{int}}) = k] = \frac{\left(\int_0^{t_{\text{int}}} \lambda(t)dt\right)^k \exp\left(-\int_0^{t_{\text{int}}} \lambda(t)dt\right)}{k!}, k = 1, 2, \cdots \tag{3}$$

3. LIFE-CYCLE HAZARD INDUCED LOSS

Considering the life-cycle of a bridge over $(0, t_{\text{int}}]$, the hazard events are recorded by an index k.

Arriving time of the k_{th} hazard is denoted as T_k with a consequent loss represented by L_k. L_k is described as loss severity. It is assumed that the bridge is repaired to the initial stage immediately after each typhoon event. Inter-arrival time is denoted as X_k, in which $X_1 = T_1$ equals to the arriving time of the first hazard event. Two random variables X_k and L_k are assumed as mutually independent. The total number of hazard events during life-cycle is represented as $N(t)$ with $k = 1, 2, 3, \ldots$ and $T_k < t$. Considering cumulative loss caused by hazard, the total lifetime typhoon-induced loss $LCL(t_{\mathrm{int}})$ over the bridge life-cycle t_{int} gives

$$LCL(t_{\mathrm{int}}) = \sum_{k=1}^{N(t_{\mathrm{int}})} L_k e^{-rT_k} \tag{4}$$

where r is the monetary discount rate to convert future loss into the present. The mean and variance of lifetime loss, as two fundamental parameters, are emphasised in this study. However, quantifying these two terms can be analytically difficult because the life-cycle loss consists of multiple random variables. In this study, renewal theory and moment generating function methods are proposed to produce a generalization of formula for the mean and variance of lifetime loss. Monte Carlo simulation is applied to validate the analytical results.

4. RENEWAL THEORY

Renewal theory can be an efficient tool to estimate HPP typhoon loss. Mathematically, a renewal process is a counting process with inter-arrival times IID, which is a generalization of an HPP. A renewal process can be applied to represent typhoon arrivals as the system resumes to the initial state after each event occurrence. This characteristic is consistent with the model assumption that the infrastructure is repaired to the initial state immediately after each typhoon arrival. One of the key components to solve lifetime loss is the renewal function. The same notation is used in this renewal derivation process as the HPP typhoon model.

The cumulative distribution function (CDF) of arriving time T_k is denoted as $F_{Tk}(t)$, representing the probability (P) when T_k is smaller than or equal to time t. The CDF can be further written as a k-fold convolution $F_X^{(k)}(t)$ of inter-arriving time X_k

$$\begin{aligned} F_{T_k}(t) &= \mathrm{P}[T_k \le t] = F_X{}^{(k)}(t) \\ &= \mathrm{P}\left[T_k = X_1 + X_2 + \cdots + X_k \le t\right] \end{aligned} \tag{5}$$

Renewal function, defined as the mean value of the number of events in the renewal process, can be written as inter-arrival time X_k (Ross 2014)

$$m(t) = E[N(t)] = \sum_{k=1}^{\infty} F_X{}^{(k)}(t) \tag{6}$$

The CDF of inter-arrival time is denoted as F_X, thus, the Laplace transform (LT) of F_X at r for a finite time interval t_{int} can be written as $LT(t_{\mathrm{int}})$

$$LT(t_{\mathrm{int}}) = \int_0^{t_{\mathrm{int}}} e^{-rs} f_X(s)ds = \int_0^{t_{\mathrm{int}}} e^{-rs} dF_X(s) \tag{7}$$

where $f_X(\cdot)$ is the probability density function (PDF) of inter-arrival time X_k. $F_X(\cdot)$ is the CDF of X_k. The Laplace Transform of X_k and L_k are assumed to exist within a set of real numbers. Following Leveille and Garrido (2001) and Van der Weide *et al* (2008), the Laplace Transform can be extended into convolution when $k \ge 1$, which gives

$$\sum_{k=1}^{\infty} LT^{(k)}(t_{\mathrm{int}}) = \int_0^{t_{\mathrm{int}}} e^{-rs} d \sum_{k=1}^{\infty} F_X{}^{(k)}(s) \tag{8}$$

According to Eq. (6), Eq. (7), and Eq. (8). The first moment of lifetime loss is the expectation

$$E[LCL(t_{\mathrm{int}})] = E[L] \int_0^{t_{\mathrm{int}}} e^{-rs} dm(s) \tag{9}$$

The second moment can be derived following similar procedures by conditioning on the first renewal time

$$\begin{aligned} &E[LCL^2(t_{\mathrm{int}})] \\ &= E[L^2] E\left[\sum_{k=1}^{N(t_{\mathrm{int}})} e^{-2rT_k}\right] \\ &+ E[L]^2 E\left[\sum_{i=1}^{N(t_{\mathrm{int}})} \sum_{j=1, j\ne i}^{N(t_{\mathrm{int}})} e^{-r(T_i + T_j)}\right] \end{aligned} \tag{10}$$

Variance can be obtained from the first and the second moment

$$V[LCL(t_{\text{int}})] = E[LCL^2(t_{\text{int}})] - E[LCL(t_{\text{int}})]^2 \tag{11}$$

The results of mean and variance of the HPP typhoon model within the life-cycle t_{int} can be obtained according to Eq. (9), Eq. (10), and Eq. (11). The first and the second moment of loss severity can be applied when its distribution type is specified. If a simple deterministic loss severity is considered with $L_k = L$, the mean and variance of lifetime loss become

$$E[LCL(t_{\text{int}})] = \frac{L\lambda}{r}(1 - e^{-rt_{\text{int}}}) \tag{12}$$

$$V[LCL(t_{\text{int}})] = \frac{L^2\lambda}{2r}(1 - e^{-2rt_{\text{int}}}) \tag{13}$$

5. MOMENT GENERATING FUNCTION

The renewal theory successfully solves the mean and variance of lifetime loss when typhoon arrivals follow an HPP. However, renewal process defines the inter-arrival time is IID. It is not applicable to a NHPP typhoon model considering climate change because the inter-arrival time is no longer IID. Therefore, moment generating function (MGF) is selected as an alternative tool to derive the mean and variance. Mathematically, MGF provides an alternate of probability distribution compared with the typical CDF or PDF. For a given random variable W, the moment generating function of W about ξ is defined as

$$M_W(\xi) = E[e^{\xi W}] \tag{14}$$

The first and second derivatives of the MGF at $\xi = 0$ provide the first and second moment of the random variable W, respectively. Accordingly, the mean and variance of W can be obtained.

5.1. HPP hazard model

The MGF of a compound HPP process has been derived mathematically by Willmot (1989) and Leveille and Garrido (2010). Following similar derivation, the MGF of lifetime loss can be derived

$$M_{LCL(t_{\text{int}})}(\xi) = \exp\left[\lambda \int_0^{t_{\text{int}}} [M_L(\xi e^{-rs}) - 1]ds\right] \tag{15}$$

in which M_L is the MGF of loss severity L based on the assumption that M_L exists. $M_{LCL(t\text{int})}(\xi)$ represents the MGF of lifetime typhoon-induced loss over a bridge life-cycle $LCL(t_{\text{int}})$. A theoretical loss distribution should be proposed for the loss severity before further derivation when using MGF. Researchers have conducted investigations on loss distribution based on existing historical loss records of natural hazard. One of the commonly used loss distribution models for natural hazard is the generalized Pareto distribution, which is a generalization of exponential distribution (Smith 2003; Read and Vogel 2016). Therefore, exponential distribution, as the simplest case of Pareto distribution, is applied to model loss severity in the life-cycle loss problem. Loss severity L follows an exponential distribution $L \sim \text{EXP}(\theta)$ with mean $1/\theta$. Therefore, the first derivatives at zero gives mean

$$E\left[LCL(t_{\text{int}})\right] = M'_{LCL(t_{\text{int}})}(0) \tag{16}$$

The second moment is obtained from the second derivative at zero

$$E\left[LCL^2(t_{\text{int}})\right] = M''_{LCL(t_{\text{int}})}(0) \tag{17}$$

Variance can be computed from the two moments

$$V\left[LCL(t_{\text{int}})\right] = E\left[LCL^2(t_{\text{int}})\right] - E\left[LCL(t_{\text{int}})\right]^2 \tag{18}$$

The characteristics of loss severity is represented by θ in the mean and variance. It can be replaced by the mean value of loss severity because of the relationship $\text{E}[L] = 1/\theta$ and second moment $\text{E}[L^2] = 2/\theta^2$, which provides identical formulation of mean and variance of lifetime loss from renewal theory.

5.2. NHPP hazard model

The estimation of life-cycle loss of a structure considering changing climate is based on NHPP involving a time-varying typhoon occurrence rate along the interested time interval. An MGF associated with NHPP was derived by Leveille and Hamel (2018). Following similar approach, the MGF of the life-cycle loss of a NHPP typhoon model can be given as

$$M_{LCL(t_{\text{int}})}(\xi) = \exp\left[\int_0^{t_{\text{int}}} \lambda(s)[M_L(\xi e^{-rs})-1]ds\right] \quad (19)$$

A linear relationship of time-variant occurrence rate is assumed as $\lambda(t) = \lambda_0(1 + at)$, where λ_0 (= λ) refers to the initial stationary typhoon occurrence rate and a (= 0.002) represents that the occurrence rate of typhoon is assumed to increase by 20% over 100 years. This linear relationship can be updated if a more advanced typhoon arriving model is proposed.

The moment can be calculated by taking a limit to the differentiation ($\lim_{\xi\to 0}$) if MGF is not differentiable at zero. Therefore, the mean of NHPP life-cycle loss is given by taking limit to the first derivative of $M_{LCL(t\text{int})}(\xi)$

$$E\left[LCL(t_{\text{int}})\right] \approx \lim_{\xi\to 0}\left(\frac{dM_{LCL(t_{\text{int}})}(\xi)}{d\xi}\right) \quad (20)$$

Variance from the second derivative

$$V\left[LCL(t_{\text{int}})\right] \approx \lim_{\xi\to 0}\left(\frac{d^2M_{LCL(t_{\text{int}})}(\xi)}{d\xi^2}\right) \quad (21)$$

5.3. Monte Carlo simulations of homogeneous and non-homogeneous Poisson process

The proposed analytical methods are verified by Monte Carlo simulations numerically. This numerical method can generate random samples to model typhoon arrival process for both HPP and NHPP. For an HPP, the inter-arrival time is simulated by exponential simulation with a stationary occurrence rate. Accordingly, the number of typhoon events can be generated randomly. For an NHPP, a time-varying occurrence rate is managed by a thinning method. The typhoon arrivals are first simulated with the maximum value of the time-varying occurrence rate in the lifetime following the same approach of HPP. Then, each event is generated when a satisfactory thinning probability is accepted. The total lifetime typhoon-induced loss can be added up by counting the number of typhoon events produced by HPP and NHPP. This process is run by satisfactory number of simulations until stable mean and variance are obtained.

6. CASE STUDY

The proposed loss estimation approaches are applied to a multi-span girder bridge in New Jersey. This bridge is 146 meters long with six equal spans. The transverse section view of deck and the geometry of girders are illustrated in Fig. 2.

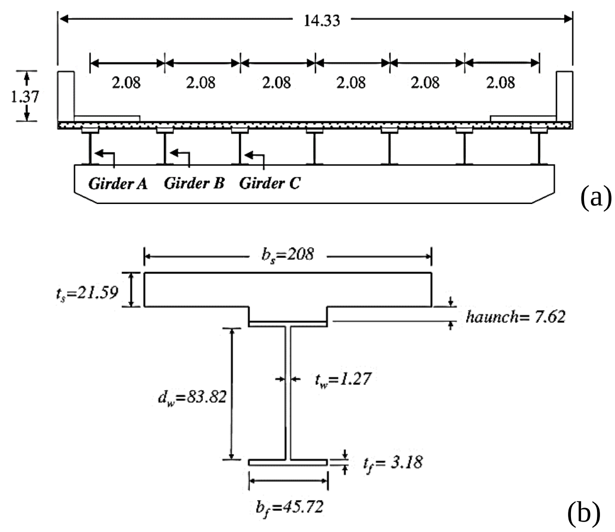

Fig. 2. (a) Transverse section of superstructure of the bridge (dimensions in meters) and (b) Geometry of the girder section (dimensions in centimetres).

The expected service life of this bridge is 75 years. The given typhoon has an occurrence rate of 0.245. The frequency of typhoon is assumed to be increased by 20% in the following 100 years. The failure mode of this bridge during typhoon hazard is deck unseating failure due to storm waves. The repair loss is assumed as 1.283×10^6 USD, which has considered the probability of the deck unseating failure of the bridge. The occurrence rate can be calculated from the number of typhoons divided by period of time, which is assumed as 0.245 for this local region. A constant monetary discount rate is assumed as 0.03. The relevant parameters are listed in Table 1. Mean and variance of lifetime loss are analytically calculated by the derived equations from renewal theory and MGF method for both HPP and NHPP typhoon models. Numerical computations are conducted by Monte Carlo (MC) simulation with 10,000 simulation times. Summarised results are shown in Table 2.

Table 1. Parameters for loss estimation.

	Notation	*Value*
Bridge life-cycle (years)	t_{int}	*75*
Mean repair loss/loss severity (10^6 USD)	$E[L]$	*1.283*
Typhoon occurrence rate(/year)	λ	*0.245*
Annual increasing rate of typhoon	a	*0.002*
Monetary discount rate	r	*0.03*

Table 2. Summarised results for lifetime typhoon-induced loss (unit: USD).

	Mean (10^6)	*Variance (10^{12})*
HPP *by renewal theory with* $E[L] = L$	*$9.373*	*$6.647*
HPP *by* MC *simulation with* $E[L] = L$	*$9.375*	*$6.532*
HPP *by renewal theory with* L ~ EXP	*$9.373*	*$13.294*
HPP *by* MGF *with* L ~ EXP	*$9.373*	*$13.294*
HPP *by* MC *simulation with* L ~ EXP	*$9.372*	*$13.332*
NHPP *by* MGF *with* L ~ EXP	*$9.833*	*$13.373*
NHPP *by* MC *simulation with* L ~ EXP	*$9.663*	*$13.334*

It shows renewal theory and MGF have provided accurate analytical results compared with the results of MC simulations. The lifetime loss for NHPP considering climate change increases about 5% in the bridge life-cycle when the occurrence is increased by 0.002 annually. The mean lifetime typhoon-induced loss increases along time. Fig. 2 shows the increasing trend of lifetime loss within the bridge life-cycle based on the results from renewal and MGF methods of an HPP model.

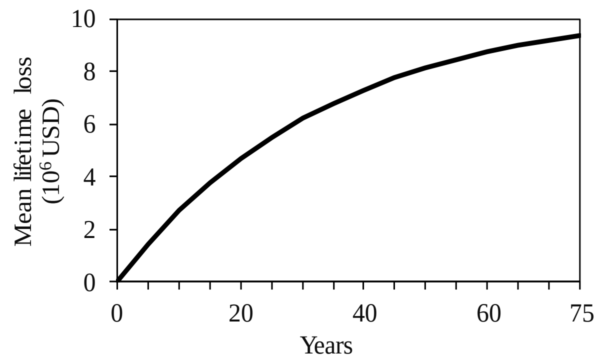

Fig. 2. The mean of lifetime typhoon-induced loss along the bridge life-cycle based on the HPP model.

In general, renewal theory is efficient on solving the HPP problems using renewal function. Compared to the renewal theory, the MGF approach is applicable to both HPP and NHPP hazard models but is more computationally demanding. The MGF method requires the distribution type of loss severity while renewal process simply needs the first and second moment of loss severity. One limitation of MGF approach is that not all the random variables have moment generating functions. The renewal theory and MC simulations can be applied if MGF is not available.

7. CONCLUSIONS

This paper has proposed the renewal theory and MGF approach to estimate the lifetime typhoon-induced loss considering with and without climate change. Climate change can lead to additional 5% loss in bridge life-cycle if the typhoon frequency is increased by 0.002 annually. MGF approach is more powerful than renewal theory because MGF can be applied to both HPP and NHPP models. MGF has involved the distribution of loss severity. However, MGF is considerable computational demanding compared to the renewal theory.

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