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Estimating Distribution of Structural Responses based on Cubic Normal Distribution and Artificial Neural Network

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ABSTRACT: The approximation of the probability density function (PDF) of the structural response is importance in structural reliability analysis. The recently developed cubic normal distribution can be used to represent the PDF of structural response due to its high flexibility and its large applicable range, whose parameters are evaluated by its first four moments. Therefore, efficient estimation of the moments is of great importance. Although some methods have been developed, evaluating the moments of structural response from the sight of balancing accuracy and efficiency remains a challenge, especially when the structural response is implicit and high-dimensional. In this paper, based on the artificial neural network (ANN), a new method is developed to efficiently estimate the statistical moments of structural response. The main procedure of the proposed method includes two steps: the structural response is approximated by the ANN and then the moments of structural response can be easily obtained. A RC frame structure with non-linear behavior is used to demonstrate the efficiency, accuracy, and applicability of the proposed method. The results show that the proposed method is of high accuracy and efficiency and provides a robust tool for representing the PDF of structural response.

1. INTRODUCTION

Uncertainty is ubiquitous in civil engineering, such as loads, material properties, geometric dimensions, and boundary conditions. In this regard, the outputs of structural response are essentially random variables. For the design more rational, it is necessary to determine the statistical distribution of structural responses, which are functions of uncertain structural parameters described as random variables. The probability density function (PDF) of structural responses can, theoretically, be derived from the probability distributions of the input random variables. However, the derivation of the PDF of structural response is generally difficult because of the nonlinear, complicated, and implicit of structural responses.

Generally, the distribution of structural response can be determined through fitting a candidate distribution to the histogram and then performing goodness-of-fit tests. The candidate distribution is generally one- or two-parameter distributions, whose parameters evaluate from the mean and standard deviation of structural response. However, those one- or two-parameters distributions may be not flexible enough. The recently developed cubic normal distribution (Zhao et al. 2018a) can be adopted to represent the distributions of structural response duo to its high flexibility and its large applicable range. The parameter of cubic normal distribution is determined by the first four moments of random variable, i.e., mean, standard deviation, skewness, and kurtosis. In order to use the cubic normal distribution to represent the PDF of structural response, computing the first four moments of structural responses with efficiency and accuracy is of paramount importance.

Many approaches can be used to numerically calculate the first four moments of structural response, such as Taylor expansion based method (Ibrahim 1987; Singh and Lee 1993), original space two- or three-point estimate method (Rosenbluenth 1975; Gorman 1980), univariate dimension reduction method (UDRM) (Zhao and Ono 2000), bivariate dimension-reduction method (BDRM) (Zhao and Lu 2011; Xu and Rahman 2004; Fan et al. 2016; Cai et al. 2018), sparse grid stochastic collocation method (SGSCM) (He et al. 2014), cubature formulae (Xu and Lu 2017), etc.. Unfortunately, these methods may not be able to achieve a good tradeoff of accuracy and efficiency to estimate the high-order central moments when the dimensions of input random variables are large.

The objective of this paper is to represent the PDF of structural response using the cubic normal distribution, in which a high efficient method for determining the first four moments of structural response is developed.

2. THE CUBIC NORMAL DISTRIBUTION

Without loss of generality, the structural response can be commonly expressed by a function $Z = G(\mathbf{X})$, in which $\mathbf{X} = [X_1, X_2, \ldots, X_n]^T$ is an n-dimensional vector of random variables representing uncertain quantities such as loads, material properties, geometric dimensions, and boundary conditions. According to the fourth-moment transformation, the standardized form of Z can be approximated by a third-order polynomial of standard normal random variable U as following (Zhao and Lu 2007; Zhao et al. 2018b):

$$Z_s = \frac{Z - \mu_Z}{\sigma_Z} = S_u(U) = a + bU + cU^2 + dU^3 \quad (1)$$

where μ_Z and σ_Z are the mean and standard deviation of Z, respectively. $S_u(U)$ is the third-order polynomial of U; and a, b, c, and d are the polynomial coefficients that can be determined by making the first four moments of $S_u(U)$ equal to those of Z_s.

Based on the normal transformation theory, the CDF of Z should be equal to that of U, i.e., $F(z) = \Phi(u)$. Taking the derivate of the left and right sides with respect to z, and using the relationship between Z and U, the PDF of Z, i.e., $f(z)$, can be formulated as (Zhao et al. 2018a):

$$f(z) = \frac{\phi(u)}{\sigma_Z(3du^2 + 2cu + b)} \quad (2)$$

where $\phi(u)$ is the PDF of standard normal random variable U. The complete monotonic expression expressed the relationship between u and z has been given by Zhao et al. (2018b).

3. STATISTICAL MOMENTS ESTIMATION

To represent the PDF of structural response by the cubic normal distribution, computing the first four moments with efficiency and accuracy is of paramount importance. Many approximate methods (e.g., UDRM, BDRM, and SGSCM) can be used to numerically determine the first four moments of structural response. However, those methods may become not feasible especially when the implicit structural response with high dimension of input random variables because the evaluation of $G(\mathbf{X})$ requires a time-consuming numerical calculation by mean of finite element analysis.

Since the artificial neural networks (ANNs) have the ability of accurately and effectively approximating the structural response. Based on ANN and Monte Carlo simulation (MCS) method, the present paper develops a new method for evaluating the first four moments of structural response.

The most widely used ANN is the multi-layer feed-forward back-propagation network, i.e., BP network. A typical BP network is shown in Figure 1, whose architecture is composed of a number of neurons situated on three or more layers, i.e., one input layer, one or more hidden layer and one output layer.

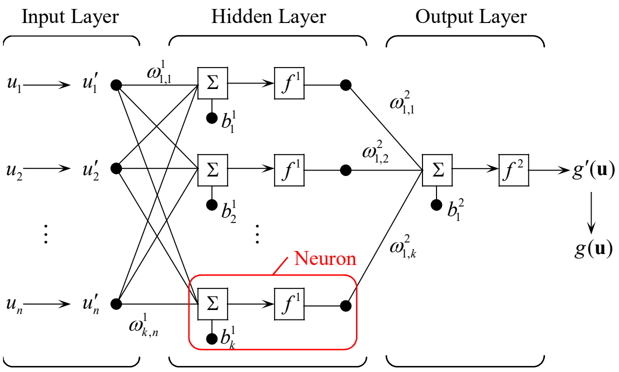

Figure 1. A three-layer feed-forward BP network with n inputs, k hidden neurons and one output

The neuron is the basic processing element with several inputs and one output, and its output can be expressed as:

$$a_k = f\left(\sum_{i=1}^{n} \omega_{k,i} u_i + b_k\right) \tag{3}$$

where u_i are the inputs; a_k is the output; $\omega_{k,i}$ and b_k are the weights and bias of the kth neuron; and $f(\cdot)$ is the activation function or transfer function. The activation functions used in the present paper are the logistic function for hidden layers and the pure linear function for output layer, respectively. The logistic function is formulated as:

$$f(a) = \frac{1}{1+\exp(-a)} \tag{4}$$

The number of neurons in input layer is the dimension of basic random variables, and that of output layer is the dimension of structural responses. However, the selection of the number of neurons in hidden layers is not a simple task and there is no general rule. In this paper, we first train several networks with different neurons and then select the best one.

The training of a network is an iterative process to update their weights and biases using optimization methods to minimize the mean square error between the predicted and the target values. Details of the ANN training algorithm can be found in Hagan et al. (1996). To overcome over-fitting during network training, the Bayesian regularization is introduced into the back-propagation algorithm. Because Bayesian regularization works best when the training data are confined within the range of [-1, 1], the inputs and outputs should be normalized. In this paper, the inputs and outputs are first normalized into [-1, 1] before presented to network by the following formulae (Lu et al. 2012):

$$u' = \frac{2(u_i - u_{i,\min})}{u_{i,\max} - u_{i,\min}} - 1 \tag{5}$$

where u_i are the original inputs; u_i' are the normalized inputs; and $u_{i,\min}$ and $u_{i,\max}$ are the original minimum and maximum of u_i, respectively.

The training data must be prepared before training network. In this paper, based on Latin hypercube sample method, the samples in independent standard normal random variables (i.e., the inputs of network) are generated. With the aid of the inverse normal transformation (e.g., Liu and Der Kiureghian 1986), the original samples can be determined.

4. NUMERICAL EXAMPLE

To illustrate the application of the proposed method, a 7-story RC frame structure with three bays is selected, as shown in Fig. 2. A non-linear structural analysis is performed. The total height of the building is 21.5 m. The height of the first story is 3.5 m, and that of upper stories is 3.0 m. The length of each bay is 6.0 m. The sizes of columns are 500×500 mm for the first four stories and 450×450 mm for 5th through 7th stories, and the sections of the beams are 200×400 mm. More detailed information about the arrangement of the reinforcements of the column and beam members is shown in Figure 2.

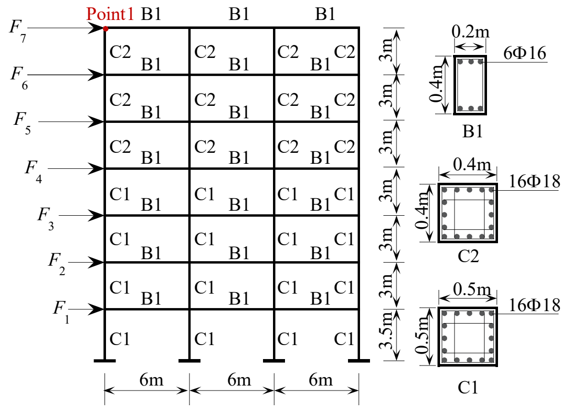

Figure 2. RC frame structure considered in example

The hysteretic model developed by Mohd Yassin (1994) and perfect elastic plastic are used to simulate the nonlinear behavior of concrete material and reinforcing steel, respectively. The parameters and their distributions of materials are listed in Table 1, in which f_{pc}, ε_c, f_{pcu}, ε_{cu}, λ, f_t, and E_{tc} denote the concrete compressive strength, strain at maximum strength, crushing strength, strain at crushing strength, ratio between unloading slope at ε_{cu} and initial slope, tensile strength, and tension softening stiffness, respectively; f_y and E_s represent the steel yield strength and Young's modulus, respectively. The RC frame structure subjected to lateral loads, which is shown in Figure 2, is considered. The lateral loads (F_1-F_7) are assumed independent Gumbel distributed random variables, and their distribution parameters are listed in Table 1.

In this example, we discuss the PDF of the horizontal displacement u_x at Point1. The PDF of u_x cannot be derived from the probability distributions of the input random variables because of the implicit structural response. To establish the network, 500 samples of independent standard normal random variables are generated by LHS, and 50 validation data are randomly selected. Several networks are trained that composed of one input layer, one output layer, and three hidden layers with 10, 20, and 30 neurons, and the best one was selected according to the mean squared error of validation data. The first four moments of horizontal displacement u_x are determined as 0.06995, 0.02424, 0.94569 and 4.53626, respectively. Using the cubic normal distribution, the PDF of $Z = u_x$ can be given as:

$$f(z) = \frac{\phi(u)}{\sigma_Z(3du^2 + 2cu + b)} \tag{6a}$$

in which $\sigma_Z = 0.02424$, $b = 0.9374$, $c = 0.14791$, and $d = 0.013312$. And u expressed as:

$$u = \left(-\frac{q}{2} + \sqrt{\Delta}\right)^{1/3} + \left(-\frac{q}{2} - \sqrt{\Delta}\right)^{1/3} - \frac{c}{3d} \tag{6b}$$

where $\Delta = 928 + 0.25q^2$; $q = -170.3 - (z - \mu_Z)/d$

Table 1. Distribution of Random Variables

Variable	Distribution	Mean	COV
f_{pc}	Lognormal	48 Mpa	0.25
ε_c	Lognormal	2.3×10^{-3}	0.25
f_{pcu}	Lognormal	40.8 Mpa	0.25
ε_{cu}	Lognormal	3.5×10^{-3}	0.25
λ	Lognormal	0.5	0.25
f_t	Lognormal	3.5 MPa	0.25
E_{ts}	Lognormal	4.0 GPa	0.25
f_y	Lognormal	400 MPa	0.05
E_s	Lognormal	200 GPa	0.05
F_1	Gumbel	60 kN	0.40
F_2	Gumbel	80 kN	0.40
F_3	Gumbel	60 kN	0.40
F_4	Gumbel	40 kN	0.40
F_5	Gumbel	30 kN	0.40
F_6	Gumbel	15 kN	0.40
F_7	Gumbel	15 kN	0.40

Note: COV denotes coefficient of variation

The PDF of the horizontal displacement u_x at Point1 are pictured in Figure 3. It is seen that the PDF of the structural response function, determined by the proposed method, is in close agreement with the histogram obtained by MCS. It should be emphasized that the evaluation of the entire distribution is just based on the 500 times of repeated deterministic finite element analysis, which is highly efficient.

5. CONCLUSIONS

Based on the cubic normal distribution, this paper proposed a method for representing the

PDF of structural response, in which the first four moments of structural response are obtained by combining the ANN and MCS. A nonlinear RC frame structure is used to validate the efficiency, accuracy, and applicability of the proposed method. The proposed method is highly efficient that only requires several hundred deterministic finite element analyses.

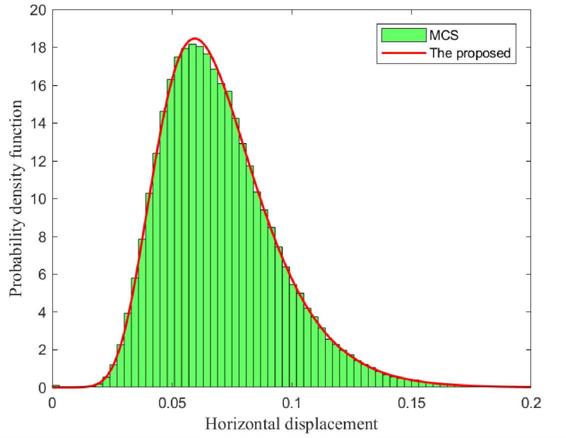

Figure 3. PDF comparisons for numerical example

6. ACKNOWLEDGMENTS

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