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Multi-dimensional fragility analysis considering structural cumulative plastic dissipation energy and its application to a NEES frame structure

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ABSTRACT: By integrating the force analogy method in the energy balance equation, the study put forward the concept of cumulative plastic strain (CPS) for seismic fragility analysis, which can be defined as the ratio of the demand of plastic dissipation energy to its capacity. The cumulative plastic strain can reflect the structural damage cumulative effect under earthquakes, which makes it especially suitable as the damage index for the structural component. Firstly, fragility curves are developed according to the maximum inter-story drift. Fragility curves of local components can be also developed assuming that a series of cumulative plastic strain thresholds are given. Further, the real threshold values of cumulative plastic strain are obtained through the degree of coincidence of these two kinds of fragility curves. The cumulative plastic strain and the floor acceleration will be determined as the quantification indices for performance limit state of the structural component and non-structural component, respectively. An innovative probabilistic seismic demand model (PSDM) following multivariate logarithmic normal distribution is constructed. Considering the uncertainty and correlation of performance limit states (PLSs), multi-dimensional PLS formula is developed to identify the structural failure domain. A full-scale 2-bay 2-story frame structure for the Network for Earthquake Engineering Simulation (NEES) project is studied to show the proposed theory. To obtain the maximum structural responses, nonlinear dynamic analysis is carried out. Consequently, the structural multi-dimensional fragility curves are derived based on CPS. In addition, the influence of PLS threshold and PLS correlation on the probability of failure is evaluated. Results show that (1) CPS damage index can fully consider the cumulative effect of damage under earthquakes, and make up for the deficiency of the inter-story drift in this aspect. (2) The multi-dimensional fragility framework can deal with the PLSs correlation and engineering demand parameters correlation simultaneously, which will generate a more precise seismic damage assessment result.

In the framework of performance-based earthquake engineering (PBEE), the degree of structural damage is quantified and the system performance levels are classified as different states, e.g., fully operational, life safety, and near collapse. Also, hazard levels are classified as frequent, occasional, rare, and very rare events for seismic fragility evaluation. Reasonable damage index and performance limit state should not only has clear physical meaning, but also should facilitate the application for the fragility analysis in PBEE framework. Considerable efforts have been devoted to the selection of the damage index for different kind of structures. The inter-story drift is widely used as the structural damage index (Bojorquez *et. al.*, 2017; Farsangi *et. al.*, 2018). Kazemi et al. (2017) invested the fragility of steel braced frames by incorporating new spectral shape indicators and a weighted damage index. Lv and Wang (2001) proposed a decision-making method of the optimal seismic fortification level for aseismic structures based on damage performance. A seismic damage index with two linearly combined weighting parameters is established and a simplified method is provided to calculate the damage index. Ding *et al.* (2005) considered the damage accumulation and the strain reinforcement effect of the steel, and established the damage mechanics model for the steel structure based on energy dissipation theory. Structural ductility is an important design parameter for the structure performance evaluation, thus ductility (e.g., ductility of rotation angle, displacement ductility, etc) is also often selected as the damage index for fragility estimation (Park, 1986; Penzien, 2015). The deformation-based damage index, e.g., peak inelastic deformation, peak roof displacement, is viewed as the simple and direct indexes reflecting the structural damage (Belejo *et al.*, 2017, Khaloo *et al.*, 2016). The advantage of this damage index is that it can be used conveniently in many respects with a clear physical meaning. However, it cannot deal with the structural damage accumulation effect under earthquakes very well.

This study will define a local damage index for the structural component by using plastic dissipation energy, and the fragility curve is used to determine the damage index threshold under different performance limit states. Within the framework of multi-dimensional fragility theory, engineering demand parameters (EDPs) respectively for the structural and non-structural component are selected to construct multi-dimensional probabilistic seismic demand model and multi-dimensional PLS function. Further, the sensitivity of multi-dimensional fragility with regard to acceleration threshold and performance limit state correlation is investigated to reveal their influence on the failure probability.

1. CUMULATIVE PLASTIC STRAIN BASED ON PLASTIC DISSIPATION ENERGY

1.1. Force analogy method

The concept of force analogy method was firstly proposed by Wong and Yang (1999), and it was originally used to solve the problem of dynamic analysis for frame structure. The force analogy method has the advantages of high efficiency, strong stability and broad applicability. The study will apply force analogy method for energy analysis of frame structure, and the plastic dissipation energy for the whole structure can be expressed as the sum of the energy dissipation of all plastic hinges, which are directly associated with the structural damage.

For a structural system with n horizontal degree of freedom, such as the frame structure with n story, assume that each story has m rotational degree of freedom, that is, there are m plastic hinges in each story. The horizontal displacement can be described by the following expression.

$$X(t) = X'(t) + X''(t) = \begin{Bmatrix} x_1'(t) \\ x_2'(t) \\ \vdots \\ x_n'(t) \end{Bmatrix} + \begin{Bmatrix} x_1''(t) \\ x_2''(t) \\ \vdots \\ x_n''(t) \end{Bmatrix} \tag{1}$$

where $X(t)$ is the total displacement vector, $X'(t)$ is the elastic displacement vector, and $X''(t)$ is the plastic displacement. At the location of plastic hinge, there exist elastic bending moment and plastic rotation angle. The total bending moment and plastic turning angle can be written as:

$$M(t) = M'(t) + M''(t) = \begin{Bmatrix} m_1'(t) \\ m_2'(t) \\ \vdots \\ m_m'(t) \end{Bmatrix} + \begin{Bmatrix} m_1''(t) \\ m_2''(t) \\ \vdots \\ m_m''(t) \end{Bmatrix} \quad \Theta''(t) = \begin{Bmatrix} \theta_1''(t) \\ \theta_2''(t) \\ \vdots \\ \theta_m''(t) \end{Bmatrix} \tag{2}$$

where $M'(t)$ is the elastic moment vector corresponding to the elastic displacement and $M''(t)$ is the plastic moment vector corresponding to the plastic displacement. Considering the equilibrium and compatibility conditions, the plastic moment and the plastic displacement can be calculated:

$$M''(t) = -(K'' - K'^T K^{-1} K')\Theta''(t) \quad (3)$$

$$X''(t) = K^{-1} K' \Theta''(t) \quad (4)$$

in which K is the overall stiffness matrix, K' is the transformation matrix between the rotation angle and the restoring force, K'' is the transformation matrix between the rotation angle and the restoring bending moment. The elastic bending moment $M'(t)$ and the elastic displacement $X'(t)$ have the following relations

$$M'(t) = K'^T X'(t) \quad (5)$$

Substitute the Equation (4) into the Equation (5)

$$M'(t) = K'^T X'(t) = K'^T [X(t) - X''(t)]$$
$$= K'^T [X(t) - K^{-1} K' \Theta''(t)] \quad (6)$$

Substitute Equations (3) and (6) into Equation (2), the control equation of the force analogy method is obtained

$$M(t) + K'' \Theta''(t) = K'^T X(t) \quad (7)$$

1.2. Energy balance equation under earthquakes

The basic equation of motion for the multi-degree-of-freedom frame structural system is shown as follows (Akiyama, Ye and Pei, 2010):

$$M\ddot{x}(t) + C\dot{x}(t) + Kx'(t) = -M\ddot{g}(t) \quad (8)$$

where M is the mass matrix; C Is the damping matrix; $\ddot{x}(t)$ represents the acceleration of the structure relative to the ground; $\dot{x}(t)$ is the relative speed; $\ddot{g}(t)$ is the acceleration of the ground. The absolute displacement can be written as $y(t) = x(t) + g(t)$. Thus, Equation (8) can be transferred to be

$$M\ddot{y}(t) + C\dot{x}(t) + Kx'(t) = 0 \quad (9)$$

Integrate Equation (9) over the time interval [0, t_s], and the following equation can be obtained:

$$\int_{t=0}^{t=t_s} \ddot{y}^T M dy + \int_{t=0}^{t=t_s} \dot{x}^T C dx + \int_{t=0}^{t=t_s} x'^T K dx = \int_{t=0}^{t=t_s} \ddot{y}^T M dg \quad (10)$$

Considering $dx = dx' + dx''$, Equation (10) is simplified as:

$$\int_{t=0}^{t=t_s} \ddot{y}^T M dy + \int_{t=0}^{t=t_s} \dot{x}^T C dx + \int_{t=0}^{t=t_s} x'^T K dx' + \int_{t=0}^{t=t_s} x'^T K dx'' = \int_{t=0}^{t=t_s} \ddot{y}^T M dg \quad (11)$$

The physical meaning of the items in Equation (11) is listed: $IE = \int_{t=0}^{t=t_s} \ddot{y}^T M dg$ is seismic input energy; $KE = \int_{t=0}^{t=t_s} \ddot{y}^T M dy$ is kinetic energy; $DE = \int_{t=0}^{t=t_s} \dot{x}^T C dx$ is damping dissipation energy; $SE = \int_{t=0}^{t=t_s} x'^T K dx'$ is elastic deformation energy; $PE = \int_{t=0}^{t=t_s} x'^T K dx''$ is plastic dissipation energy.

Thus, energy balance equation is expressed as $KE + DE + SE + PE = IE$. The equation shows that a part of the seismic input energy is stored in the form of kinetic energy and elastic deformation energy, and the other part is transferred into damping dissipation energy and plastic dissipation energy.

1.3. Cumulative plastic strain

On the basis of the plastic dissipation energy, the cumulative plastic strain is defined as the damage index for structural components. According to Equations (4) and (5), the plastic dissipation energy in Equation (11) can be written as:

$$\int_{t=0}^{t=t_s} x'^T K dx'' = \int_{t=0}^{t=t_s} x'^T K (K^{-1} K' d\Theta'') = \int_{t=0}^{t=t_s} x'^T K' d\Theta'' =$$
$$\int_{t=0}^{t=t_s} m'^T d\Theta'' = \sum_{i=1}^{m} \int_{t=0}^{t=t_s} m_i' d\theta'' \quad (12)$$

where $PE = \sum_{i=1}^{m} \int_{t=0}^{t=t_s} m_i' d\theta'' = \sum_{i=1}^{m} PE_i$. PE_i is the plastic dissipation energy at the *ith* plastic hinge, m is the number of plastic hinge. The plastic dissipation energy is expressed as the product of elastic bending moment and plastic rotation angle.

In the study, the cumulative plastic strain is defined as the ratio of the demand of plastic dissipation energy to its capacity, which has the following form:

$$\varepsilon = \underset{i}{MAX}[PE_i / (f_y \cdot A_i \cdot d_i)] \qquad (13)$$

where f_y is the yield stress of the component; A_i is the sectional area of the component; d_i is the length of the component; PE_i represents the plastic dissipation energy at the plastic hinge.

2. STRUCTURAL MULTI-DIMENSIONAL FRAGILITY ANALYSIS

2.1. Definition of multi-dimensional fragility

Seismic fragility is defined as the conditional probability that the seismic demand (or structural response) exceeds the corresponding capacity, specified for a certain performance limit state level, under given seismic intensity measures. When multiple demand parameters are considered, the traditional fragility can be extended to the multi-dimensional fragility:

$$P_f = P\{\bigcup_{i=1}^{n}(R_i \geq r_{\lim,i}) | I\} \qquad (14)$$

Where R_i represent the seismic demand parameters (e.g., deformation, stress, energy, etc.). $r_{\lim,i}$ is performance limit state threshold. I is ground motion intensity parameter, e.g., peak ground acceleration (PGA) or spectral acceleration (S_a). In this case, probabilistic seismic demand model follows multivariate logarithmic normal distribution, and its probability distribution density can be calculated by (Wang, Wu and Liu, 2018):

$$f(r_1, r_2, ..., r_n) = \frac{1}{(2\pi)^{n/2} \cdot |\Sigma|^{1/2} \cdot (r_1 \cdot r_2 \cdot ... \cdot r_n)} \exp\left\{-\frac{1}{2}(\ln r - \mu)^T \Sigma^{-1} (\ln r - \mu)\right\} \qquad (15)$$

where $\ln r = [\ln r_1, \ln r_2, ..., \ln r_n]^T$; μ is the mean vector of $Y = [\ln R_1, \ln R_2, ..., \ln R_n]^T$; Σ is the covariance matrix of Y, which represents the correlation between different seismic demand parameters.

2.2. Multi-dimensional performance limit state

Multi-dimensional performance limit state function describes the condition that the structure will be in damage state when the multiple EDPs are considered. In the function, different performance limit states are viewed to be dependent, described by correlation coefficient (N_i) (Cimellaro and Reinhorn, 2010):

$$L(R_1, \cdots, R_n) = \sum_{i=1}^{n} (\frac{R_i}{r_{\lim,i}})^{N_i} - 1 = 0 \qquad (16)$$

where R_i is the EDP, and $r_{\lim,i}$ represents the corresponding threshold of the demand parameters. When $L(R_1, \cdots, R_n) > 0$, the structure is in a specific damage state. The three-dimensional PLS is shown in Figure 1 as an example.

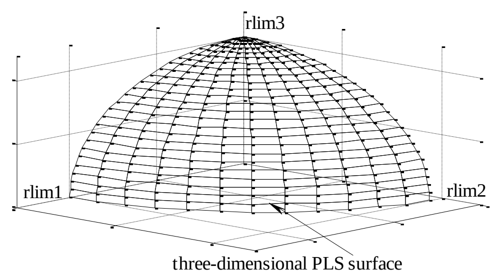


Figure 1: Three-dimensional PLS

In this study, the cumulative plastic strain (ε) and peak acceleration (Z) will be selected as the demand parameters for the structural and the non-structural component, respectively. The bi-dimensional PLS function will be obtained:

$$\frac{Z_{LS}}{Z_{LS0}} + \left(\frac{\varepsilon_{LS}}{\varepsilon_{LS0}}\right)^N - 1 = 0 \qquad (17)$$

where Z_{LS} and ε_{LS} are the maximum acceleration and cumulative plastic strain, respectively; Z_{LS0} and ε_{LS0} are the corresponding threshold values. N represent the correlation coefficient.

After the PSDM and the PLS function are determined, multi-dimensional fragility can be calculated by multi-dimensional integration. The integral function is PSDM, and the failure domain is determined by PLS function.

3. CASE STUDY

The case study structure is a 2-bay 2-story frame located in the structural laboratory at Georgia Tech for NEES (Network for Earthquake Engineering Simulation) project research (Figure 2) (Vega-Behar *et al.*, 2017), which aims to study the structural seismic performance under different reinforcement measures. The intermediate four frames in Figure 2 are not connected to each other, and the outermost frames are used to prevent lateral collapse during the laboratory test. The finite element model (FEM) for the structure was established through SAP2000 (Figure 3). The seismic precautionary intensity is 7 degrees and the design earthquake acceleration is 0.1 g. Elasticity modulus of concrete and steel bar are $E_c = 30GPa$ and $E_s = 200GPa$, respectively. Poisson's ratio of concrete and steel bar are $\mu_c = 0.2$ and $\mu_s = 0.3$, respectively. The reinforced concrete density is $2500kg/m^3$. The plastic hinges are simulated at the ends of beams and columns. First-order resonance frequency is 1.733 Hz according to the modal analysis results. On the basis of those site and structural information, the earthquake influence coefficient curve will be calculated as the target response spectrum for earthquake records selection in PEER (Pacific Earthquake Engineering Earthquake Research Center) database. Finally, a suit of 20 earthquake records, consistent with target response spectrum, are selected as the input for nonlinear dynamic time analysis.

Figure 2: NEES frame structure

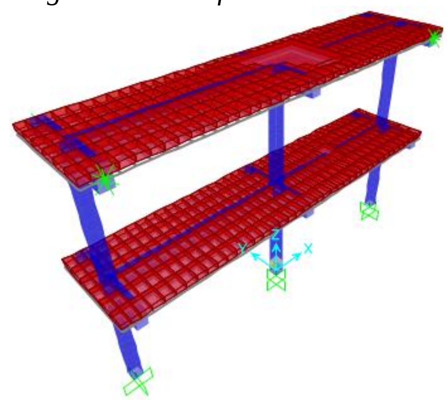


Figure 3: Three-dimensional FEM model

3.1. *Probability distribution for structural responses*

In this study, PGA is chosen as the seismic intensity parameter. The selected 20 earthquake records are scaled to be different intensity levels, i.e., 0.05 g, 0.15 g, 0.35 g, 0.55 g, 0.75 g and 0.95 g as the input for nonlinear time history analysis. Maximum structural responses including inter-story drift and cumulative plastic strain are recorded (After the rotation angle and bending moment of the plastic hinge are obtained, cumulative plastic strain can be calculated according to Equations (12) and (13)), with an assumption that their probability distributions obey the lognormal distribution, and the corresponding distribution parameters are estimated. Table 1 give the parameter estimation results for maximum CPS. μ and σ are the logarithmic mean and logarithmic standard deviation, respectively.

Table 1: Distribution parameters for CPS

PGA	0.05g	0.15g	0.35g	0.55g	0.75g	0.95g
$\hat{\mu}_\varepsilon$	-5.940	-5.382	-5.016	-4.807	-4.585	-4.338
$\hat{\sigma}_\varepsilon$	0.508	0.440	0.329	0.349	0.332	0.273

3.2. Cumulative plastic strain thresholds

According to US FEMA 273, inter-story drift thresholds for four PLS levels, i.e., Normal Operation (NO, no damage), Immediate Occupancy (IO), Life Safety (LS) and Collapse Prevention (CP), are 0.2%, 0.5%, 1.5% and 2.5%, respectively. And the probability distribution of the maximum inter-story drift is obtained. Thus, the fragility curves of the structure on the basis of maximum inter-story drift can be constructed. A series of cumulative plastic strain thresholds are assumed and the corresponding fragility curves can be obtained. These two kinds of fragility curve are drawn in the same coordinate system, shown in Figure 4. The cumulative plastic strain thresholds will be determined by comparing the similarity between these two kinds of fragility curves.

Figure 4 shows that the fragility curves with cumulative plastic strain threshold value of 0.002, 0.004, 0.008 and 0.016 have a good consistence with four fragility curves corresponding NO, IO, LS and CP. And these values will be viewed as the thresholds for these four PLSs.

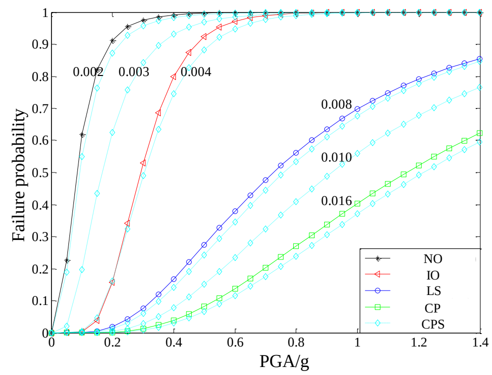


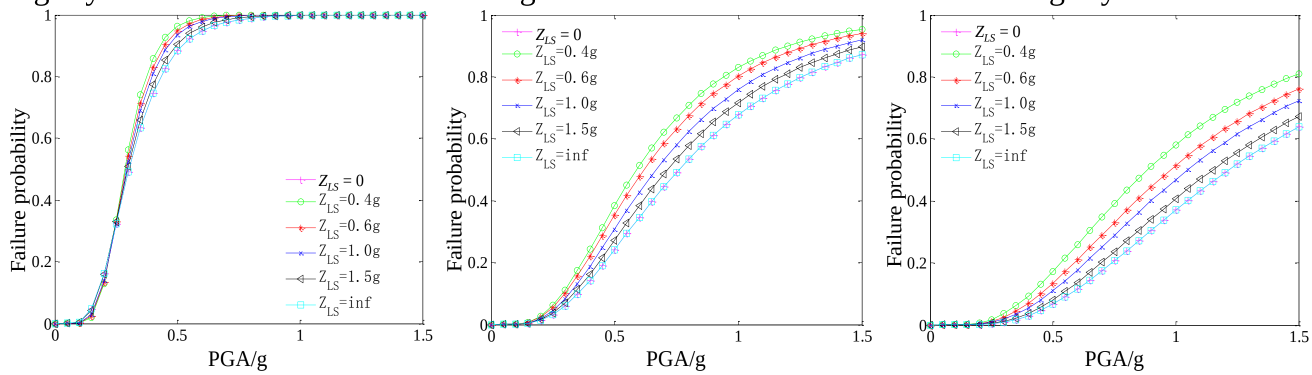
Figure 4: Comparison of two kinds of fragility curves

3.3. Multi-dimensional fragility analysis

The cumulative plastic strain and floor acceleration response are selected as quantitative performance parameters for the structural and non-structural components, respectively. Multi-dimensional probabilistic seismic demand model can be obtained according Equation 15. On the basis of PLS threshold value, multi-dimensional performance limit state function can be constructed according to Equation 17. Further, Monte Carlo will be employed to calculate the structural damage probability. Finally, Cumulative distribution function of the lognormal distribution will be used for multi-dimensional fragility curve fitting. In addition, the sensitivity of fragility to acceleration threshold and PLS correlation will be investigated.

Figure 5 shows the influence of acceleration threshold on the multi-dimensional fragility for the three PLSs, i.e., IO, LS and CP. The acceleration threshold is increased from 0.4 g to infinity, and the corresponding fragility curves are given. It is shown that with the increase of acceleration threshold, the failure probability decreases and the fragility curve moves down. When the acceleration threshold is infinity, the multi-dimensional fragility overlaps with fragility considering only the cumulative plastic strain.

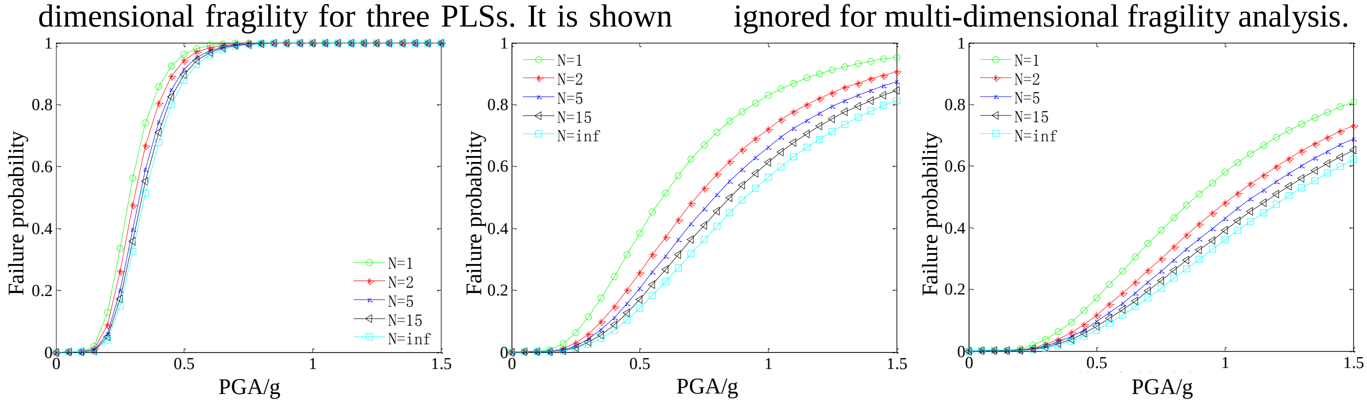
The impact of acceleration threshold on fragility curves under IO damage state is relatively small (When the threshold value changes, the fragility changes little). While, acceleration threshold has great influence on fragility curves under LS and CP damage states (When the threshold value changes, the fragility changes significantly). Thus, for these two kinds of limit states, choosing a reasonable acceleration threshold is essential for fragility assessment.



(a) IO PLS (b) LS PLS (c) CP PLS

Figure 5: Sensitivity analysis of multi-dimensional fragility curves to acceleration threshold

PLS correlation is described by the parameter N in Equation (17). N ranges from 0 to infinity, and the correlation decreases with the increase of N. For instance, when N=1, the two PLSs are linearly related. When N is infinite, the two limit states are independent. Figure 6 shows the influence of PLS correlation on multi-dimensional fragility for three PLSs. It is shown that as the value of N increase, fragility curve goes down, especially for LS and CP performance level. That is, when the PLS correlation is neglected, a lower failure probability will be obtained, resulting non-conservative estimation, which is adverse to the safety of engineering structures. Thus, the correlation between the PLSs can't be ignored for multi-dimensional fragility analysis.



(a) IO PLS (b) LS PLS (c) CP PLS

Figure 6: Sensitivity analysis of fragility curves to PLS correlation

4. CONCLUSIONS

In this study, the cumulative plastic strain damage index is proposed on the basis of plastic dissipation energy, which is high related with structural damage under earthquakes. This damage index can fully consider the damage accumulation effect under earthquakes.

The cumulative plastic strain is integrated into the framework of multi-dimensional fragility, and a NEES frame structure is employed as a case study structure to illustrate the applicability of this method. CPS and the maximum floor acceleration are selected as the structural and non-structural engineering demand parameters, respectively. On the basis of PLS threshold values, multi-dimensional PLS function can be developed. Through nonlinear time history analysis, maximum structural responses can be obtained for the construction of multi-dimensional PSDM. Finally, the multi-dimensional fragility can be estimated by multiple integral.

Result shows that multi-dimensional fragility is sensitive to the PLS threshold values and PLS correlation. As acceleration threshold increases, the failure probability decreases and the fragility curve goes down. Performance limit state correlation has great influence on multi-dimensional fragility. It is shown that as the PLS correlation weakens, fragility decrease, especially for LS and CP damage states. That is, a lower failure probability, i.e., non-conservative estimation, will be obtained, which is adverse to the safety of engineering structures.

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