Probabilistic Analysis of Ground Deformation Induced by Excavation based on Hypoplastic Constitutive Models

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ABSTRACT: Empirical model and finite element method are two commonly-used methods for prediction of ground deformation induced by excavation. Compared with the former, the finite element method can not only predict the deformation of different modes, but also predict the distributed deformation of the whole site. However, results of finite element analysis depends on the constitutive model used in the analysis. This paper uses an advanced hypoplastic constitutive model and its improved edition, which considers the small-strain effect of soil, to represent the soil behavior. Uncertainties are unavoidable in excavation engineering, such as those in soil parameters, loads, and models, etc. These uncertainties have profound effects on the prediction of deformation induced by excavation obtained from the finite element analysis. In order to consider the effect of parameter uncertainty on the prediction results, random variables are used to characterize the parameter uncertainty. Direct Monte Carlo simulation (MCS) method was used to incorporate the parameter uncertainty into reliability analysis of the deformation induced by excavation. The computational costs and convergence issues of finite element method in together with advanced constitutive model result in significant computational challenges in MCS-based reliability analysis. In order to improve the computing efficiency and robustness, artificial neural network (ANN) is adopted as a surrogate model of the finite element method to compute the soil deformation for a given set of uncertain parameters. Results show that responses predicted by the improved hypoplastic model fit the real response better.

1. INTRODUCTION

Excavation engineering is very common in urban infrastructure construction, such as subway, underground pipe network and basements of tall buildings. Excessive soil movement caused by excavation is likely to have a negative impact on surrounding buildings. This requires that the soil deformation should be controlled within a certain range. According to historical data, many literatures have studied the maximum allowable ground deformation, and local standards have been established worldwide. Therefore, the prediction of ground deformation caused by excavation, including lateral displacement and ground settlement, is a major concern in excavation engineering.

It is common practice to use empirical methods or finite element method to predict excavation-induced ground deformation. Compared with empirical methods, the finite element method may provide more accurate predictions by incorporating site-specific complex soil behaviors. The accuracy of the finite element method depends largely on the constitutive model and its associated parameters. So far, constitutive models based on different theories and hypotheses have been proposed and widely used in the field of excavation engineering. Ou and Tang (1994) utilized the hyperbolic model, proposed by Duncan and Chang (1970) to model the soil behavior in a deep excavation analysis. Finno and Calvello (2005) used Mohr-Coulomb models to describe the behavior of sand and fill layer and Hardening-Soil (HS) model (Schanz et al. 1999) to describe the behavior of clay. Hong et al (2016) used a hypoplastic model and its improved edition which considers the small strain effect to predict the soil deformation in a centrifuge test of excavation. Lim and Ou (2017) used Mohr-Coulomb model, HS model and HS small strain model to study the stress paths in deep excavations. These studies are all deterministic analyses of excavation deformation without account for parameter uncertainties.

Application of advanced constitutive models in probabilistic analysis of excavation engineering was relatively rare in the literature. This paper adopts a Hypoplastic (HP) model (von Wolffersdorff, 1996), and its modified version that considers the small strain behavior of soils (Niemunis and Herle, 1997) in probabilistic analysis of ground deformation induced by excavation. The paper integrates finite element method and direct Monte Carlo Simulation to conduct the probabilistic analysis. In order to improve the efficiency of probabilistic analysis and to avoid numerical problems in finite element calculation, artificial neural network (ANN) is applied to establish the surrogate model of finite element analysis.

2. HYPOPLASTIC MODELS AND FINITE ELEMENT ANALYSIS

2.1 *Hypoplastic constitutive models*

Finite element analysis using basic HP model (without considering small strain stiffness) and the improved HP model (considering small strain stiffness) was performed by the *Abaqus* computer software with a user-defined subroutine, which was coded in Fortran (Gudehus et al, 2008). Because these two constitutive models are well developed and documented below, only a brief description on them is given below.

Hypoplasticity is a theory that assumes the grains of granular materials aggregated to a so-called “simple granular skeleton” defined by some features (e.g., a granular material state depend on granular stress and void ratio only, grains are permanent, surface effects are absent, etc.). Based on the hypoplasticity, the nonlinear incremental model— HP model， was developed to describe the nonlinear mechanical of granular materials (von Wolffersdorff, 1996). Unlike some elastoplastic models, yield surfaces or flow rules are not needed in HP model.

According to the HP model, soil state in e-p' space is bound by upper limit e_i-p' curve and lower limit e_d-p'curve (e_i and e_d denotes the maximum and minimum void ratio, respectively; p' denotes the effective mean normal stress). The critical state line, e_c-p' curve, lies between these two curves (e_c denotes the critical state void ratio). The relationship between e and p is described below

$$\frac{e_i}{e_{i0}}=\frac{e_c}{e_{c0}}=\frac{e_d}{e_{d0}}=\exp(p'/h_s)^n \tag{1}$$

where e_{i0}, e_{c0}, and e_{d0} are the maximum void ratio, the critical state void ratio, the minimum void ratio at zero pressure; h_s and n are parameters that, respectively, control the overall slope and curvature of the three lines mentioned above. The h_s, n and e_{c0} can be obtained from the oedometer test. Generally, e_{i0} and e_{d0} can be determined by multiplying e_{c0} by two different constants. Two more parameters α and β are used to control the different aspects of mechanical behavior (e.g., shear stiffness and dilatancy) of soils, and they can be calibrated from trial and error procedures through finite element modeling of drained triaxial element tests. In summary, defining the basic HP model needs eight model parameters: critical state friction angle φ_c, h_s and n, e_{d0}, e_{c0}, e_{i0}, α and β.

Compared with the basic HP model, five additional model parameters (i.e., m_R, m_T, R, β_r, and χ) are introduced in the improved HP model to characterize the strain dependency and path dependency of soil stiffness at small strains (e.g., Niemunis and Herle, 1997; Hong et al., 2016). These five parameters can be obtained from a series of triaxial tests with different stress paths. Initial shear modulus upon 180° and 90° strain path reversal are controlled by m_R and m_T, respectively. R denotes the elastic range. The relationship between the stiffness and strain is defined by β_r and χ.

2.2 *Centrifuge test and numerical modeling*

A centrifuge model test simulating the excavation was conducted at a centrifugal gravitation of 50g (g=9.8m/s^2) by Hong et al (2016). Depth of the non-propped excavation is 8m (in prototype), width of the diaphragm wall is 600mm (in prototype), and the penetration depth of the wall into soils was equal to 0.9 times of the final excavation depth. The experimental material used in the test is medium-dense (D_r=65%) dry Toyoura sand. The excavation process is replaced by the expulsion of solution of equal density as that of soil. The lateral displacement of the wall and ground surface settlement of the test were well recorded. The excavation process is simulated in *Abaqus* (ABAQUS Inc., 2018).

Figure. 1 shows the finite-element mesh used to model the response of the centrifuge test in this study. The left and right vertical boundaries were fixed by roller supports, and pinned supports were used in the bottom boundary condition. The retaining wall was modeled as a linear elastic material with Young's modulus, Poisson's ratio, and unit weight equal to 70GPa, 0.2 and 27KN/m^3, respectively.

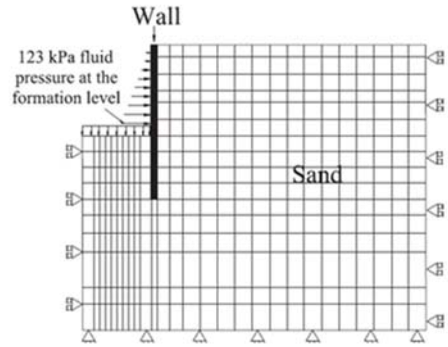


Figure 1: Finite element mesh and boundary conditions (after Hong et al, 2016)

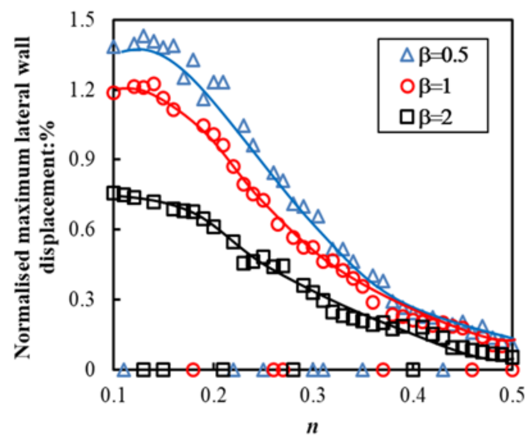


Figure 2: Illustration of the convergence issue in FEM analysis.

2.3 *Convergence issue in finite element analysis*

It is inevitable that the finite element analysis of ground deformation may not converge due to numerical issues, particularly in simulation-based probabilistic analysis where input parameters of the finite element model are randomly simulated from prescribed distributions of uncertain parameters. Take the excavation case adopted in this paper as an example. With different values for n and β, and fixed value for other five parameters, a series of finite element analyses are performed. As shown in Figure. 2, the points on the horizontal axis represent the values of n and β, with which finite element analyses fail to converge. It is not appropriate to directly discard samples that do not converge in probabilistic analysis because this may change the distribution of input parameters. Figure. 2 also shows that the maximum lateral displacement has an obvious non-linear trend with the increase of n. If a surrogate model with an explicit expression can accurately describe the non-linear relationship between model parameters and the response predicted by the finite element analysis, it can be used to as the surrogate model of the finite element model in probabilistic analysis. Besides, simulation-based probabilistic analysis might require a large number of numerical simulations to obtain reliability estimates with a reasonable accuracy. Compared with using the finite element model, using the surrogate model allows improving the efficiency of probabilistic analysis.

3. ARTIFICIAL NEURAL NETWORK

To capture the non-linear relationship between input parameters and the response obtained from the finite element analysis, Artificial Neural Network (ANN) is applied to developing the surrogate model of the finite element model in this study. The most common neural network structure consists of different layers, each of which is composed of different neurons. Neurons in different layers are connected by the directional arc with weights w_{ji} (connection weights between neuron j and i). The output x_i of the neurons in the upper layer is multiplied by different weights. Then the sum I_j of them and a threshold value θ_j is used as input to the neurons in the lower layer. The input is processed by the neuron with a transfer function f, usually a logistic sigmoid function, to get the output y_j. The whole process can be summarized below (Shahin et al, 2009)

$$I_j = \theta_j + \sum_{i=1}^{n} w_{ji} x_i \tag{2}$$

$$y_j = f(I_j) \tag{3}$$

ANNs have been successfully applied to a wide range of problems in geotechnical engineering (e.g., Goh and Kulhawy, 2003; Wang et al, 2007; Shahin, 2016). As a powerful tool to deal with non-linear problems, ANNs can capture the underlying relationship between data through repeated learning and training from the known information. The Back-propagation (BP) neural networks is used to establish the surrogate model of finite element model in this study. More details about ANNs can be referred to Gurney (1997).

Among the eight model parameters of basic HP model, φ_c=31°, five of them were regarded as uncertain, and e_{i0} and e_{d0} were calculated from e_{c0}. Table 1 summarizes the typical range of possible values of five uncertain parameters. Table 2 summarizes values of the five additional parameters of improved HP models, which are adopted from Hong et al (2016).

500 samples were uniformly and randomly generated from their respective ranges.350 samples

Table 1: Range of HP model parameters

Parameter	n	α	e_{co}	h_s(GPa)	β
Lower limit	0.1	0.1	0.85	1	0
Upper limit	0.5	0.3	1.05	6	3

Table 2: Values of five additional parameters of the improved Hypoplastic model

Parameter	m_R	m_T	R	β_r	χ
Value	8	4	0.00002	0.15	1.0

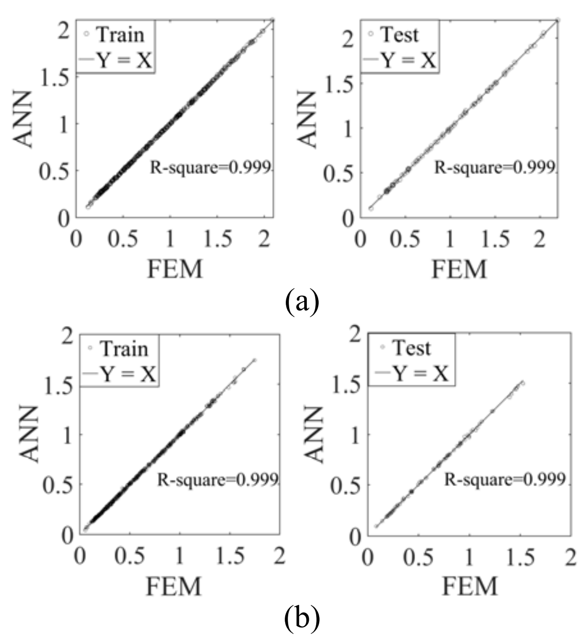


Figure 3: Comparison of predictions by ANN and FEM using basic HP model :(a) Normalized maximum lateral displacement. (b)Normalized maximum ground surface settlement.

were used to train the ANN model. After the training was completed, additional 150 samples, which were not used in the training process, were used to validate the ANN model. The ANN model is trained in an iterative manner until the validation error is reasonably small, allowing the ANN model to fit the finite element model well without overfitting. Figure. 3 and Figure. 4 compare responses, which are normalized by excavation depth, computed by basic and improved HP model of all the samples with those calculated from the ANN model. It is shown that results from the finite element analysis and the ANN model are in good agreement, which validates the surrogate model.

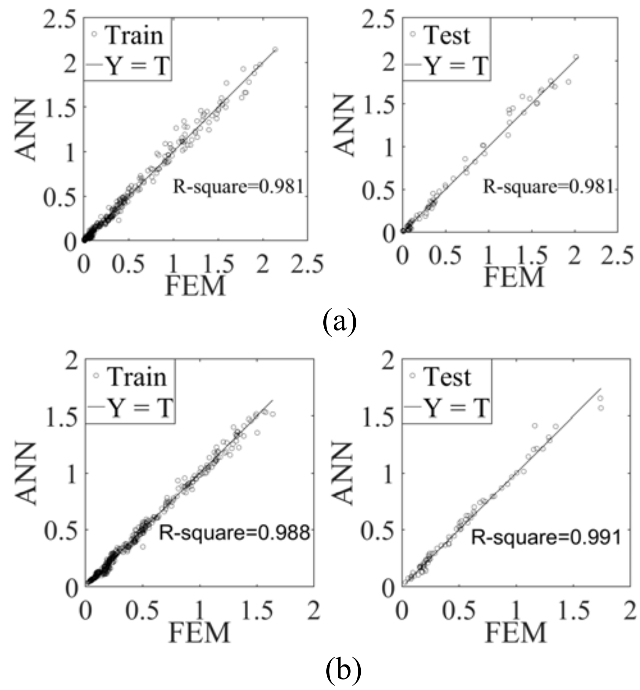


Figure 4: Comparison of predictions by ANN and FEM using improved HP model:(a)Normalised maximum lateral displacement. (b)Normlised maximum ground surface settlement.

4. PROBABILISTIC ANALYSIS OF GROUND DEFORMATION INDUCED BY EXCAVATION

This section combines direct Monte Carlo simulation (MCS) with the ANN model to perform the probabilistic analysis of ground deformation induced by the excavation. Table 3 summarizes probability distributions of uncertain parameters involved in the analysis. Assume that the n, α, h_s, and β follow the normal distribution. Their mean values were consistent with the values of these parameters calibrated by Herle and Gudehus (1999), and their coefficient of variation (COV) is assumed to be 0.1. The e_{c0} is assumed to follow the uniform distribution with the range shown in Table 1.

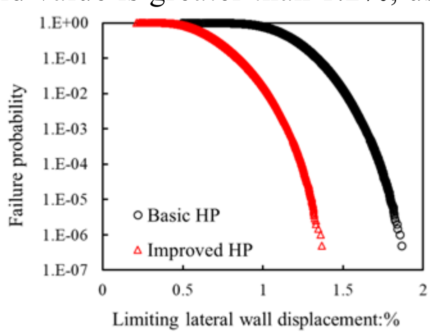
Table 3: Probability distributions of hypoplastic model parameters.

Parameter	Distribution	Mean	COV
n	Normal	0.27	0.1
α	Normal	0.18	0.1
h_s (GPa)	Normal	2.6	0.1
β	Normal	1.1	0.1
e_{co}	Uniform		

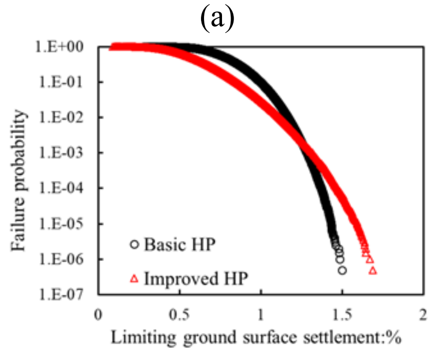
A MCS run with 200,000 samples is performed to carry out the probabilistic analysis based on the ANN models trained based on finite element analysis using basic and improved HP models in the previous section. The maximum lateral wall displacement and ground surface settlement are two responses in excavation analysis of the most concern. The probability distributions of these two responses are investigated. The confidence intervals of the displacement curve are also evaluated. In addition, this section also compares results obtained from MCS analyses based on the basic and improved HP models.

The serviceability limit state requires that both the maximum wall displacement and the ground surface settlement induced by excavation do not exceed their respective threshold values for judging unsatisfactory performances. The serviceability failure is then defined if the responses exceed the threshold values. The probability of serviceability failure is calculated using the samples generated by MCS.

Figure. 5(a) shows the variation of the failure probability based on the MCS analysis with basic and improved HP model as a function of the threshold value of lateral wall displacement. It is shown that the probability of failure decreases with the increase of the threshold value (or limiting value), and its values calculated from the ANN model based on the basic HP model are always larger than those computed using the ANN model based on improved HP model for a given threshold value. This means that the failure probability of maximum lateral wall displacement predicted by basic HP model is more conservative than that predicted by improved HP model in this example. Figure. 5(b) shows the variation of the failure probability based on the MCS analysis with basic and improved HP model as a function of the threshold value of ground surface settlement. As the threshold value of ground surface settlement is smaller than 1.2% of the excavation depth, the probability of failure of the maximum ground surface settlement predicted by basic HP model is more conservative than that predicted by improved HP model. As the threshold value is greater than 1.2%, using



(a)



(b)

Figure 5: Failure probability curves computed using basic and improved HP models: (a) normalized maximum lateral wall displacement; (b) normalized maximum ground surface settlement.

improved HP model is more conservative in this example.

Figure. 6 shows the two responses estimated from 200,000 MCS samples. It is observed that the maximum lateral displacement and ground surface settlement obtained from basic and improved HP model-based ANNs are positively correlated. That means that a larger maximum lateral wall displacement generally corresponds to a lager maximum ground surface settlement. The same observations were also reported in the literature based on observational data obtained in real excavation cases (e.g., Luo et al, 2018). It can also be found that the maximum ground surface settlement estimated using basic HP model is smaller than the maximum lateral wall displacement. In contrast, the two responses in one simulation using the improved HP model are close to each other. It is apparent that the responses estimated based on two models deviate from each other. The measurement value obtained from the centrifuge test is more consistent with responses estimated from the improved HP model. This suggests that it is possible to predict both responses more accurately based on improved HP model. Based on the basic HP model, it difficult to, simultaneously, predict the maximum lateral wall displacement and ground surface settlement accurately. When parameters of the basic HP model are adjusted or back analyzed to estimate the prediction of ground deformation induced by the excavation, it is difficult to identify a set of parameters of the basic HP model that can provide consistent estimates of both maximum lateral wall

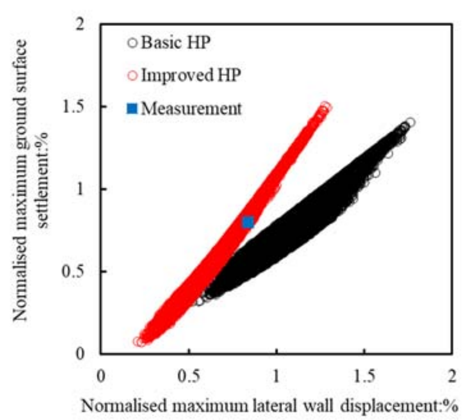
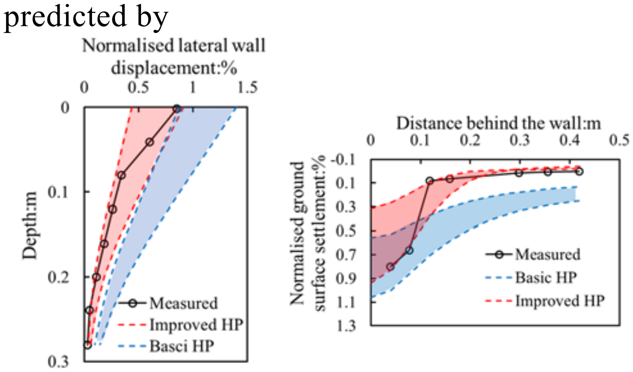


Figure 6: Sample estimates of ground deformation

displacement and ground surface settlement in this example. From this point of view, improved HP model outperform the basic one.

Figure. 7(a) shows the 95% confidence interval of the prediction of the lateral wall displacement profile at different depth based on two constitutive models. It can be observed that the confidence interval of improved HP model is on the left side of the interval of basic HP model. This means that the predictions based on improved HP model of the wall displacement at different depths are smaller than those obtained from the basic HP model. It is shown that the entire displacement curve is enveloped in the confidence interval of improved HP model. This suggests the performance of improved HP model is better than the basic HP model in terms of the prediction of the lateral wall displacement at different depth.

Figure. 7(b) shows the 95% confidence interval of prediction of the ground surface settlement at different distance behind the wall based on two constitutive models. Except for some overlap near the wall, it shows that the confidence interval of improved HP model generally plots above the confidence interval of basic HP model. This means that the prediction of ground surface settlement based on basic HP model is more conservative than that based on improved HP model. This is consistent with the observation obtained from Figure. 6. The ground surface settlement decreases with the increase of distance behind the wall, while the curve predicted by basic HP model is flatter. Similar to Figure. 7(a), it can be seen that the measured profile of the ground surface settlement obtained from the centrifuge test is more consistent with the confidence interval of the ground surface settlement profile estimated from the improved HP model than that estimated from the basic HP model. This, again, suggests the performance of improved HP model is better than the basic one in terms of the prediction of the ground surface settlement at different distances behind the wall.



(a) (b)

Figure 7: 95% confidence interval of the overall responses computed by ANN using basic and improved HP model: (a) lateral wall displacement computed with basic HP model; (b) lateral wall displacement computed with improved HP model;

5. SUMMARY AND CONCLUSIONS

This paper integrates the direct Monte Carlo simulation (MCS) method and finite element method to conduct the reliability analysis of ground deformation induced by excavation (including lateral wall displacement and ground surface settlement) using the hypoplastic model and its modified edition that considers the strain dependency and path dependency of soil stiffness at small strains. Artificial Neural Network (ANN) were used to establish the surrogate model of finite element model. Based on the results, the following conclusions can be drawn:

(1) Probabilistic analysis results based on finite element analyses depend on the constitutive model. In this study, the ground deformation induced by excavation estimated using the basic HP model is more conservative than that based on the improved HP model.

(2) Results show that ANN is able to capture the underlying nonlinear relationship between the input parameters and output of the finite element model using hypoplastic model. ANNs is a powerful tool in the finite element model-based probabilistic analysis. It not only helps overcome the numerical convergence issue, but also improve the computational efficiency of probabilistic analysis.

(3) It was also found that the measurements of ground deformation induced by excavation in the centrifuge test are more consistent with those estimated from the ANN based on the

improved HP model than those estimated from the ANN based on the basic HP model.

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