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Two-dimensional simulation of large-scale wind field via a joint wavenumber-frequency power spectrum

Yongbo Peng
State Key Laboratory on Disaster Reduction in Civil Engineering, Tongji University, Shanghai, China

Weijie Zhao
College of Civil Engineering, Tongji University, Shanghai, China

Jianbing Chen
College of Civil Engineering, Tongji University, Shanghai, China

ABSTRACT: The demand of large span spatial structures is booming in developing countries as their urbanization processes keep steady pace. These structures are characterized with long-span roofs, which probably will suffer wind-induced damages in their lifecycles because of their flexible natures and harmful aerodynamic actions caused by complicated forms of roof surface and stochastic wind field around them. Relevant studies are yet relatively rare in history. The recently proposed wavenumber-frequency joint PSD based spectral representation method (WN-SRM) has greatly reduced the computational burden of traditional wind field simulation methods, which makes possible for performing stochastic wind field simulation for large size structures. This paper presents a two-dimensional, homogeneous wind field simulation for a long-span, unevenly curved roof structure. The results show that the improved spectral representation method, i.e. WN-SRM works well in wind field simulation for flexible, long-span roof structure in terms of efficiency and effectiveness.

1. INTRODUCTION

As the pace of urbanization speeds up in China and other developing countries, the demand of large-scale public buildings grows rapidly, e.g. large-scale railway stations, terminal buildings, gymnasiums and exhibition halls etc. These buildings are usually characterized with large spans due to the demand of wide space inside, and large-span roofs naturally follow. Compared to high-rise buildings or long-span bridges, the length, width and height of large-span buildings are very even, which means wind loads might not be top concerns for the main bodies of these structures. However, for roof of large-span building alone, it is more of a thin piece of paper which is subject to aerodynamic actions from wind easily considering the distinct difference between its height and its length/width. Moreover, roofs of these buildings are often designed as complex three-dimensional curved surfaces for structural and aesthetic reasons, and this leads to a more complicated stochastic wind field around themselves, which probably could result in wind-induced roof damage problems (Cao and Wang 2013; Yang et al. 2018). In other words, wind loads for long span roof structures await further sophisticated studies and careful determination.

In order to perform wind-induced vibration response analysis on a given structure and meanwhile sufficiently embody the stochastic nature of wind loads, it makes sense to obtain a large number of stochastic wind speed samples. This can be achieved by field measurements carried out to monitor wind field (Au and To 2012; Peng et al. 2018; Shen and Li 2010; Wang et al. 2010; Yang et al. 2010). Nevertheless, this method typically involves high cost, tremendous workload and lack of efficiency. Thus, present stochastic wind field studies usually resort to numerical simulation methods with the help of computer technology, which has been verified to be an effective and efficient way to generate stochastic wind fields for structures (Shields and Deodatis 2013; Togbenou et al. 2016; Togbenou et al. 2018; Ubertini and Giuliano 2010).

Typically, wind speed is decomposed into two components: the mean wind speed and the fluctuating wind speed. And the fluctuating wind speed is firstly considered as an ergodic, zero-mean and Gaussian stochastic process that can be simulated more easily. A series of different wind field simulation methods have been proposed during the past seventy years(Gersch and Yonemoto 1977; Rice 1944; Yamada and Ohkitani 1991). As an important division of the system of stochastic wind field simulation methods, the spectral representation method (SRM) considers fluctuating wind speed as the superposition of a battery of cosine functions with a series of discrete angular frequencies. It is a classic, and meanwhile popular stochastic process simulation method thanks to its simplicity and logicality in theory and producing unconditionally stable simulated stochastic wind field with good precision. This method has been developed to simulate multi-variate, multi-dimensional, nonstationary, non-Gaussian, or combination of these, stochastic process (Deodatis and Micaletti Raymond 2001; Liang et al. 2007; Peng et al. 2017; Shinozuka 1971; Shinozuka and Deodatis 1991). Nevertheless, when the SRM deals with wind field simulation involving multiple spatial points, the computational burden becomes outrageously heavy because of the Cholesky decomposition of cross-PSD matrix at all discrete frequencies involve in the solution process. As some scholars have pointed out, the SRM would break down easily when simulating wind field with more than 20-30 spatial points. Recently, a wavenumber-frequency joint PSD based RSM , i.e. WN-RSM has been developed and applied to circumvent the horrible Cholesky decomposition step in stochastic wind field simulation (Benowitz and Deodatis 2015; Peng et al. 2016; Song et al. 2018).

For a long span, unevenly curved roof structure, the wind field around it is a stochastic one with theoretically countless three-dimensional spatial points which need to be simulated. However, it is no way to accomplish that without simplifying the problem. Considering that long span roofs are relatively flat, a two-dimensional, homogeneous, stochastic wind field around the mentioned roof structure is explored using the WN-SRM in this paper. Section 2 gives an overview of the mentioned roof, which is part of an unfinished practical engineering program. Section 3 goes to the procedure of two-dimensional, homogeneous wind field simulation using the WN-RSM. Numerical simulation is carried out in Section 4. Section 5 presents the concluding remarks to this paper.

2. OVERVIEW OF THE LONG-SPAN ROOF

The objective of concern is a spatial structure with long-span curved steel truss roof, which takes the shape of bird's wings when viewing it from the sky. The main body of the terminal building is about 215 meters long, 684 meters wide, and 55 meters in height. The roof shape takes the form of a relatively flat, bi-directional free-form surface, whose length is around 325 meters, width is about 714 meters, and the difference in height between the highest point and the lowest point of the roof surface is only approximately 7 meters; see Figure 1.

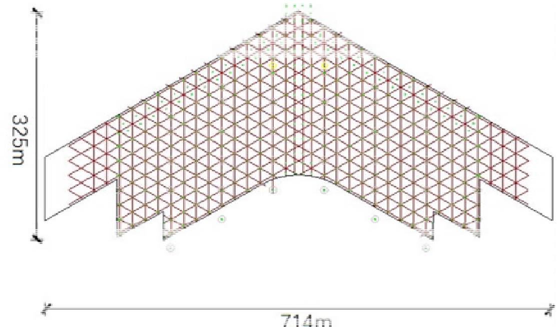

Figure 2: Plan of steel truss roof.

Considering the dimensions feature the large-span roof exhibits, a two-dimensional wind field simulation, which is based on the horizontal plane, against the roof needs to be conducted to obtain fluctuating wind loads, which are essential for its future wind-induced vibration response analysis. From theoretical perspective, the fluctuating wind speed time series at any two points of the roof should be different, and every spatial point on the roof should be applied with corresponding fluctuating wind loads. However, performing wind field simulation on every spatial point of the roof is not practical in engineering. Thus a partition scheme of the roof is adopted. Total 33 subareas were finally obtained, as shown in Figure 2.

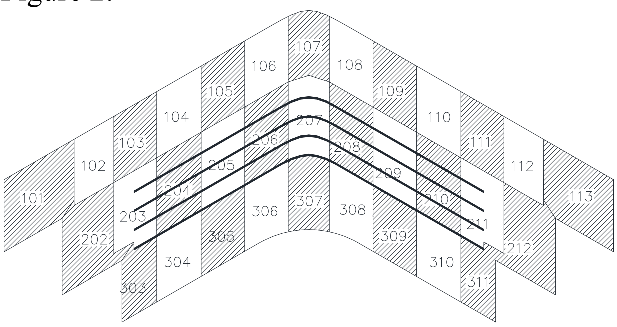

Figure 2: Partitioning scheme for steel truss roof.

3. TWO-DIMENSIONAL WIND FIELD SIMLATION METHOD

Consider wind speed processes at two arbitrary spatial points (x_1, y_1) and (x_2, y_2), where x represents the distance from a reference point on vertical axis, while y represents the distance from the same reference point on abscissa axis. The fluctuating components of these two wind speed processes can be denoted as $\tilde{u}_1(x_1, y_1, t)$ and $\tilde{u}_2(x_2, y_2, t+\tau)$, respectively, where t stands for time, and τ stands for time difference. If the expectation of $\tilde{u}_1$ and $\tilde{u}_2$ were calculated, the results should be both zero. For homogeneous wind field, the correlation function of $\tilde{u}_1$ and $\tilde{u}_2$ depends on the space difference and time difference of the two points rather than each coordinates or time. Thus, the correlation function $R_{\tilde{u}_1\tilde{u}_2}$ can be expressed as (Vanmarcke 2010):

$$\begin{aligned} R_{\tilde{u}_1\tilde{u}_2}(x_1, y_1, x_2, y_2, t, t+\tau) &= E\left[\tilde{u}_1(x_1, y_1, t)\tilde{u}_2(x_2, y_2, t+\tau)\right] \\ &= R_{\tilde{u}_1\tilde{u}_2}(\xi_1, \xi_2, \tau) \end{aligned} \tag{1}$$

where $\xi_1 = |x_1 - x_2|$ and $\xi_2 = |y_1 - y_2|$ denote the space differences on vertical axis and on abscissa axis, respectively. The cross-PSD is derived when perform Fourier transform to the correlation function on τ

$$\begin{aligned} S_{\tilde{u}_1\tilde{u}_2}(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} R\left(\xi_x, \xi_y, \tau\right)e^{-\mathrm{i}\omega\tau}d\tau \\ &= S^{(F)}\left(\xi_x, \xi_y, \omega\right) \end{aligned} \tag{2}$$

where i represents the unit of imaginary number $\sqrt{-1}$; ω represents the circular frequency.

Noted that cross-PSD of fluctuating wind speeds can also obtained by the equation below (Simiu and Scanlan 1996):

$$S_{\tilde{u}_1\tilde{u}_2}(\omega) = \sqrt{S_{\tilde{u}_1}(x_1, y_1, \omega)S_{\tilde{u}_2}(x_2, y_2, \omega)} \cdot \rho_{\tilde{u}_1\tilde{u}_2}\left(\xi_x, \xi_y, \omega\right) \tag{3}$$

where $S_{\tilde{u}_1}(x_1, y_1, \omega)$ and $S_{\tilde{u}_2}(x_2, y_2, \omega)$ are the PSDs of fluctuating wind speed at the two points; $\rho_{\tilde{u}_1\tilde{u}_2}$ is the coherence function of the two points in homogenous wind field.

In practice, $S_{\tilde{u}_1}(x_1, y_1, \omega)$ and $S_{\tilde{u}_2}(x_2, y_2, \omega)$ are often independent of spatial coordinates, namely they can be written as:

$$S_{\tilde{u}_1}(x_1, y_1, \omega) = S_{\tilde{u}_2}(x_2, y_2, \omega) = S_0(\omega) \tag{4}$$

Consequently, Eq. (3) can be expressed as:

$$S_{\tilde{u}_1\tilde{u}_2}(\omega) = S_0(\omega) \cdot \rho_{\tilde{u}_1\tilde{u}_2}\left(\xi_x, \xi_y, \omega\right) \tag{5}$$

Now, consider the Davenport spectrum, who is independent of height(Davenport 1961):

$$S(\omega) = 2.0u_*^2 \frac{\left(\dfrac{1200\omega}{2\pi U_{10}}\right)^2}{\omega\left(1 + \left(\dfrac{1200\omega}{2\pi U_{10}}\right)^2\right)^{4/3}} \tag{6}$$

where u_* is friction velocity; U_{10} is mean wind velocity at 10 meters above the ground.

The coherence function refers to one that depends on spatial differences (Simiu and Scanlan 1996):

$$\rho\left(\xi_x,\xi_y,\omega\right)=\exp\left(-\frac{|\omega|\sqrt{C_{1x}^2\xi_x+C_{1y}^2\xi_y}}{2\pi U_{10}}\right) \tag{7}$$

where C_{1x} and C_{1y} are decay parameters on x direction and on y direction (Peng et al. 2018).

Substitute S_0 and $\rho_{\tilde{u}_1\tilde{u}_2}$ in Eq. (5) with that in Eq. (6) and in Eq. (7), respectively, and it yields:

$$\begin{aligned} S^{\mathrm{F}}(\xi_x,\xi_y,\omega) &= \sqrt{S_1(\omega)S_2(\omega)}\rho(\xi_x,\xi_y,\omega) \\ &= 2.0u_*^2\frac{\left(\dfrac{1200\omega}{2\pi U_{10}}\right)^2}{\omega\left(1+\left(\dfrac{1200\omega}{2\pi U_{10}}\right)^2\right)^{4/3}}\cdot \\ &\quad \exp\left(-\frac{|\omega|\sqrt{C_{1x}^2\xi_x{}^2+C_{1y}^2\xi_y{}^2}}{2\pi U_{10}}\right)\end{aligned} \tag{8}$$

Finally, perform Fourier transform to S^F in terms of spatial differences ξ_x and ξ_y, and it produces an explicit wavenumber-frequency joint power spectrum (Benowitz and Deodatis 2015):

$$\begin{aligned} S^{(W-F)}(\kappa_x,\kappa_y,\omega) &= \frac{u_*^2}{\pi C_{1x}C_{1y}\left(\frac{1}{2\pi U_{10}}|\omega|\right)^2}\cdot\frac{\left(\frac{1200}{2\pi U_{10}}\omega\right)^2}{|\omega|\left(1+\left(\frac{1200}{2\pi U_{10}}\omega\right)^2\right)^{4/3}}\cdot \\ &\quad \left(1+\frac{\left(\kappa_x/C_{1x}\right)^2+\left(\kappa_y/C_{1y}\right)^2}{\left(|\omega|/(2\pi U_{10})\right)^2}\right)^{-3/2}\end{aligned} \tag{9}$$

where κ_x and κ_y respectively denote wave numbers on x axis and on y axis on the horizontal plane.

Based on the wavenumber-frequency joint power spectrum in Eq. (9), spectral representation method can be used to generate the two-dimensional stochastic fluctuating wind field for long-span roof structure using the following formula (Mantoglou and Wilson 1982):

$$\begin{aligned} u(x,y,t) &= \sum_{i=1}^{N_{\kappa_x}}\sum_{j=1}^{N_{\kappa_y}}\sum_{k=1}^{N_\omega}\sqrt{4S^{(W-F)}(\kappa_i^{(x)},\kappa_j^{(y)},\omega_k)\Delta\kappa_x\Delta\kappa_y\Delta\omega}\cdot \\ &\quad \left[\cos(\kappa_i^{(x)}x+\kappa_j^{(y)}y+\omega_k t+\phi_{ijk}^{(1)})+\cos(\kappa_i^{(x)}x+\kappa_j^{(y)}y-\omega_k t+\phi_{ijk}^{(2)})+\right. \\ &\quad \left.\cos(\kappa_i^{(x)}x-\kappa_j^{(y)}y+\omega_k t+\phi_{ijk}^{(3)})+\cos(\kappa_i^{(x)}x-\kappa_j^{(y)}y-\omega_k t+\phi_{ijk}^{(4)})\right]\end{aligned} \tag{10}$$

where N_κ and N_ω are numbers of discretized points in wavenumber domain and in frequency domain respectively; $\Phi_{ijk}^{(r)}$ are mutually independent random variables who are uniformly distributed on $[0,2\pi]$, and $r=1,2,3,4$.

4. NUMERICAL SIMULATION

To illustrate the effectiveness of the WN-SRM on generating two-dimensional stochastic fluctuating wind field for long-span roof structure, first, select typical spatial points on the horizontal plane of the roof, namely 107, 113, 307, as shown in Figure 2; then, perform fluctuating wind speed simulations at these three points and generate three simulated wind speed time series. Relevant parameters being used in the simulation of two-dimensional homogeneous stochastic wind field are listed in Table1 below.

Table 1: Relevant parameters being used in simulation procedure.

parameter	value
u_*	2 [m/s]
U_{10}	33.5 [m/s]
C_{1x}	10
C_{1y}	10
T	200 [s]
Δt	0.125 [s]
F_s	8π [rad/s]
$\Delta\omega$	$2\pi/T$ [rad/s]
W_n	1.5π [rad/m]
$\Delta\ell$	0.0015 [rad/m]

Note: T is the time length of simulated wind speed series; Δt is the discretized time interval of wind speed series; F_s is the upper limit of frequency domain; $\Delta\omega$ is the partitioned frequency interval; W_n is the upper limit of wavenumber domain; $\Delta\ell$ is the partition wavenumber interval.

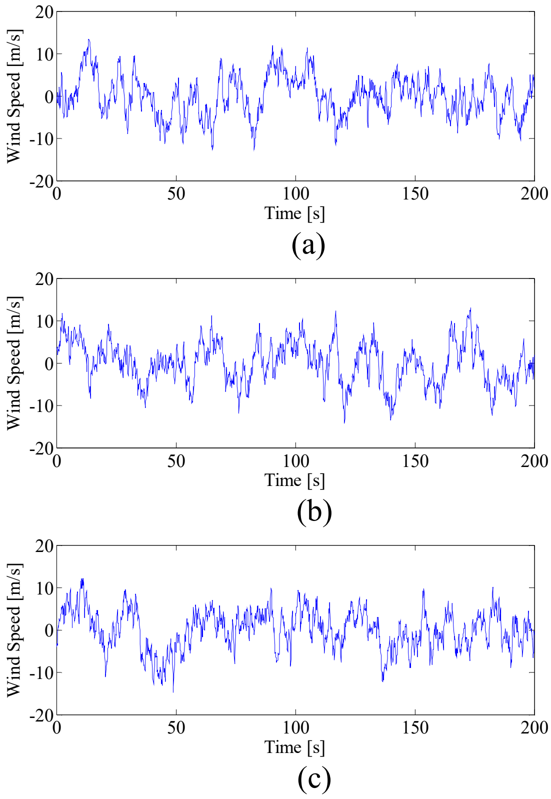

Figure 3: Simulated wind speed series at: (a) 107; (b) 113; (c) 307.

Figure 3 shows the simulated wind speed series at the spatial points 107, 113 and 307. It is seen that the differences from the wind speed series are obvious due to spatial variation of the simulated points. In general, characteristics of simulated stochastic wind speed field show a good consistency with the nature of fluctuating winds.

5. CONCLUSIONS

A wavenumber-frequency joint PSD based spectral representation method (WN-SRM) has been adopted to simulate two-dimensional, homogeneous, stochastic wind field around a long span curved roof. Numerical results reveal the applicability of this method. However, there are three issues worth of notice in the simulation.

- The WN-SRM is carried out by performing threefold summation on two wavenumber domains and one frequency domain, which is numerical integration in essence. Accordingly, careful partitioning scheme on all wavenumber and frequency domains should be considered.
- Two-dimensional simulation of stochastic wind field still encounters with a large amount of spatial points in terms of calculation workload. Effects such as bringing in Fast Fourier Transform (FFT) to reduce computational cost will be meaningful.
- The vertical component of stochastic wind field plays an important role in wind-induced vibration of long-span roof structures. The nonhomogeneous stochastic wind field simulation considering the vertical component of fluctuating wind speed simultaneously has to be conducted for attaining more accurate information of stochastic wind field around long span roof structures.

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