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Estimation of Small Time-Dependent Failure Probability in High Dimensions using Subset Simulation

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ABSTRACT: Time-dependent reliability analysis of deteriorating structures is significant in their performance assessment and maintenance. Various methodologies have been used by researchers to predict the time-dependent reliability of structures. However, it is still a challenge to estimate the small time-dependent failure probability in high dimensions. In the present study, based on subset simulation an adaptive stochastic simulation procedure is proposed considering the stochastic nature of the occurrence of time-dependent random variables. Moreover, a modified Metropolis-Hastings algorithm is developed to reduce repeated trajectories and suit the property of time-dependent reliability problem. Fourth-moment transformation is utilized in the study for without the exclusion of random variables with unknown probability distributions. The proposed method is illustrated by a cantilever tube subjected to external forces and torsion. The methodology can be used as a tool for structural engineers and asset managers to assess small time-dependent failure probability of a deteriorating structure in high dimensions and make decisions with regard to its maintenance and rehabilitation.

1. INTRODUCTION

Structural reliability during service life is in essence a time-dependent property due to aging under the aggressive service environment and varying service and environmental demands. Therefore, time-dependent reliability problems for structural safety and serviceability during the entire service life have to be solved. Consequently, various approximated methods (Melchers and Beck 2018; Mori and Ellingwood 1993; Li et al. 2005; Duprat 2007; Li et al. 2015) have been developed to assess the time-dependent reliability.

Among these methods, the simplest one is the point-in-time method (Duprat 2007; Val 2007), which discretizes the time-dependent problem into a series of time-independent reliability problems at preset time points. The stochastic of occurrence of time-varying random variables, e.g. live load, is not considered in point-in-time method. Assume that the occurrence of live load events is described by the Poisson point process, Mori and Ellingwood (1993) proposed a closed-form solution based on conditional probability theory for structural time-dependent reliability analysis considering both the randomness of resistance and the stochastic nature of load. Li et al. (2015) improved the

Mori and Ellingwood's work to account for the non-stationarity in load process in structural reliability assessment. However, the conditional-probability-based method is not applicable for the case with implicit performance function or multiple random variables. Assume that both the resistance and the load effect are continuous stochastic processes, the time-dependent failure probability of a structure can be estimated directly from the first-passage probability (Melchers and Beck 2018; Li et al. 2005). Unfortunately, analytical solutions for the first-passage probability only exist for Gaussian and lognormal processes.

All above methods could not perfectly deal with the high dimensional and complicated reliability problems which are likely encountered in real application. The objective of this study is to propose a simulation-based method to estimate the small time-dependent failure probability (e.g. $P_f < 10^{-3}$) in high dimensions.

Monte Carlo simulation (MCS) is well known to be robust. Its main drawback, however, is poor efficiency for calculating small probability. A relatively efficient sampling, importance sampling techniques (Melchers 1989), are used to shift the underlying distribution towards the failure region. The success of this method relies on prudent choices of importance sampling density that requires knowledge of failure region. Subset simulation (Au and Beck 2001; Au and Beck 2003; Ching et al. 2005; Wang et al. 2014) has been recently developed as an efficient simulation method for computing small failure probabilities for general and dynamic reliability problems in high dimensions. Its efficiency derives from introducing intermediate failure events to express the low failure probability as a product of larger conditional failure probabilities.

In application of subset simulation for dynamic reliability problem, the responses are computed at the preplanned discrete time instants. In general time-dependent reliability analysis, the occurrence of time-dependent random variables is always stochastic. Accordingly, it is necessary to investigate how to adjust or improve subset simulation to evaluate time-dependent reliability.

In this study, an adaptive stochastic simulation procedure based on subset simulation for efficiently evaluating the small time-dependent failure probability of a deteriorating structure in high dimensions is proposed considering the stochastic nature of the occurrence of time-dependent random variables. To suit the property of time-dependent reliability problem, a modified Metropolis-Hastings algorithm is developed. Differing from one-to-one mapping used in dynamic reliability problems, a mother sample vector is randomly chosen from load events of mother trajectory in the conditional level to generate an offspring sample vector. Moreover, to reduce the repeated trajectories candidate sample vector is generated randomly until candidate sample vector is different with mother sample vector. In this study, fourth-moment transformation is utilized for without the exclusion of random variables with unknown probability distributions.

2. SUBSET SIMULATION

This section provides a brief introduction to subset simulation and Markov Chain Monte Carlo (MCMC) which is used to generate the conditional samples. For a more detailed discussion, the reader is referred to Au and Beck (2001), Ching et al. (2005), and Wang et al. (2014).

Given a failure event F, let $F_1 \supset F_2 \supset \cdots \supset F_r = F$ be a nested sequence of failure regions. Using the definition of conditional probability, one has (Au and Beck 2001)

$$\begin{aligned}\mathrm{Prob}(F) &= \mathrm{Prob}(F_r \mid F_{r-1})\,\mathrm{Prob}(F_{r-1}) = \cdots \\ &= \mathrm{Prob}(F_1)\cdot\prod_{i=1}^{r-1}\mathrm{Prob}(F_{i+1} \mid F_i)\end{aligned} \tag{1}$$

Eq. (1) indicates that the problem of simulating rare events in the original probability space is replaced by a sequence of simulations of more frequent events in the conditional probability spaces. Prob(F_1) can be readily estimated by crude MCS, while Prob($F_{i+1}|F_i$) (i =1, 2, …, r-1) should be estimated by MCMC which can generate samples conditional on the failure region F_i. The Metropolis–Hastings algorithm is used in MCMC for sampling from conditional distributions (Au and Beck 2001). To generate MCMC sample functions, we perturb a mother sample function to generate an offspring sample functions. If the offspring trajectory is belong to the F_i domain, the offspring trajectory will be accepted; otherwise, rejected and the mother trajectory is repeated as the next sample trajectory (Ching et al. 2005).

Subset Simulation with MCMC may be summarized as follows (Ching et al. 2005). First, we simulate m sample trajectories by crude MCS to approximately compute Prob(F_1). From these MCS sample trajectories, we can readily obtain some mother trajectories that lie in F_1. With the Metropolis-Hastings MCMC algorithm, these mothers can be used to simulate more offsprings, also lie in F_1. The offsprings can be further used to estimate Prob($F_2|F_1$). Observe that the offsprings that lie in F_2 so are used as mothers for simulating more offsprings to estimate Prob($F_3|F_2$). Repeating this process, we compute the conditional probabilities of the higher conditional levels until the failure event of interest, F, has been reached.

For dynamic reliability problem, the responses are computed at the preplanned discrete time instants in practice. Thus, there is one-to-one mapping between mother sample and offspring sample as time goes on. In general time-dependent reliability analysis, it is assumed that the occurrence of time-dependent random variables is stochastic. In other words, the number and time of occurrence of time-dependent random variables are different in mother and offspring trajectories. Accordingly, the stochastic simulation procedure based on subset simulation for evaluating time-dependent failure probability should be developed to suit the property of time-dependent reliability problems.

3. TIME-DEPENDENT RELIABILITY ASSESSMENT USING SUBSET SIMULATION

3.1. Time-dependent reliability evaluation

In view of the different property of random variables about time, random variables can be divided into time-independent and time-dependent random variables (Hong 2000). The former includes parameters of dead load, original material strength, and initial geometry variables etc., while the latter includes in-service loads and environmental actions, corrosion rate, and so on. Letting $\mathbf{X}$ and $\mathbf{Y}(t)$ denote the vectors of time-independent and time-dependent random variables, respectively, the general time-dependent performance function of a deteriorating structure can then be expressed as $G[\mathbf{X}, \mathbf{Y}(t), t]$, in which $G[\mathbf{X}, \mathbf{Y}(t), t] > 0$ represents the safe domain.

Time-dependent reliability is the probability that a structure will perform its intended function successfully for a specified time interval. Accordingly, the time-dependent failure probability within a given period (0, T] can be formulated as

$$P_f(T) = \text{Prob}\left\{G\left[\mathbf{X}, \mathbf{Y}(t), t\right] \le 0, \exists t \in (0, T]\right\} \quad (2)$$

in which Prob($\cdot$) denotes the probability of the event in the bracket. Eq. (2) indicates that the structure is failure as long as there exists $G[\mathbf{X}, \mathbf{Y}(t), t]$ lying in failure domain at one instant within interval (0, T].

Suppose that the occurrence of time-dependent random vector is modeled as a Poisson point process (Mori and Ellingwood 1993), Eq. (2) can be rewritten as

$$P_f(T) = \text{Prob}\left\{\min_{k=1,2,\ldots,N}\left\{G\left[\mathbf{X}, \mathbf{Y}(t_k), t_k\right]\right\} \le 0\right\} \quad (3)$$

where N is the number of occurrence of $\mathbf{Y}(t)$ and is a random variable followed with Poisson distribution with parameter $\lambda(t)$, and t_k is the occurrence time of $\mathbf{Y}(t)$. When the mean occurrence rate $\lambda(t)$ is constant, t_k is followed with uniform distribution on the interval (0, T] .

If $\lambda(t)$ is related with time, t_k can be generated from its cumulative distribution function $F_{t_k}(t)$, which is expressed as

$$F_{t_k}(t) = \frac{\int_0^t \lambda(\tau)d\tau}{\int_0^T \lambda(\tau)d\tau} \tag{4}$$

3.2. Time-dependent reliability assessment using subset simulation

According to Eq. (3) and subset simulation, the intermediate failure event, F_i, of the time-dependent reliability problem is defined as

$$F_r = \left\{ \min_{k=1,2,\ldots,N_r} \left\{ G\left[\mathbf{X}, \mathbf{Y}(t_k), t_k\right] \right\} \le b_r \right\} \tag{5}$$

in which b_r is the threshold value that is selected such that Prob($F_r|F_{r\text{-}1}$) = p_0. The choice of the value p_0 is discussed in many researches (Au and Beck 2001; Au and Beck 2003; Ching et al. 2005). Number of trajectory per level, m, should be selected large enough to give accurate estimates of p_0 (Ching et al. 2005; Wang et al. 2014).

The flowchart of time-dependent reliability assessment using subset simulation is showed in Figure 1. First, we simulate m sample trajectories within the interval (0, T] by MCS to approximately compute Prob(F_1). With the MCMC algorithm, mother trajectories that lie in F_1 chosen from MCS sample trajectories are then used to simulate more offsprings, also lying in F_1, and Prob($F_2|F_1$) is obtained. The offsprings that lie in F_2 so are used as mothers for simulating more offsprings to estimate Prob($F_3|F_2$). Repeat this process until $b_r \le 0$ has been reached.

To bypass the problem that original Metropolis-Hasting algorithm is not applicable in high dimensions as the acceptance ratio systematically tends to zero, samples are first generated in independent standard normal space and then transformed into original space to calculate the values of performance function (Au and Beck 2001). In view of the stochastic nature of time-dependent random variables, a modified Metropolis-Hasting algorithm is developed in section 3.3. Fourth-moment transformation (Zhao and Lu 2007) presented in section 3.4 is utilized to transform samples in independent standard normal space into original space for random variables with unknown probability distributions.

The main procedure of time-dependent reliability assessment using subset simulation can be detailed summarized as follows.

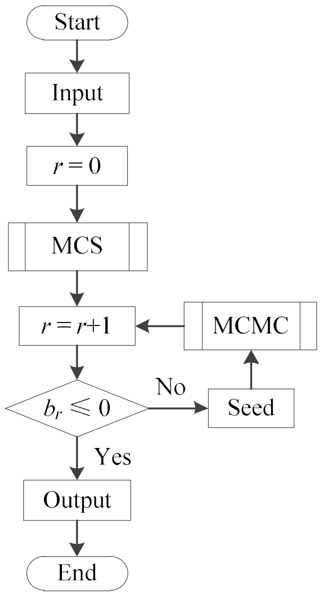

Figure 1 Flowchart

1. Initialize algorithmic parameters, such as level (conditional) probability, p_0, and number of trajectory per level, m.
2. Set $r = 1$.
3. Generate samples, $\{\mathbf{u}_{\mathbf{x}}^{(i)}: i = 1, 2, \ldots, m\}$, of independent standard normal distribution corresponding to $\mathbf{X}$.
4. Obtain the numbers of occurrence of $\mathbf{Y}(t)$, $\{N^{(i)}, i = 1, 2, \ldots, m\}$ during the given period (0, T] that are samples of Poisson distribution with parameter $\lambda_{\mathbf{Y}}$.
5. Determine the times of occurrence of $\mathbf{Y}(t)$, $\{t_k^{(i)}: i = 1, 2, \ldots, m, k = 1, 2, \ldots, N^{(i)}\}$ from Eq. (4).
6. Generate samples vectors, $\{\mathbf{u}_{\mathbf{y}_k}^{(i)}: i = 1, 2, \ldots, m, k = 1, 2, \ldots, N^{(i)}\}$, of independent standard

normal distribution corresponding to $\mathbf{Y}(t)$ at t_k.

7. With the aid of fourth-moment transformation (see section 3.4), transform $\mathbf{u}_{\mathbf{x}}^{(i)}$ and $\mathbf{u}_{\mathbf{y}_k}^{(i)}$ into $\mathrm{x}^{(i)}$ and $\mathbf{y}_k^{(i)}$ in original space.
8. Calculate the responses $\{g_k^{(i)} = G\left(\mathbf{x}^{(i)}, \mathbf{y}_k^{(i)}, t_k^{(i)}\right)$: $k = 1, 2, \ldots, N^{(i)}, i = 1, 2, \ldots, m\}$.
9. Find the minima $g_{\min}^{(i)}$ = min{ $g_k^{(i)}$ = $G\left(\mathbf{x}^{(i)}, \mathbf{y}_k^{(i)}, t_k^{(i)}\right)$: k = 1, 2, …, $N^{(i)}$}. Sort the minimums { $g_{\min}^{(i)}$: i = 1, 2, …, m} in ascending order, and record the indices of sequence. According to the indices, sort the samples { $\mathbf{u}_{\mathbf{X}}^{(i)}$, $\mathbf{u}_{\mathbf{y}_k}^{(i)}$: i = 1, 2, …, m} as { $\mathbf{u}_{\mathbf{X}}^{(s)}$, $\mathbf{u}_{\mathbf{y}_k}^{(s)}$: s = 1, 2, …, m}.
10. Choose the intermediate threshold value b_r as the mp_0th value in the ascending order of { $g_{\min}^{(i)}$: i = 1, 2, …, m}. Note that it has been assumed that mp_0 is an integer value.
11. If $b_r \le 0$, proceed to step 15 below. Otherwise, identify the first mp_0 samples { $\mathbf{u}_{\mathbf{X}}^{(s)}$, $\mathbf{u}_{\mathbf{y}_k}^{(s)}$: s = 1, 2, …, mp_0}, whose response lie in the region $F_r = \{g_{\min} \le b_r\}$.
12. The samples { $\mathbf{u}_{\mathbf{X}}^{(s)}$, $\mathbf{u}_{\mathbf{y}_k}^{(s)}$: s = 1, 2, …, mp_0} provide seeds for applying the MCMC simulation to generate $m(1-p_0)$ additional conditional samples, so that it obtains a total of m conditional samples which lie in region F_r at conditional level r + 1. (The detail of MCMC based on the modified Metropolis-Hastings algorithm used in the study is described as below)
13. Set $r \leftarrow r$+1.
14. Return to step 9 above.
15. Stop the algorithm.

3.3. Modified Metropolis-Hastings algorithm

Since the occurrence of time-dependent random variables is modeled as a Poisson point process, the number and times of occurrence of time-dependent random variables may be inconsistent in the mother and offspring trajectories. That is to say, one-to-one mapping used in dynamic reliability problems for generating an offspring trajectory is impractical for time-dependent reliability problems. In this study, we randomly choose a mother sample vector from N_r load events of mother trajectory in r-th conditional level to generate an offspring sample vector. In addition, to reduce the repeated trajectories candidate sample vector is generated randomly until candidate sample vector is different with mother sample vector.

The modified Metropolis-Hastings algorithm for simulating samples in the r+1-th level is summarized as follows.

1. Start from an initial (mother) trajectory { $\mathbf{u}_{\mathbf{x}}$, $\mathbf{u}_{\mathbf{y}_k}$: k = 1, 2, …, N_r}.
2. l from 1 to $n_{\mathbf{x}}$ ($n_{\mathbf{x}}$ is the number of time-independent random variables):
   2.1 Generate a pre-candidate sample ω following the proposal PDF, e.g., in this study $\omega \sim$ uniform $[u - w, u + w]$ where w is the half-width of the uniform PDF, and can be set as 1 (Santoso et. al 2011).
   2.2 Generate $\varpi \sim$ uniform [0, 1]. If $\varpi <$ min[1, $\phi(\omega)/\phi(u_{x_l})$], take ω as the candidate sample, $u'_{x_l} = \omega$; otherwise, u'_{x_l} = u_{x_l} .
3. If the candidate sample vector, $\mathbf{u}'_{\mathbf{x}}$, and the mother sample vector, $\mathbf{u}_{\mathbf{x}}$, are the same, return to step 2.
4. Obtain the number of occurrence of $\mathbf{Y}(t)$, N_{r+1}, followed with Poisson distribution. Determine the times of occurrence, {t_k: k = 1, 2, …, N_{r+1}}, from Eq. (4).
5. j from 1 to N_{r+1}:
   5.1 Randomly choose a mother sample vector, $\mathbf{u}_{\mathrm{y}}$, from { $\mathbf{u}_{\mathrm{y}_k}$: k = 1, 2, …, N_r}.
   5.2 l from 1 to $n_{\mathbf{y}}$ ($n_{\mathbf{y}}$ is the number of time-dependent random variables):
      5.2.1 Generate a pre-candidate sample ω following the proposal PDF.

5.2.2 Generate $\varpi \sim$ uniform [0, 1]. If $\varpi <$ min[1, $\phi(\omega)/\phi(u_{Y_l})$], take ω as the candidate sample, $u'_{y_l} = \omega$; otherwise, $u'_{y_l} = u_{y_l}$.

5.3 If the candidate sample vector, $\mathbf{u}'_{\mathbf{y}_j}$, and the mother sample vector, $\mathbf{u}_{\mathbf{y}}$, are the same, return to step 5.2.

6. With the aid of fourth-moment transformation (see section 3.4), transform $\mathbf{u}'_{\mathbf{x}}$ and $\mathbf{u}'_{\mathbf{y}_j}$ into $\mathbf{x}$ and $\mathbf{y}$ in original space.
7. Calculate the responses $\{g_j = G(\mathbf{x}, \mathbf{y}_j, t_j) : j = 1, 2, \ldots, N_{r+1}\}$. Find the minima $g_{\min}$ in the vector $\{g_j : j = 1, 2, \ldots, N_{r+1}\}$. If $g_{\min} \leq b_r$, take $\{\mathbf{u}'_{\mathbf{x}}, \mathbf{u}'_{\mathbf{y}_j} : j = 1, 2, \ldots, N_{r+1}\}$ as the trajectory in conditional level $r + 1$; Otherwise, take $\{\mathbf{u}_{\mathbf{x}}, \mathbf{u}_{\mathbf{y}_k} : k = 1, 2, \ldots, N_r\}$ as. (see Figure 2)

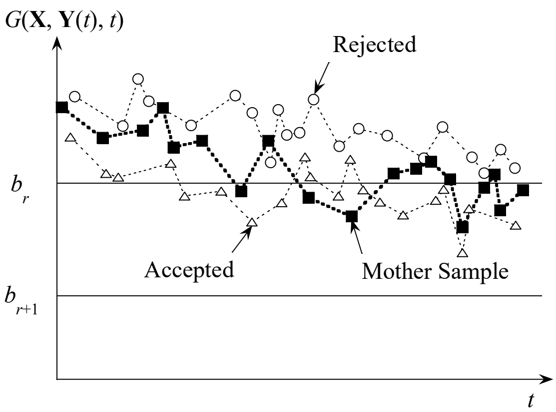

Figure 2 Accepted and rejected candidate trajectories

3.4. Fourth-moment transformation

To conduct time-dependent reliability analysis without the exclusion of random variables with unknown probability distributions, the fourth-moment transformation (Zhao and Lu 2007) is used. The random variable X is directly expressed as a third-order polynomial of a standard normal random variable U, which is generally formulated as follows

$$X_s = S_u(U) = a_1 + a_2 U + a_3 U^2 + a_4 U^3 \tag{6}$$

where X_s is the standardized random variable of X, $X_s = (X-\mu_X)/\sigma_X$; μ_X and σ_X are the mean and standard deviation of X, respectively; $S_u(U)$ is the third-order polynomial of U, and a_1, a_2, a_3, and a_4 are the polynomial coefficients that can be obtained by making the first four central moments of $S_u(U)$ equal to those of X_s (Fleishman 1978). The explicit expressions of the four coefficients suggested by Zhao and Lu (2007) are expressed as

$$a_1 = -a_3 = -\frac{\alpha_{3X}}{6(1+6k)} \tag{7a}$$

$$a_2 = \frac{1-3k}{1+a_1^2-k^2} \tag{7b}$$

$$a_4 = \frac{k}{1+a_1^2+12k^2} \tag{7c}$$

where k is given by

$$k = \frac{1}{36}\left(\sqrt{6\alpha_{4X} - 8\alpha_{3X}^2 - 14} - 2\right) \tag{8}$$

where α_{3X} and α_{4X} are the skewness and kurtosis of the random variable X, respectively.

4. EXAMPLES

To investigate the efficiency and accuracy of the proposed method for time-dependent structural reliability assessment, a numerical example is presented in this section.

A cantilever tube (Du 2008) showed in Figure 3 is subjected to external forces F_1, F_2, F_3, and torsion W. The failure criterion is reached when the maximum Von Mises stress $\sigma_{\max}$ exceeds the material's yield strength S_y. Accordingly, the limit-state function is defined by

$$G(\mathbf{X}, \mathbf{Y}(t), t) = S_y - \sigma_{\max} \tag{9}$$

The maximum Von Mises stress occurs at the top point of the section at the origin, and $\sigma_{\max}$ is given by

$$\sigma_{\max} = \sqrt{\sigma^2 + 3\tau^2} \tag{10}$$

where σ is the normal stress, formulated as

$$\sigma = \sigma_1 + \sigma_2 \tag{11}$$

$$\sigma_1 = \frac{F_1 \sin\alpha_1 + F_2 \sin\alpha_2 + F_3}{\pi\left[d_1^2 - \left(d_1 - 2d_2\right)^2\right]/4} \tag{12a}$$

$$\sigma_2 = \frac{\left(F_1 L_1 \cos\alpha_1 + F_2 L_2 \cos\alpha_2\right) d_1}{\pi\left[d_1^4 - \left(d_1 - 2d_2\right)^4\right]/32} \tag{12b}$$

in which d_2 is related with time, $d_2 = d_0(1\text{-}0.015t)$.

The torsional stress τ is calculated by

$$\tau = \frac{W d_1}{\pi\left[d_1^4 - \left(d_1 - 2d_2\right)^4\right]/16} \tag{13}$$

In this example, S_y, d_0, d_1, L_1, and L_2 are time-independent random variables, while F_1, F_2, F_3, and W are time-dependent random variables. All random variables are mutually independent, and their uncertainty properties are given in Table 1. The mean occurrence rate of external forces and torsion, λ, is assumed as 1 per year. The service life is considered as 30 years.

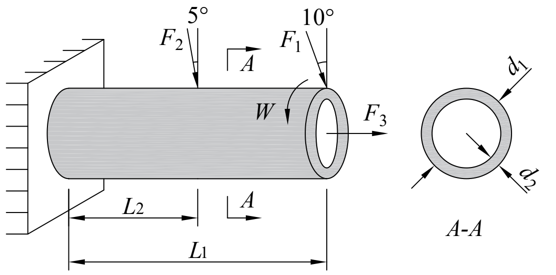

Figure 3 Cantilever tube

Table 1 Statistical properties of random variables

Parameters	*Mean*	*COV*	*Distribution*
S_y*(MPa)*	*220*	*0.1*	*Normal*
d_0 *(mm)*	*6*	*0.02*	*Normal*
d_1 *(mm)*	*45*	*0.012*	*Normal*
L_1 *(mm)*	*120*	*0.0012*	*Uniform*
L_2 *(mm)*	*60*	*0.0024*	*Uniform*
F_1 *(N)*	*3,000*	*0.1*	*Normal*
F_2 *(N)*	*3,000*	*0.1*	*Normal*
F_3*(N)*	*12,000*	*0.1*	*Gumbel*
W(N•mm)	*90,000*	*0.1*	*Normal*

In this example, the probability of each level is assumed as 0.1, and the number of trajectories of each level is selected as 8,000. Figure 4 provides detailed results for the four independent subset simulation runs. The black dotted bold line represents the average of the four runs. It also shows the MCS estimates. The estimated indices obtained from MCS with deterministic trajectories approximately ranged from 160,000 to 2,191,370,000 (COVs of MCS range from 2% to 5.0%) are used as a baseline. Figure 4 shows that (1) the average of reliability index of the four runs is close to the MCS estimates; (2) the smaller time-dependent reliability index, the more stable of the results.

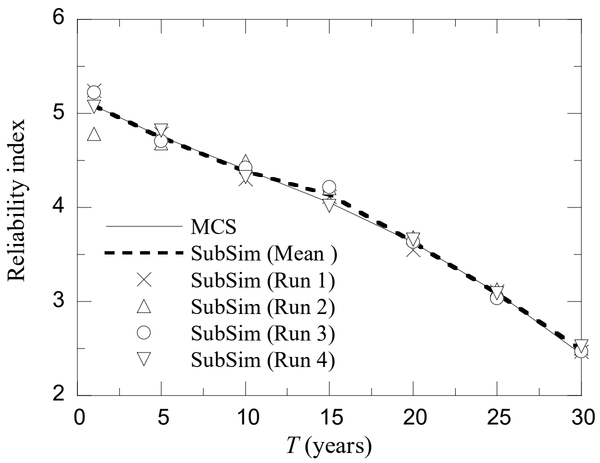

Figure 4 Time-dependent reliability indices

Since the proposed method is simulation based method, the computational effort will hardly increase as the increase of dimension of random variables. Therefore, the proposed method can be extended to higher-dimensional time-dependent reliability problems. It can be observed from this example that: for small time-dependent failure probability problems (e.g., $P_f \in (10^{-3}, 10^{-6})$), the proposed method can provide relatively stable results; for tiny time-dependent failure probability problems (e.g., $P_f < 10^{-6}$), the stability of the proposed method need to be further investigated. Theoretically, the proposed method can be extended for the high-dimensional engineered systems with small time-dependent failure probabilities, which will be tested and verified in further works due to the limited space.

5. CONCLUSIONS

In the study, an adaptive stochastic simulation procedure based on subset simulation for efficiently evaluating the small time-dependent failure probability of a deteriorating structure in high dimensions is proposed. The stochastic nature of the occurrence of time-dependent random variables is considered and is modeled as a Poisson point process. A modified Metropolis-Hastings algorithm is presented to reduce repeated trajectories and suit the property of time-dependent reliability problems. Fourth-moment transformation is utilized for without the exclusion of random variables with unknown probability distributions. In future works, we will investigate the stability of the developed method for tiny failure probability problems and the application for complex engineered systems.

6. ACKNOWLEDGMENTS

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