A New Hazard-Agnostic Finite Element Model for Community Resilience Assessment

Hussam Mahmoud
Associate Professor, Dept. of Civil and Env. Engineering, Colorado State University, Fort Collins, CO, USA

Akshat Chulahwat
Graduate Research Assistant, Dept. of Civil and Env. Engineering, Colorado State University, Fort Collins, CO, USA

ABSTRACT: Mitigating the impact of disasters on communities requires not only a deep understanding of the essential features of infrastructure, social, and economical components that make a community resilient, but also the development of mathematical models that can seamlessly integrate these features. In this study, we present a new and novel theoretical dynamic model for quantifying community resilience. The model is founded on mathematically integrating infrastructural, social, and economic sectors of the community of interest. The underlying fundamentals of the proposed theory hinges on assuming the behavior of a community in response to a hazard is equivalent to the response of a vibrating mass of finite stiffness and damping. The dynamic model is implemented through the development of a finite element formulation capable of quantifying resilience both temporally and spatially. The finite element model is further utilized to devise a new hazard-agnostic definition of community resilience, which is demonstrated through logical verification tests conducted on a testbed city. Through various analysis and sensitivity studies, it is observed that the model can be used to identify vulnerable areas in a community as well as provide a spatial and temporal measure of community resilience for various types of hazards such as physical disruptions and even social disorder.

Natural disasters have been increasing in frequencies and their imapact in the past decade. These events have caused subtaitnial losses to communties, particularly those in large urban areas. The initial losses are sustained by the infrastrcture and are described in terms of direct economic losses. The conseuqnetial losses are often substantial as well and include social and economic consequences. It is therefore no longer acceptable to rely solely on performance engineering as a way to control infrastructral losses with no regards, or an attempt to understand, the social and economic consequences. Moreover, understanding recovery from losses is critical to the functiolaity of the various infrastrcture and the community as a whole. That is a community should not be just capable of minimizing damage against a hazard but should also be stable enough to recover quickly and efficiently from the damage sustained. The concept of 'Resilience' is described as the ability of a community to withstand external shocks to its population and/or infrastructure and to recover from such shocks efficiently and effectively (Timmerman 1981; Pimm 1984). A community in itself is quite complex as it cannot be considered a single entity; instead, it is a collaboration of several essential units which work together to sustain the inhabitants. Each of these units is being studied extensively and some researches have provided a sound foundation for future developments in the direction of community resilience.

There are fundamental studies regarding community resilience (Miles and Chang 2006; Twigg 2009 Cutter et al. 2010 McAllister 2015);

however most of the studies target only a specific part of community resilience. The reasons for studying specific aspects of resilience is the notion that a community “gets to decide” what makes them reslient. For exmple, the current notion is that communtiy leaders or poplultion decide collectively on what is the most imprtnat goal to recover, which could be either infrastrcture, social, or economical, or a combination of such. However, in this study we argue that a complex community can not deterimie or decide on what is most important for them; instead a holistic approach is needed for deteriminging what is critical for a community. A community can be considered analogous to a multi-cellular organism as it also comprises of several sub-units which work in tandem with each other to ensure proper functioning. In this study, we present a novel spatial and temporal model of studying community resilience. We devise a new model that allows resilience to be quatified while integrating all resilience goals together to deterime the overall resilince of the community. The model is essentially a finite element analysis of resileicne (FEAR).

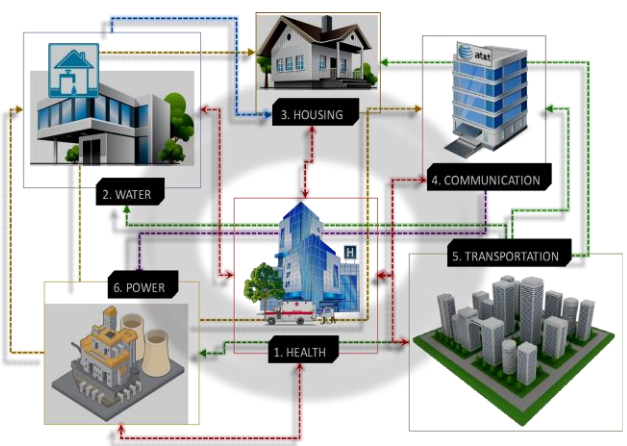

Figure 1: Inter-dependencies of infrastructure lifelines with each other in a community with the direction of arrow determining lifeline being supported

1. FINITE ELEMENT ANALYSIS OF RESILIENCE (FEAR)

In formulating the finite element model, the community of interest should be divided into grids or elements at the resolution required. This is shown in Figure 2 where a map of Gotham city, the infamous city of Batman, is shown alongside the finite element mesh created from the map. The layout of Gotham city is represented by 4 distinct regions - Uptown, Midtown, Downtown and Arkham Asylum, with each region comprising of different social and infrastructure properties. Each region can be considered as an independent ecosystem connected by means of bridges.

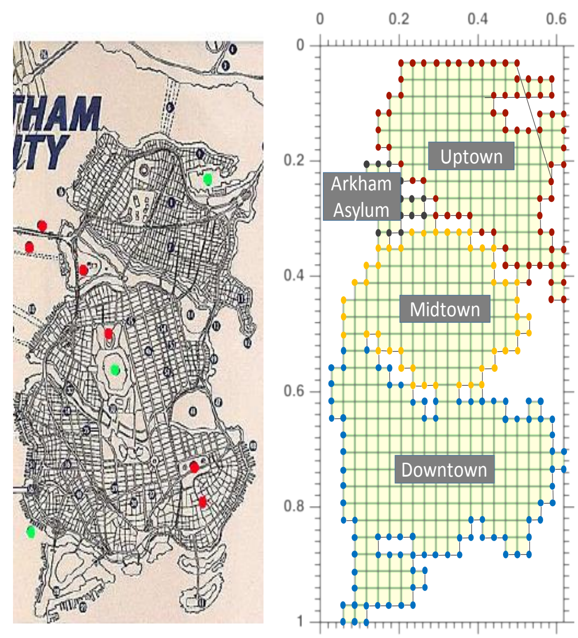

Figure 2: Layout of Gotham City and its corresponding finite element mesh

The FEM framework was developed by first formulating a set of base differential equations describing the variation in behavior of the lifeline systems, both temporally and spatially. Eq. 1 shows the generalized coupled second order differential equation for nth degree of freedom or lifeline system. The concept of this generalized differential equation is derived from the general 2-D wave propagation equation and it resembles the differential equation of a 2-D vibrating plate. The left-hand side term is the Laplacian of the disturbance in the nth lifeline system which varies equally in both x and y-directions in proportion to the effective stability/functioning ($\sum_{l=1}^{N} K_{nl}$) of the respective lifeline system. The effective stability is the integral stability of the system reduced by the sum of the interdependency terms. The right-hand side terms are the force, damping and mass terms, which represents the relative damage, long-term economic investment and a combination of social vulnerability index and short-term economic investment of each lifeline system. Detailed discussion for the specificity of this representation can be found in Mahmoud and Chulahwat (2018).

All performance parameters involved were formulated to be dimensionless. The independent variables of the equations were normalized by the maximum damage incurred to all lifelines (in terms of $). The independent variables x and y were normalized by the maximum distance in x and y directions, and time t was normalized by a reference time measure.

$$-\sum_{l=1}^{N}[K_{nl}(t).\nabla^2 u_l(x,y,t)] = F_n(x,y,t) + \sum_{l=1}^{N}[C_{nl}.\dot{u}_l(x,y,t)] + \sum_{l=1}^{N}[M_{nl}(T).\ddot{u}_l(x,y,t)] \quad (1)$$

The coupled differential equation given by Eq. 1 was used to derive the 'weak form' using the Ritz-Galerkin method for the FE formulation of the resiliency model. The weak form was solved by discretization, using a custom 4-node planar iso-parametric element approximation. The custom 4-node element represents 6 degrees of freedom at each of the 4 boundary nodes. On discretization, the respective stiffness, damping and mass matrices for N lifeline systems were derived by Eq. 2, 3, 4 and the force/disruption matrix was derived by Eq. 5. In these equations, ψ_i^{nl} and ψ_j^{nl} are the i^{th} and j^{th} shape functions, and M_{nl} , C_{nl} , K_{nl} and F_n are the economic vulnerability, economic investment, infrastructure robustness and interdependencies, and Monetary damage values of disruption, respectively, for nth lifeline. The local element matrices are assembled into a global matrix for each parameter to obtain a set of coupled differential equations representing each node. These are solved using the Newmark method (Newmark, 1959), to obtain the normalized nodal disruption curve for each node (X_i/F).

$$M_{ij}^{nl} = \iint\left[M_{nl}\psi_i^n\psi_j^l\right]dxdy \quad (2)$$

$$C_{ij}^{nl} = \iint\left[C_{nl}\psi_i^n\psi_j^l\right]dxdy \quad (3)$$

$$K_{ij}^{nl} = \iint\left[K_{nl}\left[\frac{\partial\psi_i^n}{\partial x}\frac{\partial\psi_j^n}{\partial x} + \frac{\partial\psi_i^l}{\partial y}\frac{\partial\psi_j^l}{\partial y}\right]\right]dxdy \quad (4)$$

$$F_i^n = \iint[F_n\psi_i^n]dxdy \quad (5)$$

The initial displacements/disruptions, required for solving the coupled equations are obtained from Eq. 6, where $[K_{global}]$, $[F_{global}]$ and $[X_I]$ are the global infrastructure matrix, initial disruptions vector for each node, and global damage vector for each node. The initial velocity is assumed to be zero to keep the analysis on the conservative side and the initial acceleration is obtained from the initial disruption using Eq. 7.

$$[X_I] = [K_{global}]^{-1}[F_{global}] \quad (6)$$

$$\left[\frac{d^2X_I}{dt^2}\right] = \frac{1}{[M_{global}]}\left([F_{global}] - [K_{global}][X_I]\right) \quad (7)$$

2. DEFINITION OF RESILIENCE

In a single node representation of community in the proposed model, we combine the Infrastructure, Social and Economic features of a community to evaluate the cumulative recovery/disruption curve (X) where the area under this curve represents the Resilience Index $(RI)_i$ of a specific lifeline. The model is dynamic in nature so the recovery curve is free to stabilize to a new state different than the one it started from. As a result, the area under the recovery curve could be either positive or negative. The sign of the new state does not matter since the recovery curve is only a measure of disturbance in the community. Only its separation from the initial state is of relevance. Hence, resilience is defined as the absolute area under the disruption curve to the point T_∞, which is the time taken for a lifeline to stabilize, as shown in Eq. 11.

$$(RI)_i = \int_0^{T\infty} \left|\frac{X_i(t)}{F_{max}}\right| dt \quad (8)$$

The measure of resilience defined in Eq. 8, is that of a given lifeline for a specific location. Resilience of a specific lifeline for a whole community is given by the volume under the disruption surface (S_i), which is obtained from the disruption vector (X) by linear interpolation. The volume is evaluated for the whole community for each time step from $t = 0$ to $t = T_\infty$ using Eq. 9, where X_l and Y_l are the normalized dimensions of the community in x and y-directions. Furthermore, the cumulative sum of resilience for all lifelines ($(RI)_i$) yields resilience of the entire community ($(RI)_{Total}$) as given by Eq. 10.

$$(RI)_i = \int_0^{T\infty} \int_0^{Y_l} \int_0^{X_l} \left|\frac{X_i(t)}{F_{max}}\right| dxdydt \quad (9)$$

$$(RI)_{Total} = \sum_{i=1}^{N} (RI)_i \quad (10)$$

3. RESULTS

A test was conducted on Gotham city to demonstrate the feasibility of the framework. The parameters in this test were assumed to vary with time to introduce non-linearity in the analysis by using amplification and functionality curves for M and K matrices (note: the entries for the matrices are not shown for space limitation).

The damage pattern for the tests considered was such that a magnitude $F_i(t = 0)/F_{max} = -1$ was considered only in the housing lifeline of the districts of Gotham – Uptown, Midtown and Downtown only. $F_i(t = 0)/F_{max}$ was assumed to be -1 for each element of a lifeline marked in Figure 3, hence a total of -9 for a single lifeline of each district.

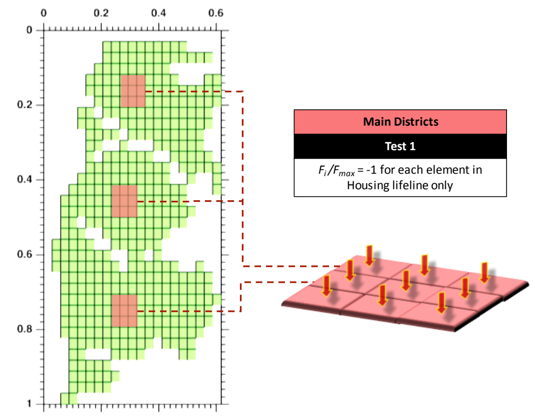

Figure 3: Disruption pattern of Gotham for the three districts

Figure 4 shows the disruption surface of entire Gotham for the housing lifeline and Figure 5 the respective time to stabilization (T_∞) for each node of Gotham. Downtown stabilized the fastest, followed by Midtown and lastly by Uptown. The respective Total Resilience Index ($(RI)_{Total}$) values for the three districts - Uptown, Midtown and Downtown were calculated to be – 1.165, 0.648 and 0.498, respectively.

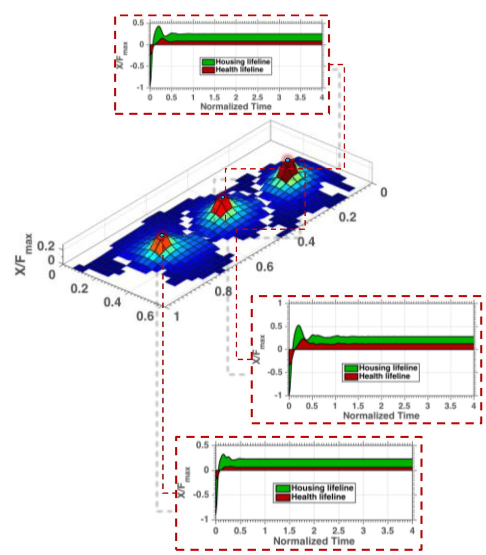

Figure 3: Test 1 results. Disruption surface (S_i) of Housing lifeline of Gotham shown at $t = 4$. Along with individual plots for specific nodes of Downtown, Midtown and Uptown showing variation of X/F_{max} for Housing and Health lifeline with respect to time.

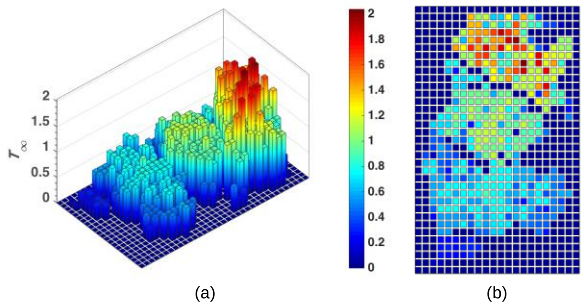

Figure 4: Time to stabilization (T_∞) values obtained in Test 1 for Housing lifeline of each region of Gotham (a) Side-view (b) Top-view

4. CONCLUSIONS

In this paper, we proposed a novel dynamic finite element model for resilience (FEAR) to quantify resilience at the community level both temporally and spatially using traditional mechanics. The model considers the governing systems of a community and their correlations with each other. Unlike, previous resilience models, the dynamic model not only considers Physical Infrastructure Stability but also 2 other key factors of resilience – Social Stability and Economic Investment, as well. The unique aspect of the model is in that it integrates infrastructure, social, and economic fabric of a community to quantify its resilience.

In the scope of this study, the proposed resilience model could not be verified because of lack of data, however the above-mentioned test (among other tests conducted by Mahmoud and Chulahwat (2018) gave a hint of the immense capabilities of the model. Quantification of resilience is quite a complex problem and the current models of resilience lack in their ability of capturing the complete picture. Certain detailed models are also being worked to quantify resilience which consider a plethora of factors, however therein also lies limitations. These models are so intricate in nature that they can only be used by highly trained individuals and the amount of input data required increases the pre-processing time substantially in addition to the processing time required, as a result they cannot be used for emergency purposes. The FEAR model on the other hand is a FE based model, hence it follows the same working principle as an FE software. This makes the proposed model highly user-friendly and in addition, the input data required is not too significant as the model utilizes a presbyopic point of view i.e. it looks at the bigger picture and does not consider minute factors, or rather does not differentiate between them.

5. REFERENCES

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