Updating Probabilistic Model of Traffic Loads on Bridges Using In-Service WIM Data

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ABSTRACT: These days, a Weigh-In-Motion (WIM) system enables us to estimate traffic loads on a bridge based on site-specific traffic environment. However, since the traffic environment of a bridge may change significantly during its service life, it is necessary to monitor the in-service traffic environment and to update the probabilistic model of traffic load. This study aims to develop a methodology to update distribution parameters of random variables in the probabilistic traffic load model by Bayesian inference. Three main methods are used together to establish the updating methodology: conjugate prior distributions, Bayesian linear regression, and Gibbs sampling. The proposed method is demonstrated by numerical examples using WIM data from two sites in South Korea.

1. INTRODUCTION

Traffic loads on bridges generally have large variability and uncertainties because of significant influences by the traffic environment. Therefore, it is essential to consider these uncertainties through a probabilistic framework for accurate estimation of traffic load effects. However, most of the existing design codes and assessment specifications do not consider these site-specific traffic conditions, and thus defined based on conservative assumption and engineer's judgment (Moses 2001).

Recently, to collect information regarding passing vehicles on roads and bridges, e.g. axle weight, axle spacing, passing time, and velocity, a large amount of Weigh-In-Motion (WIM) data have been collected. In structural engineering, many researchers have tried to estimate the traffic load effects on bridges using WIM data for cost-effective design or accurate evaluation of a bridge in operation (O'Brien et al. 2015). To this end, Monte Carlo (MC) simulation based approach has been often adopted. This approach can provide accurate estimation of traffic load effects by extrapolation from results of a long-run simulation (Enright and O'Brien 2013).

Kim and Song (2018) developed a probabilistic model of bridge traffic loads using an MC simulation approach based on statistical investigations of WIM data measured in South Korea and theories of transportation engineering. The developed model includes two major probabilistic models which respectively represent (1) vehicle characteristics, e.g. weight and length of vehicles, and (2) traffic flow characteristics for simulating the pattern of traffic flow. This traffic load model can generate artificial WIM data for an operation period of interest by employing procedure illustrated in Figure 1, based on the aforementioned two major models. Next, traffic load effects are computed using the generated WIM data and influence lines of the target bridge to investigate important characteristics of the traffic load effects through extrapolation process toward the service life of the bridge.

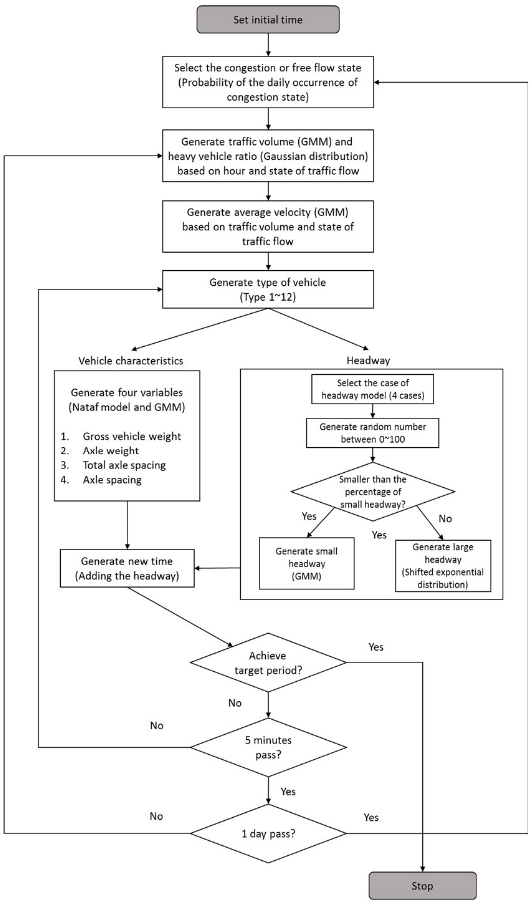

Figure 1: Procedure for generating artificial WIM data based on the traffic load simulation model (Kim and Song 2018)

However, since the traffic environment of a bridge keeps changing due to many causes in surroundings of a bridge, e.g. construction of a factory near the bridge, or industrial development of the city, it is necessary to monitor the change of traffic environment and continuously update the probabilistic model of traffic load so that traffic load effects on the bridge can be estimated more accurately. This requires the development of a methodology that allows the parameters of variables in traffic load model to be continuously updated through a probabilistic framework. In addition, since a WIM system is not always installed or operational for in-service bridges, WIM data may not be available at all, or can only be obtained for a short period of time. Therefore, only brief site-specific conditions such as traffic volume or the ratio of vehicle type could be available, but this limited information may hamper accurate estimation of the traffic load effects. To address this, the traffic load model developed by the authors (Kim and Song, 2018) is used as a generic traffic load model, i.e. a model a priori. If only limited data are available, we update the corresponding parameters of the generic model using the available traffic data while the parameters of the variables that cannot be obtained from the related data are supplemented by the parameters of the generic model. This makes it possible to more accurately estimate the site-specific traffic load effect for a particular bridge.

This study aims to develop a methodology to update the distribution parameters of random variables appearing in the probabilistic traffic load model based on Bayesian inference. In particular, the approach combines prior knowledge with in-service WIM data employing Bayes' theorem (Gelman et al. 2013). In the following section, we briefly introduce the main random variables of the developed traffic load model, and then explain the Bayesian inference methods used for the sub-models: conjugate prior distribution, Bayesian linear regression, and Gibbs sampling. In Section 3, numerical examples are investigated to apply the developed updating methods and test them, e.g. in terms of the influence of hyper-parameters of the prior model. Finally, the conclusion is presented.

2. UPDATING METHODS FOR PROBABILISTIC MODEL OF TRAFFIC LOADS BASED ON BAYESIAN INFERENCE

The variables used in the probabilistic traffic load model (Kim and Song, 2018) can be categorized into three groups in terms of the type of probabilistic model used for fitting. The first one is the Gaussian distribution, which is used to represent the heavy vehicle ratio. The second type is linear regression model which is used to fit the percentage of small headway for a given traffic volume. The last one is the Gaussian mixture model which is used to describe vehicle characteristics, traffic volume and, average velocity. We have developed the updating methodology for the traffic load model by using appropriate Bayesian inference methods for each of the three models.

2.1. Gaussian distribution: conjugate prior

In Bayesian inference, the parameters of the probability distribution are treated as random variables rather than fixed values. The prior distribution of parameters θ is characterized by hyper-parameters α, i.e.

$$\theta \sim p(\theta|\alpha) \tag{1}$$

The main goal of Bayesian inference is to obtain the posterior probability distribution for the parameter by updating the prior distribution based on the Bayes rule with newly available observations. This posterior probability distribution can be calculated as

$$p(\theta|\boldsymbol{X},\alpha) = \frac{p(\boldsymbol{X}|\theta)p(\theta|\alpha)}{p(\boldsymbol{X}|\alpha)} = \frac{p(\boldsymbol{X}|\theta)p(\theta|\alpha)}{\int p(\boldsymbol{X}|\theta)\mathrm{p}(\theta|\alpha)\mathrm{d}\theta} \propto p(\boldsymbol{X}|\theta)p(\theta|\alpha) \tag{2}$$

where $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\}$ is a set of n observations and $p(\boldsymbol{X}|\theta)$ is the conditional probability density of $\boldsymbol{X}$ given θ called as likelihood function. However, since it is generally difficult to calculate the integral of the denominator in Eq. (2), a conjugate prior distribution, which makes the posterior distribution a closed-form, is preferred if applicable (Raiffa and Schlaifer 1974).

Among the random variables in the probabilistic traffic load model, the heavy vehicle ratio (x) is fitted with a Gaussian distribution with mean μ and variance σ^2, i.e.

$$x|\mu,\sigma^2 \sim N(\mu,\sigma^2) \tag{3}$$

When both parameters (μ,σ^2) are considered as random variables, the normal (N) and inverse-gamma (Γ^{-1}) distribution form the conjugate prior distributions consisting of four hyper-parameters, i.e.

$$\mu|\sigma^2 \sim N\left(\mu_0, \frac{\sigma^2}{\lambda}\right),\ \sigma^2 \sim \Gamma^{-1}(\alpha,\beta) \tag{4}$$

where μ_0 and λ are the hyper-parameters of the prior distribution of the mean parameter μ, and α, β are the hyper-parameters of the prior distribution of the variance parameter σ^2. The joint probability density function (PDF) of the normal-inverse-gamma distribution is expressed as

$$f(\mu,\sigma^2|\mu_0,\lambda,\alpha,\beta) = \frac{\beta^\alpha\sqrt{\lambda}}{\Gamma(\alpha)\sqrt{2\pi\sigma^2}}\left(\frac{1}{\sigma^2}\right)^{\alpha+1} exp\left(-\frac{2\beta+\lambda(x-\mu)^2}{2\sigma^2}\right) \tag{5}$$

When we obtain a set of n observations $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\}$, $p(\boldsymbol{X}|\theta)$ can be calculated by

$$\begin{aligned} p(\boldsymbol{X}|\theta) &= \textstyle\prod_{i=1}^{n} p(x_i|\,\mu,\sigma^2) \\ &\propto \textstyle\prod_{i=1}^{n} \frac{1}{\sigma}\exp\left[-\frac{1}{2\sigma^2}(x_i-\mu)^2\right] \\ &\propto \frac{1}{\sigma^n}\exp\left[-\frac{1}{2\sigma^2}\textstyle\sum_{i=1}^{n}(x_i-\mu)^2\right] \end{aligned} \tag{6}$$

Using the normal-inverse-gamma prior distribution in Eq. (5) and the likelihood function in Eq. (6), the posterior probability distribution in Eq. (2) is represented by the same form of distribution, i.e. normal-inverse-gamma distribution, whose hyper-parameters are updated as follows (Bernardo and Smith 2001):

$$\begin{gathered} \mu_0 \to \frac{\lambda\mu_0 + n\bar{x}}{n+\lambda}, \\ \lambda \to \lambda + n \\ \alpha \to \alpha + \frac{n}{2} \\ \beta \to \beta + \frac{1}{2}\left(ns + \frac{\lambda n(\bar{x}-\mu_0)^2}{n+\lambda}\right) \end{gathered} \tag{7}$$

where $\bar{x}$ and s are the sample mean and variance. Thus, the posterior probability distribution of the parameters μ,σ^2 can be obtained simply by updating the parameters of the conjugate prior distribution. When desired, during the Bayesian inference, one can determine the relative importance of information embedded in the prior information and the currently measured data by adjusting the values of the hyper-parameters of the prior distribution model.

2.2. Linear regression model: Bayesian linear regression

In the probabilistic model describing traffic flow characteristics, the percentage of small headway is used as an important criterion regarding whether generating small or large headway when simulating the headway between two adjacent vehicles in MC simulations. Kim and Song (2018) developed the linear regression model of the percentage on the traffic volume for each of the traffic volume ranges as shown in Figure 2.

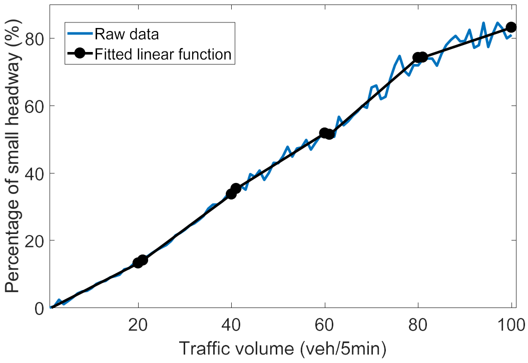

Figure 2: Predicting percentage of small headway for a given traffic volume (Kim and Song 2018)

The linear regression model developed for each group of traffic volume can be expressed as

$$y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_p x_{ip} = \boldsymbol{x}_i^{\mathrm{T}} \boldsymbol{\beta} + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma^2) \tag{8}$$

where y_i, $i = 1, \ldots, n$ is the response variable, $\boldsymbol{x}_i$ is $(p+1)\times 1$ predictor vector, $\boldsymbol{\beta}$ is $(p+1)\times 1$ parameter vector, and ϵ_i is independent and identically normally distributed random variable representing the model error. Bayesian linear regression is an appropriate method for linear regression within the context of Bayesian inference so this study employs this method for updating the linear regression model in the traffic load model.

As shown above, the posterior probability distribution for the parameter of linear regression is calculated using the conjugate prior distribution to facilitate calculation of posterior distribution. Since the model error variable follows the Gaussian distribution, a normal-inverse-gamma distribution is selected as a conjugate prior distribution which is defined as follows. In this case, by contrast, the prediction vector consists of multiple random variables so multivariate Gaussian distribution is used as a conjugate prior distribution, i.e.

$$\boldsymbol{\beta} | \boldsymbol{\Sigma}, \boldsymbol{\lambda} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}\boldsymbol{\lambda}^{-1}),$$
$$\boldsymbol{\mu} = \begin{pmatrix} \mu_0 \\ \vdots \\ \mu_p \end{pmatrix}, \quad \boldsymbol{\Sigma}\boldsymbol{\lambda}^{-1} = \begin{pmatrix} \sigma^2/\lambda & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma^2/\lambda \end{pmatrix},$$
$$\sigma^2 \sim \Gamma^{-1}(\alpha, \gamma) \tag{9}$$

where $N(\boldsymbol{\mu}, \boldsymbol{\Sigma}\boldsymbol{\lambda}^{-1})$ is a multivariate Gaussian distribution of $(p+1)$ dimensional random vector $\boldsymbol{\beta}$ with the mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}\boldsymbol{\lambda}^{-1}$ and β_i and β_j $(i \neq j)$ are assumed to be uncorrelated.

When n samples $(\boldsymbol{y}, \boldsymbol{X})$ consisting of $n\times 1$ vector $\boldsymbol{y}$ and $n\times(p+1)$ vector $\boldsymbol{X}$ are observed, the posterior probability distribution is obtained as follows using Bayes rule. More details about the derivation can be found in Seber and Lee (2012).

$$\boldsymbol{\beta} | \boldsymbol{\Sigma}, \boldsymbol{\lambda} \sim N\left((\boldsymbol{\lambda} + \boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X})^{-\mathbf{1}}\left(\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X}\widehat{\boldsymbol{\beta}} + \boldsymbol{\lambda}\boldsymbol{\mu}\right), \boldsymbol{\Sigma}(\boldsymbol{\lambda} + \boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X})^{-\mathbf{1}}\right) \tag{10}$$

$$\sigma^2 \sim \Gamma^{-1}\left(\alpha + \frac{n+p+1}{2}, \left[\gamma^{-1} + \frac{1}{2}(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta})^{\boldsymbol{T}}(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}) + \frac{1}{2}(\boldsymbol{\beta} - \boldsymbol{\mu})^{\boldsymbol{T}}\boldsymbol{\lambda}(\boldsymbol{\beta} - \boldsymbol{\mu})\right]^{-1}\right) \tag{11}$$

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X})^{-\mathbf{1}}\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{y} \tag{12}$$

$\widehat{\boldsymbol{\beta}}$ is the estimate of $\boldsymbol{\beta}$ by ordinary least squares (OLS), which is one of the methods to minimize the sum of squared residuals and obtained by Eq. (12). In this study, the Bayesian linear regression method described in this section is used to update the linear regression models for percentage of small headway using the mean values of the posterior distributions of the parameters.

2.3. Gaussian mixture model: Gibbs sampling

A Gaussian mixture model (GMM) is a probabilistic model consisting of multiple Gaussian distributions, and is suitable for representing random variables with multiple peaks or modes. Therefore, Kim and Song (2018) modeled various random variables showing multiple peaks, i.e. axial weight, axle spacing, traffic volume, average velocity of the developed traffic load model using GMMs. The PDF of GMM of each of the random variables is described as

$$f(x|\boldsymbol{\theta}) = \sum_{j=1}^{J} \pi_i f_j\left(x \middle| \mu_j, \sigma_j^2\right) \tag{13}$$

where $\boldsymbol{\theta} = \{\boldsymbol{\pi} = (\pi_1, \dots, \pi_J), (\mu_1, \sigma_1^2), \dots, (\mu_J, \sigma_J^2)\}$ denotes all parameters of GMM, π_j is the relative weight of the j-th Gaussian component, satisfying $0 < \pi_j < 1$ and $\sum_{j=1}^{J} \pi_j = 1$, and $f_j\left(x \middle| \mu_j, \sigma_j^2\right)$ is the PDF of the j-th Gaussian distribution whose parameters are μ_j, σ_j^2.

To facilitate parameter estimation of GMM models, a classification vector $Z = (z_1, \dots, z_n)$ consisting of latent variables z_i is introduced. This is because the GMM models have weight parameters unlike other probability distributions. The latent variable $z_i = j$ means that the i-th observation is classified into the j-th component (Bilmes 1998).

In the Bayesian inference of the GMM, a conjugate prior distribution appropriate for the likelihood function of each parameter is desired. In this study, a normal-inverse-gamma distribution shown in Eq. (3) and (4) is used as conjugate prior distribution of the parameters of each Gaussian distribution. The weight $\boldsymbol{\pi} = (\pi_1, \dots, \pi_J)$ is modeled by a Dirichlet prior distribution because the likelihood function of $\boldsymbol{\pi}$ is a multinomial distribution which can represent the probability of the number of samples belonging to each component when they are observed (Gauvain and Lee 1994). A Dirichlet distribution can be expressed as follows:

$$\boldsymbol{\pi} = (\pi_1, \dots, \pi_J) \sim D(\alpha_1, \dots, \alpha_J),$$
$$f(\boldsymbol{\pi}|\boldsymbol{\alpha}) = \frac{\Gamma(\alpha_1 + \cdots + \alpha_J)}{\Gamma(\alpha_1) \cdots \Gamma(\alpha_J)} {\pi_1}^{\alpha_1 - 1} \cdots {\pi_J}^{\alpha_J - 1} \tag{14}$$

where α_j is the hyper-parameter of Dirichlet distribution.

When n samples $\boldsymbol{y}$ are observed, the above conjugate prior distributions and the likelihood function are used to calculate posterior distributions of parameters from Eq. (2). First, the posterior distribution of the Gaussian distribution for j-th component is calculated as follows to have a form which is similar to Eq. (7):

$$\mu_0 \rightarrow \frac{\lambda \mu_0 + n_j \overline{x}_j}{n_j + \lambda},$$
$$\lambda \rightarrow \lambda + n_j$$
$$\alpha \rightarrow \alpha + \frac{n_j}{2}$$
$$\beta \rightarrow \beta + \frac{1}{2}\left(n_j s_j + \frac{\lambda n_j (\overline{x}_j - \mu_0)^2}{n_j + \lambda}\right) \tag{15}$$

The difference from Eq. (7) is that the number of samples classified into j-th component, n_j and the sample mean $\overline{x}_j$, variance s_j are used instead.

Second, the posterior distribution of Dirichlet distribution $\boldsymbol{\pi}$ can be defined as

$$\boldsymbol{\pi}|\boldsymbol{y} \sim D\Big(\alpha_1 + \sum_{i=1}^{n} I(z_i = 1), \dots, \alpha_J + \sum_{i=1}^{n} I(z_i = J)\Big) \tag{16}$$

where $\sum_{i=1}^{n} I(z_i = j)$ represents the process to count the number of samples belonging to the j-th component. Also, one needs to obtain the classification vector $Z = (z_1, \dots, z_n)$ that describes which component the corresponding sample y_i belong to. To obtain the classification vector Z, the posterior probability t_{ij}, i.e. the probability that the i-th sample belongs to the j-th component is calculated as

$$t_{ij} = p(z_i = j|y_i) = \frac{\pi_j f_j\left(y_i \middle| \mu_j, \sigma_j^2\right)}{\sum_{j=1}^{J} \pi_j f_j\left(y_i \middle| \mu_j, \sigma_j^2\right)} \tag{17}$$

Unlike the above two cases (Gaussian distribution, linear regression), GMM introduces latent variable z_i for the parameter estimation so Gibbs sampling is used to Bayesian inference in this study. Gibbs sampling is one of the Markov Chain Monte Carlo simulation (MCMC) methods, which is useful particularly when it is difficult to extract a sample directly from the joint distribution of parameters while it is easy to extract a sample from the conditional probability distribution of the parameters of interest given the values of other parameters (Gelfand 2000). The samples of parameters sequentially extracted one by one are used as conditional values when sampling other parameters in the next step. We set the initial values and then generate the samples of parameters from the conditional posterior distribution through the following four steps (Franzén 2008):

1) Generate the sample for variance σ_j^2, j=1,…,J of each component from the posterior inverse gamma distribution, given $\boldsymbol{y}$ and $Z = (z_1, \dots, z_n)$
2) Generate the sample mean μ_j, j=1,…,J of each component from the posterior Gaussian distribution, given $\boldsymbol{y}$, $Z = (z_1, \dots, z_n)$ and new sample of variance σ_j^2 from Step 1
3) Generate the sample for the weight of components $\boldsymbol{\pi} = (\pi_1, \dots, \pi_J)$ from the posterior Dirichlet distribution given $\boldsymbol{y}$ and $Z = (z_1, \dots, z_n)$
4) Calculate the new classification vector $Z = (z_1, \dots, z_n)$ based on the new posterior probability t_{ij} which is calculated using $\boldsymbol{y}$ and new samples of parameters $\pi = (\pi_1, \dots, \pi_J), (\mu_1, \sigma_1^2), \dots, (\mu_J, \sigma_J^2)$

These four steps are repeated until the samples of the parameter can well represent that of the posterior distribution.

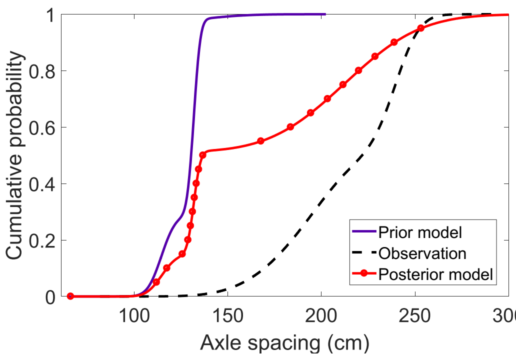

Figure 3: The CDF of prior (original), observation and posterior (updated) probability distribution

The initial steps of sampling process which do not show convergence to the posterior distribution are called burn-in period (Gelman et al. 2013). The corresponding samples are discarded before the sample-based Bayesian inference. In this study, the GMM of second axle spacing in Type 5 is updated by artificially simulated observations. Figure 3 shows the cumulative distribution function (CDF) of the prior (original) GMM, posterior (updated) GMM and the cumulative frequency function of the observations. Figure 4 shows the histories of the estimated weights of the three components. It is shown that the samples of the parameter converge to the posterior distribution after around 20 iterations.

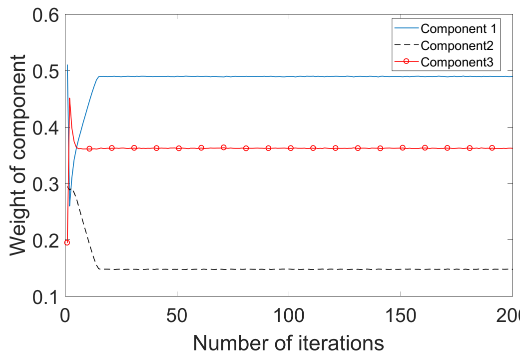

Figure 4: Convergence histories of the estimated weights of three components in GMM

3. VERIFICATION OF UPDATING METHODS

To verify the Bayesian inference-based updating methodology introduced in Section 2, the probabilistic model of traffic load (Kim and Song 2018) is updated by actual WIM data. After each updating, a total of 10 sets of artificial WIM datasets are generated for one-year period from the updated model, and traffic load effects were obtained using the influence line loading. The daily maximum load effect was selected and mean values of 10 sets are plotted in the Gumbel probability paper to examine whether the traffic load effects from updated models converge to the traffic load effects from observations as the number of updatings increases.

The probabilistic model constructed from the WIM data of the primary lane of Gimcheon (Kim and Song 2018) is assumed to be a prior model (original) and the measured WIM data from the primary lane of Sunsan is assumed to be the new observations. For the updating, the hyper-parameters of the prior model are set to the value which has the same weight as that of observations. In this paper, this is expressed as the relative weight of prior model being 1 and this weight is an indicator of how much we believe the prior information compared to the new observations. In addition, the *total load* on a bridge is used to assure general validity regardless of the bridge's span length. The results are shown in Figures 5(a) and (b) for two span lengths: 100 m (short span) and 800 m (long span).

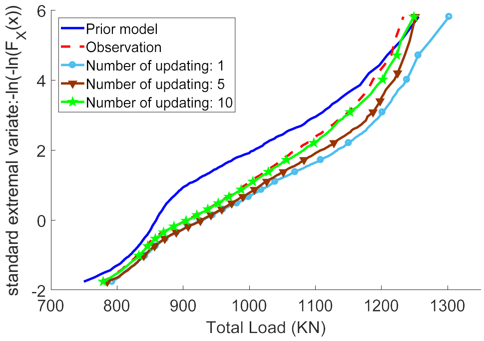

Figure 5(a): Gumbel probability paper of daily maximum total load for 100m span (the weight of prior model: 1)

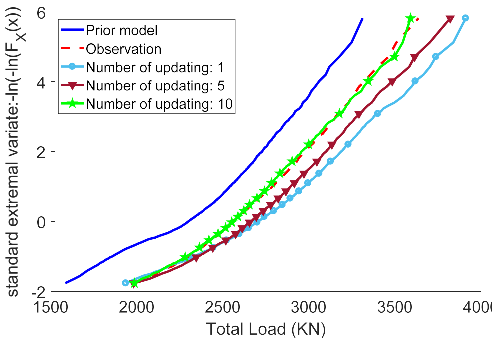

Figure 5(b): Gumbel probability paper of daily maximum total load for 800m span (the weight of prior model: 1)

As the number of updatings increases, it is shown that the CDF of the total load of updated models converge to that by the observations, and it is seen that convergence is achieved after the updating is performed about 10 times.

In addition, to confirm the effect of the hyper-parameter of the prior model, the updating of a traffic load model is performed the same way except that we changed the value of hyper-parameters corresponding the relative weight of prior model to 2 and 0.5, respectively.

Figures 6 and 7 show the result of updating for the two different weights. It is seen that the required number of updating for convergence increases as the weight of the prior model increases, which is a reasonable result considering the characteristics of Bayesian updating. Through these numerical examples, it is verified that the proposed updating methods can effectively update the probabilistic model for traffic load based on Bayesian inference.

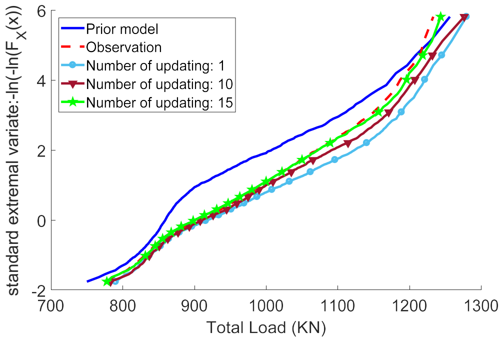

Figure 6(a): Gumbel probability paper of daily maximum total load for 100m span (the weight of prior model: 2)

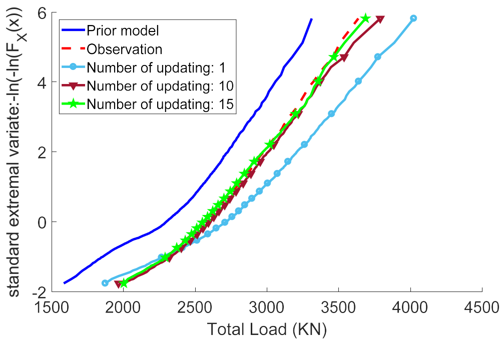

Figure 6(b): Gumbel probability paper of daily maximum total load for 800m span (the weight of prior model: 2)

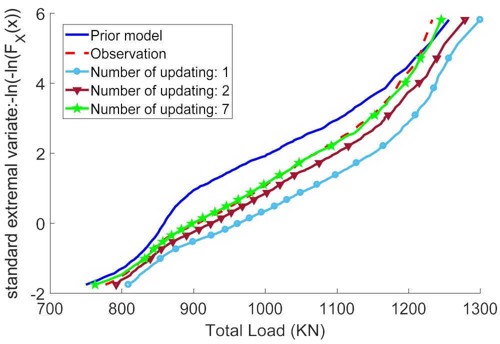

Figure 7(a): Gumbel probability paper of daily maximum total load for 100m span (the weight of prior model: 0.5)

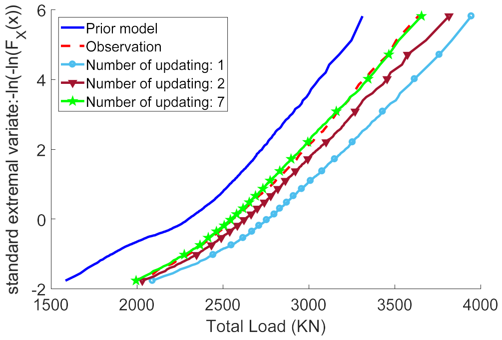

Figure 7(b): Gumbel probability paper of daily maximum total load for 800m span (the weight of prior model: 0.5)

4. CONCLUSIONS

In this study, we proposed a methodology to update the probabilistic model recently developed for traffic load (Kim and Song 2018) based on Bayesian inference. The three main methods used to establish the Bayesian updating method are conjugate prior distribution, Bayesian linear regression, and Gibbs sampling. Numerical examples of employing the developed updating method are provided to verify that the traffic load effects from prior model converge to those from new observations as the number of updating increases. Additionally, it was confirmed that the convergence speed slows down by adjusting the hyper-parameter value of the prior model to increase the relative weight of the prior distribution. Through these examples, the developed Bayesian updating methodology was successfully demonstrated. Through further developments and applications, the effects of Bayesian updating based on actual in-service WIM data and other site-specific traffic conditions on traffic loads and the corresponding reliability of bridges will be thoroughly investigated.

ACKNOWLEDGMENT

The authors would like to gratefully acknowledge the support by the research project, “Development of Life-cycle Engineering Techniques and Construction Methods for Global Competitiveness Upgrades of Cable Bridges” of the Ministry of Land, Infrastructure and Transport (MOLIT) of the Korean Government (Grant No. 19SCIP-B119960-04).

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