

# Robust design and optimization of stochastic wind-excited systems: an adaptive kriging-based approach

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**ABSTRACT:** This research proposes a robust design framework for wind-excited systems in which performance is estimated at a system level in terms of state-of-the-art performance-based design metrics. In particular, the robust design problem is formulated as a stochastic optimization with objective the minimization of the variance of the performance metric. Constraints are also imposed on the initial cost of the system and expected value of the performance metric. To effectively treat the performance metrics within the optimization problem, adaptive kriging models of the deaggregated loss metrics are defined in terms of the second order statistics of the demands. By then relating the demand statistics to the design variables through the concept of the Auxiliary Variable Vector, a deterministic optimization sub-problem is defined that can handle high-dimensional design variable vectors and general stochastic excitation. By solving a sequence of sub-problems, each formulated in the solution of the previous, solutions to the original robust design problem are found. A case study consisting in a large-scale system subject to stochastic wind excitation is used to illustrate the applicability of the proposed framework.

## 1. INTRODUCTION

Recent trends in wind engineering are leading to the acceptance of performance-based design (PBD) as a means for obtaining structures that meet the safety requirements demanded by society (e.g. Bernardini et al., 2015; Chuang and Spence, 2017). To achieve this, the inevitable uncertainties affecting both the system parameters and the external wind excitations must be fully modeled. This has led to the development of a number of probabilistic wind PBD frameworks that describe performance in terms of probabilistic metrics such as expected repair cost and time (e.g. Chuang and Spence, 2017). The greater complexity, as compared to traditional design approaches, that these frameworks are introducing has created a need for efficient optimization methods that can inform decision-makers of the optimal trade-offs between cost and design ro-

bustness. Within this context, this paper introduces a robust design framework in which performance is estimated at a system level in terms of state-of-the-art wind PBD metrics. In particular, the robust design problem is formulated as a stochastic optimization with objective the minimization of the variance of the performance metric. Constraints are also imposed on the initial cost of the system and expected value of the performance metric. To effectively treat the performance metrics within the optimization problem, adaptive kriging models of the deaggregated loss metrics are defined in terms of the second order statistics of the demands. By then relating the demand statistics to the design variables through the recently introduced concept of the Auxiliary Variable Vector, a deterministic optimization sub-problem is defined that can handle

high-dimensional design variable vectors and general stochastic excitation. By solving a sequence of sub-problems, each formulated in the solution of the previous, solutions to the original robust design problem are found.

## 2. PROBLEM SETTING

In designing wind-sensitive systems, one of performance metrics that is of interest to stakeholders and society is the anticipated economic loss given that a severe windstorm has occurred. Hence, the performance metric considered in this work is defined as the system-level repair cost  $L$ . For robust wind-excited systems, it is of interest here to identify systems that have minimal variability in this performance metric, while still ensuring acceptable initial cost and expected system loss. This robust design optimization problem may be formulated as:

$$\begin{aligned} \text{Find } \quad & \mathbf{x} = \{x_1, \dots, x_m\}^T \\ \text{to minimize } \quad & \sigma_{L|im}^2(\mathbf{x}) \\ \text{subject to } \quad & V(\mathbf{x}) \leq V_0 \\ & \mu_{L|im}(\mathbf{x}) \leq \mu_0 \\ & x_i \in \mathbb{X}_i \quad i = 1, \dots, m \end{aligned} \quad (1)$$

where  $\mathbf{x}$  is a design variable vector containing the deterministic parameters used to define the structural system (e.g., beam and column dimensions);  $\sigma_{L|im}^2$  is the optimization objective function defined in terms of the conditional variance of the loss measure  $L$  given an extreme windstorm of intensity  $im$  (e.g., variance of the system repair cost);  $V$  represents a function associated with the initial cost of the structural system (e.g., material volume of the structural system);  $\mu_{L|im}$  is the conditional expected value of the loss measure (e.g., expected total repair cost);  $V_0$  and  $\mu_0$  are the threshold values that  $V$  and  $\mu_L$  shall not exceed; and  $\mathbb{X}_i$  is the interval to which the  $i$ th component of the design variable vector must belong.

To account for various sources of uncertainty (e.g. uncertainties in structural properties, random nature of stochastic wind loads, and epistemic uncertainties in the analysis models), the variance and the expected value of the system loss measure can be determined through the following probabilistic

integrals based on recently introduced wind PBD frameworks (Chuang and Spence, 2017)):

$$\sigma_{L|im}^2 = \iiint (l - \mu_L)^2 \cdot p(l|dm) \cdot p(dm|edp) \cdot p(edp|im) \cdot dl \cdot ddm \cdot dedp \quad (2)$$

$$\mu_{L|im} = \iiint l \cdot p(l|dm) \cdot p(dm|edp) \cdot p(edp|im) \cdot dl \cdot ddm \cdot dedp \quad (3)$$

where  $DM$  is the damage measure identifying the extent of component damage (e.g., window cracking);  $EDP$  is the engineering demand parameter responsible for initializing the damage (e.g., inter-story drifts);  $IM$  is the intensity measure of the severe wind event (e.g., wind speed with a return period of 1700 years); and  $p(a|b)$  denotes the conditional probability of  $A$  assuming the value  $a$  given  $b$ . This paper adopts the standard notation that uppercase letters denote random variables while lowercase letters represent realizations.

Due to the large number of uncertain parameters necessary for describing the problem of interest (i.e. in the order of thousands), the integrals of Eqs. (2) and (3) have high dimensions. Therefore, in this work,  $\sigma_{L|im}^2$  and  $\mu_{L|im}$  are estimated through methods based on stochastic simulation.

## 3. SIMULATION-BASED LOSS ASSESSMENT

To estimate  $\sigma_{L|im}^2$  and  $\mu_{L|im}$  through simulation methods, realizations of the system-level loss measure,  $L$ , are needed. For a particular design  $\mathbf{x}$  subject to a wind hazard of given intensity  $im$ , anticipated system loss can be obtained through the following four logical analysis steps: (1) wind load estimation; (2) structural demand estimation; (3) damage estimation; and (4) loss estimation.

### 3.1. Wind Load Estimation

In this work, the intensity measure of the wind event is defined in terms of a reference wind speed with a mean recurrence interval (MRI) of  $w$  years,  $\bar{v}_w$ . The reference wind speed is generally obtained from data collected at meteorological stations. To obtain the wind speeds at the site of interest from this data, this work adopts the probabilistic transformation scheme proposed in Minciarelli et al. (2001) for transforming the wind speed

data of nearby meteorological stations to the site-specific wind speeds,  $\bar{v}_H$ .

To generate a realization of the vector-valued stochastic wind process  $\mathbf{f}(t)$  while accounting for complex aerodynamic phenomena such as vortex-induced vibration, a wind tunnel-informed proper orthogonal decomposition (POD)-based model (e.g. Chen and Kareem, 2005) is adopted in this work. Within this context, the  $k$ th component of  $\mathbf{f}(t)$  may be estimated as:

$$f_k(t; \bar{v}_H, \boldsymbol{\varepsilon}) = \sum_{l=1}^{N_l} \sum_{b=1}^{N_b-1} \left\{ |\Phi_{kl}(\omega_b)| \sqrt{2\Lambda_l(\omega_b; \bar{v}_H)\Delta\omega} \cdot \cos(\omega_b t + \vartheta_{kl}(\omega_b) + \varepsilon_{bl}) \right\} \quad (4)$$

where  $\bar{v}_H$  is the site-specific wind speed at the building top;  $\boldsymbol{\varepsilon}$  is a vector containing the random variables used in the stochastic wind model;  $N_l$  is the number of loading modes considered;  $\Delta\omega$  denotes the frequency increment (hence, the Nyquist frequency is  $N_b\Delta\omega/2$ , with  $N_b$  the total number of discrete frequencies considered), while  $\omega_b = b\Delta\omega$ ;  $\varepsilon_{bl}$  is an independent random variable that characterizes the stochastic nature of the wind and is distributed uniformly over  $[0, 2\pi]$ ;  $\vartheta_{kl} = \tan^{-1}(\mathbf{Im}(\Phi_{kl})/\mathbf{Re}(\Phi_{kl}))$ ; while  $\Phi_{kl}(\omega)$  and  $\Lambda_l(\omega)$  are components of  $\Phi(\omega)$  and  $\Lambda(\omega)$  obtained from solving the following eigenvalue problem:

$$[\mathbf{S}_f(\omega; \bar{v}_H) - \Lambda(\omega; \bar{v}_H)\mathbf{I}]\Phi(\omega) = 0 \quad (5)$$

where  $\mathbf{S}_f$  is the cross power spectral density matrix of the full-scale loading processes obtained from classic wind tunnel testing. Once  $\Lambda$  and  $\Psi$  are estimated at wind tunnel speed, they can be rapidly scaled to any wind speed,  $\bar{v}_H$ , of interest.

### 3.2. Structural Demand Estimation

To estimate the vector-valued response process  $\mathbf{d}(t)$  of the system due to the stochastic excitation  $\mathbf{f}(t)$ , the load-effect model outlined in (Spence and Kareem, 2014) is adopted in this work. Hence, the  $k$ th component of  $\mathbf{d}(t)$  is given by:

$$d_k(t; \mathbf{v}) = v_1 \Gamma_{d_k}^T [\mathbf{f}(t) + \mathbf{K}\Psi_n^T \mathbf{q}_{R_n}(t; \mathbf{v})] \quad (6)$$

where  $v_1$  is a random variable taking into account the epistemic uncertainty in using the load-effect model of Eq. (6) and is a component of the random vector  $\mathbf{v}$  that collects all uncertain parameters in the system (e.g. uncertainty in the damping ratios and natural frequencies);  $\Gamma_{d_k}$  is a vector of influence functions defined as the the response in  $d_k$  due to a unit load acting at each degree of freedom of the system;  $\mathbf{K}$  is the stiffness matrix of the system;  $\Psi_n = [\boldsymbol{\psi}_1, \dots, \boldsymbol{\psi}_n]$  is the mode shape matrix of order  $n$ ; while  $\mathbf{q}_{R_n}(t) = \{q_{R_1}(t), \dots, q_{R_n}(t)\}^T$  is the vector collecting the resonant modal responses.

Having obtained the wind-induced response process in  $T$ , a realization of the corresponding engineering demand parameter can be estimated as the absolute maximum response in  $[0, T]$  as follows:

$$edp_k(\mathbf{v}) = \max_{t \in [0, T]} |d_k(t; \mathbf{v})| \quad (7)$$

### 3.3. Damage Estimation

Each engineering demand parameter  $edp_k$  can cause damage initialization for components susceptible to  $edp_k$ . For efficiency, components having similar properties and subject to the same  $edp_k$  can be grouped together in a performance group (PG). Consider a component having  $m$  possible damage states, a damage measure associated with the  $c$ th component in the  $k$ th PG can be defined as:

$$dm_k^{(c)}(\theta) = \begin{cases} 0 & \text{if } \text{Fr}_k(1|edp_k) < \theta_k^{(c)} \\ 1 & \text{if } \text{Fr}_k(2|edp_k) < \theta_k^{(c)} \leq \text{Fr}_k(1|edp_k) \\ \dots & \\ m & \text{if } \theta_k^{(c)} \leq \text{Fr}_k(m|edp_k) \end{cases} \quad (8)$$

where  $\text{Fr}_k(m|edp_k) = p(dm_k = m|edp_k)$  denotes a fragility function defined as the conditional probability that the  $c$ th component assumes the damage state  $m$  given the demand level  $edp_k$ ; while  $\theta_k^{(c)}$  is a realization of a random variable, uniformly distributed between 0 and 1. It is assumed here that the damage states follow a sequential logic and are uncorrelated between different components.

### 3.4. Loss Estimation

In order to restore functionality to the system, all damaged components must be repaired. Hence, the

total wind-induced loss may be calculated as the sum of all component losses. In repairing each component, the uncertain repair cost can be modeled as the probability of repair cost given the damage state  $j$ ,  $F_{C_{k|j}}$ . A realization of  $L$  can be estimated through the following summation over all  $N_c$  components and all  $N_g$  performance groups:

$$l(\boldsymbol{\kappa}) = \sum_{k=1}^{N_g} \sum_{c=1}^{N_c} F_{C_{k|j}}^{-1}(\boldsymbol{\kappa}_k^{(c)} | dm_k^{(c)} = j) = \sum_{k=1}^{N_g} g_k(\boldsymbol{\kappa}_k) \quad (9)$$

where  $F_{C_{k|j}}^{-1}$  is the inverse cumulative probability density function of the component-level loss given the damage state  $j$ ;  $\boldsymbol{\kappa}_k^{(c)}$  is a realization of a uniformly distributed random variable between 0 and 1; and  $g_k$  is the loss associated with the  $k$ th PG.

In determining  $\sigma_{L|im}^2$  and  $\mu_{L|im}$ , a Monte Carlo simulation can be used. Within this context,  $\sigma_{L|im}^2$  may be estimated as:

$$\sigma_{L|im}^2 \approx \frac{1}{N_s - 1} \sum_{i=1}^{N_s} \left[ \sum_{k=1}^{N_g} g_k^{(i)}(\mathbf{u}^{(i)}, \bar{v}_w) - \mu_{L|im} \right]^2 \quad (10)$$

where:  $N_s$  is the number of samples;  $\mathbf{u}^{(i)} = \{\boldsymbol{\varepsilon}^T, \mathbf{v}^T, \boldsymbol{\theta}^T, \boldsymbol{\kappa}^T\}^T$  collects the  $i$ th realization of all uncertain variables; while  $\mu_{L|im}$  is given by:

$$\mu_{L|im} \approx \frac{1}{N_s} \sum_{i=1}^{N_s} \left[ \sum_{k=1}^{N_g} g_k^{(i)}(\mathbf{u}^{(i)}, \bar{v}_w) \right] \quad (11)$$

**4. KRIGING-BASED OPTIMIZATION STRATEGY**  
To efficiently solve the robust optimization problem of Eq. (1), this section introduces a sequential optimization strategy that is based on approximately decoupling the probabilistic analysis from the optimization process. The proposed decoupling strategy involves the following three steps: (1) approximation of the second order demand statistics; (2) system loss approximation; and (3) optimization sub-problem formulation.

#### 4.1. Approximation of the Demand Statistics

Once the Monte Carlo simulation of Sec. 3 has been carried out in  $\mathbf{x}_{MC}$ , the second order statistics of the engineering demand parameters can be

efficiently and effectively approximated for moves of  $\mathbf{x}$  away from  $\mathbf{x}_{MC}$  through the Auxiliary Variable Vector (AVV) approach (Spence et al., 2016; Bobby et al., 2016). Within this context, the second order demand statistics are approximated as:

$$\mu_{EDP_k}(\mathbf{x}) \approx \Gamma_{d_k}^T(\mathbf{x}) \hat{\Upsilon}_k(\mathbf{x}_{MC}) \quad (12)$$

$$\sigma_{EDP_k}(\mathbf{x}) \approx \Gamma_{d_k}^T(\mathbf{x}) \check{\Upsilon}_k(\mathbf{x}_{MC}) \quad (13)$$

where  $\hat{\Upsilon}$  and  $\check{\Upsilon}$  are the AVVs corresponding to  $\mu_{EDP_k}$  and  $\sigma_{EDP_k}$  and that can be estimated from the results of a single simulation carried out in  $\mathbf{x}_{MC}$ . In particular, Eqs. (12) and (13) are exact in  $\mathbf{x}_{MC}$ .

#### 4.2. Approximation of the Loss Statistics

To estimate the loss statistics, a prediction strategy is needed that can efficiently approximate  $\sigma_{L|im}^2$  and  $\mu_{L|im}$  based on the demand statistics of Sec. 4.1. It is proposed here that kriging models can be constructed and used to predict the standard deviation,  $\sigma_{G_k}$ , and the mean,  $\mu_{G_k}$ , of the group-level loss associated with the  $k$ th PG in the space of the demand's second order statistics.

Let  $\tilde{\sigma}_{G_k}$  represent a kriging model for estimating  $\sigma_{G_k}$ , the kriging prediction at a given point  $(\mu_{EDP_k}, \sigma_{EDP_k})$  is given by (Sacks et al., 1989):

$$\tilde{\sigma}_{G_k}(\mu_{EDP_k}, \sigma_{EDP_k}) = \hat{\boldsymbol{\mu}} + \boldsymbol{\Omega}^T(\mu_{EDP_k}, \sigma_{EDP_k}) \mathbf{R}^{-1}(\mathbf{y} - \mathbf{1}\hat{\boldsymbol{\mu}}) \quad (14)$$

where  $\hat{\boldsymbol{\mu}}$  is the maximum likelihood estimate for the mean of the random field assuming that the observations  $\mathbf{y} = \{\sigma_{G_k}^{(1)}(\mu_{EDP_k}^{(1)}, \sigma_{EDP_k}^{(1)}), \dots, \sigma_{G_k}^{(h)}(\mu_{EDP_k}^{(h)}, \sigma_{EDP_k}^{(h)})\}^T$  are realizations of a Gaussian process;  $\boldsymbol{\Omega}$  is a vector collecting basis functions; while  $\mathbf{R}^{-1}(\mathbf{y} - \mathbf{1}\hat{\boldsymbol{\mu}})$  are the corresponding weights assigned to the basis functions of  $\boldsymbol{\Omega}$ . In selecting the  $h$  support points of the sampling plan  $\mathbf{y}$ , the Morris-Mitchell optimal Latin hypercube sampling (Morris and Mitchell, 1995) is used to ensure an optimal space-filling property. In obtaining  $h$  observations, stochastic simulation is carried out at each point to estimate the nonlinear mapping between the loss and demand statistics while taking into account variability in the system as well as in stochastic loads. Once

a full simulation is carried out for the first observation,  $\sigma_{G_k}(\mu_{EDP_k}^{(1)}, \sigma_{EDP_k}^{(1)})$ , the other observations only require the execution of a damage and loss assessment in terms of the following transformed samples:

$$\bar{ed}p_k^{(i)} = \frac{\sigma_{EDP_k}^{(j)}}{\sigma_{EDP_k}^{(1)}}(edp_k^{(i)} - \mu_{EDP_k}^{(1)}) + \mu_{EDP_k}^{(j)} \quad (15)$$

where  $edp_k^{(i)}$  is the original  $i$ th demand sample while  $i = 1, \dots, N_s$  and  $j = 2, \dots, h$ .

A similar procedure can be carried out for calibrating a kriging model,  $\tilde{\mu}_{G_k}$ , for predicting the group-level expected loss,  $\mu_{G_k}$ . Consequently,  $\sigma_{L|im}^2$  and  $\mu_{L|im}$  can be replaced with the following kriging-based approximations,  $\tilde{\sigma}_L^2$  and  $\tilde{\mu}_L$ , as:

$$\sigma_{L|im}^2(\mathbf{x}) \approx \tilde{\sigma}_L^2(\mathbf{x}) = \sum_{k=1}^{N_g} \sum_{j=1}^{N_g} \rho_{kj}(\mathbf{x}_{MC}) \tilde{\sigma}_{G_k}(\mathbf{x}) \tilde{\sigma}_{G_j}(\mathbf{x}) \quad (16)$$

$$\mu_{L|im}(\mathbf{x}) \approx \tilde{\mu}_L(\mathbf{x}) = \sum_{k=1}^{N_g} \tilde{\mu}_{G_k}(\mathbf{x}) \quad (17)$$

where  $\rho_{kj}$  is the correlation coefficient between  $G_k$  and  $G_j$ .

#### 4.3. Sequential Optimization

To decouple the simulation from the optimization loop, if it is assumed that the AVVs,  $\hat{\mathbf{Y}}$  and  $\check{\mathbf{Y}}$ , and the correlation coefficient,  $\rho_{kj}$ , do not vary as the design  $\mathbf{x}$  changes during the optimization process, the following deterministic optimization sub-problem can be defined:

$$\begin{aligned} \text{Find } \quad & \mathbf{x} = \{x_1, \dots, x_m\}^T \\ \text{to minimize } \quad & \tilde{\sigma}_L^2(\mathbf{x}; \hat{\mathbf{Y}}(\mathbf{x}_{MC}), \check{\mathbf{Y}}(\mathbf{x}_{MC}), \rho_{kj}(\mathbf{x}_{MC})) \\ \text{subject to } \quad & V(\mathbf{x}) \leq V_0 \\ & \tilde{\mu}_L(\mathbf{x}; \hat{\mathbf{Y}}(\mathbf{x}_{MC}), \check{\mathbf{Y}}(\mathbf{x}_{MC})) \leq \mu_0 \\ & x_i \in \mathbb{X}_i \quad i = 1, \dots, m \end{aligned} \quad (18)$$

This deterministic sub-problem can be solved using any gradient-based optimization technique. The Optimality Criteria algorithm is adopted here. Because the solution to this sub-problem is only exact in  $\mathbf{x}_{MC}$ , the Monte Carlo simulation is carried out

again in the solution of the previous sub-problem, and a new sub-problem is formulate. This process is denominated a design cycle (DC) and needs to be repeated until solutions of two consecutive DCs are identical therefore ensuring an exact solution to the original robust design problem of Eq. (1).

## 5. APPLICATION

The main purpose of this section is to examine the applicability of the proposed framework.

### 5.1. Description of the system

The structural system considered in this example is a 37 stories and six bay steel moment-resisting frame envisaged as part of 3-D building. The first floor height is 6 m while the height of all other floors is 4 m. The bay width is 5 m. Thus, the total height and total width are 150 m and 30 m respectively, while the total depth is taken as 60 m. All beams belong to the family of AISC (American Institute of Steel Construction) W24 steel profiles. All columns are square box sections whose mid-line diameters,  $b_i$  must belong to the discrete set  $\{0.20, 0.21, \dots, 3.99, 4.00\}$  m with the corresponding wall thickness taken as  $b_i/20$ . A total of 259 design variables are considered as shown in Figure 1. In the initial design, all beams were assigned W24×176 profile, while the mid-line diameter of all columns was set to 0.60 m. The floor systems are assumed to be rigid diaphragms having an area density of 100 kg/m<sup>2</sup>. The first three modes (with initial circular frequencies  $\omega_1 = 0.926$  rad/sec,  $\omega_2 = 2.964$  rad/sec, and  $\omega_3 = 5.535$  rad/sec) were used in estimating the resonant response of the system. The mean modal damping ratios were assumed to be 1.5%. The distributions used to described the uncertain parameters associated with the response estimation can be found in Table 2 of Suksuwan and Spence (2018).

### 5.2. Hazard and wind model description

In modeling the hurricane wind hazard, the meteorological wind speeds were estimated from a Type II distribution of mean 32 m/s and standard deviation of 4.7 m/s. This wind speed data was assumed to correspond to an averaging time  $\tau = 60$  s, roughness length  $z_{01} = 0.01$  m, and height at the meteorological station  $H_{met} = 10$  m. In transforming

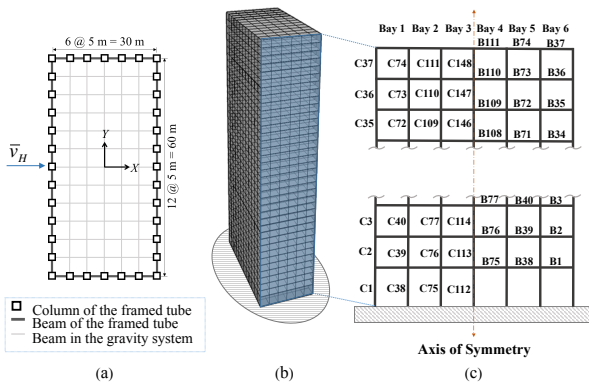


Figure 1: (a) 37-story building plan; (b) Isometric view; (c) Frame layout showing beam and column assignments.

$\bar{V}_{Hmet}$  to  $\bar{V}_H$ , the transformation scheme proposed in Minciarelli et al. (2001) was adopted. In particular, the averaging time  $T$  was taken as 3600 s while the roughness length at the site of interest was taken as  $z_0 = 1$  m. Distributions associated with the basic random variables related to the wind speed transformation can be found in Table 3 of Suksuwan and Spence (2018). The distribution of  $\bar{V}_{Hmet}$  was used to derive the site-specific wind speed associated with an MRI of 1700 years,  $\bar{v}_{1700}$ . The stochastic wind loads were generated based on wind tunnel tests carried out on a rigid 1:300 scale model of the aforementioned building (Tokyo Polytechnic University, 2008) using three loading modes. Only wind blowing down the  $X$ -direction was considered.

### 5.3. Calibration of the loss model

The system-level repair costs associated with damage to the cladding system of the building is considered as the loss metric. The cladding components are taken as midrise stick-built curtain wall and grouped together in 37 PGs. Each PG consists of 60 components that are susceptible to the same  $EDP_k$ . Within this context,  $EDP_k$  for  $k = 1, \dots, 37$  are taken as the maximum inter-story drift ratios. Three damage states are considered for each cladding component. In determining the component damage state and consequence, fragility curves with associated consequence functions were obtained from the fragility database contained in Federal Emergency

Management Agency (FEMA) (2012).

### 5.4. Optimization objective and adaptive search scheme

The main objective of this example is to minimize the variance of the total repair cost given that a windstorm of return period 1700 years has occurred, while still ensuring that the total volume of the structural system does not exceed  $V_0 = 150$  m<sup>3</sup> and the expected value of the total repair cost does not exceed  $\mu_0 = \$1000000$ . A total of  $N_s = 4000$  samples were used in the Monte Carlo simulation. In updating kriging surrogates, the following adaptive search scheme was considered:  $\mu_{EDP_k} \in [(1 - \Delta_\mu)\mu_{EDP_k}^{(DC-1)}, (1 + \Delta_\mu)\mu_{EDP_k}^{(DC-1)}]$  and  $\sigma_{EDP_k} \in [(1 - \Delta_\sigma)\sigma_{EDP_k}^{(DC-1)}, (1 + \Delta_\sigma)\sigma_{EDP_k}^{(DC-1)}]$  where  $\Delta_\mu = \Delta_\sigma = 0.95$  for the initial DC while  $\Delta_\mu = \Delta_\sigma = 0.5$  for successive DCs. If  $\mu_{EDP_k}$  and  $\sigma_{EDP_k}$  changed by less than 1%, only the center point was updated as long as the current center point of the adaptive search space moved less than a total of 5% from the center point of the last fully updated model.

### 5.5. Results and discussion

Figure 2 presents the convergence history of the objective function. The steady and smooth convergence properties of the method is clearly seen. Regarding the system-level constraints, Figures 3 and 4 present the convergence histories of the structural volume and the expected value of the total repair cost. From Figures 3 and 4, it can be observed that useful designs that satisfy both constraints were obtained within only three design cycles, leaving the remaining design cycles for further minimization of the objective function.

With respect to the approximation of the demand statistics using the AVV approach, Figure 5 reports the convergence histories of the second order demand statistics associated with the 5th performance group. The high accuracy of approximate demand statistics as compared to the actual values obtained from the Monte Carlo simulation is clearly observed. Regarding the adaptive updating algorithm, Figures 6 and 7 illustrate how the kriging surrogates were progressively updated during the optimization process. In particular, the large initial

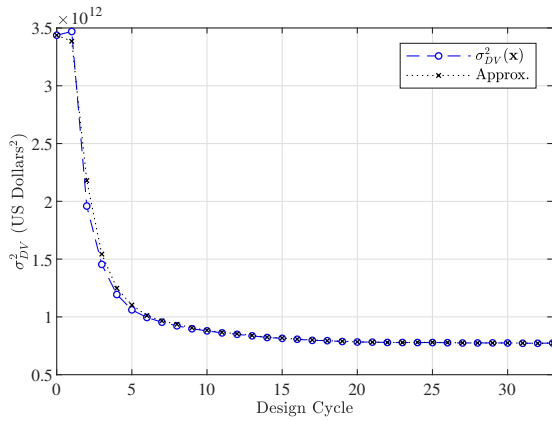


Figure 2: Convergence history of the objective function.

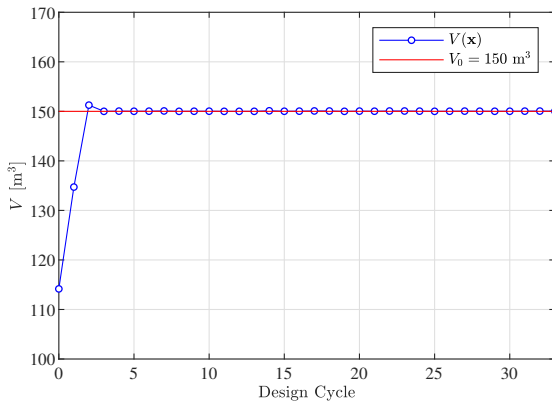


Figure 3: Convergence history of the constraint on the material volume of the system.

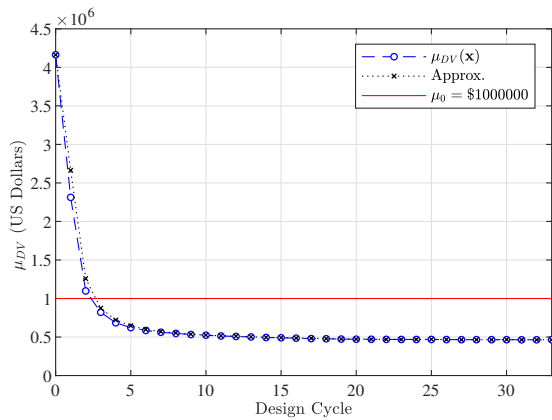


Figure 4: Convergence history of the constraint on the expected system-level loss.

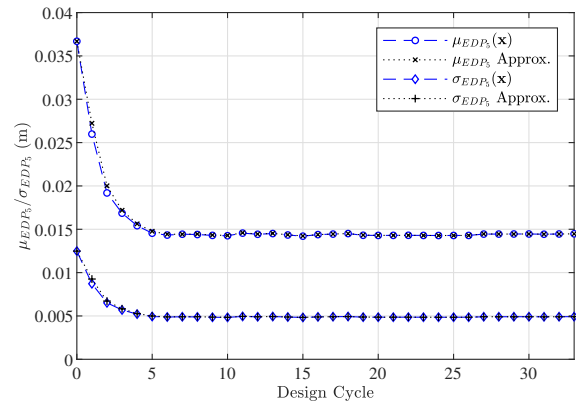


Figure 5: Illustration of the convergence histories of the second order statistics of  $EDP_5$ .

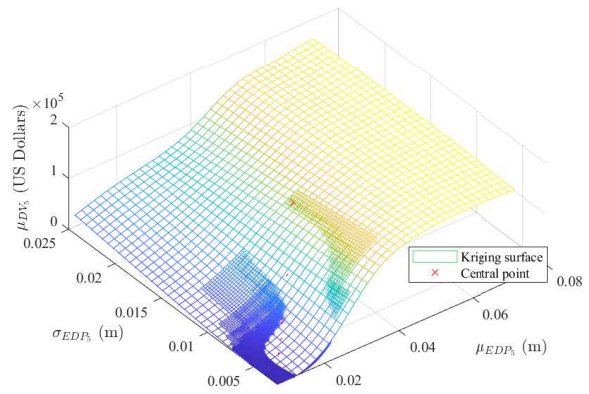


Figure 6: Illustration of the adaptively updated kriging surrogate for  $\mu_{DV_5}$ .

kriging surfaces of DC=1 allows for a global search for the optimal solution, while the subsequent updated kriging surfaces fine-tune the loss approximation in the optimal region of the design space. The proposed updating scheme provides accurate performance approximations as shown in Figures 2 and 4.

## 6. CONCLUSIONS

This paper presented a robust design optimization framework for wind-excited systems in which performance is estimated at a system level in terms of performance-based design metrics. A stochastic optimization is formulated with objective to minimize the variance of the performance metric. Con-

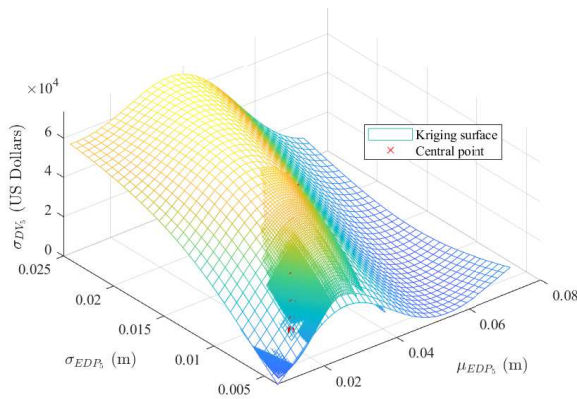


Figure 7: Illustration of the adaptively updated kriging surrogate for  $\sigma_{DV_s}$ .

straints are imposed on the initial cost of the system and expected value of the performance metric. The framework is based on developing a low-dimensional kriging model of the deaggregated loss metrics within the space of the second order statistics of the demands. By then relating the demand statistics to the design variables through the Auxiliary Variable Vector approach, a deterministic optimization sub-problem is defined that can handle high-dimensional design variable vectors and general stochastic excitation. By solving a sequence of sub-problems, each formulated in the solution of the previous, solutions to the original robust design problem are found. The applicability of the proposed framework was demonstrated through a large-scale case study.

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