A Novel Markovian Framework for Optimum Maintenance of Deteriorating Bridges in Earthquake-prone Areas

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ABSTRACT: The performance of bridges can deteriorate during their lifetime due to aging, traffic load-induced fatigue, and environmental corrosion. In earthquake-prone areas, structural deterioration increases the seismic vulnerability of the bridges, which means a rise of potential economic and social losses in the future. Therefore, it is of critical importance to determine the optimum maintenance strategies considering bridge deterioration. To this end, the present paper proposes an infinite-horizon hybrid Markov decision process model, where the processes of both earthquake occurrence and structural deterioration are integrated into a unified Markovian framework. The structural deterioration process is modeled by a Markov chain, and a simplified earthquake-induced state transition probability matrix is adopted. For the purpose of demonstration, the proposed model is applied to a simple case study.

The performance of bridges is degrading over time due to various deterioration factors, including aging, traffic load-induced fatigue, and environmental corrosion, among others. The performance degradation of bridges will impair their safety under extreme traffic loads or natural hazards (e.g. earthquakes or hurricanes). At present, a large number of bridges all over the world are approaching the end of their designed life. For example, according to the 2017 Infrastructure Report Card (ASCE (2017)), of the overall 614,387 bridges in the United States, almost four in 10 are 50 years or older, and an additional 15% are between the ages of 40 and 49 in 2016. The average age is 43 years old. 9.1% of these bridges were structurally deficient. It is very important to manage aging bridges properly in order to extend their service life.

Performance of a bridge is determined by the condition states of its elements, the assessment of which relies on visual inspection (Gattulli and Chiaramonte (2005)). Specifically, in bridge management systems such as the Pontis (Thompson et al. (1998)), visual inspection data are employed to give the condition rating indices of bridge elements to characterize their deterioration levels. It is assumed that the degree of deterioration along an element is uniform. There is much uncertainty in the prediction of element deterioration. After a period of time, an element will degrade from the original condition state to lower states according to certain probabilities. By assuming that deterioration rates are stationary over time, the transition of condition states can be modeled as Markov chains (Cesare et al. (1992); Morcous (2006)). Other more complicated models have also been proposed, such as stochastic duration models (Mauch and Madanat (2001); Mishalani and Madanat (2002)).

The Markov decision process (MDP) model is a powerful mathematical tool for making sequential decisions under uncertainty and has already received numerous applications in the optimum maintenance of bridges. To name a few here: Scherer and Glagola (1994) explored the use of MDP for bridge management systems, with emphasis on the issues of state-space explosion and Markovian property; Tao et al. (1995) proposed a reliability-based state space to capture both serviceability and safety attributes, and applied MDP to the optimal design of a composite five-girder bridge; Robelin and Madanat (2007) developed an MDP framework for bridge maintenance optimization using a deterioration model that takes into account aspects of the history of the bridge condition and maintenance. In situations when not all of the model parameters and system states are exactly known, the partially observable Markov decision process (POMDP) model is adopted instead. For example, Ellis et al. (1995) presented a POMDP model for bridge inspection, maintenance and repair by recognizing that inspections do not yield perfect estimates of the true internal state of structural components. In these cases, decisions are made at fixed time points. Accordingly, they are named discrete-time Markov decision processes (DTMDPs). There are also other cases in which the time interval between two consecutive decisions is random. Continuous-time Markov decision processes (CTMDPs) or semi-Markov decision processes (SMDPs) have been widely applied to this class of problems in a variety of contexts (e.g. Beutler and Ross (1987); Buchholz and Schulz (2011)). As a critical part of bridge maintenance in earthquake-prone areas, the post-earthquake decision-making for restoring damaged bridges is also a CTMDP or SMDP in essence, depending on the probabilistic distribution of the occurrence time of earthquakes. However, as far as the authors know, little work has been carried out in this aspect. In this paper, we propose a hybrid MDP model which can integrate the preventive maintenance at fixed epochs and the essential maintenance after earthquakes into a unified framework.

1. STATES AND ACTIONS

Condition rating indices are assigned to elements in bridge management systems. In the Pontis, each element is visually inspected by a trained inspector and classified into one of four or five condition states (Thompson et al. (1998)). For example, the Colorado Department of Transportation suggested condition state ratings for painted open steel girders (Estes and Frangopol (2003)), as shown in Table 1.

Table 1: Condition State Ratings for Painted Open Steel Girders

Condition state	*Description*
1	*No evidence of active corrosion. Paint system is sound and protecting the girder.*
2	*Slight peeling of the paint, pitting, or surface rust, etc.*
3	*Peeling of the paint, pitting, surface rust, etc.*
4	*Flaking, minor section loss (<10% of original thickness).*
4	*Flaking, swelling, moderate section loss (>10% but <30% of the original thickness). Structural analysis not warranted.*
5	*Flaking, swelling, moderate section loss (>10% but <30% of the original thickness). Structural analysis not warranted due to the location of corrosion on the member.*
5	*Heavy section loss (>30% of original thickness), may have holes through base metal.*

The damage degree of elements intensifies as the condition rating index increases. For a bridge, the condition states of all elements are then integrated into its overall performance by structural analysis. Moreover, earthquakes could cause additional damage to the bridge, which further reduces its performance indicator. A variety of performance indicators have been proposed, such as reliability, risk, and sustainability, among others (Frangopol et al. (2017)). However, due to limited space, we don't intend to specify the indicator and elaborate on the procedure for calculating it. Instead, an abstract indicator is used for demonstration purposes.

The continuous range of performance can be discretized into a series of values. Without loss of generality, we define ten mutually exclusive bridge states from *State* 1 to *State* 10. Note that *State* 1, *State* 3, and *State* 5 are the target performance levels corresponding to the high, normal, and low design standard, respectively. In addition, *State* 10 represents the worst situation when the bridge loses its traffic load capacity and has to be rebuilt.

Given a bridge state, the optional restoration actions are as follows: If it is above the prescribed threshold, which is taken as *State* 8 in this paper, the bridge can continue to work in the current state or be restored to one of the three target states. Otherwise, the bridge has to be shut down for restoration or reconstruction.

2. RANDOM TRANSITION OF STATES

For a bridge, its state changes over time due to progressive deterioration. Besides, an earthquake could also change its state, which is a big issue in earthquake-prone areas. In this section, the state transition models of these two mechanisms are described separately.

2.1. Deterioration-induced State Transition

The random transition process of states caused by progressive deterioration is modeled as a Markov chain, in which state transitions occur at a series of time points, and the transition probabilities to future states are independent of the past states. In addition, if the Markov chain is stationary, as assumed in this paper, the transition probabilities are constant over time.

Assume that transitions happen only between two subsequent states in one year. Thus, a one-year deterioration-induced transition probability matrix has the following form:

$$\mathbf{TP}_D = \begin{bmatrix} p_{1,1} & 1-p_{1,1} & \cdots & 0 & 0 \\ 0 & p_{2,2} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & p_{9,9} & 1-p_{9,9} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{1}$$

where $p_{i,i}$ denotes the probability that the bridge remains in the *i*th state. *State* 10 is considered to be an absorbing state. Further, according to the Markovian property, the *t*-year transition probability matrix can be expressed as $\mathbf{TP}_D^t, t \in \mathrm{N}$.

2.2. Earthquake-induced state transition

Earthquake-induced damage further weakens the performance of a bridge. For an earthquake event, there are many uncertainties in its occurrence, epicenter, magnitude, propagation, site effect, and the resulting structural response. It is quite complicated to consider all these factors in determining the earthquake-induced transition probability matrix. Due to limited space, we adopt a simplified method instead.

1. Normalize the performance indicator so that its difference between two consecutive states is equal to one.
2. Given a bridge state, assume a lognormal distribution for the random decrease *D* of the normalized performance indicator caused by an earthquake. That is
$$\ln D \sim N(\mu_i, \sigma_i^2) \tag{2}$$
3. Divide the range of *D* into discrete values, and calculate the corresponding probability masses
$$\begin{aligned} p_d^{(i)} &= \Pr\left(D = d \middle| State\ i\right) \\ &= \begin{cases} F_D\left(0.5\middle|\mu_i,\sigma_i\right), d = 0 \\ F_D\left(d+0.5\middle|\mu_i,\sigma_i\right) - F_D\left(d-0.5\middle|\mu_i,\sigma_i\right), d > 0 \end{cases} \end{aligned} \tag{3}$$
where $F_D\left(\cdot\middle|\mu_i,\sigma_i\right)$ denotes the cumulative distribution function of *D* corresponding to *State i*.
4. Calculate the earthquake-induced transition probabilities by definition

$$\mathbf{TP}_E = \begin{bmatrix} p_0^{(1)} & p_1^{(1)} & \cdots & p_8^{(1)} & 1-\sum_{k=0}^{8} p_k^{(1)} \\ 0 & p_0^{(2)} & \cdots & p_7^{(2)} & 1-\sum_{k=0}^{7} p_k^{(2)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & p_0^{(9)} & 1-p_0^{(9)} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{4}$$

3. HYBRID INFINITE-HORIZON MDP MODEL

As mentioned above, DTMDPs and CTMDPs are applicable to the decision-making of optimum maintenance in the context of progressive deterioration and earthquakes, respectively. In this section, we propose a hybrid infinite-horizon MDP model which is able to take both hazards into account at the same time.

The basic idea of the new Markovian framework is as follows: (1) A bridge is seen as an infinite-horizon dynamic system which repeatedly experiences degradation, restoration, and even reconstruction. (2) The occurrence of earthquakes is assumed to be a Poisson process. (3) If no significant earthquakes occur during a pre-defined time interval, the bridge will be inspected at the end of the interval, followed by a maintenance decision based on the inspection result. The end of the previous time interval is taken as the start of the next time interval. However, if a significant earthquake occurs during the time interval, inspections and maintenances are carried out immediately, and the time point is taken as the start of the next time interval.

Based on the above description, the formula of the hybrid infinite-horizon MDP model is derived as follows:

$$\begin{aligned} V(s) &= \min_{a\in A}\Bigg\{ C(s,a) + \sum_{s'\in S}\left[\int_0^T \nu e^{-(\nu+\lambda)\tau} p\left(s',\tau\middle|s^a\right)\mathrm{d}\tau\right]V(s') \\ &\quad + \sum_{s'\in S}\left[e^{-\lambda T}\left(1-\int_0^T \nu e^{-\nu\tau}\mathrm{d}\tau\right)p\left(s',T\middle|s^a\right)\right]V(s')\Bigg\} \\ &= \min_{a\in A}\Bigg\{ C(s,a) + \sum_{s'\in S}\left[\sum_{i=0}^{T-1}\int_i^{i+1} \nu e^{-(\nu+\lambda)\tau} p_i\left(s'\middle|s^a\right)\mathrm{d}\tau\right]V(s') \\ &\quad + \sum_{s'\in S}\left[e^{-\lambda T}\left(1-\int_0^T \nu e^{-\nu\tau}\mathrm{d}\tau\right)p_T\left(s'\middle|s^a\right)\right]V(s')\Bigg\} \\ &= \min_{a\in A}\Bigg\{ C(s,a) + \sum_{s'\in S}\left[\frac{\nu}{\nu+\lambda}\sum_{i=0}^{T-1}\left(e^{-(i-1)(\nu+\lambda)} - e^{-i(\nu+\lambda)}\right)p_i\left(s'\middle|s^a\right) + e^{-(\nu+\lambda)T}p_T\left(s'\middle|s^a\right)\right]V(s')\Bigg\} \end{aligned} \tag{5}$$

where ν denotes the mean occurrence rate of earthquakes; λ denotes the discount rate; T denotes the pre-defined time interval for regular inspections; $p\left(s',\tau\middle|s^a\right)$ is the state transition probability from s^a to s' at time τ, which is a step function of time, as follows:

$$p_i\left(s'\middle|s^a\right) = \begin{cases} \mathbf{TP}_D^i\left(s^a,:\right)\times\mathbf{TP}_E\left(:,s'\right),\ 0\le i\le T-1 \\ \mathbf{TP}_D^i\left(s^a,s'\right),\ i=T \end{cases} \tag{6}$$

For more details on MDP theory, please refer to related monographs, such as Powell (2007).

4. CASE STUDY

The proposed hybrid MDP model is applied to a hypothetical bridge exposed to deterioration and earthquakes.

The mean values and standard deviations in Eq. (3) are listed in Table 2.

Table 2:Mean Value and Standard Deviation

State	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
μ	0.30	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39
σ	0.3									

The one-year deterioration-induced state transition probability matrix is

$$\begin{bmatrix} 0.950 & 0.050 & & & & & & & & \\ & 0.945 & 0.055 & & & & & & & \\ & & 0.940 & 0.060 & & & & & & \\ & & & 0.935 & 0.065 & & & & & \\ & & & & 0.930 & 0.070 & & & & \\ & & & & & 0.925 & 0.075 & & & \\ & & & & & & 0.920 & 0.080 & & \\ & & & & & & & 0.915 & 0.085 & \\ & & & & & & & & 0.910 & 0.090 \\ & & & & & & & & & 1.000 \end{bmatrix}$$

Restoration or reconstruction losses include direct economic costs and indirect losses due to traffic delay. The direct economic costs are listed in Table 3. The indirect losses are converted equivalently to economic costs. For simplicity, it is assumed that the indirect costs incurred when the bridge is in *State* 8, 9, and 10 are \$7 million, \$8 million, and \$20 million, respectively. In other cases, indirect losses are not considered.

Other parameters involved in the analysis are as follows: v=0.05; λ=4%; T=5 yr.

Table 3: Direct Economic Cost (10^3 \$)

	Post-decision state		
	State 1 (High standard)	*State 3 (Normal standard)*	*State 5 (Low standard)*
State 1	*0*	*-*	*-*
State 2	*200*	*-*	*-*
State 3	*400*	*0*	*-*
State 4	*600*	*200*	*-*
State 5	*800*	*400*	*0*
State 6	*1,000*	*600*	*200*
State 7	*1,200*	*800*	*400*
State 8	*1,400*	*1,000*	*600*
State 9	*1,600*	*1,200*	*800*
State 10	*3,400*	*3,000*	*2,600*

The optimum maintenance decisions in three cases are obtained by DTMDP, CTMDP, and the proposed model, respectively, as shown in Table 4. We can see that the coupling of deterioration and seismic hazards leads to a higher requirement for target performance in restoring the bridge.

Table 4: Optimum Maintenance Decisions

State	*Deterioration only (by DTMDP)*	*Earthquake only (by CTMDP)*	*Deterioration+Earthquake (by the proposed model)*
1	*no restoration*	*no restoration*	*no restoration*
2	*no restoration*	*no restoration*	*no restoration*
3	*no restoration*	*no restoration*	*no restoration*
4	*no restoration*	*no restoration*	*no restoration*
5	*no restoration*	*no restoration*	*restored to State 3*
6	*restored to State 5*	*restored to State 5*	*restored to State 3*
7	*restored to State 5*	*restored to State 5*	*restored to State 3*
8	*restored to State 5*	*restored to State 5*	*restored to State 3*
9	*restored to State 5*	*restored to State 5*	*restored to State 3*
10	*rebuilt to State 5*	*rebuilt to State 5*	*rebuilt to State 3*

5. CONCLUSIONS

A hybrid MDP model is creatively proposed in this paper for making optimum maintenance decisions in the environment of coupled structural deterioration and earthquake hazards. It integrates DTMDP and CTMDP into a unified Markovian framework. A simple case study is performed to demonstrate its application. There is still some work to be done before the model can be applied to practical problems. For example, the derivation of a more reliable earthquake-induced state transition probability matrix from probabilistic seismic hazard analysis and structural fragility analysis.

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