Quantifying the value of structural monitoring for decision making

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ABSTRACT: A life-cycle approach to infrastructure design and management involves decisions pertaining to operation, maintenance, intervention, and rapid response measures. Such an approach may only be conceived when formulated on the basis of observations during the life-cycle of these systems. Structural Health Monitoring (SHM) offers a tool to such an end, with sensors employed to generate information on the state of structural systems, which may then be exploited to derive performance indicators. A fundamental, practical question regarding monitoring of structural systems is however the quantification of any gains, monetary or otherwise, for infrastructure owners if they choose to install a monitoring system to their structures, in place of, or in addition to, other available choices, such as structural inspection visits. This essentially comprises a Value of Structural Health Monitoring (VoSHM) problem, which poses important mathematical and computational challenges related to several infrastructure system uncertainties, stochastic observations and their Value of Information (VoI), and any uncertain action outcomes. In this work, we implement optimal stochastic control approaches for infrastructure management in the form of Partially Observable Markov Decision Processes (POMDPs), which inherently possess the notion of the VoI into their formulation and, in fact, automatically utilize it at every decision step for decision-making. In addition, we show that based on POMDPs the VoSHM can be efficiently estimated, allowing for informative decisions by the structural owner, based on quantitative metrics in relation to the expected benefits of the SHM system. A representative application is shown in this regard for a multi-component engineering system, showcasing the wide applicability and effectiveness of the suggested approach and its practical merits.

1. INTRODUCTION

Structures and infrastructure systems are exposed to adverse operational conditions, aging and deterioration throughout their life-cycle. Recent catastrophic events, such as the Morandi bridge collapse in Genoa, evidence the urgency for utilization of schemes that can inform about structural integrity while supporting Operation & Maintenance (O&M) actions. Sensor technologies have sufficiently matured today to allow for reliable and diversified measurements of structural response (e.g. accelerations, strains, loads, temperature, etc.), which serve as evidence of structural condition. Structural Health Monitoring (SHM) techniques are developed for processing raw data and translating these into structural performance indicators, or "structural health". SHM systems may therefore support decision-making regarding the management of infrastructures throughout their life-cycle, with decision policies for inspection, maintenance and intervention actions. In this direction, new methods and tools are needed, capable of quantifying the Value of Information (VoI) and similarly the Value of Structural Health Monitoring (VoSHM) (Straub et al. 2017).

In existing literature, several methods attempt to tackle decision-making for optimal management of infrastructure. A common approach lies in casting this as an optimization problem, with the aim of satisfying either individual objectives, e.g. condition, availability, safety, durability or reliability (e.g. Liu et al., 1997; Miyamoto et al., 2000; Furuta et al., 2004), or multiple conflicting objectives, as a combination of the above (Bocchini & Frangopol, 2013). Multi criteria objectives may be tackled by multi-objective optimization schemes, which will typically deliver a set of compromising Pareto solutions (Liu & Frangopol 2005).

Key to the success of such optimization tools is the adequate incorporation of uncertainties, which may be of aleatory or epistemic nature. According to Faber (2005), engineering decision problems may be classified into three main categories, namely those of prior, posterior, or pre-posterior decision problems. The problem of inspection and maintenance planning falls in the latter class, (Sørensen, 2009, Goulet et al., 2015), and may be treated by means of dedicated tools, such as the Value of Information (VoI). The VoI may be defined as the amount a decision maker is willing to pay for information prior to making a decision. Within the context of SHM, the VoI may be quantified as the difference between the expected operational cost of the structure in absence of relevant information, and the cost upon availability of monitoring information (Zonta et al., 2014), whereas VoSHM can provide a broader definition to describe relative costs between default observational schemes (e.g. inspection visits) and SHM-aided plans (Andriotis et al. 2019). Various works (Pozzi & Der Kiureghian, 2011; Straub & Faber 2005) exploit the VoI to quantify the value of monitoring/inspections in support of maintenance interventions within a Bayesian framework. In (Papakonstantinou et al., 2016a) the value of permanent monitoring is quantified in the context of Partially Observable Markov Decision Processes (POMDPs) for a non-stationary corroding structure.

POMDPs offer an extension of Markov Decision Processes (MDP), having as objective the determination of an optimal sequence of actions (policy) that maximizes rewards or respectively minimizes costs. While MDPs offer a setup that is particularly suited for structural O&M, they are principally formulated under the assumption that the system's conditions can be observed perfectly at each time step. Relaxing this assumption, POMDPs allow for decision-making under partial (uncertain or incomplete) observations. In an early work, Madanat & Ben-Akiva (1994) adopt POMDPs for decision-making for highway-pavement networks. Ellis et al. (1995) and Corotis et al. (2005) demonstrate use of POMDPs for bridge inspection planning. Discrete POMDP formulations, rendering such an implementation suitable for non-stationary infrastructure maintenance and inspection planning in real time has been proposed in (Papakonstantinou & Shinozuka 2014a, 2014b; Papakonstantinou et al., 2016a). Schöbi & Chatzi (2016) on the other hand adopt a continuous formulation for treating problems described by linear and/or nonlinear transition functions, whereas adept formulations for cases of mixed observability have been presented in (Papakonstantinou et al., 2018).

POMDPs offer a comprehensive and flexible modeling framework for life-cycle analysis and may further be extended to tackle problems of multiple components or systems, as exemplified in (Memarzadeh & Pozzi, 2016). Fereshtehnejad & Shafieezadeh (2017) have proposed the use of point-based value iteration combined with a counting process to optimize decisions in structural POMDP systems. Recently, to efficiently address the known curse of dimensionality and model unavailability in largescale POMDP system applications, Andriotis and Papakonstantinou (2018) developed a Deep Reinforcement Learning (DRL) framework, proposing an actor-critic architecture within off-policy DRL, tailored to long-term maintenance and inspection planning in generic deteriorating engineering environments, and structural domains with high-dimensional state and action spaces.

In this work, we present a method for calculating the VoSHM, within the framework of POMDPs. The described method is applied and demonstrated on a three-component system, which is analyzed under two different inspection scenarios; one with optional inspection visits and one with continuous availability of observations pertaining to permanent monitoring. The underlying POMDP problems are solved here using point-based value iteration on the reachable subset of the belief space, however the delineated steps for the VoSHM quantification are general and not specific to any particular POMDP solution scheme.

2. PARTIALLY OBSERVABLE MARKOV DECISION PROCESSES

POMDPs provide a theoretically sound decision framework for environments with uncertain action outcomes, operating under partial observability. Inspection techniques and monitoring devices in structural settings commonly provide noisy observations that do not reveal the true state of the system with certainty. In such cases, within a POMDP context, a belief **b** over the possible states of the system, *S*, can only be obtained, which is a probability distribution over *S* and a sufficient statistic of the history of actions and observations. That is, given **b** the agent (or decision-maker) has all required information to take an action, regardless of the entire sequence of past actions and observed states. Beliefs at every decision step sum up to 1, thus forming a $|S|-1$ dimensional simplex. The agent is equipped with actions, *a*, drawn from a finite set of actions *A*, which have a certain instant cost or negative reward *R*(*s*,*a*), typically also dependent on the state of the system. Following the selected action, *a*, and the underlying transition dynamics, the environment switches to a new state s' according to a known stochastic model, with transition probability $P\left(s'\middle|s,a\right)$. The new belief $\mathbf{b}'$ at the new time step can be readily obtained by a Bayesian update:

$$b'\left(s'\right)=\frac{p\left(o\middle|s',a\right)}{p\left(o\middle|\mathbf{b},a\right)}\sum_{s\in S}p\left(s'\middle|s,a\right)b\left(s\right) \qquad (1)$$

where $p\left(o\middle|\mathbf{b},a\right)$ is the normalizing constant:

$$p\left(o\middle|\mathbf{b},a\right)=\sum_{s'\in S}p\left(o\middle|s',a\right)\sum_{s\in S}p\left(s'\middle|s,a\right)b\left(s\right) \qquad (2)$$

Thus, a POMDP is defined as a 6-tuple (*S, A, P, O,* P_{obs}*, R*) where, *S*, *A* and *O* finite set of states, actions and possible observations respectively, *P* state transition probabilities, P_{obs} observation probabilities modeling, and *R* rewards.

The objective in a POMDP sequential decision problem is to find an optimal sequence of actions that minimizes the total expected cost over the entire life-cycle (or planning horizon). This sequence of actions defines the agent's policy, *π*, which is a function, mapping beliefs to actions. The total expected cost collected under policy *π*, defines the POMDP value function, $V \rightarrow \mathbb{R}$. For every belief, V^* estimates the amount of discounted reward the decision-maker can gather when acting according to the optimal policy *π**. The optimal value function for a discounted, infinite horizon POMDP is written as (Papakonstantinou & Shinozuka, 2014a):

$$V^*\left(\mathbf{b}\right)= \max_{a\in \mathrm{A}}\left[\sum_{s\in S}b\left(s\right)R\left(s,a\right)+\gamma\sum_{o\in O}p\left(o\middle|\mathbf{b},a\right)V^*\left(\mathbf{b}'\right)\right] \qquad (3)$$

where *γ* is the discount factor, a positive scalar less than 1, associated to the future discounted state and action values, to relate them to the present.

POMDPs can be seen as belief MDPs, however solving a POMDP is a much more challenging problem. Although the reachable belief space is typically a small subset of the entire belief simplex, the different permissible belief and action and observation sequences generate large policy trees that are very difficult to determine. A significant feature of POMDP models is that their optimal value functions are piecewise linear and convex, thus they can be approximated arbitrary well by a set of vectors, also called α-vectors. Owing to this property, adjacent belief points can be supported by the same vector, which precisely represents the optimal value function. Hence, the gradient of the value function at any belief point is given by a corresponding α-vector and based on a set of α-vectors the value function is written:

$$V^*(\mathbf{b}) = \max_{\{\alpha^i\}_i} \sum_{s\in S} b(s)\,\alpha^i(s) \tag{4}$$

With this representation, the value function over the continuum of points of the belief state-space is described by a finite set of α-vectors, and each vector is associated with a specific optimal action.

2.1. Point-based solvers

Point-based value iteration is an approximate method for solving POMDPs and constitutes the central feature of point-based POMDP solvers. Point-based POMDP solvers share three main steps: (i) they use a simple lower bound initialization of the value function, (ii) they collect permissible belief points that are likely to describe the possible action and observation outcomes, and (iii) they perform Bellman backups for the α-vectors on subsets of the belief space.

A number of point-based algorithms exists in the literature. Based on previous works by Papakonstantinou et al. (2016b; 2018) on evaluating and assessing the performance of a great variety of point-based solvers, in this paper three point-based solvers are utilized for the numerical examples, namely FRTDP, SARSOP and Perseus. FRTDP and SARSOP concurrently maintain a lower and an upper bound to guide the belief trajectories over the belief space and to monitor convergence. Perseus utilizes a randomly selected set of belief points over which α-vector and value function backups are performed. Lower bounds are piece-wise linear whereas upper bounds are determined by sawtooth approximations or linear programming.

3. VALUE OF STRUCTURAL HEALTH MONITORING

VoSHM can efficiently inform decision-makers regarding the possible gains from investing in life-long SHM devices and practices, instead of planning inspection visits at discrete times during the structural life-cycle. As such, VoSHM relates to the critical decision regarding the nature of the monitoring scheme that needs to be adopted, in essence quantifying the benefits of continuous data collection and inflow of information in the decision-support system.

Following this logic within the premises of POMDPs, the VoSHM can be defined as the difference between the value function of a system, V_2, featuring permanent monitoring, and the value function of the same system, V_1, without permanent monitoring, but with the option of inspection visits with known possible costs at certain time steps:

$$VoSHM(\mathbf{b}) = V_2^*(\mathbf{b}) - V_1^*(\mathbf{b}) \tag{5}$$

Using Eq. (5), we can compute the VoSHM at every possible belief point that the system can visit throughout the planning horizon. Typically, the belief of foremost interest is the root belief, $\mathbf{b}_0$, which reflects the probability distribution over all possible states at the initial conditions, i.e. for current step t=0. In this case, VoSHM quantifies the life-cycle value of the monitoring system. For $t \neq 0$, which generally corresponds to $\mathbf{b} \neq \mathbf{b}_0$, Eq. (5) describes the remaining VoSHM from that time. The notion of remaining VoSHM can be of particular practical importance in cases where determination of the optimal salvage time of the monitoring system needs to be defined.

4. RESULTS

4.1. Settings 1 and 2

For our numerical investigation, we consider a three-component system, operating under partial observability. Stochastic deterioration of the components, for all $i \in \{1, 2, 3\}$, is defined by independent transition matrices, P_i, as:

Table 1: Base rewards for individual components.

State	*1*	*2*	*3*
Repair	*-12.0*	*-18.0*	*-30.0*
Inspection	*-1.0*	*-1.0*	*-1.0*
State loss	*0.0*	*-5.0*	*-12.0*

$$P_1 = \begin{bmatrix} 0.82 & 0.13 & 0.05 \\ & 0.87 & 0.13 \\ & & 1.00 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0.72 & 0.19 & 0.09 \\ & 0.78 & 0.22 \\ & & 1.00 \end{bmatrix} \tag{6}$$

$$P_3 = \begin{bmatrix} 0.79 & 0.17 & 0.04 \\ & 0.85 & 0.15 \\ & & 1.00 \end{bmatrix}$$

As indicated by Eq. (6), each component has three states with stationary transition dynamics, i.e., transition from state i to j is independent of time and component age. Overall, the examined system can be fully specified by 27 states.

In order to quantify the VoSHM for this three-component system, two POMDP control settings are evaluated. For Setting 1, 4 inspection and maintenance control actions are available for each component, including the possibility of structural inspection visits at belief points suggested by the POMDP solution. These actions are *'no-inspection and no-repair'*, *'inspection and no-repair'*, *'no-inspection and repair'*, and *'inspection and repair'*. The total number of system actions is 64. For Setting 2, observations are available by default at every decision step, corresponding to a permanent monitoring system. Accordingly, 2 maintenance control actions are available, i.e., *'no-repair'*, and *'repair'*. Based on the possible action combinations, there are 8 system actions in Setting 2. In both settings, for all components, a choice of repair action yields the following transitions:

$$P_{rep} = \begin{bmatrix} 0.90 & 0.10 & \\ 0.80 & 0.20 & \\ & 0.70 & 0.30 \end{bmatrix} \tag{7}$$

Observation matrices, for all components, read:

$$P_{obs} = \begin{bmatrix} 0.90 & 0.05 & 0.05 \\ 0.05 & 0.90 & 0.05 \\ 0.05 & 0.05 & 0.90 \end{bmatrix} \tag{8}$$

Eq. (8) assigns a 90% observation accuracy every time an inspection is made, meaning that there is 0.90 probability of observing the correct state, and 0.10 probability of uniformly observing either one of the other states.

Base negative rewards (or costs) for individual components are given in Table 1 for different states and actions. System level interdependence among components is established though the reward function, with certain penalties added to the base total costs at different system state configurations. That is, for states in {(2,2,1)}, {(2,2,2),(1,2,3),(2,2,3)}, {(3,3,1),(3,3,2)}, and {(3,3,3)}, penalties are -5.0, -10.0, -14.0, and -18.0, respectively, where vector (i, j, k) denotes component state combinations, i.e. (3, 3, 1) indicates that there are 2 components in state 3 and one component in state 1.

4.2. Policy evaluations and VoSHM

For both POMDP settings, the three point-based algorithms discussed in Section 2.1 are implemented. The relevant results can be seen in Tables 2 and 3. The values correspond to 3,600s analyses, as it was noticed that longer runs did not provide substantial precision improvement. As also shown in Figures 1 and 2, Setting 1 practically converges after 1,000s, whereas Setting 2 after 110s for all algorithms. It can be seen that the precision of the solution of Setting 1 is somewhat lower that the precision of Setting 2, for FRTDP and SARSOP. This can be attributed to the fact that the system in Setting 1 operates in a much more challenging POMDP environment with more actions and, consequently, larger reachable belief space. Apart from that, low

Table 2: Performance of different point-based POMDP solvers in Setting 1.

Solver	Lower bound	Upper bound	Precision gap
FRTDP	-529.125	-512.952	3.06%
SARSOP	-528.585	-512.881	2.97%
Perseus	-527.592	N/A	N/A

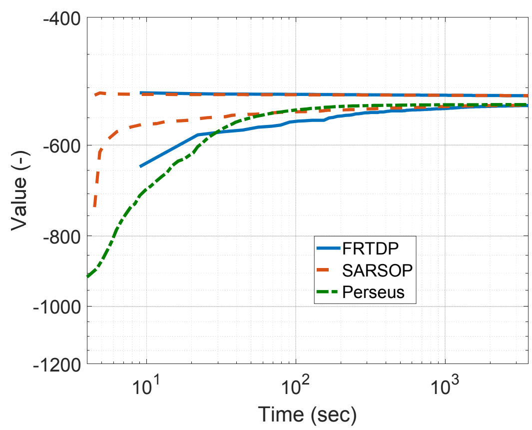

Figure 1: Anytime performance of point-based POMDP solvers in Setting 1.

Table 3: Performance of different point-based POMDP solvers in Setting 2.

Solver	Lower bound	Upper bound	Precision gap
FRTDP	-473.245	-470.556	0.57%
SARSOP	-473.244	-470.764	0.52%
Perseus	-473.244	N/A	N/A

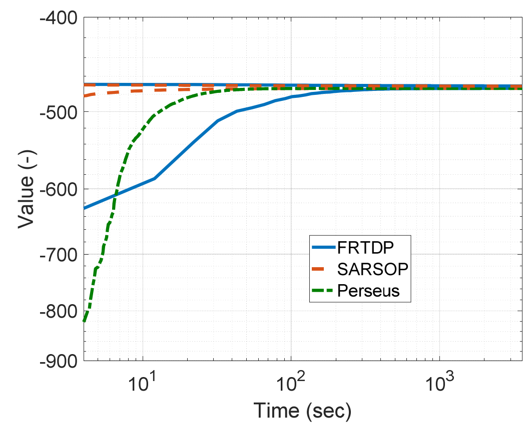

Figure 2: Anytime performance of point-based POMDP solvers in Setting 2.

precision can also be triggered by a rough approximation of the upper bound. FRTDP and SARSOP utilize approximate upper bounds, determined by a sawtooth approximation. The bound that actually contains all the information of the optimal policy is the lower bound and this is shown to be reached with great agreement among all different solver runs.

In Figure 1, the progress of both the lower and the upper bounds, where applicable, is depicted. SARSOP converges faster, thus exhibiting a better anytime performance, as also discussed in (Papakonstantinou et al. 2018). Perseus, although starting from a cruder initial lower bound, eventually reaches the best value, slightly outperforming counterparts. The same features are also noticed in Figure 2. In this plot, the overall convergence is much faster for all algorithms, due to the simpler nature of the decision problem, and SARSOP demonstrates considerable strengths in early convergence, practically converging before 10s. Perseus has a clear anytime performance advantage compared to FRTDP, in this case, whereas all algorithms reach identical lower bounds after 3,600s.

A realization of the converged policies is shown in Figures 3 and 4. For Setting 1, each component needs to perform different policies in order to collectively minimize the total expected cost of the system. Component 1 requires an inspection visit roughly every two years, whereas its repair actions are mostly combined with inspections. Component 2 does not choose the no-inspection/no-repair action, predominantly requires actions that involve inspection, and it is also the component with the most repair actions throughout its life-cycle. Component 3 policy combines features of the other two policies. This is anticipated as the transition dynamics of component 3 are in-between the other two cases, defined by components 1-2. Figure 4, illustrates a life-cycle policy realization for Setting 2, with the same random seed as for the realization of Figure 3. In this case, inspections are always available at no cost, due to the permanent monitoring system assumption and in order to evaluate its value. Policies follow similar trends for all components, as in Setting 1. The VoSHM is shown for all solvers in Figure 5. The VoSHM in this example is near 10% of the life-cycle cost of Setting 1. This

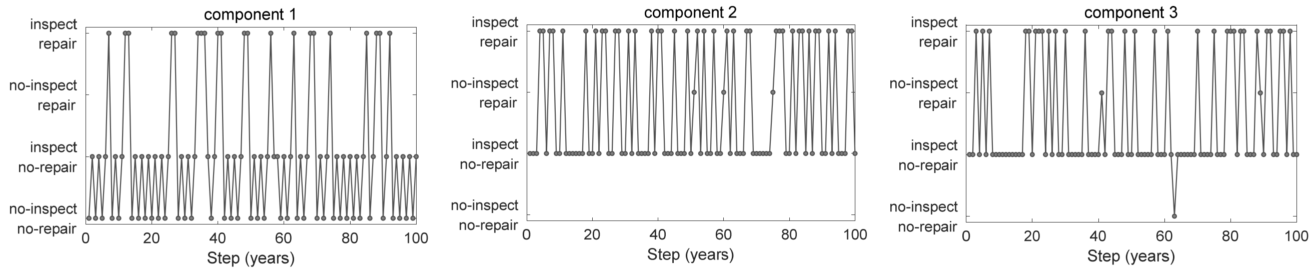

Figure 3: Policy realization for all system components operating in Setting 1, for 100 years.

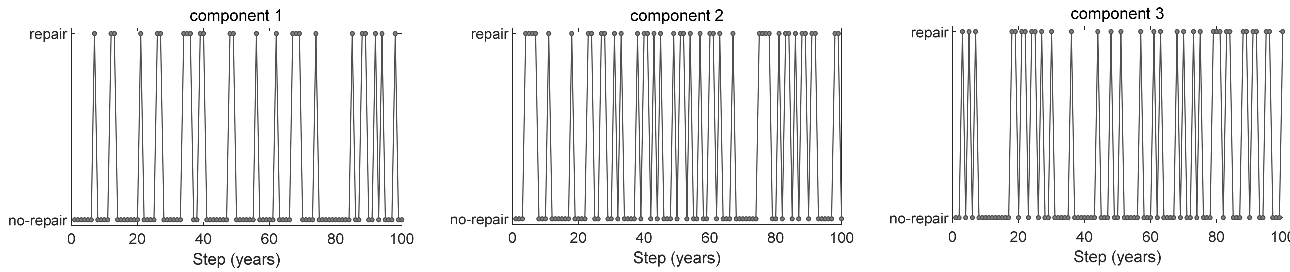

Figure 4: Policy realization for all system components operating in Setting 2, for 100 years.

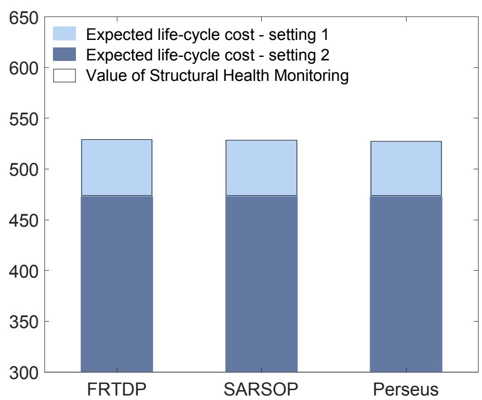

Figure 5: Expected life-cycle costs for Settings 1, 2 and corresponding VoSHM estimates.

means that any permanent monitoring system with lifetime cost lower than this amount should be preferred for SHM, in place of any inspection visits plan, including the optimal one.

In cases of large multi-component systems, point-based solvers may not be straightforwardly applicable, if at all, as they require full offline models for interstate transitions and cost functions, for all possible state and action combinations. In such large systems, deep reinforcement learning POMDP solutions provide an efficient alternative (Andriotis & Papakonstantinou, 2018) for quantifying the VoI and VoSHM, using exactly the same steps outlined in this paper.

5. CONCLUSIONS

A POMDP-based methodology for quantifying the Value of SHM is presented. We compute the VoSHM in deteriorating environments with uncertain action outcomes and incomplete information about the actual system state, which is probabilistically determined through noisy real-time observations and Bayesian updates. We determine the VoSHM based on the optimal POMDP policies of two different inspection settings. The first setting involves optional inspection visits, whereas the second setting operates under the assumption of continuous observations throughout the entire operational life, thus representing a permanent monitoring system. Both theory and results indicate that the presented methodology provides a simple and straightforward way to quantify the value of different SHM alternatives.

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