Maximum Response Estimation Method of an Oscillator Based on Natural Period-Dependent Spectrum Intensity

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ABSTRACT: This study proposes a maximum response estimation method based on natural period-dependent spectrum intensity, $SI_{n.p}$, for an oscillator with a bilinear hysteresis curve. The $SI_{n.p}$ value proposed by Kitahara and Itoh (1999) is defined as an integration of the elastic response spectrum not from 0.1 s to 2.5 s as defined by Housner (1952) but from the elastic first natural period of a structure to the first natural period elongated by its inelastic behavior, T_{el}. For simplicity, Kitahara and Itoh treated T_{el} as a constant value for a target structure; however, it is apparent that a proper T_{el} value depends on the response of the structure for each ground motion. Thus, this study determines the integration range assuming that T_{el} is optimally estimated on the basis of the secant slope at the maximum response of an oscillator for each ground motion. The accuracy of the estimation method based on the new $SI_{n.p}$ is demonstrated by using the nonlinear dynamic analysis results of various models of oscillators subjected to 1419 ground motions.

1. INTRODUCTION

For reliability-based seismic performance assessment and design of a structure, the structural demand needs to be evaluated in a probabilistic sense. Such information for a given structure can be obtained via a simulation-based approach (e.g. Collins et al., 1996) with nonlinear dynamic analysis (NDA). However, the seismic demand must be estimated for hundreds or thousands of earthquake records, which requires intensive computational effort. Thus, a predictor or an estimate of seismic structural demands that are less time-consuming than NDA, such as inter-story drifts, can be useful.

The Calculation of Response and Limit Strength method was introduced in the Japanese Building Code in 2000 as a seismic design rule for ordinary building structures. In this method, inter-story drifts are evaluated considering the inelastic first-modal response, although the higher-order modal response is neglected. Luco and Cornell (2001) proposed a predictor that uses the first two elastic modes and the square-root-of-sum-of-squares rule of modal combination. Originally used as a measure of ground motion intensity, their method considers the first-mode inelastic spectral displacement. Mori et al. (2004, 2006) proposed a predictor based on Luco and Cornell's predictor that considers a post-elastic deflected shape for the first modal vector. Naturally, each of these predictors has unique levels of simplicity and accuracy.

These techniques often use the maximum response computed via NDA of the inelastic oscillator that is equivalent to the original frame. Alternatively and more practically, the response of the equivalent oscillator can be estimated on the basis of several simpler methods such as the equivalent linearization technique (ELT), energy conservation rule, and displacement conservation rule using a single spectrum or an ensemble of elastic response spectra at the site. However, these techniques are not sufficiently accurate and may provide an optimistic estimate of the seismic structural demands.

Measurement factors of earthquake ground motion intensity such as peak ground velocity (PGV), elastic spectral response, and spectrum intensity (SI) (Housner, 1952) have also been used to predict the response of the equivalent oscillator by clarifying the relationship between the intensity index and the response. Numerous studies on ground motion intensity have shown that the appropriate intensity index for predicting the response of a structure is strongly dependent on the natural period of the target structure.

In view of the above problem, Kitahara and Itoh (1999) proposed a new earthquake ground motion intensity value known as natural period-dependent SI (noted as $SI_{n.p.}$ hereafter) that is applicable to bridge piers with a wide range of natural period. This value is defined as an integration of an elastic velocity response spectrum, Sv, not from 0.1 s to 2.5 s as defined by Housner but from the elastic first natural period of a structure, T_1, to the first natural period elongated by its inelastic behavior, T_{el}. Kitahara and Itoh conveniently treated T_{el} as a constant value for a target structure; however, it is apparent that proper T_{el} depends on the response of the structure for each earthquake ground motion. Kadas et al. (2011) proposed a modified $SI_{n.p.}$, also noted as $SI_{n.p.}$ for simplicity, for reinforced concrete (RC) frames. This value is calculated by integration of an elastic response spectrum between T_1 and T_{el} estimated for each ground motion. However, they did not thoroughly investigate the effects of hysteresis curve parameters such as post-yield stiffness on the estimate of the maximum response. Thus, the applicability and versatility of the integration ranges proposed by these studies are in question.

It is necessary to clarify the general relationship between the maximum response and $SI_{n.p.}$ for the maximum response estimation method based on $SI_{n.p.}$ with higher applicability and versatility. Thus, in this study, a new $SI_{n.p.}$ value is proposed, noted hereafter as SI_{μ}. The integration range of SI_{μ} is determined on the basis of T_{el} estimated by the maximum ductility factor, μ, of the structure calculated via NDA for each ground motion. Naturally, SI_{μ} cannot be used to predict the response directly; however, as proposed below, the maximum response can be predicted without NDA by modeling the relationship between the maximum response and SI_{μ} in advance.

The objective of this research is to propose a maximum response estimation method based on SI_{μ} for an oscillator with a bilinear hysteresis curve. The accuracy and applicability of the proposed method are investigated by using the NDA results of various models of oscillators subjected to 1419 ground motions.

2. PAST STUDIES ON NATURAL PERIOD-DEPENDENT SI

Kitahara and Itoh proposed $SI_{n.p.}$ with $Sv(T;h=0.2)$, in which T is the natural period and h is the damping factor of a structure, using integration ranges of $0.9T_1$ to $1.2T_1$ for a steel pier and $1.0T_1$ to $2.8T_1$ for an RC pier. The upper and lower limits of the integrations were determined on the basis of NDA results using 15 piers and 72 ground motions so that the correlation coefficient between $SI_{n.p.}$ and the maximum response of the piers was the highest. The selected correlation coefficient was 0.90–0.95, whereas the correlation coefficient between the PGVs and the maximum response was about 0.7–0.9.

Kadas et al. proposed the use of a weighted $SI_{n.p.}$ with an elastic acceleration response spectrum Sa at an integration range of T_1–T_{el} as

$$SI_{n.p.} = \int_{T_1}^{T_{el}} \frac{Sa(T;h=0.05)}{C_y \cdot g} \cdot \frac{T-T_1}{T_{el}-T_1} dT, \quad (1)$$

in which C_y is a yield-base shear coefficient, and g is gravity acceleration. They proposed to estimate T_{el} by Eq. (2), which is modeled on the basis of NDA results using 7 RC frames and 100 ground motions assuming that the T_{el} in Eq. (1) is optimally estimated with the secant slope at the maximum inter-story drift ratio of the structure, which is calculated via NDA.

$$T_{el} = 1.07 \cdot T_1 \cdot \left\{Sa(T_1; h = 0.05)/(C_y \cdot g)\right\}^{0.45} \quad (2)$$

The correlation coefficient between the maximum inter-story drift ratio and the $SI_{n.p.}$ proposed by Kadas et al. was found to be 0.792–0.992.

3. RELATIONSHIP BETWEEN MAXIMUM RESPONSE AND SI_μ

3.1. *Estimate of SI_μ*

In this study, SI_μ is estimated using the maximum ductility factor, μ, of an oscillator calculated via NDA as Eqs. (3) and (4). On the basis of the preliminary analysis results, an acceleration response spectrum with a damping factor equal to 0.20 was selected for estimating SI_μ.

$$SI_\mu = \int_{T_1}^{T_{el}} Sa(T; h = 0.20)\, dT \quad (\mathrm{mm/s}) \quad (3)$$

$$T_{el} = T_1 \cdot \sqrt{\frac{\mu}{1 - \alpha + \alpha \cdot \mu}} \quad (\mathrm{s}) \quad (4)$$

In these equations, α is post-yield stiffness ratio, and T_{el} was estimated on the basis of the secant slope at maximum displacement.

3.2. *Analytical model*

3.2.1. *Inelastic oscillator*

The oscillators with a bilinear hysteresis curve and the following characteristics were considered:

- Elastic natural period
  $T_1 = 0.1, 0.2, 0.35, 0.5, 1.0$, or 1.5 s
- Yield base shear coefficient
  $C_y = 0.2, 0.3, 0.4, 0.5, 0.6$, or 0.7
- Post-yield stiffness ratio
  $\alpha = 0.0, 0.1, 0.3, 0.5, 0.7$, or 0.9
- Damping factor
  $h_1 = 0.05$

3.2.2. *Ground motion record*

To consider the effect of the characteristics of various ground motions and the related uncertainties, 219 observed ground motions recorded mostly in the United States and Japan were used in addition to 1400 simulated ground motions.

Of the 219 observed ground motions, (Furukawa et al., 2017) 108 were intraplate earthquakes of moment magnitude $M = 6.0$–7.7, and the other 111 were interplate earthquakes ($M = 7.1$–8.0) include 91 Tohoku Region Pacific Coast earthquakes that occurred on March 11, 2011 (K-NET).

The simulated ground motions consist of 6 sets of 50 ground motions, which consider different types of earthquakes such as interplate or intraplate and soil conditions such as hard, medium, or soft (Mori et al. (2018)). The ground motions in each set were normalized such that their PGVs equaled 0.5, 1.0, 1.5, or 2.0 (m/s). The durations of the ground motions were set to 40.96 s and 163.84 s for intraplate and interplate earthquakes, respectively.

3.3. *Regression of maximum response on SI_μ*

Figure 1(a) shows the relationship between the maximum ductility factor, μ, of the oscillators with $T_1 = 0.5$, $\alpha = 0.0$, and $C_y = 0.2$, 0.4 and 0.6 calculated via NDA using the ground motions described in Section 3.2.2 and SI_μ in the forms of $\ln(\mu - 1)$ and $\ln(SI_\mu)$. It should be noted that of the results, only those with $1 < \mu < 20$, which are generally of concern in structural engineering, were plotted in the figure. Because they plotted closely along a linear line, the relationship can be modeled by a linear function. In the figure, the regression lines are also presented by solid lines as well as the standard error of the estimate, σ_{SI}. It is interesting to note that in Figure 1(a), the slopes of three regression lines

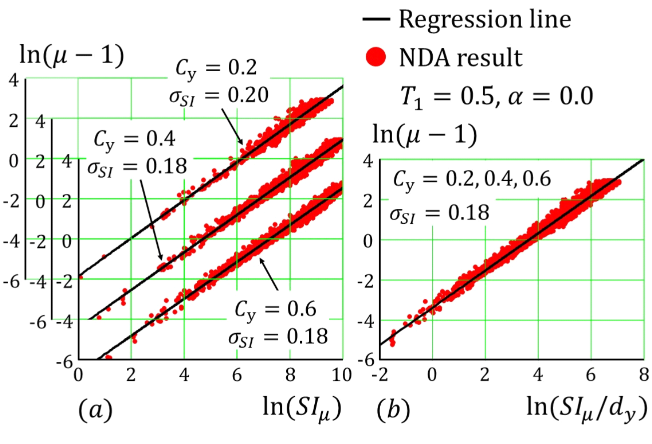

Figure 1: Relationship between $\ln(\mu - 1)$ *and* (a) $\ln(SI_\mu)$, (b) $\ln(SI_\mu/d_y)$, $(1 < \mu < 20)$

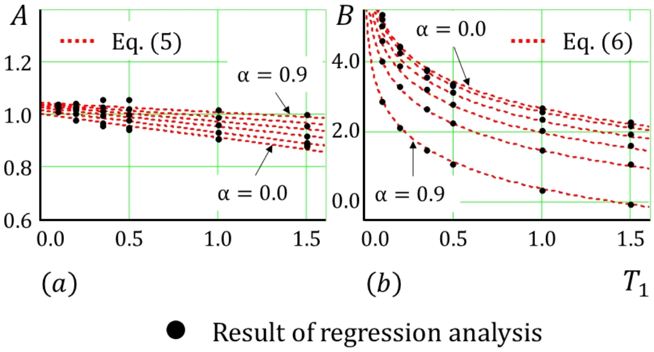

Figure 2: Regression coefficients and Eqs. (5) *and* (6)

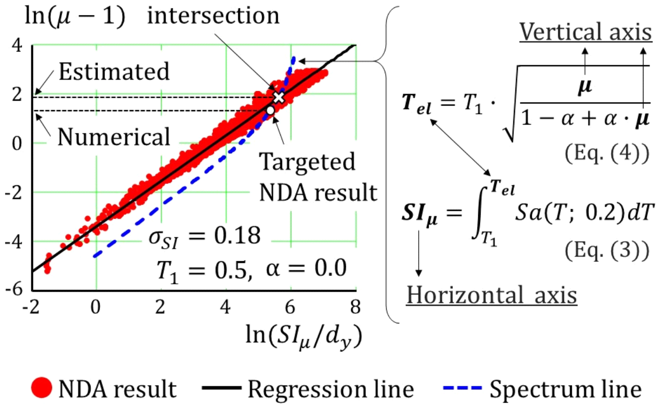

Figure 4: Maximum response estimation method

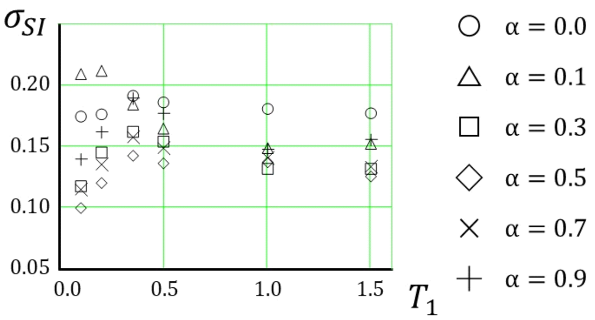

Figure 3: σ_{SI} *using Eqs.* (5) *and* (6)

are approximately equal to each other, and their intercepts tend to decrease proportionally as the yield displacement of the oscillators, d_y, according to C_y.

Considering these observations, the results plotted in Figure 1(a) were re-plotted in Figure 1(b) with new horizontal axis, $\ln(SI_\mu/d_y)$; all of the results then plotted closely to a single linear line regardless of C_y.

Similar results were obtained for the other oscillators. The slopes, A, and intercepts, B, of the regression lines as shown in Figure 1(b) are summarized in Figures 2(a) and 2(b), respectively. On the basis of regression analyses, the slopes and intercepts are modeled below.

$$A = (0.07\cdot\alpha-0.1)\cdot T_1+0.05\sqrt{\alpha}+1 \quad (5)$$
$$B = -1.1\cdot\ln(T_1)+\ln(14.3\cdot(1-\alpha)) \quad (6)$$

Figure 3 shows the standard error of the estimate, σ_{SI}, using Eqs. (5) and (6). The results are slightly larger than those using parameters directly obtained by the regression analysis as shown in Figure 1(b). However, the maximum was about 20% and generally small.

4. PROPOSED ESTIMATION METHOD OF MAXIMUM RESPONSE

The maximum response of an oscillator subjected to ground motion can be approximately estimated as the intersection of the regression line of $\ln(SI_\mu/d_y)$ on $\ln(\mu-1)$ and the relationship between SI_μ and μ via Eqs. (3) and (4) for each ground motion. Hereafter, this relationship is referred to as a spectrum line. The spectrum line can be drawn for each ground motion by treating μ in Eq. (4) as a variable and SI_μ in Eq. (3) as a function of μ. Figure 4 shows the NDA results and the regression line presented in Figure 1(b). The maximum response of the oscillator with $T_1=0.5$, $\alpha=0.0$ and $C_y=0.2$ subjected to one of the ground motions is shown by a white circle, and the spectrum line of the ground motion is shown by a dashed line. By definition, the NDA result of an oscillator to ground motion is always located on the spectrum line of the ground motion. The μ of the estimate and the μ calculated via NDA, hereafter referred to as μ_{pro} and μ_{NDA}, respectively, are equal to each other when σ_{SI} is equal to 0. This is because the NDA results of the oscillator to any ground motions are located on the regression line. It should be noted that σ_{SI} is less than about 0.2.

5. ACCURACY OF PROPOSED METHOD

5.1. *Bias and dispersion*

The accuracy of the proposed estimation method is expressed by its bias, a, defined by the median (or geometric mean) of μ_{pro}/μ_{NDA}, and its dispersion, σ, defined by the standard deviation of the

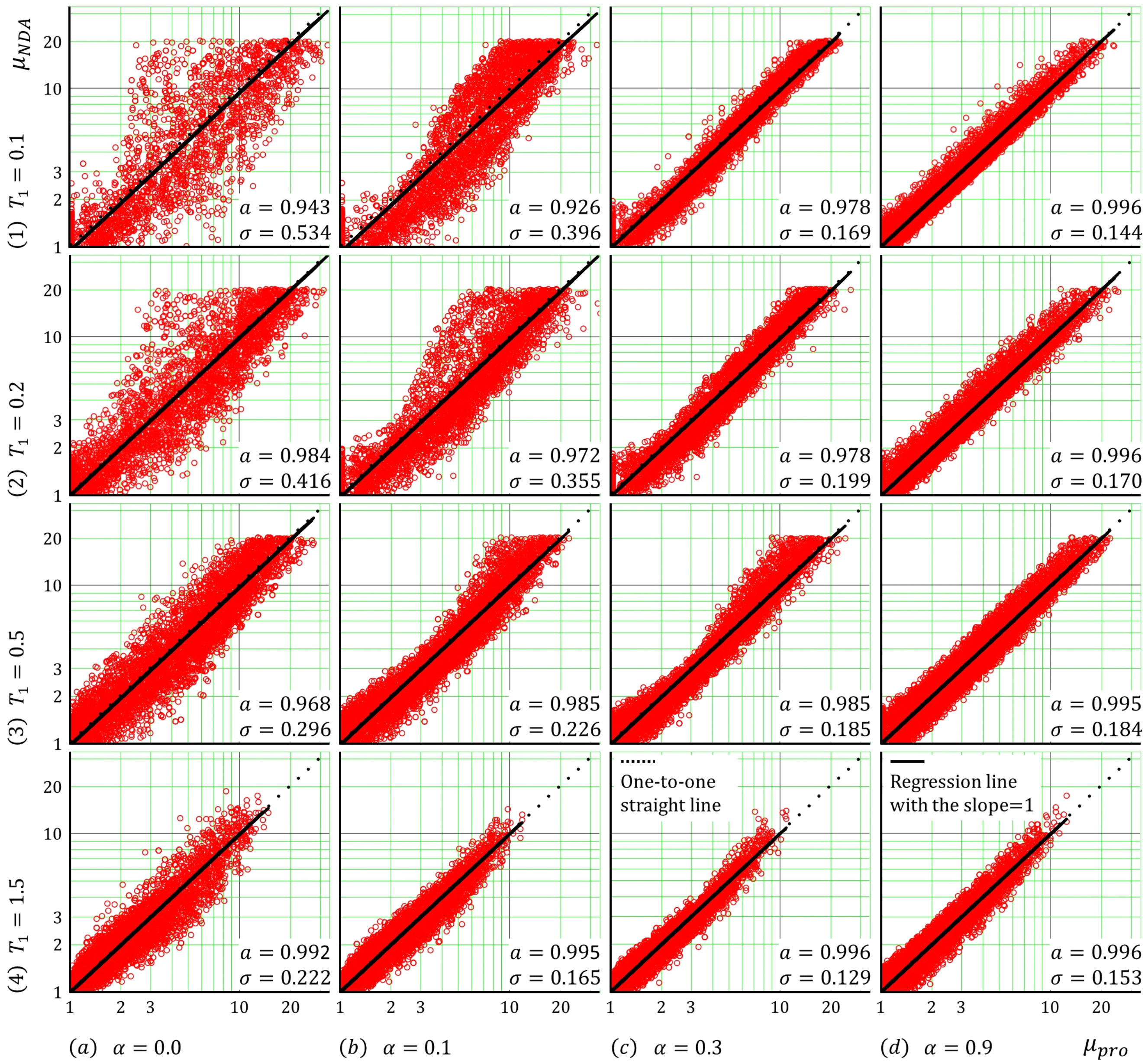

Figure 5: Comparison between μ_{NDA} and μ_{pro}, $(1 < \mu_{NDA} < 20)$

natural logarithms of μ_{pro}/μ_{NDA}. The bias and dispersion are equivalently obtained by performing a one-parameter log–log linear least-squares regressions of μ_{NDA} on μ_{pro}. The regression model is expressed by

$$\ln(\mu_{NDA}) = \ln(a) + \ln(\mu_{pro}) + \ln(\varepsilon), \qquad (7)$$

in which ε is the multiplicative random error in the model $\mu_{NDA} = a\mu_{pro}\varepsilon$ with (by definition) a median equal to unity and dispersion (standard deviation of $\ln(\varepsilon)$) σ.

5.2. *Results*

Figures 5(a-1)–5(d-4) illustrate the regressions of μ_{NDA} on the estimator μ_{pro} in the log–log scale for the oscillators with $T_1 =$ (1) 0.1, (2) 0.2, (3) 0.5, and (4) 1.5 and $\alpha =$ (a) 0.0, (b) 0.1, (c) 0.3, and (d) 0.9 subjected to the ground motions described in Section 3.2.2. Here, μ_{pro} is estimated by using regression lines calculated directly by regression analysis rather than by using Eqs. (5) and (6) to investigate the factors that could affect the dispersion of the estimator. The results of all oscillators with different C_y are presented together in each part of Figure 5 because similar results were obtained regardless of C_y, given a combination of T_1 and α. In each figure, the regression line with the slope equal to unity and the one-to-one straight line are presented by a solid line and a dotted line, respectively, along with "a" and "σ" values. Because a spectrum line is always located above a regression line of $\ln(SI_\mu/d_y)$ on $\ln(\mu-1)$, the lines do not cross each other. In this case, μ_{pro} was set to be equal to unity.

For all of the oscillators shown in Figure 5, a was fairly close to unity. On the contrary, σ became large when T_1 was short and α was small. When T_1 was 0.1 and α was 0.0, σ took the maximum value of 0.534, as shown in Figure 5(a-1). σ_{SI} presented in Figure 3 can be one of the factors that increases the dispersion of the estimate as described in Section 4. However, σ_{SI} for the oscillator with $T_1 = 0.1$ and $\alpha = 0.0$ was not significantly larger than those of the other oscillators. The difference between slopes of the regression line of $\ln(SI_\mu/d_y)$ on $\ln(\mu-1)$ and the spectrum line could be another factor. Figures 6(a-1)–6(b-2) shows the regression lines of $\ln(SI_\mu/d_y)$ on $\ln(\mu-1)$ and the spectrum lines of six randomly chosen ground motions for oscillators with $T_1 =$ (1) 0.1, (2) 1.5 and $\alpha =$ (a) 0.0, (b) 0.9. As shown in the figures, the slopes of the regression and spectrum lines were close to each other when T_1 and α were small. A similar tendency was observed for other ground motions described in Section 3.2.2. The estimator could have a large error when the slopes of these lines are similar to each other, and the small difference in location of the spectrum line could lead to a large difference in the point at which these two lines intersect.

The accuracy of the proposed method was investigated by comparing the estimation method using $SI_{n.p.}$, an ELT, the energy conservation rule, and the displacement conservation rule. Of the three estimates of $SI_{n.p.}$ described in Section 2, the $SI_{n.p.}$ proposed by Kitahara and Itoh for a steel pier was used here because its accuracy was the highest. The maximum response was estimated by substituting $SI_{n.p.}$ for a regression line of $\ln(SI_{n.p.}/d_y)$ on $\ln(\mu-1)$ obtained in this study directly by regression analysis. Among the ELTs proposed previously, the equivalent natural period and the equivalent damping factor proposed by Shimazaki (1999) and a damping reduction factor for response spectra proposed by Kasai et al. (2003) were used in this research.

Figures 7(a)–7(f) and 8(a)–8(f) show the bias, a, and the dispersion, σ, of the estimators as a function of the natural period of the oscillators with $\alpha =$ (a) 0.0, (b) 0.1, (c) 0.3, (d) 0.5, (e) 0.7, and (f) 0.9. For the proposed method, these figures present a and σ, obtained by using regression lines calculated directly by regression analysis for all oscillators described in Section 3.2.1 (●), and those by using approximate regression lines obtained using Eqs. (5) and (6) (▲).

As shown in Figure 7, the bias, a, of the proposed method based on direct regression analysis was approximately equal to unity for all of the oscillators. In addition, as shown in Figure 8, the dispersion, σ, of the method was smaller than that of the other methods when α was less than or equal to 0.5. Both a and σ of the proposed method using Eqs. (5) and (6) were approximately equal to those of the proposed method based on regression analysis except a for $\alpha = 0.0$ and 0.1. However, even for these two cases, a of the proposed method using Eqs. (5) and (6) was closer to unity than the a values of the ELT, energy conservation rule, and displacement conservation rule for most of the oscillators. a of the esti-

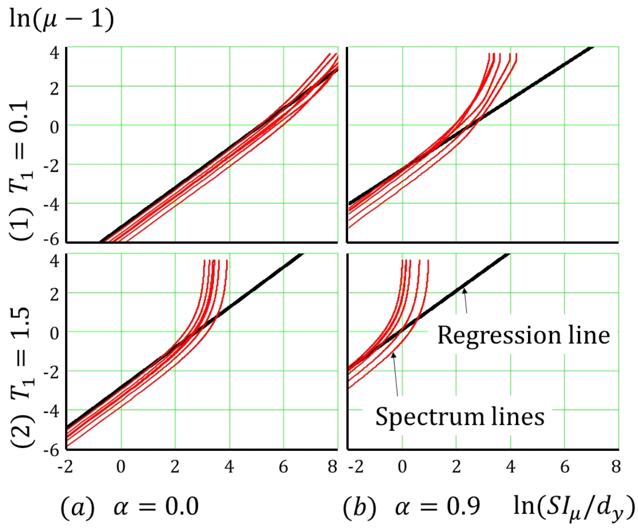

Figure 6: Regression lines and spectrum lines of six ground motions

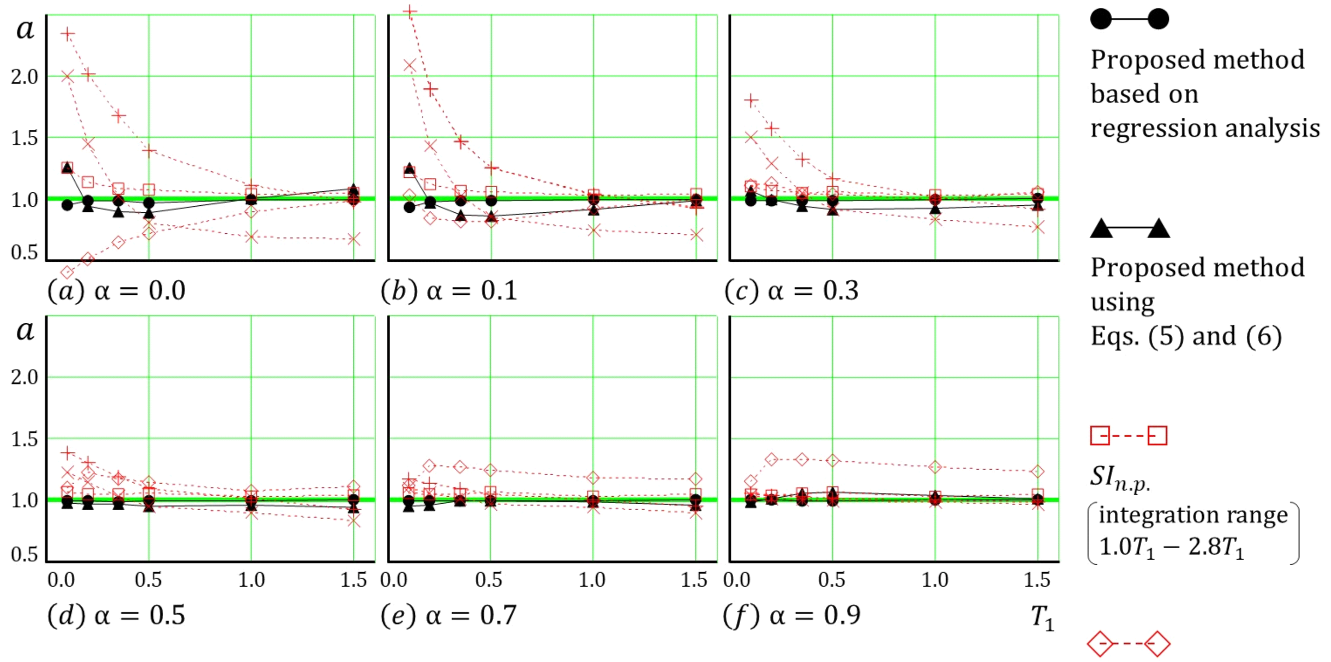

Figure 7: "a" for each estimation method of maximum response, $(1 < \mu_{NDA} < 20)$

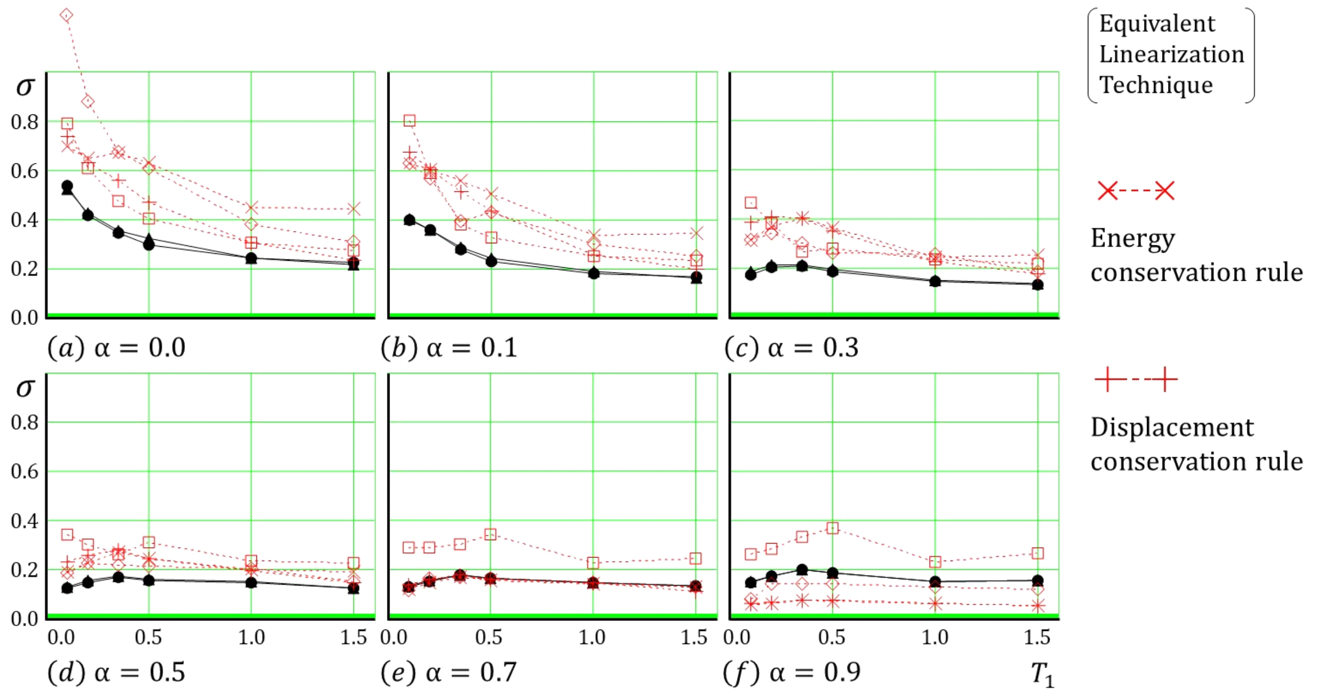

Figure 8: "σ" for each estimation method of maximum response, $(1 < \mu_{NDA} < 20)$

mation method using $SI_{n.p.}$ was also relatively close to unity for all of the oscillators. On the contrary, σ of the method was the largest among all estimation methods when α was equal to 0.7 and 0.9. The σ values of the ELT, energy conservation rule, and displacement conservation rule were similar to that of the proposed method when α was equal to 0.7. These values became smaller when α was equal to 0.9. This tendency can be inferred owing to the following fact: The σ values of these methods were 0 when α was equal to unity because these methods estimate the elastic maximum response with the elastic response spectrum. On the contrary, σ of the proposed method did not change significantly when α was larger than 0.3 because σ of the method depends on σ_{SI}.

6. CONCLUSIONS

This study proposed an estimation method for the maximum displacement of an oscillator with a bilinear hysteresis curve by using new natural period-dependent spectrum intensity with an integration range determined on the basis of T_{el} estimated for each ground motion. By using the proposed method, the effects of the spectral characteristics of earthquake ground motion and the parameters of the hysteresis curves on the maximum response can be considered. However, they cannot be considered in the estimation method based on $SI_{n.p.}$, in which the integration range is fixed for a target structure. The bias of the proposed method is generally closer to unity than that of the estimation method using $SI_{n.p.}$, an ELT, the energy conservation rule, and the displacement conservation rule. The dispersion, σ, of the proposed method was smaller than that of other methods expect for the oscillators with $\alpha = 0.9$.

Further investigations considering additional general hysteresis curves will be conducted to improve the applicability and versatility of the proposed method. Moreover, the seismic hazard at a site expressed in terms of $SI_{n.p.}$ as functions of T_1 and T_{el} will be studied to estimate the spectrum line.

7. ACKNOWLEDGMENTS

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