A new method for load and resistance factor design

Jiao Wang
Lecturer, Department of Architecture, Guangzhou University, China

Xue-Feng Wei
Graduate Student, Department of Architecture, Guangzhou University, China

Yan-Gang Zhao
Professor, Department of Architecture, Kanagawa University, Japan

ABSTRACT: The reliability-based load and resistance factors design (LRFD) has been widely used in the structural design codes. In almost all of the current reliability methods for the determination of the load and resistance factors, the basic random variables are assumed to have known probability distributions The third-moment method has been proposed to overcome the shortcomings (e.g. requirement of probability density functions (PDF) of random variables, inevitable iterative computation, requirement of design points) of other methods. However, in the existing third-moment method, the iterative computation is required. In this paper, the application of the existing third-moment method is inspected and a simpler method is proposed. In the computation of the target mean resistance, one-time iteration in the existing third-moment method is further simplified to no iteration, by changing the equation of the target reliability to another one. In addition, there is not any mathematical limitation in the new equation of the target reliability. From the proposed method in this paper, it can be concluded that: 1) the present method gives good improvement upon the method based on the third-moment method; 2) the computation of the existing third-moment method is further simplified to no iteration; 3) the limitations of applicable range in exiting third-moment methods are avoided; 4) the accuracy of the proposed method is proved to be higher than the exiting third-moment methods. With several examples, the comparison of the existing third-moment method, the ASCE method and the proposed method is given .The results show that the proposed method is accurate, simple and safe.

1. INTRODUCTION

In the past three decades the reliability-based load and resistance factors design (LRFD) has been widely used in the structural design codes (2010). The load and resistance factors are generally determined using the first order reliability method (FORM) in which, the design point must be firstly determined and then derivative-based iterations have to be used. Some simplified methods (2002; Zhao et al., 2000 and 2001; Lin et al., 2001; Lin et al., 2003; Nowak et al., 2000; Brranco et al., 2009) are proposed in order to avoid iteration computation. However, in almost all of the current methods, the basic random variables are assumed to have known distribution function. In reality the CDF/PDFs of some of the basic random variables are often unknown due to the lack of the statistical data. (Mori et al., 2002; Ugata et al., 2000)

Then the third-moment (3M) method, with no need for the design point or any assumption of the PDFs of random variables, was proposed. (Zhao et al., 2011 and 2012). In this method the computation of the target mean resistance is simplified to one iteration.

In this paper, the application of the existing 3M method is reviewed and a simpler method without iteration is proposed. In the computation of the target mean resistance, the iteration in the existing 3M method is eliminated, Moreover, there is not any mathematical limitation in the new equation of the target reliability. Compare the applicability of the ASCE method, the existing 3M method and the proposed method, the result shows that the proposed method is saving material, safe, accurate and simultaneously easier.

2. COMPUTATION PROCEDURE OF LOAD AND RESISTANCE FACTORES

2.1. Determination of Load and Resistance Factors

The LRFD format may be expressed as the follows:

$$\emptyset R_n \geq \sum \gamma_i Q_{ni} \tag{1}$$

where ϕ is the resistance factor, γ_i is the partial load factor to be applied to load S_i, R_n is the nominal value of the resistance, Q_{ni} is the nominal value of load Q_i.

In reliability-based design, the load and resistance factors ϕ and γ_i should be determined with a specified reliability, called the target reliability. Therefore, Eq. (1) should be probabilistically to the following equations：

$$G(X) = R - \sum Q_i \tag{2}$$

where R and Q_i are random variables representing uncertainty in the resistance and load effects, respectively.

For a given target reliability β_T or target probability of failure P_{fT}, Eq. (2) can be expressed in terms of probability:

$$\beta \geq \beta_T \quad \text{or} \quad P_f \leq P_{fT} \tag{3}$$

where β and P_f are the reliability and the probability of failure, respectively.

If R and Q_i are mutually independent normal random variables, the second-moment (2M) method is correct and the design formula is expressed as：

$$\beta_{2M} \geq \beta_T \tag{4}$$

where

$$\beta_{2M} = \frac{\mu_Z}{\sigma_Z}$$
$$\mu_z = \mu_R - \sum \mu Q_i \, , \tag{5a}$$

$$\sigma_Z = \sqrt{\sigma_R^2 + \sum \sigma_{Q_i}^2} \tag{5b}$$

where β_{2M} is the 2M reliability index; μ_Z and σ_Z are the mean value and standard deviation of the performance function $G(x)$, respectively; μ_R and σ_R are the mean value and standard deviation of R, respectively; and μ_{Qi} and σ_{Qi} are the mean value and standard deviation of Q_i, respectively.

Substituting Eq. (5) in Eq. (4), the load and resistance factors can be expressed as：

$$\mu_R(1-\alpha_R V_R \beta_T) \geq \sum \mu_{s_i}(1+\alpha_{Q_i} V_{Q_i} \beta_T) \tag{6}$$

Comparing Eq. (6) with Eq. (1), the load and resistance factors can be expressed as：

$$\phi = (1-\alpha_R V_R \beta_T)\frac{\mu_R}{R_n} \tag{7a}$$

$$\gamma_i = (1+\alpha_{Q_i} V_{Q_i} \beta_T)\frac{\mu_{Q_i}}{Q_{ni}} \tag{7b}$$

where V_R and V_{Qi} are the coefficient of variation for R and Q_i, respectively; and α_R and α_{Qi} are the senstivity coefficients of R and Q_i, respectively, where

$$\alpha_R = \frac{\sigma_R}{\sigma_Z}, \alpha_{S_i} = \frac{\sigma_{Q_i}}{\sigma_z} \tag{8}$$

As introduced above, the 2M method is based on the assumption of all the variables obey normal distribution and are independent of each other. In the case of R and Q_i are other random variables, the 2M reliability in Eq. (5) is incorrect. Therefore , other methods were proposed, typically, the FORM. [11] The load and resistance factors can be obtained as：

$$\phi = \frac{R^*}{R_n}, \quad \gamma_i = \frac{Q_i^*}{Q_{ni}} \tag{9}$$

where R^* and Q_i^* are the values of the variables R and Q_i, respectively, at the design point of the FORM.

2.2. Existing 3M Method for the Computation of Load and Resistance Factors

In the existing 3M method based on 3P-lognormal distribution the two steps recursive optimization is used to avoid the iteration computation:

$$\mu_{RT} = \sum \mu_{Q_i} + \beta_{2T}\sigma_Z \quad (10)$$

$$\mu_{R_0} = \sum \mu_{Q_i} + \sqrt{\beta_T^{3.5} \sum \sigma_{Q_i}^2} \quad (11)$$

where μ_{RT} is the target mean resistance; μ_{R0} is the original target mean resistance; σ_Z is the standard deviation of $G(\mathbf{X})$; and β_{2T} is the target 2M reliability, which is obtained by the 3M method

$$\beta_{3M} = -\frac{\alpha_{3Z}}{6} - \frac{3}{\alpha_{3Z}} \ln(1 - \frac{1}{3}\alpha_{3z}\beta_{2M}) \quad (12)$$

The inverse function of Equation (12) is expressed as：

where

$$\beta_{2T} = \frac{3}{\alpha_{3Z}} \left\{ 1 - \exp\left[\frac{\alpha_{3Z}}{3}(-\beta_{3T} - \frac{\alpha_{3Z}}{6}) \right] \right\} \quad (13)$$

$$\alpha_{3Z} = \frac{1}{\sigma_Z^3}(\alpha_{3R}\sigma_R^3 - \sum \alpha_{3s_i}\sigma_{Q_i}^3) \quad (14)$$

The steps for determining the load and resistance factors using this method are as follows

a. Calculate μ_{R_0} using Eq. (11).
b. Calculate σ_Z, α_{3Z} and β_{2T} using Eq. (5), Eq. (14) and Eq. (13), respectively.
c. Calculate μ_{RT} with Eq. (10).
d. Repeat step b with μ_{RT}. Then with the values of σ_Z, α_{3Z} and β_{2T}, calculate α_R and α_{S_i} with Eq. (8).
e. Determine the load and resistance factors with Eq. (7).

The shortcoming of the existing 3M method is that one iteration calculation of σ_Z, α_{3Z} and β_{2T} is inevitable. And Equation (13) is complicated. When Equation (12) is used for the calculation of 3M reliability, there is a mathematical limitation in as：

$$1 - \frac{1}{3}\alpha_{3z}\beta_{2M} > 0$$

which is

$$\alpha_{3z} < \frac{3}{\beta_{2M}}$$

3. PROPOSITION OF THE NEW METHOD

3.1. Computation Process of the Proposed Method

In order to overcome the shortcomings of Equation (12) and (13), a new model is proposed to replace Equation (12):

$$\beta_{3M} = \frac{1}{3}\beta_{2M}\left[2 + e^{\frac{1}{2}\alpha_{3Z}\left(\beta_{2M} - \frac{1}{\beta_{2M}}\right)} \right] \quad (15)$$

Regard β_{3M} and β_{2M} as the target 3M reliability index β_T and target 2M reliability index β_{2T} , β_{2T} can be expressed as the inverse function of Equation (15):

$$\beta_{2T} = \frac{3.8}{\alpha_{3Z}}\left[1 - e^{\frac{-\alpha_{3z}\beta_T}{3.8}} \right] \quad (16)$$

For the mathematic inversefunction of Equation (15) is inexistent, here Equation (16) is an approximate inverse function, which is used to replace Equation (13). Obviously, Equation (16) is simpler than Equation (13). In existing research Equation (15) is proved more applicable and accurate than the inversefunction of Equation (13). Moreover, there is not any mathematical limitation in Equation (15).

The convergence of the two steps recursive optimization Eq. (10) and Eq. (11) is inspected and then the following formula, with better convergence, is proposed to replace Eq. (10) and Eq. (11)

$$\mu_{RT-new} = \sum \mu_{s_i} + \left(\beta_T + \frac{1}{2\beta_T} \right)^{1.7} \sqrt{\sum \sigma_{S_i}^2} \quad (17)$$

The steps for determining the load and resistance factors using the new method are as follows:

1. Calculate μ_{RT} using Eq. (17).
2. Calculate σ_Z, α_{3Z} and β_{2T} using Eq. (5), Eq. (14) and Eq. (16), respectively. Then calculate α_R and α_{Si} with Eq. (8).
3. Determine the load and resistance factors with Eq. (7).

4. COMPARISION OF ASCE METHOD, THE EXISTING 3M METHOD, AND THE PROPOSED METHOD

In order to compare the application of the proposed method, the following example is considered (ASCE 7-10, C2.3.6)

$$G(X) = R - (D + L + S) \tag{18}$$

where R, D, L and S are the resistance, dead load, live load and snow load, respectively.

The load combination is the same as ASCE 7-10, combination 2 of Section 2.3.2. The details (the mean value, coefficient of variation and third moment) of the basic variables are shown in Table 1. Although the distribution types of loads and resistance are not necessary in the calculation of load and resistance factors, the distribution types are given in Table 1 for the Monte-Carlo (MC) simulation .

Table 1. *Basic random variables for Eq. (18)*

RVs	μ_{Qi}/D_n	V_i	$\sigma_{Qi} = \sigma_{Qi}/D_n \cdot V_i$	α_{3i}	μ_R/R_n or μ_{Qi}/Qi_n	Qi_n/D_n	Distribution
R	−	0.09	−	0.27	1.06	−	Lognormal
D	1	0.25	0.25	0	1.0	1	Normal
L	0.175	0.59	0.103	1.18	0.35	0.5	Gamma
S	0.6874	0.21	0.144	1.14	0.982	0.7	Gumbel

The results of load and resistance factors in different methods are listed in Table 2. The results show that the resistance factors ϕ of three methods are in great agreement. And the results of the live load factor γ_L and the snow load factor γ_S are also close, while the dead load factor γ_D in ASCE is slightly greater than that in existing and proposed 3M method.

Table 2. *Results of load and resistance factors in different methods*

	ϕ	γ_D	γ_L	γ_S
ASCE method	0.877	1.600	0.598	1.229
Existing 3M method	0.865	1.439	0.500	1.191
Proposed method	0.848	1.383	0.481	1.164

In order to compare the accuracy of three methods, with the load and resistance factors in Table 2, MC simulation (100,000 times) is used to calculate the reliabilities. For MC simulation, the calculation of μ_R/D_n is necessary (e.g. the proposed method):

$$\frac{\mu_R}{D_n} = \frac{\mu_R}{R_n} \cdot \frac{R_n}{D_n}$$
$$= \frac{\mu_R}{R_n} \cdot \frac{1}{\phi}\left(\gamma_D \cdot \frac{D_n}{D_n} + \gamma_L \cdot \frac{L_n}{D_n} + \gamma_S \cdot \frac{S_n}{D_n}\right)$$
$$= 3.05$$

With the values of μ_R/D_n, μ_D/D_n, μ_L/D_n and μ_S/D_n, the results of MC simulation are shown in Table 3, reliability 3.01 of the proposed method is closest to the target reliability 3.0. Therefore, the proposed method is considered accurate enough. The existing 3M method is also accurate and safe, but in this method, there are a lot of limitations and iteration is inevitable. The ASCE method is much simple, but the reliability by this method is much greater than the target reliability, which is safe but waste of structural materials.

Table 3. *Reliability of MC simulation with different methods*

	ASCE method	Existing 3M method	Proposed method
β	3.53	3.08	3.01

The above example is based on the assumption of $\beta_T = 3$. In ASCE 7-10 Section 1.5.1, building and other structures are classified to four different risk categories based on the risk to human life, health, and welfare associated with their damage or failure by nature of their occupancy or use. For different risk category and damage type, in ASCE 7-10 C1.3.1 the acceptable reliability indexes (β_T = 2.5 - 4.5) are provided for a 50-year service period. In this paper, β_T = 1.0 - 4.5 is chose to analyze the application of three different methods.

As shown in Figure 1, load and resistance factors calculated with the existing method, ASCE method and the proposed method are inconsistent. And the difference increases with the increase of the target reliability β_T. For resistance factor, the difference of three methods is slight.

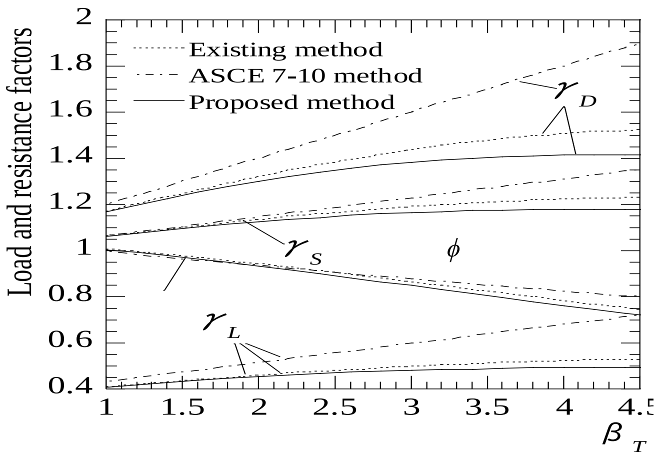

Figure 1. *Load and resistance factors calculated with different methods for* β_T *= 1.0 - 4.5*

5. CONCLUSION

a. The proposed method for load and resistance factors is simpler than the existing 3M method. The iteration in the computation of the target mean resistance in the existing 3M method is eliminated in the proposed method.

b. There is no mathematical limitation in the computation of the target reliability in the proposed method, while in the existing 3M method, the mathematical limitation is inevitable.

c. Compared with the ASCE method, the proposed method is considered safe and saving material.

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