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Probabilistic Analysis of Cantilever Sheet Pile Walls Using Copula Models

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ABSTRACT: Stability check of cantilever sheet pile walls by considering the variation in the soil properties shows that a minor change in value of properties can cause of failure of the whole structure. Factor of safety is used in deterministic analysis to account for the variability, but the probabilistic design is more practical compare to deterministic design, as the inherent variability is considered in probabilistic method. The objective of this paper is that the probabilistic analysis of cantilever sheet pile walls against overturning failure mode considering variability of soil parameters and water table position. To depict the real condition on cantilever sheet pile walls, the soil properties (including cohesion, friction angle, unit weight) and water table depth ratio (ratio between water table depth from the top of the wall and wall height above dredge line) are considered as random variables. The correlation of soil parameters likes cohesion, friction angle and unit weight is measured by multivariate copula models. Normal copula under elliptical class and Students' 't', Gumbel, Frank copula under Archimedean class are used. The parameters of the random variables are collected from the existing literature. The probability of failure against overturning mode is calculated by Monte Carlo simulation technique. In this paper a probabilistic model accounting correlation is proposed based on copula theory for designing cantilever sheet pile walls. The proposed method can be very effective in performing a site specific probabilistic analysis of it embedded in different soil conditions and variable water table position.

1. INTRODUCTION

Sheet pile walls are widely used as waterfront wall structures, river protection walls, excavation and temporary supports in foundation with high ground water table. Stability of the sheet pile generally depends on lateral passive resistance of the soil. The resistance depends on soil properties and their variations, surcharge loads, fluctuations of water table etc.

Uncertainties and imprecision are very much involved with geotechnical engineering which mostly deals with the natural material soil. The geotechnical properties of soil such as cohesion (c), friction angle (ϕ), unit weight (γ) can be measured by laboratory tests. But the testing shows a large variation because of its dependency upon so many things like borehole location, no of samples, the method of sample collection, boring methods, instrumental and human errors. The traditional deterministic analysis where single values assigned to the mechanical properties of the soil and a single factor estimating the safety of the whole system, regardless of the uncertainties pertaining to the variability of soil properties cannot fully explain the safety of the system.

The greatest challenge is to identify and quantify the uncertainty of soil properties. The probabilistic analysis offers the framework to encounter this challenge where the variables included in the analysis are expressed in probabilistic terms. This becomes possible when the statistical data i.e mean, standard deviation and distribution of each variable are well known.

The evolution of techniques to establish the correlations without compromising the desired accuracy is key for probabilistic analysis.

The application of traditional Pearson's correlation coefficient to consider the dependence of variables has been criticized by some researchers in regards to the statistical aspects. To obtain the correlation between two random variables joint probability distribution is needed and to construct join probability distribution of multivariate or bi-variate data copula theory is widely used. Copulas are functions that join the multivariate distribution to their one-dimensional marginal distribution. So many copulas are there in the literature such as Gaussian, Frank, Plackett, Gumbel, Clayton etc.

This study focuses on the reliability of the cantilever sheet pile in overturning failure mode. The probability of failure is obtained including the variation of soil properties (here ϕ and γ, assuming the back fill and foundation soil is sandy soil), fluctuation of the water table and penetration depth ratio using the Monte Carlo Simulation Technique. Firstly the probability of failure is calculated assuming that the ϕ and γ are uncorrelated and later the correlation is introduced to investigate the influence of correlation. The copula theory have been used to characterize the dependency in bi-variate context. Gaussian, Student's't', Frank, Gumbel copulas are used to study the effect of various copula on the probability of failure. Reliability indices are also obtained for various copulas.

2. LITERATURE REVIEWS

Generally, the traditional deterministic analysis of cantilever sheet pile can be done by using Rankine's earth pressure theory where the variation of the soil properties is not considered. To account the uncertainties associated with these soil properties a Factor of Safety (F.O.S) is used in this deterministic approach. But this F.O.S does not consider the sources and amount of uncertainty associated with the system. To accurately evaluate the safety of the system it is necessary to have the knowledge of the uncertainties associated with soil properties which are required in the design equation.

To show the real condition on the sheet pile, Basma (1990) first introduced the probabilistic techniques. The soil properties, water table heights and surcharge loads are treated as uncorrelated random variables. The variation in the penetration depth and maximum moment are evaluated and presented in a graphical form. The final design process involves the combination of the mean and variance of the design requirements based on a given probability of failure.

Cherubini (1998) suggested probabilistic approach to the design of sheet pile walls. The importance of probability density distribution and the fluctuation scale associated with soil variability in the probabilistic approach is stated. The probabilistic approach proposed as a means of estimating penetration depth. The author showed that F.O.S varies with the variations of the dimensionless ratio (H+D)/H and reliability influenced by the fluctuation scale and by the coefficient of variation of the geotechnical parameter involved (here, ϕ).

Baidya et al. (2013) analysed cantilever sheet pile walls in different soil conditions based on risk factor. Probability of failure (P_f) for overturning failure of the sheet pile walls is analyzed considering the soil properties as random variables. But correlation between the variables is ignored. The statistical data i.e. mean, co-efficient of variation (CoV) and the distribution of the variables are taken from the previous literature. For different water table heights F.O.S is calculated and probability of failure is also obtained using a performance function by Monte Carlo Simulation and presented graphically corresponding to coefficient of variations of different friction angle. Variation of unit weight has less effect compared to internal friction angle on stability of the sheet pile wall. Design recommendations (D/H ratio) for different variations of friction angle are provided.

Wu (2013) introduced the correlation among these soil properties (cohesion, friction angle, unit weight) and proposed a copula based methodology for prediction modelling. Two existing observed soil data sets from river banks are used to fit a trivariate Gaussian copula and a trivariate fully nested Frank copula. The ranking correlation coefficient Kendall's and the copula model parameters are estimated and the best-fitting model is chosen based on goodness-of-fit test. A series of triplet samples (i.e. cohesion, friction angle and unit weight) simulated from the trivariate normal copula with flexible marginal distributions are used as input parameters to determine the uncertainties of soil properties and to evaluate their correlations. The influence of the cross-correlation of these soil properties on reliability-based geotechnical design is showed with two simple geotechnical problems: (a) the bearing capacity of a shallow foundation resting on a clayey soil and (b) the stability of a cohesive-frictional soil in a planar slope. The sensitivity analysis of their correlations of random variables on the influence of the reliability index provides a better view into the role of the dependence structure in the reliability analysis of geotechnical engineering problems.

Wu (2015) found the dependence structure between shear strength parameters is negative and asymmetric. A set of paired samples of shear strength components simulated from the different bi-variate copulas. The Gaussian copula leads to an overestimation of the reliability index whereas the Gumbel copula yields the lowest reliability index.

3. VARIABILITY, DISTRIBUTION AND CORRELATION OF GEOTECHNICAL PARAMETERS

The soil parameters friction angle(ϕ) and unit weight(γ) are considered as random variables here. In the Table 1, statistical data of these random variables are given which are taken from GuhaRay and Baidya (2014).

Table 1: Statistical data of random variables.

Variables	unit weight(kN/m^3)	friction angle
Mean	19	40°
CoV(%)	3-7	10°-50°
Distribution	Gaussian	Log-normal

Water table depth ratio (i.e the ratio of water table depth from ground to sheet pile height above dredge line) and penetration depth ratio(i.e. penetration depth to sheet pile height above dredge line) of the sheet pile are also considered as random variables here. The maximum and minimum values are given in the Table 2, which are taken from Das (2009). Water table depth ratio and penetration depth are denoted as α and β respectively. Distribution is assumed as uniform distribution for these two variables.

Table 2: Ranges of values of α and β

Variables	Max.	Min.
α	1	0
β (for sandy soil)	2.5	1

The dependence between the random variables is mostly determined by the Pearson's coefficient of linear correlation. Negative correlation is mainly reported against the laboratory measurements. The friction angle of soil has a weak negative correlation with the unit weight (Wu, 2013).

For the simplification here CoV of unit weight (γ) considered as 7 % and CoV of friction angle (ϕ) considered as 20 % (GuhaRay and baidya, 2014).

4. DETERMINISTIC ANALYSIS OF CANTILEVER SHEET PILE

A typical cantilever sheet pile wall having a height Z above dredge line and penetration depth D is considered for the investigation. Water table is considered at depth Z_1 from the top. The dimensions are in metre. The earth pressure distributions for cohesion less soil are presented in the Figure 1 (Bowles, 1988).

Subsoil conditions are taken for analysis:

Cohesion less back fill and foundation soil with $\gamma = 19\ kN/m^3$ and $\phi = 40°$

The F.O.S against overturning based on Rankine's Method are given in the Eq. 1.

$$F.O.S = \frac{1/6P_4Z_4^2}{R(\bar{Z}+Z_4)+1/6(P_3+P_4)Z_5^2} \tag{1}$$

F.O.S obtained by deterministic analysis for different positions of water table are summarized in Table 3, assuming Z = 6m (GuhaRay and Baidya, 2014).

It is clear from Table 3 that F.O.S decreases with rise of water table. The F.O.S is found to be minimum for $Z_1/Z = 0$. Also it can be seen that the

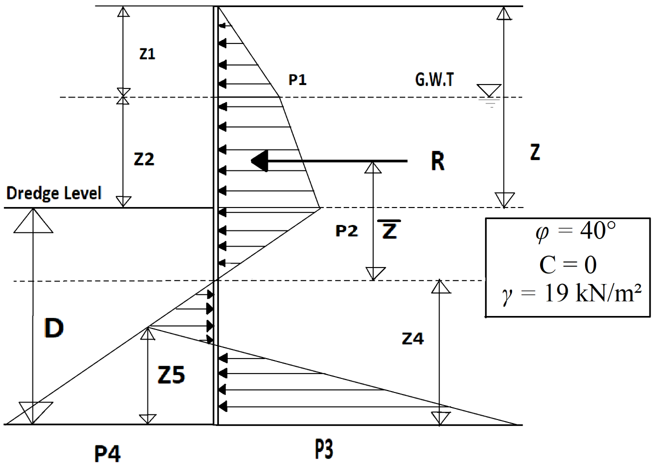

Figure 1: Earth pressure diagram of cantilever sheet pile in cohesion-less soil.

Table 3: F.O.S for various position of water table

Z_1/Z	0	0.2	0.6	0.8	1.0
F.O.S	1.658	1.665	1.699	1.723	1.749

lowest F.O.S for each case is greater than 1.5, that means the structure is safe.

5. COPULA APPROACH

In many cases of statistical modelling, it is essential to obtain the joint pdf between two or more random variables. Even though the marginal distributions of each of the random variables are known, their joint distributions may not be easy to derive from these marginal distributions.

A copula, C, is a function that joins or couples multiple distribution functions to their one dimensional marginal distribution functions (Nelsen, 2006). Application of copula to probability and statistics is achieved through Sklar Theorem (Sklar, 1959), which states that, if $H_{X,Y}$ is a joint distribution function, then there exists a copula C(u,v) such that for all x, y in $\overline{R} \in (-\infty, \infty)$,

$$H_{X,Y}(x,y) = C(F_X(x), F_Y(y)) \tag{2}$$

Where $F_X(x)$ and $F_Y(y)$ are the marginal distributions of X and Y respectively.

Copulas are functions that join or couple multivariate distribution functions to their one-dimensional marginal distribution functions. Alternatively, copulas are multivariate distribution functions whose one-dimensional marginal distributions are uniform in the interval of [0, 1]. Sklar's theorem is the foundation of almost all applications of the copula theory.

6. MODELLING OF SOIL PROPERTIES

The previous section presents the basic idea behind the copula theory. In this section, the copula approach is employed to model the distribution of soil parameters. The angle of friction ϕ, unit weight γ, water table depth ratio α and penetration depth ratio β are considered as random variables. To find out the correlation between angle of friction ϕ and unit weight γ, copula theory is used.

This copula approach includes the following three steps:

1. According the statistical data of the random variables generate random numbers.
2. Using the different copula, copula parameters are estimated and correlation between the desired paired of variables is found out.
3. Now generate the correlated random numbers to put them into the design equation to find out the probability of failure (P_f)

Here, four different copulas i.e. Gaussian, Student's't', Gumbel, Frank are used to check the effect of copulas on system probability.

7. MONTE CARLO SIMULATION METHOD

In today's literature of reliability, many reliability methods such as first order reliability method (FORM), second order reliability method (SORM) and monte carlo simulations (MCS) are available for determining the probability of failure. But FORM and SORM may result in error due to linearization at design point, to avoid this MCS is adopted to obtain probability of failure.

In geotechnical practice, geotechnical structures with different failure modes, such as slopes, retaining walls and footings are often studied as a series system . For such a system with k failure modes, incidence of any failure mode would lead to system failure. To formulate the series system reliability problem, g_i(X) (i = 1, 2, ... ,k) denoted as the performance function corresponding to the i-th failure mode, then the probability of failure of a series system, P_{fs} can be expressed as

$$P_{fs} = P[g_1(X) \leq 0 \cup g_2(X) \leq 0 \cup \ldots \cup g_k(X) \leq 0] \tag{3}$$

Random vector X may include other random variables such as geotechnical parameters friction angle and unit weight which is investigated in this study. The performance function, adopted for most geotechnical reliability problems is shown in the Eq. 4.

$$g_i(X) = F.O.S_i(X) - 1 \tag{4}$$

$F.O.S_i(X)$ is the factor of safety corresponding to the i-th failure mode.

Details of the procedure are summarized as follows: (1) Draw m physical samples $X_{m\times n}$ = [X_1,X_2,...,X_n] from the joint probability density function f(x_1,x_2,....,x_n) of X . This process can be further divided into three steps. First, m samples of independent standard uniform variables $V_{m\times n}$ = (V_1,V_2,...,V_n) are produced by computing software: $V_{m\times n}$ = rand(m, n). Second, the vector $V_{m\times n}$ are transformed into the correlated standard uniform variables $U_{m\times n}$ = (U_1,U_2,...,U_n) for a specified copula. The algorithms for transforming an independent standard uniform vector V into a correlated standard uniform vector U for a specified copula are referred (Nelsen, 2006). Li et al. (2015) presented the algorithms for converting V to U for the selected four bi-variate copulas step by step, which will be used in this study. Third, the physical samples $X_{m\times n}$ = [X_1,X_2,...,X_n] can be easily obtained using the isoprobabilistic transformation. Set U_1 = $F_1(X_1)$,U_2 = $F_2(X_2)$,..,U_n = $F_n(X_n)$, then $X_{m\times n}$ = [X_1,X_2,..,X_n] = [$F_1^{-1}(U_1)$, $F_2^{-1}(U_2)$,..., $F_n^{-1}(U_n)$] in which F_1^{-1}(.), F_2^{-1}(.),.., F_n^{-1}(.) are the inverse CDFs of X_1, X_2,..., X_n respectively. (2) Substitute the physical samples $X_{m\times n}$ into the performance function $g_i(X)$ corresponding to failure modes 1, 2, ... ,k, respectively. For a specified sample $X_{j\times n}$ (j = 1, 2, ... ,m), the system failure will occur if any of $g_i(X_{j\times n})$ is less than or equal to zero. (3) Calculate the number (Num) that the overturning failure occurs over m samples. Then, the probability of failure for the overturning failure is obtained as P_f = Num/m.

8. RESULTS AND DISCUSSION

Estimated probability of failure (P_f) without considering correlation is shown in Table 4. It is determined by using MCS by using Eq. 1.

Table 4: Probability of failure without correlation

Iteration No	P_f
1,00,000	0.0295
2,00,000	0.0281

Estimated probability of failure (P_f) considering correlation for different types of copula (i.e. Gaussian, Students' 't', Gumbel, Frank) is summarized in the Table 5 for 1,00,000 and 2,00,000 iterations respectively.

Table 5: Probability of failure for different types of copula

	Probability of Failure	
Iteration no / Copulas	1,00,000	2,00,000
Gaussian	0.0697	0.068
Students' 't"	0.0713	0.0692
Gumbel	0.0735	0.0707
Frank	0.695	0.0711

It is clear that the probability of failure produced by different copulas does not differ considerably. The probability of failure for Students' 't' copula is the smallest among the that of other four selected copulas. On the other hand, it is the highest for Frank copula.

Scatter plots of friction angle and unit weight from the bi-variate Gaussian copula are shown in Fig. 2. 2,00,000 samples were selected.

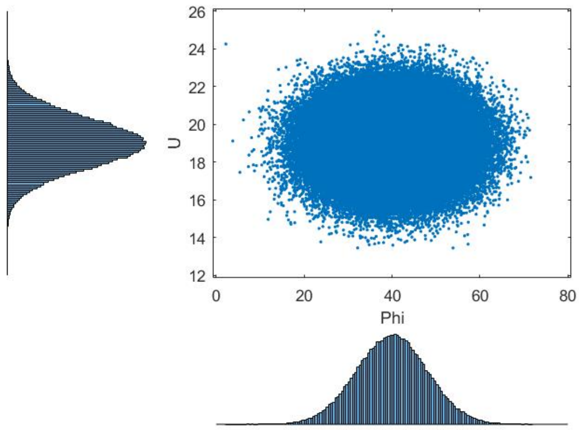

Figure 2: Scatter plot of correlated values of ϕ and γ using Gaussian copula.

Cornell (1971) proposed an index for the evaluation of safety, called the reliability index (β), which is given by the ratio of the safety margin expected value, to its standard deviation.

To calculate the reliability index, the following steps should be taken into consideration:

1. Find out the number of simulations.

2. For each simulation, generate values of ϕ, γ from the statistical data, including the correlation also.

3. Calculate F.O.S and assume the F.O.S as a normally distributed random variable.

4. Calculate the mean and standard deviation of F.O.S. The reliability indices can be calculated for different types of copula using Eq. 5.

$$Reliability \quad Index = \frac{\mu(F.O.S) - 1}{\sigma(F.O.S)} \tag{5}$$

To show the effect of correlation between soil parameters (ϕ, γ) on system reliability the reliability index values without considering correlation are summarized in the Table 6 and with considering correlation for different types of copulas are summarized in Table 7.

Table 6: Reliability Index without correlation

Iteration No	Reliability Index
1,00,000	2.5926
2,00,000	2.62

Table 7: Reliability indices for different types of copula

Copulas \ Iteration no	Reliability Index	
	1,00,000	2,00,000
Gaussian	1.8558	1.8560
Students' 't"	1.8412	1.8515
Gumbel	1.8195	1.8124
Frank	1.8473	1.8381

It is clear from the tables that reliability indices differ significantly when correlation is taken into account.

9. CONCLUSION

Cantilever sheet pile walls has been analyzed with the help of copula theory in a bi-variate context to characterize the dependency between soil parameters (ϕ, γ for cohesion less soil). The water table depth ratio, penetration depth ratio, friction angle (ϕ), unit weight (γ) are considered as random variable. The correlation between ϕ and γ is modeled by Gaussian, Students' 't', Gumbel, Frank copula.

From the results, the importance of the correlation is very much clear. Without considering correlation, the reliability index is significantly high comparing to when that is considered. The actual probability of failure is high when the correlation is considered between soil parameters. So, it is really important to consider the correlation between the soil properties to get the clear picture of system failure. From this study, it is known that reliability index for different copulas is close to each other. So, in that case any kind of copula can be used to model the dependency between soil parameters. But for the site specific data, particular copula may be best fitted, that should be checked.

10. REFERENCES

Cherubini, C. (2000). "Probabilistic approach to the design of anchored sheet pile walls." *Computers and Geotechnics*, 26(4), 309-330.

Bowles, J.E. (1988). "Foundation Analysis and Design." *McGraw-Hill International Editions*, 4th edn.

Basma, A.A. (1990). "Reliability-based design of sheet pile structures." *Reliability Engineering and System Safety*, 33(2), 215–30.

GuhaRay, A. and Baidya, D.K. (2012). "Reliability coupled Sensitivity based Design Approach for Gravity Retaining Walls." *Journal of the Institution of Engineers, India: Series A, Springer*, 93(3), 193-201.

Harr, M.E. (1984). "Reliability-based design in Civil Engineering." *Dept. of Civil Engineering, School of Engineering, North Carolina State University, Raleigh, N.C.*

Kulhawy, F. H. (1992). "On the evaluation of static soil properties, stability and performance of slopes and embankments II." *Seed RB, Boulanger RW, editors. GSP 31, ASCE, Berkeley, California*, 95–115.

GuhaRay, A. and Baidya, D. K. (2014). "Risk factor based analysis of sheet pile walls in different soil conditions." *Indian Geotechnical Conference, Kolkata Chapter, Kolkata*.

Wu, X. Z. (2013). "Trivariate analysis of soil ranking-correlated characteristics and its application to probabilistic stability assessments in geotechnical engineering problems." *Soils and Foundations*, 53(4), 540–556.

Cornell, C.A. (1971). "First order uncertainties analysis of soils deformation and stability." *Proc. ICASP1*, 129-44.

Wu, X. Z. (2015). "Modelling dependence structures of soil shear strength data with bi-variate copulas and applications to geotechnical reliability analysis." *Soils and Foundations*, 55(5), 1243-1258.

Sklar, A. (1959). "Fonctions de Répartition à n Dimensions et Leurs Marges." *Institut Statistique de l'Université de Paris*, Paris, 229-231.

Nelsen, R. B. (2006). *An Introduction to Copulas, Lecture Notes in Statistics*. Springer, NewYork.

Das, B. M. (2009). "Principles of Foundation Engineering." *Design Guides, Temporary Sheet pile design, Cengage Learning*, 7th edition, 3.13.1 - 6.

Li. D. Q, Tang. X. S, Zhou. C. B, Phoon. K. K, (2015). "Characterization of uncertainty in probabilistic model using bootstrap method and its application to reliability of piles." *Applied Mathematical Modelling*, 39, 5310–5326.