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Application of continuous gamma process for structural capacity evaluation in the presence of diverse, site specific,data; GAP-ID

Tatyana, Micic
Dept. of Civil Engineering, City, University of London, London, UK

ABSTRACT:
This paper reports on application of stochastic modeling for capacity loss, GAP-ID, procedure to establish critical failure mode. Using the example of an existing simply supported reinforced concrete solid slab this paper will address application of continuous gamma process to include information from monitoring sources. At the selected decision time, continuous gamma process parameters are established using available prior data. This feature provides for account of accuracy and nature of prior data and establishes prediction model for structural capacity. As a result, GAP-ID provides a consistent and rigorous estimate of capacity using current data that is particularly useful as a long term planning tool. This methodology also enables inclusion of future data such as updated climate scenarios or from implementation of new monitoring techniques.

1. INTRODUCTION

Modern infrastructures such as bridges, tunnels, masts, etc. are expected to provide uninterrupted service for increasing numbers of users while being exposed to changing environmental conditions in comparison with their construction time. New technology is providing facilities for increasing monitoring of multiple factors that can affect infrastructure such as environmental parameters, however, with variable accuracy and in the absence of benchmarks from past performance the use of newly available information is challenging. As a consequence costly decisions in terms of repair or replacement of infrastructure components can arise. Models that integrate emerging quantitative and qualitative data from various monitoring sources with structural capacity models associated with established codified design and inclusion of site specific data for ageing structures are still in development.

There are many aspects that are specific to existing infrastructure:

- Performance has been acceptable in the past
- Gross errors associated with design and construction can be ignored
- Some physical measurements can be obtained
- Site specific deterioration could have occurred
- Distinct change in operational conditions might have occurred
- Future performance requirements might be different from the design stage
- Planning for maintenance and repair might be constrained
- Remedial action might be required and could initiate large costs

This is why there are many alternative approaches for quantification of capacity such as, VanNoortwijk, (2009) and Frangopol et al. (2004).

2. INFRASTRUCTURE AGEING

It has been long accepted Pandey et al. (2009) that uncertainties associated with infrastructure vary over the life-cycle.

1. It is well known that in design uncertainties associated with site specific parameters such as environmental conditions, composition of the soil are accompanied by modelling uncertainty associated with analysis and design, uncertain quality of construction, etc. However, qual-

ity control procedures areincreasingly in place and modern codes of practice are available. Initial costs imposed by modern construction practice are large but predictable, human error effects could be significant.

2. Over the life-cycle the repair and maintenance records are site specific and for long life-cycle environmental conditions are changing. Changes in environmental conditions could be a result of pollution, urbanization, climate change, etc. Uncertainty associated with site specific parameters is normally increasing however impact of human error, for example, is reducing. The change in infrastructure is a continuous process and parameters such as loss of capacity are progressive. Due to changes in policy, economic development and local factors some agreed practice for repair, maintenance, assessment might be in place but unlikely to have been consistent over the time. Regulatory documentation might also be available but often generic rather than detail targeted , JCSS. Costs of intervention for existing structures are small per annum but cumulatively could be significant. The cost of loss of facility is often significantly larger than the cost of life cycle processes.
3. Assessment of structure is associated with a specific time so uncertainties are associated with cumulative effects that are site specific in respect to capacity, prior repair, maintenance, retrofit, etc. Assessment requires a deterministic outcome on the basis of uncertain information and therefore methodologies are needed that enable inclusion of uncertainties that are site specific. Regulatory documentation is difficult to put in place for numerous highly uncertain parameters. Due to high costs and significant consequences of decisions in assessment processing of data to obtain quantitative data is critical for owners.
4. Since the 80's as a consequence of rapid infrastructure development post World War II the volume of infrastructure that is subject to assessment, monitoring, repair, etc is continuously increasing. Coupled with lack of data from the previous years the challenges for authorities and owners are not abating.

2.1. *Data in Assessment*

Data in assessment is distinctly site specific. Certain physical properties, such as cover to reinforcement for reinforced concrete structures is likely to have higher variability than at the time of construction. This variability is likely to be spatial so for a reinforced concrete slabs that might be exposed to external effects, drainage is a significant factor and the effect of reinforcement cover variability can become critical. Statistical samples for site specific parameters are small, for a reinforced concrete bridge deck measurement of reinforcement cover can only be carried out for a specific number of locations rather than continuously. These measurements have to be carried out at site specific locations, dependent on site specific deterioration, maintenance records, repair records, etc. Performance requirements are site specific as well, for a highway bridge the composition of the traffic, in terms of weights and frequencies, might have changed, surrounding area development might have generated particular requirements that were not present at the time of design and construction. As far as the loading is concerned, monitoring techniques can generate more substantial data samples and therefore the notional balance of uncertainties is likely to have changed between initial time (at completion) and current time (time of assessment or similar).

2.2. *Capacity Data in Assessment*

Capacity data for infrastructure is site specific and time dependent. Due to inevitable ageing process, that is a natural phenomena, influenced by repair and maintenance, it's uncertainty is time-dependent and, as a consequence, the potential that there is insufficient capacity for required performance is potentially hazardous. While in design capacity is a function of characteristic parameters established by regulatory documentation in assessment these parameters have to be identified on the basis of site specific conditions and available information. Different models have been implemented Marsh and Frangopol (2008).

- Deterministic values using expert judgment
- Random variable/ random field models
- Stochastic process models

Justification for use of deterministic properties for an existing infrastructure is often difficult, due to uncertainties identified already. Random variable models have been used extensively in design Pandey et al. (2009) but the justification for use of such models for assessment implies establishment of random variable properties on the basis of limited data. Increasingly stochastic process models are being seen as a viable methodology to characterize capacity of existing infrastructure. Stochastic processes are a suitable model for time variant properties as they reflect incremental, non-negative nature of the deterioration progress. Several authors have identified that continuous stochastic processes are more appropriate than discrete ones, VanNoortwijk, (2009). In recent years due to increasing focus on effects of changing climate, continuous stochastic process models for capacity are seen as suitable for probabilistic prediction models.

3. CAPACITY AGEING MODELLING

In particular, continuous gamma process has emerged as an effective probabilistic model to represent loss of structural capacity due to ageing processes. It is well known that many bridge owners rely on regular visual inspections but that they are also introducing new sensors for monitoring. GAP-ID is a methodology for inclusion of such diverse, site specific information.

3.1. Continuous Gamma process model for capacity

The Gamma process is a stochastic process with independent non-negative increments having gamma distribution with a constant scale parameter and a time dependent shape function for modelling gradually accumulating damage in a sequence of small increments, VanNoortwijk, (2009). We consider a variable X with gamma distribution characterized with shape factor $\alpha > 0$ and scale parameter $\beta > 0$, VanNoortwijk, (2009), Ohadi and Micic (2011).

$$G(x|\alpha,\beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} \exp(-\beta x)$$

where Γ function is for $\alpha > 0$

$$\Gamma(\alpha) = \int_{z=0}^{\infty} z^{\alpha-1} e^{-z} dz$$

4. REINFORCED CONCRETE SLAB PERFORMANCE MODELLING

Reinforced concrete slabs are frequently found in framed structures but also in short span highway bridges. In the presence of uncertainties, such as areas of reinforcement, cover to the reinforcement, extent of corrosion, etc. their performance is significantly affected by environmental conditions. In this paper we consider a square solid slab that is fully built in on two adjacent edges an investigate the effect of ageing on critical failure modes. Such slabs are often orthotropic and with different exposure conditions on the bottom and top face. for example when used as balconies on residential framed buildings, Thoft-Christensen and Pirzada (1984).

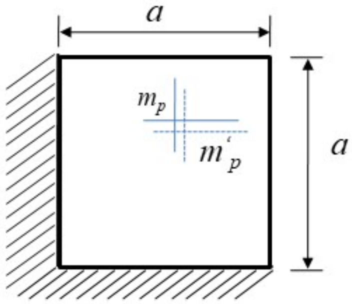

Figure 1: Sample square reinforced concrete slab built-in on two adjacent edges.

For the expression of relevant limit state the yield line approach is often implemented. This is a standard analytical technique for slabs but it is particularly useful for structural analysis in the presence of uncertainties as data for relevant random variables such as strength of materials are obtained at yield (failure). Assuming a yield line pattern such as the one shown in Figure 1 if a unit virtual displacement is imposed at a point on the slab, rotations of yield lines are expressed as functions of displacements, slab geometry and the support conditions. In general, for a valid yield line pattern, deterministic formulation of governing virtual work equation includes reference to a proportional load.

When the structure is subject to ageing it is exposed to the load that due to different levels of uncertainty cannot be characterized as proportional and in addition, the capacity is variable, thus for a valid yield line pattern the virtual work equation can be formulated:

$$M = g(X,\gamma) = \sum_{yl_l} \mathbf{m_p}(\mathbf{x},\mathbf{y})\theta(x,y,\gamma_i) -$$

$$\sum_{reg} \int\int \mathbf{w_j}(\mathbf{x},\mathbf{y})\delta(\mathbf{x},\mathbf{y},\gamma_i)\mathbf{dxdy} = \mathbf{0}$$

where m_p is the flexural resistance, θ rotation about the yield line, w_j represents the applied load, δ is the displacement field. The set of factors γ_i is introduced to define the geometry of the yield line pattern. This formulation is effectively a safety margin for a slab subject to lateral load and can be used to evaluate the reliability index for the structure.

This formulation is suitable for analysis of structures as it is linear in terms of random variables, m_p and w_i and it is acceptable to implement the Cornell reliability index formulation, Haldar and Mahadeven (1999). We can write:

$$\beta(X,\gamma) = \frac{\mu_g}{\sigma_g} = \frac{f_{M_p}(\gamma_i)\mu_{M_p} - \sum f_{w_j}(\gamma_i)\mu_{w_j}}{\sigma_g}$$

$$i,j = 1,2,...,n$$

where μ_g is the mean value of governing failure function and σ_g its standard deviation. It is evident that the most relevant is the minimum reliability index that is a function of configuration of the yield pattern given by γ_i parameters. We can write:

$$\frac{\partial\beta(X,\gamma_i)}{\partial\gamma_i} = \frac{\partial}{\partial\gamma_i}\left(\frac{\mu_g}{\sigma_g}\right) = \frac{\frac{\partial\mu_g}{\partial\gamma_i}\sigma_g - \frac{\partial\sigma_g}{\partial\gamma_i}\mu_g}{\sigma_g^2}$$

By implementing simple optimization and transformation of the above equation we obtain the expression for the minimum reliability index:

$$\frac{\partial\mu_g}{\partial\gamma_i}\sigma_g - \frac{\partial\sigma_g}{\partial\gamma_i}\mu_g = 0 \qquad i = 1,2,....,n \qquad (1)$$

This is a set of equations that can provide the solution for the critical failure mode (form of the yield line pattern) that is independent of random variable characteristics in the simplest case of one capacity and one loading random variable. However, in assessment, multiple uncertainties emerge and this differentiation is more complex.

4.1. Safety margin for slab supported on two adjacent edges

For the reinforced concrete orthotropic slab shown in Figure 1 there are two distinct viable yield line patterns that can develop when the slab is subject to lateral load as shown in Figure 2. One pattern results in sagging yield line and the other one results in hogging yield line. The ratio between hogging and sagging capacities determines which pattern would govern. The linear safety margin for pattern 1 is:

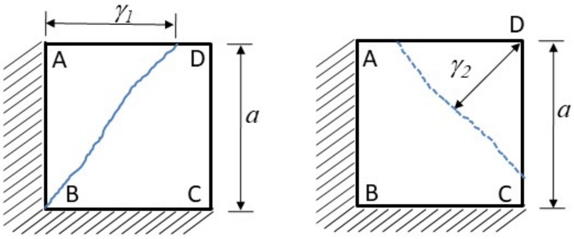

Figure 2: Yield line patterns for square solid slab

$$M_1 = 6M_p\left(\frac{a}{\gamma} + \frac{\gamma_1}{a}\right) - Pa(3a - \gamma_1) = 0$$

where M_p is the random variable that represents the sagging capacity, γ_1 is the parameter defining the yield line pattern and P is the applied load. For simplicity, using the Cornel formulation we find that the minimum reliability index is found for:

$$\left(-\frac{a}{\gamma_1^2} + \frac{1}{a}\right)(3a - \gamma_1) + \left(\frac{a}{\gamma_1} + \frac{\gamma 1}{a}\right) = 0$$

This function is independent of random variables and the critical $\gamma_1 = 0.721a$. For pattern 2 we have:

$$M_2 = M_p^{'}\frac{a}{\gamma_2} - \frac{1}{6}Pa\gamma_2 = 0$$

where $M_p^{'}$ is a random variable representing hogging capacity. Using Cornel approach the minimum reliability index is associated with $\gamma_2 = 0.707a$.

Thus, for such slab, critical yield line pattern will depend on the ratio between the hogging and sagging capacities. Namely when $M_p^{'}$ is less than $0.46M_p$ the Pattern 2 governs and otherwise Pattern 1 is governing, Thoft-Christensen and Pirzada (1984) . This is significant as the exposure conditions could be different for top and bottom face of the slab. Therefore, due to increasing complexity of the optimization in Eq. (1) application of FOSM approach becomes cumbersome.

4.2. *Ageing effects for RC slab supported on two adjacent edges*

For the reinforced concrete slab, such as the one described above the effects of ageing could be significant as the flexural capacity determines the critical pattern that can develop. We consider that the slab is subject to ageing and identify that effects of ageing on the bottom and top reinforcement are represented through loss of respective flexural capacities. In many circumstances, such as external balcony application the exposure is different for bottom and top reinforcement and equally measurement of reinforcement loss is likely to be of different accuracy. A sample of outcomes is described in Table 1 where all capacities are expressed in terms of assumed $M_{p,o}$. The assumed flexural capacity $M_{p,o}$ is a convenient format to relate the flexural capacities provided by bottom and top reinforcement. For a slab of this format it is expected that sagging capacity is higher than hogging one, however, from the consideration above it is evident that in case of ageing dominant failure mode could change from sagging to hogging one. For ageing structure it would be beneficial to have quantitative information about the dominant failure mode. Information in this format can be used to obtain parameters of the continuous gamma process for characterization of the capacity loss. Several references have detailed descriptions for evaluation of these parameters, such as, VanNoortwijk, (2009). Here, using the method of moments, and data from Table 1 we obtain capacity projections for sagging and hogging flexural capacities. For sagging flexural capacity three projections for capacity loss are shown in Figure 3. This information can be used to establish sagging capacity loss with certain probability at different times. This has been evaluated for both sagging and hogging capacities using 50% probability of exceedance. Equally, for the hogging capacity, probability distribution functions are shown for the same projections. It should be borne in mind that the initial hogging capacity is assumed in this example to be lower than the sagging one. From data in the illustrative example above we can identify critical yield line patterns for projected time horizons as shown in Table 3. In this illustrative ex-

Table 1: Slab capacity data sample(in terms of $M_{p,o}$)

yr	M_p	$M_{p_{post}}$	$M_p^{'}$	$M_{p_{post}}^{'}$	D_{M_p}	$D_{M_p^{'}}$
0	1.00	1.00	0.50	0.500		
5	1.00	1.00	0.50	0.495		0.005
10	1.00	0.99	0.49	0.490	0.01	0.010
15	0.99	0.97	0.49	0.482	0.03	0.018
20	0.97	0.94	0.482	0.466	0.06	0.034

Table 2: Continuous gamma process parameters for sagging and hogging flexural capacity at year 20

Capacity	β	α_{30}	α_{40}	α_{50}
M_p	0.5	4.5	6.0	7.5
$M_p^{'}$	0.47	2.4	3.2	4.0

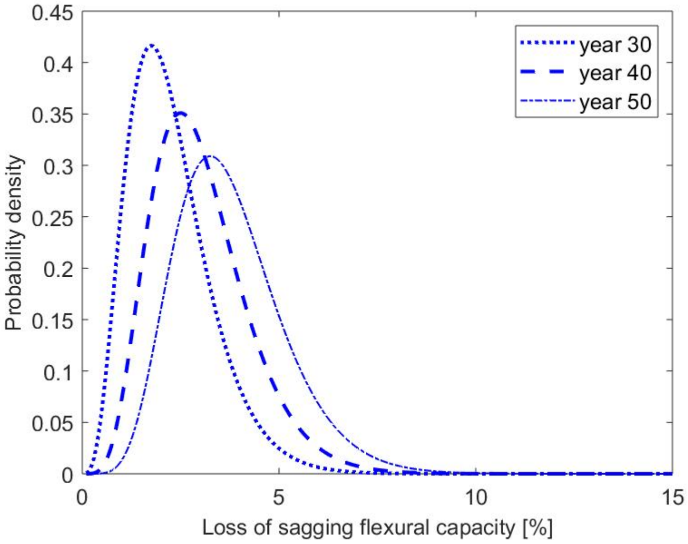

Figure 3: Projected loss of sagging capacity using data from year 20 (as a percentage of $M_{p,o}$)

Table 3: Critical yield line patterns

	$y = 0$	$y = 10$	$y = 30$	$y = 40$
M_p'/M_p	0.5	0.495	0.467	0.476

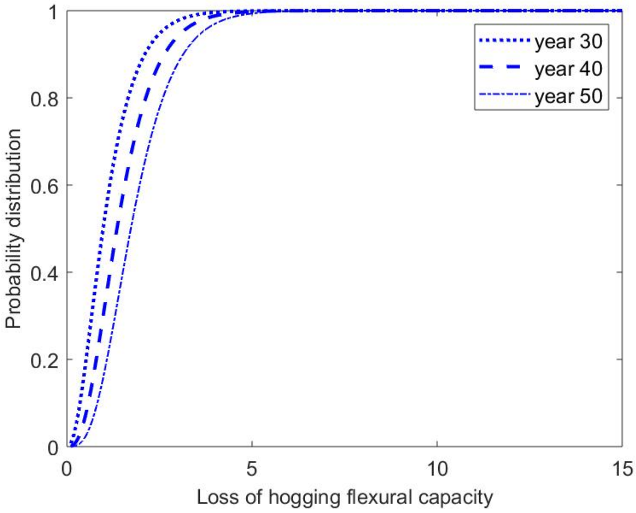

Figure 4: Projected loss of hogging capacity using data from year 20 (as a percentage of $M_{p,o}$)

ample, it can be observed that due to particular loss of hogging flexural capacity pattern 2 could develop between years 20 and 40. In this example that is the case in year 24 however this outcome is highly site specific. This information can be used to identify repair/maintenance action in anticipation of ageing effects. Furthermore, with targeted preventative action development of either of the patterns can be prevented. Information in Figure 3 can be used to establish probability of critical capacity loss at relevant times. In summary, rather then carrying out FOSM reliability analysis GAP-ID provides a methodology that incorporates site specific capacity loss and its notional values can be updated on the basis of quality of available data and frequency of data acquisition. Thus,

- Site specific data is used to update projections for capacity
- On the basis of site monitoring relevant loads can be established using statistical samples generated through application of site specific technologies
- Once uncertainty is quantified through stochastic gamma process and characteristic loads established using monitoring techniques, further analysis can be deterministic
- The outcomes of the analysis remain notional but site specific analysis enables planning for future action, repair, monitoring, replacement, etc. In this particular case yield line pattern governs if there is a cracking ceiling or floor.

In comparison with FOSM methodologies and FOSM combined with optimization to establish the critical limit state as described above, application of stochastic gamma process is significantly simpler.

5. CONCLUSIONS

This paper has considered development of the critical yield line pattern in the presence of capacity loss for ageing structures. It has been demonstrated that the critical yield line pattern for such slabs could be established using hybrid methodology that involves Cornell/Optimization approach, however only for cases with single resistance and sing load random variable. In more complex cases that characterize ageing structures such representation is too simplistic and more complex representations represent challenging numerical approach. The continuous gamma process is proposed as an efficient approach to represent capacity loss in a sample structure. The application of such models is gaining popularity due to its straightforward modelling. Using an illustrative example it has been confirmed in this paper that by using the method of moments it was possible to establish scale and shape parameters for continuous gamma process representation for hogging and sagging flexural capacity. Using assumed inspection outcomes in the first 20 years of structural life cycle projected capacity losses are established. It was identified that using such approach it is possible to identify governing failure mode (here represented with viable yield line pattern). These outcomes provide excellent base for inclusion of up to date, site specific, loading parameters as well. The proposed analysis can reflect quality of site-specific data and, thus, include uncertainties associated with inspection outcomes progressively. The methodology is relatively straightforward, and therefore very powerful, when authorities or owners have large and diverse portfolios of structures. While the outcomes of the probabilistic analysis are, as ever, notional, it is feasible that benchmarks for particular structural types could be established.

The outcomes could provide guidance on priority for repair and maintenance as development of alternative failure modes brings different problems as has been demonstrated in the illustrative example.

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