13th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP13
Seoul, South Korea, May 26-30, 2019

Performance of Variable Partial Factor approach in a slope design

Mika Knuuti
M.Sc, Dept. of Civil Engineering, Tampere University of Technology, Tampere, Finland

Tim Länsivaara
Professor, Dept. of Civil Engineering, Tampere University of Technology, Tampere, Finland

ABSTRACT: Most of the design codes have moved from traditional total factor of safety method to the partial factor approach, aiming to cover the uncertainties better. The target has been to reach more consistent safety levels, but it has not always obtained. This has raised more interest towards reliability based design and its applications. In this paper, the performance of two partial factor approaches were compared from the reliability point of view; eurocode 7 design approach 3 and proposed Variable Partial Factor approach. The results show that the partial factor method with fixed partial factors cannot fully cover the uncertainties related to the design. The partial factors should be dependent on the level of uncertainty of the parameters. The results also shows that RBD can be applied in designer friendly way. In addition, some challenges in the determination of the characteristic values were pointed out.

1. INTRODUCTION

The traditional total factor of safety approach has much been used in geotechnical design. However, it has also been widely criticized as there is no link between total factor of safety and probability of failure. In other words, high total factor of safety does not always imply a low probability of failure, and vice versa. The probability of failure is greatly influenced by the uncertainties related to the design. Partly this is the reason why most design codes, including eurocode 7, are nowadays using the partial safety factor approach, aiming to cover these uncertainties better. However, the performance of these methods from reliability point of view has been questionable.

This has been led to growing attention towards reliability based design (RBD). Even though the use of rather complex full reliability based design seems to be still far from everyday engineering, it can be utilized in several, more designer friendly ways. One way is to use partial factor method with factors that are determined based on RBD. This way the factors would also depend on number and accuracy of the site investigations.

In this paper, a simple earth slope is evaluated with two different partial factor methods. The other method applies fixed partial factor and the other varying partial factors, calibrated with RBD. The aim is to compare how well these methods can cover the true uncertainties in the design, by comparing the design results to those obtained with probabilistic analyses. Also the challenges in the determination of a characteristic value of a parameter is discussed.

2. THE PROBABILISTIC METHODS FOR SLOPE STABILITY ANALYSES

Multiple probabilistic methods can be used for slope stability calculations. Each method has certain characteristics, but usually they can be divided into two larger groups: approximate methods and fully probabilistic methods. Even though in many cases approximate methods are suitable because of their ease of use, small computational effort and sufficient accuracy, in this paper fully probabilistic methods are used, because those are already implemented in our slope stability analysis package. The two most used fully probabilistic methods are the Monte Carlo simulation (MC) and Latin Hypercube sampling (LH) (McKay 1979).

The Monte Carlo simulation is a method that seeks to simulate stochastic processes by random selection of input values to a limit state function (model) based on their joint probability density function. The main advantages of this method are that it is rather easy to use, it is powerful solving method and it is applicable to both linear and non-linear problems. However, it may sometimes require a large number of simulations to provide a reliable distribution for the depending parameter, especially if the probability of failure is very small.

Latin Hypercube sampling is also a fully probabilistic method to create nearly random response of the limit state function. The method is based on stratified sampling where cumulative density function is divided into equal partitions, which usually equals the number of samples. Then a random point is chosen from each partition ensuring that the distribution is sampled evenly. The advantages of this method are that it reduces the computational time compared to Monte Carlo simulation and if the output is dominated by some input parameters, the method ensures that each of those input variables are represented in a fully stratified manner (McKay 1979).

3. RELIABILITY BASED DESIGN BACKGROUND

3.1. Limit state function G

Limit state function defines the boundary between acceptable and unacceptable behavior of the system or mechanism. Limit state function can be for example an ultimate limit state (ULS) (bearing capacity) or a serviceability limit state (SLS) (acceptable settlement). One way of presenting the limit state function is as shown in Eq. (1), where R represents resistances and S represents actions or action effects.

$$G(R,S)=R-S \qquad (1)$$

Limit state function is considered differently depending on design methodology. In traditional factor of safety design (i.e. EN1997-1), variables R and S are considered as deterministic values which partial factors are applied to. This may lead to very unambiguous results if the system is stable or not (single value).

$$R/S>1 \qquad (2)$$

In probabilistic design approach, variables R and S are considered as random variables, which accounts the uncertainty related to each parameter This approach leads to nondeterministic answer having no longer a single answer. Results are presented as a probability of failure (p_f) of the system.

$$p_f=P(R-S<0) \qquad (3)$$

Furthermore, in the sense of convenience, interpretation and better understanding these probabilities of failure can be converted to more representative form, the reliability index (β).

$$p_f=\varphi(-\beta) \qquad (4)$$

3.2. Random variables

A random variable is typical parameter in RBD calculations. Random variable can be for example a soil parameter, action or some uncertainty (model uncertainty, tolerance etc.). In definition, random variable means a function that assigns a certain real value for each outcome with a certain probability in sample space S. In other words, single deterministic parameters are described with their probability density functions or cumulative distribution functions where the parameter gets “random” value.

3.3. Target reliability

The aim of the reliability based design is to obtain a certain acceptable target reliability (β_T) for a given design problem. In eurocode 0 annex B (EN1990), guideline values for target reliabilities are given for 1-year and 50-years reference periods and for different reliability classes (RC). For 1-year reference period the values are 5.2; 4.7 and 4.2 and for 50-years reference period 4.3; 3.8 and 3.3, corresponding reliability classes RC1, RC2 and RC3 respectively. These values are recommendations and it has been left for each country to choose appropriate values. In this study, the recommended values are used as the basis of calculations.

4. FOR SLOPE STABILITY

4.1. Eurocode 7 design approach 3 (EC7 DA3)

Eurocode 7 provides three different design approaches for ULS design (EN1997-1). The design approaches 1 (DA1) and 3 (DA3) seems to be the most chosen methods for slope stability design among European countries (Bond 2013). In both methods, the partial factors are applied to soil strength and to actions. The recommended values for these factors are $y_{ø}$'=y_{c}'=1.25 on soil strength in a effective stress analysis and y_{cu}=1.4 for total stress analysis. Subscripts ø', c' and cu refer to effective friction angle, effective cohesion and undrained shear strength respectively. On the actions side, only variable loads are factored with y_{Q}=1.3 whereas permanent loads are factored with y_{G}=1.0 (EN1997-1).

4.2. Variable partial factor approach (VPF)

The Variable Partial Factor approach is based on the same design idea than that in previously described design approach 3, but now with a different set of partial factors. In this approach, the partial factors are calculated based on reliability theory. A full description of the Reliability Based Design based partial factors for slope stability are presented in the paper of Länsivaara and Poutanen (2013). However, the main points are included herein;

1. All safety is put into the material partial factor
2. The material partial factor depends on the uncertainty of the material
3. The consequence of failure should influence the material partial factor (not load)

The partial factors for the soil strength can be chosen from figure 1. The partial factors presented in the figure are as a function of coefficient of variation (COV), reliability class and a target reliability index (RI) (Länsivaara and Poutanen 2013). Also the calculated values include the following assumptions;

1. the soil weight is left unfactored, but the uncertainty involved can be accounted for by including it in the material factor. The COV- value for permanent load was set to 0.1.
2. for the variable load a COV-value of 0.25 was used
3. normal distributions with dependent combination were used for permanent and variable loads
4. a lognormal distribution was used for material strength

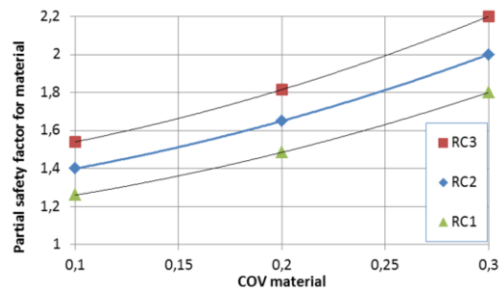

***Figure 1**. New partial factors for soil strength for different reliability classes and coefficient of variations. All load factors are equal to 1.0.*

The proposed method aims for a better coverage of the true uncertainties, resulting in a more consistent probability of failure. The advantage of the proposed method is that it covers better the true uncertainties related to initial parameters and considers the target reliability level. This will lead to more safe and economic designs. However, it is unrealistic to assume, that in everyday geotechnical problems the variability in soil parameters can be determined exactly based on conducted soil investigations, or even that all sources of uncertainty are known. Therefore, the design standards should include the basic requirements for typical design cases if there is no better information available. Few countries have already implemented this kind of basic guidelines succesfully into their design standards: Canada (Fenton et al 2016), USA (Allen 2013), Japan (Honjo et al 2009,2010) and the Netherlands (Vrouwenvelder et al 2013).

4.2.1. Partial factors for Variable Partial Factor approach

Considering the difficulties that might encounter, it is not convenient to determine partial factors independently for each design case in everyday practice. A better approach would be to create for example a classification system (i.e. Fenton et al 2016) depending on some factors (e.g. coefficient of variation, soil investigation methods) from which the designer can choose the appropriate partial factors

For example, effective strength parameters determined from triaxial tests, usually have a low variability. In this case values corresponding to COV=0.1 could be chosen from the graphs (Länsivaara and Poutanen 2013). However, if the parameters are determined solely based on soundings, higher values of COV would be a more appropriate choice, depending on the sounding method.

4.3. Characteristic value

Characteristic value is often chosen as a "cautious estimate of the value affecting the occurrence of the limit state" (EN1997-1). This definition leaves lot of room for geotechnical engineers to choose appropriate value based on for example soil investigations, literature, experience and structure characteristics and is therefore a popular yet vague choice.

Another recommendation in eurocode 7 is the statistical approach, which is getting lot of attention nowadays. This method proposes the selection of characteristic value statistically, if sufficient data is available. The characteristic value corresponds the value that "the calculated probability of a worse value governing the occurrence of limit state under consideration is not greater than 5%". This 5 % fractal value for the mean value with confidence level of 95%, can be calculated with Eq. (5):

$$x_k = x_{mean} - k_n \cdot \sigma_x / \sqrt{(n)} \quad (5)$$

,where x_k is the characteristic value, x_{mean} is the mean value of parameter x, σ_x is the standard deviation of the parameter x, k_n is a statistical multiplier and n is the number of observation points (e.g. soil investigation points). According to the eurocode the choice of the characteristic value covers the variation of uncertainty of the property, and a fixed partial factor can be used.

It is good to note that the simple statistical procedure given by Eq. (5) does not cover everything related to a cautious estimate. For example, it does not consider the limit state in questions and its zone of influence, considerations about how representative a measured value is, nor the possible correlation between parameters.

5. CALCULATION EXAMPLES

5.1. Studied case example

To study the performance of the presented design methods, and to compare these methods to probabilistic design, a parametric case example is conducted.

The case example is taken from literature as a benchmark case, which is already studied widely for example in Bhattacharya et al (2003), Hassan and Wolff (1999) and Li and Lumb (1987). In these papers, different probabilistic methods were used in order to calculate the reliability index of the slope and/or to verify new calculation methods. A short summary from results in previous studies is given in table 1, where different abbreviations are used to separate the critical deterministic slip surface (*cdss*) results from the critical probabilistic slip surface (*cpss*) results. FS_{min} and β_{FS} stands for the minimum factor of safety and the reliability index of the *cdss* whereas FS_β and β_{min} are the corresponding factors of the *cpss*. These results are not discussed here more precisely and the reader is advised to look at the reference papers.

Table 1. Previous results for the studied design example.

Method	β_{FS}	β_{min}	FS_{min}	FS_β
Hassan and Wolff (1999)	*2.336*	*2.293*	*1.33*	-
Li and Lumb (1987)	-	*2.500*	-	-
Bhattacharya et al. (2003)	*2.306*	*2.239*	*1.32*	*1.33*

The case example consists of a simple sand slope shown in figure 2. The parameters affecting to the slope stability are the effective friction angle *φ'*, effective cohesion *c'*, the pore water pressure ratio r_u and the unit weight *y*. From these parameters, the effective friction angle and the effective cohesion are treated as independent random variables whereas the pore water pressure ratio r_u and the unit weight *y* are considered as deterministic values. Lognormal distributions are chosen for the effective cohesion *c'* and the effective friction angle *φ'*. Numerical values for parameters used in calculations are given in table 2.The search for the most critical *cdss* and *cpss* was done with Spencer's method, which satisfies all equilibrium conditions. Moreover, for simplicity, the search was done with circular slip surfaces.

Table 2. Statistical properties of the soil parameters.

Param.	Mean	COV [%]	Std.Dev.
c'	18.0 kPa	20	3.6 kPa
φ'	30°	10	3°
r_u	0.2	10	0.02
y	18 kN/m^3	5	0.9 kN/m^3

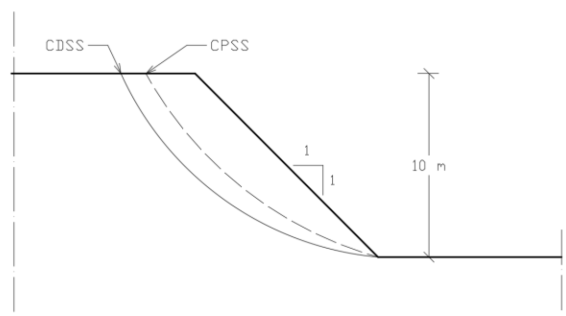

Figure 2. Homogenous sand slope with the slope inclination of 1:1.

5.2. Comparison to previous calculations

The reliability indices and factors of safety for the *cdss* and *cpss* were calculated with different reliability methods and slope analysis methods. For Monte Carlo simulation and Latin Hypercube sampling 10 000 samples was found to be satisfactory to produce stable results. The results for comparison calculations are shown in table 3.

Table 3. Results from probabilistic analyses. (LEM=limit equilibrium method, FEM=finite element method)

Method	β_{FS}	β_{min}	FS_{min}	FS_{β}
Determin.	-	-	1.33	-
LEM, MC	2.342	2.297	1.33	1.34
LEM, LH	2.327	2.283	1.33	1.36

The obtained results with different design methods are similar to those presented in table 1, although some differences exists due to different assumptions made regarding probability distribution type, choice of reliability method etc. The most beneficial result is that the reliability indices are similar between *cdss* and *cpss*. This could lead to decreased computation time in further calculations since it is faster to find the *cdss* instead of *cpss*. However, in this study this information was not been used since in cases where the variation in initial parameters is large, the location of *cpss* could differ a lot from that of *cdss*.

5.3. Partial factor design

After the probabilistic analyses, the slope was analysed with two partial factor methods; the eurocode 7 design approach 3 and the Variable Partial Factor approach. For both methods, the characteristic values of the effective cohesion and the effective friction angle were determined by using the statistical approach presented in eurocodes. The characteristic values were calculated with a lognormal version of the Eq. (5) (EN1990) by assuming that ten soil investigations points are available from the site. The k_n- value is 1.645 assuming that the COV- value is known.

The recommended partial factors are used for eurocode 7 design approach 3, whereas for Variable Partial Factor approach, the partial factors were chosen based on figure 1 and the corresponding COV- value of the parameter.

5.4. Case 1

The first calculation case is to analyse the slope with the information shown in table 2. The calculated characteristic values for the effective cohesion and effective friction angle are c'_k*=16.2* *kPa* and $\varphi'_k=28.5°$ respectively. The results of the stability calculations are shown in table 4.

Table 4. Calculation results for the two partial factor methods.

Design method	*ODF-value*	*β*
EC7 DA3	*0.98*	*≈2.4 (table 5)*
VPF	*0.80*	

For both design methods, the ODF-values remain under the threshold of 1.0. This means that further actions are needed in order to secure the stability of the slope. Further actions could be reinforcements, comprehensive risk analysis if the current design values are acceptable after all or conduct vaster and more precise soil investigations. The comprehensive risk analysis could be possible for the EC7 DA3 since the ODF- value is close to the threshold. This would lead to reliability index of around 2.4 (table 4).

The ODF-value obtained with Variable Partial Factor approach is much smaller than the required 1.0. This implies that more extensive actions are required compared to the eurocode result, in order to obtain the defined safety level.

5.4.1. Case 2

In the second case, the performance of partial factor methods is studied from the reliability point of view. This is done by assuming three different COV- values for the effective stress parameters; COV=0.1, COV=0.2 and COV=0.3. These COV-values represents the possible varying uncertainty related to the data set derived with different soil investigation methods and/or from other sources. Both strength parameters have the same coefficient of variation in each design situation, which differs from case 1. This assumption is based on the presumption that both parameters are derived from a triaxial test. In addition, the effective stress parameters are assumed uncorrelated since the statistical approach (EN1990 Annex D) does not account for correlation between multiple properties.

The calculated characteristic values (Eq. (5)) for both effective strength parameters with different COV- values are presented in table 5. It can be noticed that the characteristic values decrease as the uncertainty related to the parameter increases. After the characteristic values has been determined, the slope is analysed with the two design approaches. The results are shown in table 6.

Table 5. Characteristic values of the effective strength parameters for different values of COV

Parameter	*Characteristic values*		
	COV=0.1	*COV=0.2*	*COV=0.3*
c'_k *[kPA]*	*17.1*	*16.2*	*15.5*
φ'_k *[°]*	*28.5*	*27.1*	*25.8*

Table 6. ODF- values for different design approaches as the COV-value of the stress parameters varies.

Design method	*ODF-values*		
	COV=0.1	*COV=0.2*	*COV=0.3*
EC7 DA3	*1.00*	*0.95*	*0.90*
VPF	*0.90*	*0.71*	*0.56*

Then the reliability of the slope was calculated with varying COV- value of the stress parameters. The reliability indices and factors of safety obtained are shown in table 7. A graph, which combines the information from tables 6 and 7, is presented in figure 3.

Table 7. Reliability indices and factors of safety for the slope, calculated by using MC in LEM.

LEM,MC	*Probabilistic analyses results*		
	COV=0.1	*COV=0.2*	*COV=0.3*
β_{min}	*3.610*	*1.747*	*1.130*
FS_β	*1.34*	*1.35*	*1.35*

Table 7 shows that the statistical approach of determining the characteristic values in this situation has a rather small effect to the overall safety of the slope. ODF- value for eurocode 7 design approach 3 decreases just a little from 1.0 to 0.9, even though the uncertainty in the initial stress parameters increases rather greatly. The design remains near the required safety level despite the fact that the reliability of the slope decreases drastically with increasing uncertainty. This can be seen from figure 3. The reliability index of the slope decreases from 3.61 to 1.18, which means an increase in probability of failure from 0.015% to 11%. Even though some improvements are made for the case where COV=0.2 and COV=0.3, a partial factor method with fixed partial factors, in this case EC7 DA3, together with statistical determination of the characteristic values, cannot fully cover the uncertainties related to the design. The reliability indices would still be quite far from the target reliabilities presented in eurocode 0 (EN1990).

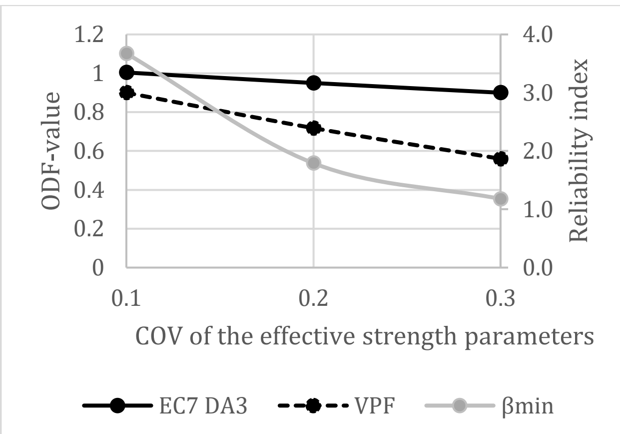

Figure 3. Comparison of design results between different partial factor methods as a function of COV of the stress parameters. Also the reliability indices from probabilistic design calculations with different COV-values are presented.

The Variable Partial Factor approach performs better, by leading to design that is much more conservative in situations where the uncertainties are high (table 7). This can be seen also from figure 3, where the ODF-value decreases clearly as the reliability of the slope decreases. Now by doing the necessary improvements for the slope requiring that the ODF- value increases, in case of COV=0.3 from 0.56→1.0, also the reliability indices would be highly affected. The probabilities of failure would be much smaller for the VPF than those obtained with EC7 DA3.

6. CONCLUSIONS

6.1. Characteristic value

The statistical approach presented in eurocode 0 annex D is problematic to use when dealing with soil material. First, the variation in soil parameters is much greater than that in manmade construction materials. Secondly, the amount of data is usually limited to one or few soil investigations points making it difficult to determine a certain fractal value from these. Thirdly, it is assumed that the choice of the characteristic value accounts for the variability of the material parameter. However, a simple approach like the one used, based on spatially average mean, is not always sufficient to account for uncertainty related to the parameter for the specific limit state considered. This was seen from the calculation case 2.

On the other hand, a 5% fractal for a single value might well lead to a very conservative design. A prudent method should cover at least spatial averaging considering the limit state in question, multiple correlated parameters (like *c'* and *φ'*) and any trend the parameters might have. In addition, the approach should account for a priori knowledge, for example using Baeysian statistics.

6.2. Partial factors

The calculation cases showed that the partial factors should be somehow dependent on the level of uncertainty of a certain parameter. The fixed factors cannot always cover the uncertainties in a required manner, especially if we are aiming to consistent level of reliability of the designs. The proposed Variable Partial Factor approach accounts the variation in initial parameters better, but this is just a one way to deal with the uncertainties in a more consistent way. Other aspects that should be considered are what the required target reliability is and what the definition of the characteristic value is. These are needed in order to calibrate the partial factors.

The target reliability does not need to be a certain value, but rather a range where we are aiming to obtain more or less similar reliabilities with similar design situations. Also the definition for characteristic value should be more fixed so that in similar design with similar data sets the designers would end up in similar choices for characteristic values.

6.3. Comments about assumptions made in calculation cases

In case 2, it could also be reasonable to assume greater variance for the effective cohesion than for the effective friction angle. This would worsen the situation by leading to smaller reliability index as the ODF- value would remain almost the same.

Further, in this paper, the effective cohesion and the effective friction angle were considered as independent random variables, but typically, there could be correlation between these parameters. Especially, if those are determined from the same sample with the same lab test; e.g. triaxial test. The correlation between parameters would decrease their variances, as they would be dependent one from another. By accounting the correlation in the reliability calculations, it would lead to increased reliability indices. As well, the correlation affects to the determination of the characteristic value by increasing it, so it should be accounted also in partial factor methods.

Another factor influencing the calculation results is the spatial correlation of the parameters. This could be accounted with methods like Random Finite Element Method (RFEM) (Fenton and Griffiths 2008), but due to the difficulties of determining vertical and horizontal correlation lengths, and a due to rather new and advanced calculation methods this is yet seldom done in practical engineering.

REFERENCES

Allen, T.M. (2013) AASHTO geotechnical design specification development in the USA. Modern Geotechnical Design Codes of Practice: Implementation, Application and Development. Amsterdam, IOS Press. pp. 243–260.

Bhattacharya, G., Jan, D., Ojha, S. and Chakraborty, S. (2003). Direct search for minimum reliability index of earth slopes, COmputers and Geotechnics 30 (2003), 455-462.

Bond, A. (2013). Simplifying Eurocode 7's design approaches. Task for Evolution Group 8 Harmonization at the SC7 meeting in Madrid in April 2013

CEN. 2002. EN 1990: Eurocode - Basis of structural design. Comité Européen de Normalisation

CEN. 2004. EN 1997-1: Geotechnical design – Part 1: General rules. Comité Européen de Normalisation

Fenton, G.A., Naghibi, F., Dundas, D., Bathurst, R.J. & Griffiths, D.V. (2016) Reliability-based geotechnical design in the 2014 Canadian Highway Bridge Design Code. Canadian Geotechnical Journal, 53 (2), 236–251

Fenton G.A and Griffiths, D.V. (2008). Risk assessment in geotechnical engineering. John Wiley&Sons Inc.

Hassan AM. Wolff TF. Search algorithm for minimum reliability index of earth slopes Journal of geotechnical and geoenviron-mental engineering, ASCE 1999;125(04):301–8.

Honjo, Y., Kieu Le, T.C., Hara, T., Shirato, M., Suzuki, M. & Kikuchi, Y. (2009) Code calibration in reliability based design level I verification format for geotechnical structures. In: Proceedings, Second International Symposium on Geotechnical Safety & Risk. Leiden, CRC Press/Balkema. pp. 435–452.

Honjo, Y., Kikuchi, Y. & Shirato, M. (2010) Development of the design codes grounded on the performance-based design concept in Japan. Soils and Foundations, 50 (6), 983–1000.

Li KS, Lumb P. Probabilistic design of slopes. Canadian geotechnical journal 1987;24(04):520–35.

Länsivaara, T. and Poutanen, T. (2013). Safety concepts for slope stability. Tampere University of Technology

McKay, M.D., Beckman, R.J., Conover, W.J. (1979). "A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output from a Computer Code". American Statistical Association. 21 (2): 239–245.

Vrouwenvelder, T., van Seters, A. & Hannink, G. (2013) Dutch approach to geotechnical design by Eurocode 7, based on probabilistic analyses. Modern Geotechnical Design Codes of Practice: Implementation, Application and Development. Amsterdam, IOS Press. pp. 128–139.