A fuzzy stochastic correlation model for geometric imperfections of cylindrical shells

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ABSTRACT: Geometric imperfections are the major part of the disagreement between theoretically and experimentally determined buckling loads of thin walled cylindrical shells. In a common probabilistic approach the spatial varying imperfections are modeled as Gaussian random fields. Due to the underlying uncertainties like a small sample size or imprecise measurements it is practically impossible to define crisp input parameter for a random field representation, e.g., correlations. In this paper, the classical probabilistic approach is therefore extended to a fuzzy stochastic approach by using a polymorphic uncertainty model (fp-r) from Graf et al. (2015). This allows to take into account natural variability and incompleteness aiming to consider aleatory and epistemic uncertainties in a decision making process.

1. Introduction

In the past, many researchers have discussed the disagreement between theoretically and experimentally determined buckling loads. The initial imperfections are mainly the reason for the wide experimental scatter. The uncertainty of their magnitudes and shapes have a major influence on the stability loads. Nowadays, several researches try to represent imperfections as homogeneous Gaussian or non-Gaussian random fields. Due to the lack of measurements, random fields are mostly assumed homogeneous and the correlation parameters are chosen to simulate a 'worst case' scenario. Especially, the correlation parameters have a great influence on the imperfection shape, hence on the scatter of the buckling loads. Lauterbach et al. (2018) have investigated this influence quantitatively. Thus, a general statement about correlation parameters, which is based only on a few measurements is very risky. The aim of this paper is the extension of the classical probabilistic approach to a fuzzy stochastic approach in cylindrical shell design for a consideration of data uncertainties. Therefore, the model 'fuzzy probability based random variable (fp-r)' from Graf et al. (2015) is used to consider variability and incompleteness. As a first task, a correlation model is obtained from real measurements using Delft's imperfection data bank from Arbocz and Abramovich (1979). Then, the uncertain correlation functions, which are varying from shell to shell are described by suitable functions. The main idea is to define the corresponding correlation parameters for this functions as fuzzy variables. This leads to a representation of spatial varying geometric imperfections as fuzzy random fields. Finally, the statistical moments of the stability loads are presented as fuzzy sets for decision making.

2. Correlation model

In the present paper, the need of a fuzzy stochastic approach is shown on the evaluation of an extensive imperfection data bank from Arbocz and Abramovich (1979). Here, the A-shells as one shell type of the data bank are chosen to build a correlation model for a fuzzy stochastic analysis. The seven A-shells are unstiffened, isotropic copper shells, which are manufactured by electroplating. The first task is the representation of geometric imperfections as half wave cosine Fourier series, as given in Arbocz and Abramovich (1979). Averaged shell dimensions, material properties like the Young's modulus E and the Poisson's ratio ν, the maximum evaluated amplitude $|w(x,y)|$ and the number of data points $N_C \times N_R$ for the measurements are shown in Table 1. The next

R $[mm]$	101.60
L $[mm]$	202.29
t $[mm]$	0.1160
max $\|w(x,y)\|$ $[mm]$	0.3672
E $[N/mm^2]$	104410
ν $[-]$	0.3
$N_C \times N_R$	49x31

Table 1: Averaged geometry and material properties

step is to obtain the statistical properties like the covariances from the given Fourier series. As an efficient random field discretization technique the EOLE-method (Expansion Optimal Linear Estimation) from Li and Kiureghian (1993) is used, which allows to represent the random field with only a few random variables by minimizing the variance error. As a main advantage the covariance matrix is only required on a sub-set of field nodes, the so-called 'random field mesh'. The series is given by

$$\hat{w}(\boldsymbol{x},\theta) = \mu(\boldsymbol{x}) + \left(\sum_{i=1}^{M} \frac{\xi_i(\theta)}{\sqrt{\lambda_i}} \, \boldsymbol{\varphi}_i(\boldsymbol{x}^S) \right) C(\boldsymbol{x}^S,\boldsymbol{x}) \quad , \tag{1}$$

where the vector $\boldsymbol{x}^S = [\boldsymbol{x}_1 \ldots \boldsymbol{x}_i^S \ldots \boldsymbol{x}_M^S]$ contains the M nodes of the random field and $\boldsymbol{x} = [\boldsymbol{x}_1 \ldots \boldsymbol{x}_j \ldots \boldsymbol{x}_N]$ the N nodes in full space (e.g. FE-nodes). Consequently, $C(\boldsymbol{x}^S,\boldsymbol{x})$ leads to a covariance matrix containing the covariances from random field nodes with the FE-nodes,

$$C(\boldsymbol{x}^S,\boldsymbol{x}) = \tag{2}$$

$$\begin{bmatrix} C(\boldsymbol{x}_1^S,\boldsymbol{x}_1) & \ldots & C(\boldsymbol{x}_1^S,\boldsymbol{x}_j) & \ldots & C(\boldsymbol{x}_1^S,\boldsymbol{x}_N) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ C(\boldsymbol{x}_i^S,\boldsymbol{x}_1) & \ldots & C(\boldsymbol{x}_i^S,\boldsymbol{x}_j) & \ldots & C(\boldsymbol{x}_i^S,\boldsymbol{x}_N) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ C(\boldsymbol{x}_M^S,\boldsymbol{x}_1) & \ldots & C(\boldsymbol{x}_M^S,\boldsymbol{x}_j) & \ldots & C(\boldsymbol{x}_M^S,\boldsymbol{x}_N) \end{bmatrix} .$$

Furthermore, $\xi_i(\theta)$ is an uncorrelated Gaussian random variable with zero mean and unit standard deviation. $\boldsymbol{\varphi}_i(\boldsymbol{x}^S)$ and λ_i are the eigenfunctions and eigenvalues of a given autocovariance function $C(\boldsymbol{x}_i^S,\boldsymbol{x}_j^S)$ on the random field mesh. The expected value $\mu(\boldsymbol{x})$ of the imperfection field is set constant to zero. However, the separation of the coarser random field mesh from the FE-mesh allows keeping the eigenvalue problem as small as possible. This makes the EOLE-method very interesting in a fuzzy stochastic approach, where with a changing correlation structure the eigenvalue problem of the covariance matrix must be solved several times. To get an autocovariance function from the measurements the following assumptions to the random field and correlation structure have to be made:

- Gaussian
- Homogeneity
- Separability
- Ergodicity .

In addition to the assumption of Gaussian random fields and homogeneity, Schenk and Schuëller (2003) proposed a fully separable correlation structure for geometric imperfections:

$$C_n(\Delta x,\Delta y) = \sigma_n^2 \, \rho_n(\Delta x) \, \rho_n(\Delta y) \quad , \tag{3}$$

where $\rho_n(\Delta x)$ and $\rho_n(\Delta y)$ are the one-dimensional autocorrelation functions along the axial and circumferential direction with the corresponding lags Δx and Δy. σ_n^2 is here the sample variance of an imperfection field of the n^{th} test shell defined by

$$\bar{\sigma}_n^2 = \frac{1}{M} \sum_{k=1}^{M} (w(\boldsymbol{x}_k) - \bar{\mu}_n)^2 \quad , \tag{4}$$

with $\bar{\mu}_n$ the sample mean of the full imperfection field of one test shell and M is the number of random field nodes, holding the number of data points $M = N_C \times N_R$, see Table 1. Following to the assumption of separability the variation of imperfections describe independent one-dimensional stochastic processes $w(x_r)$ in axial direction with $r = 1 \ldots N_R$ observations and $w(y_c)$ in circumferential direction with $c = 1 \ldots N_C - 1$ observations, where the seam nodes have to be deleted for a cylinder as a closed structure. Consequently, the sample autocorrelation function with respect to axial lags can be defined as:

$$\rho_c(\xi \Delta x_0) = \frac{1}{N_R} \sum_{r=1}^{N_R-\xi} (w(x_r + \xi \Delta x_0, y_c) - \bar{\mu}_n)(w(x_r, y_c) - \bar{\mu}_n) \quad , \tag{5}$$

with $\xi = 0 \ldots N_R - 1$ denoting the multiple of the constant lag Δx_0. Similarly, the autocorrelation function respect to circumferential lags holds

$$\rho_r(\eta \Delta y_0) = \frac{1}{N_C - 1} \sum_{c=1}^{N_C-1} (w(x_r, y_c + \eta \Delta y_0) - \bar{\mu}_n)(w(x_r, y_c) - \bar{\mu}_n) \quad , \tag{6}$$

with $\eta = 0 \ldots N_C - 1$ times the constant lag Δy_0. Here, it should be noted, that the number of lags, the upper sum limit, must be set constant to $N_C - 1$ for deleting the seam nodes. Finally, to get the sample autocorrelation function of one test shell, the method of ensemble averaging is used, assuming that each stochastic process is ergodic. This means that one sample with its stochastic information represents the whole set. The assumption of ergodicity is allowable if the stochastic process can be divided into independent parts. The individual parts are the variation of imperfections in axial and circumferential direction of each row of nodes. Thus, averaging the autocorrelation function across the axial samples with

$$\rho(\xi \Delta x_0) = \frac{1}{N_C - 1} \sum_{c=1}^{N_C-1} \rho_c(\xi \Delta x_0) \quad , \tag{7}$$

and across circumferential samples with

$$\rho(\eta \Delta y_0) = \frac{1}{N_R} \sum_{r=1}^{N_R} \rho_r(\eta \Delta y_0) \quad , \tag{8}$$

leads to an ensemble autocorrelation function of one test shell. The results of the estimated autocorrelation functions of the A-shells from measurements are depicted in Fig. 1. Here, the functions

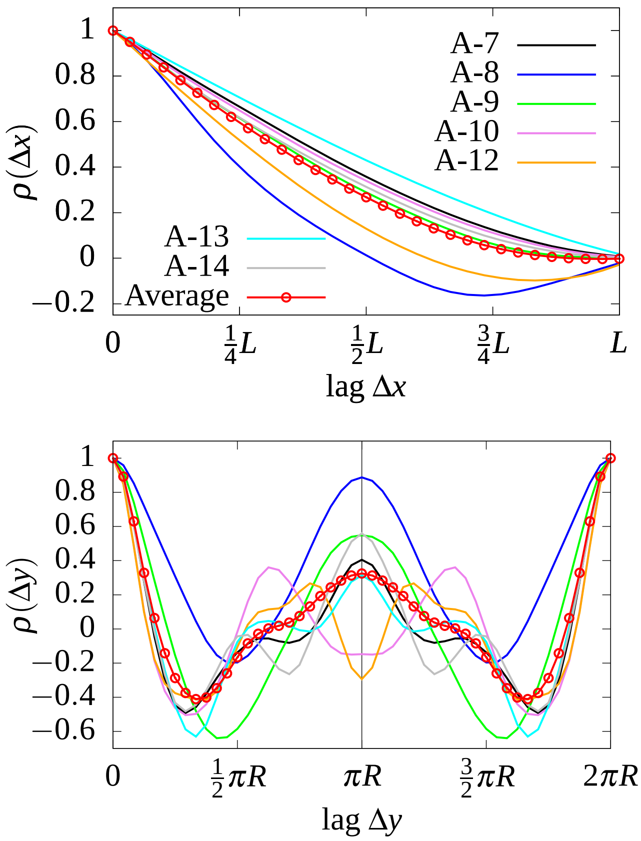

Figure 1: Estimated autocorrelation functions in axial (above) and circumferential direction (below)

vary from shell to shell and show a large scatter. Furthermore, the autocorrelation functions in axial direction tending to zero for large lags Δx. The circumferential functions have different wave numbers, but all of these functions are symmetrical with regard to the half circumferential axis. This is due to the fact that the circumferential distance is always the shorter way around the cylinder. In addition, if a representative autocorrelation function of the full test series is needed, an averaging across all test shells is possible, illustrated by the 'Average'-curve in Fig. 1. The proposed fuzzy approach requires a functional representation of the correlation structure controlled by correlation parameters or the so-called correlation lengths. The estimated autocorrelation functions in Fig. 1 allow to find representative functions for a data fitting. Several functions have been tested, where the quadratic exponential function gives a good fit for the axial direction,

$$\rho(\Delta x, l_{c,h}) = exp\left(-\frac{\Delta x^2}{l_{c,h}}\right) \quad , \tag{9}$$

with the correlation length $l_{c,h}$. As a good fit for the circumferential direction, a linear-cosine form is chosen:

$$\rho(\Delta y, l_{c,u}, T) = \left(1 - \frac{\Delta y}{l_{c,u}}\right) \cdot cos\left(\frac{2\pi\Delta y}{T}\right) \quad , \tag{10}$$

with the correlation length $l_{c,u}$ and the period length T. The parameters $l_{c,h}$ and $l_{c,u}$ are regarded as fitting parameters and the period length T is kept constant for each shell type to minimize later the number of fuzzy input variables. The unknown fitting parameters appear nonlinearly in a fitting model, hence the nonlinear least square method (NLLSQ) is used. In this case the definition of the NLLSQ is to find a minimizer $l_{c,h}$ or $l_{c,u}$, where the middle nonlinear problem is solved by the Gauss-Newton method. The circumferential correlation functions are fitted on the half function $\Delta y \in [0, \pi R]$ and are then mirrored at $\Delta y = \pi R$. The results of the fitting of the correlation functions and sample variance are presented in Table 2. The period length is obtained

shell	$\bar{\sigma}^2$ $[mm]$	$l_{c,h}$ $[mm]$	$l_{c,u}$ $[mm]$	T=const. $[mm]$
A-7	0.0070	9234	210	217
A-8	0.0100	2744	180	217
A-9	0.0035	7179	199	217
A-10	0.0029	8630	327	217
A-12	0.0070	4211	307	217
A-13	0.0113	11862	234	217
A-14	0.0041	7733	187	217

Table 2: Fitting results and sample variance

from the fitting of the average correlation curve, but has been taken equal for all shells in the series.

3. Fuzzy stochastic approach

The present paper focuses on taking into account different types of uncertainties in this context of cylindrical shells, namely aleatoric and epistemic uncertainties. Aleatoric uncertainty is the natural variability and is mostly modeled with a classical stochastic approach. It is clear that this type of uncertainty cannot be reduced. This means that a manufacturing process of a shell is a random process itself with unavoidable uncertainties. Epistemic uncertainty includes incompleteness and impreciseness of available data and can be modeled by fuzzy approaches. In contrast to aleatoric uncertainty, incompleteness and impreciseness is due to the lack of knowledge and can be reduced, e.g., by collecting more data or defining stricter tolerance values for the manufacturing process. Graf et al. (2015) introduced the concept of 'polymorphic uncertainty modeling', which describes different models considering more than one uncertainty characteristic: natural variability, incompleteness and impreciseness. Here, the model of fuzzy probability based randomness (fp-r) is used. This model allows taking into account variability and incompleteness in the uncertain correlation structure for geometric imperfections. It is based on random variables and fuzzy sets. The theoretical background of fuzzy sets and their numerical treatment is given for example in Möller and Beer (2004). A normalized fuzzy set $\tilde{A}$ is fully defined as follows:

$$\begin{aligned} \tilde{A} &= \{(x, \mu_A(x)) \mid x \in \mathbb{R}\} \quad , \\ \mu_A(x) &: \mathbb{R} \to [0,1] \quad , \end{aligned} \tag{11}$$

where $\mu_A(x)$ is the membership function, which maps a fundamental set $\mathbb{R}$ onto the interval $[0,1]$. The numerical treatment requires the α-discretization, where the membership function is discretized into r α-levels:

$$A_{\alpha_k} = \{x \in \mathbb{R} \mid \mu_A(x) \geq \alpha_k\}, \quad k = 1, ..., r \quad , \tag{12}$$

which are crisp subsets of the support $S(\tilde{A})$ defined by

$$S(\tilde{A}) = \{x \in \mathbb{R} \mid \mu_A(x) > 0\} \quad . \tag{13}$$

Following the definition of fuzzy sets, the here used fuzzy triangular number can be expressed as follows:

$$\tilde{a} = \langle x_l, x_m, x_r \rangle \quad , \tag{14}$$

defined by the smallest and largest value x_l and x_r as well as the value x_m related to the membership $\mu_A(x) = 1$, also referred to as 'trend value'. The specification of such a fuzzy number is called fuzzification and is often based on a histogram as a first point of reference, where the sample size is potentially small. The main idea of the proposed approach is to transform the correlation structure in Eq. (3) to a fuzzy stochastic correlation model:

$$\tilde{C}(\Delta x, \Delta y) = \tilde{\sigma}^2 \, \tilde{\rho}(\Delta x) \, \tilde{\rho}(\Delta y) \quad , \tag{15}$$

by representing the variance $\tilde{\sigma}^2$ and the correlation parameters $\tilde{l}_{c,h}$ in $\rho(\Delta x)$ and $\tilde{l}_{c,u}$ in $\rho(\Delta y)$ as fuzzy triangular numbers. This introduction of fuzziness extends the common random field representation to a fuzzy random field definition. A fuzzy random field is here a collection of fuzzy probability based random variables (fp-r):

$$\{\tilde{w}(\boldsymbol{x}, \theta) : \boldsymbol{x} \in \Omega, \theta \in \Theta\} \quad , \tag{16}$$

where $\boldsymbol{x}$ contains the shell surface coordinates and Ω denotes the set of all possible outcomes θ. A fuzzy probability based random variables (fp-r) describes the probability measure of a random number with a set of probability functions, see Graf et al. (2015). In case of 'fuzzy random geometric imperfections', this means that for the fuzzy correlation structure, defined in Eq. (15), the probability measure of the event 'geometric imperfection' has a fuzzy characteristic. Thus, every event at a location $\boldsymbol{x}_0$ is represented by the fuzzy values $\tilde{\sigma}^2$, $\tilde{l}_{c,h}$ and $\tilde{l}_{c,u}$. The so-called bunch parameter representation has proven to be beneficial for the numerical implementation, where the fuzzy random field describes in a parametric form a function bunch, see, e.g., Möller and Beer (2004)

$$\tilde{w}(\boldsymbol{x}, \theta) = w(\tilde{\boldsymbol{s}}, \boldsymbol{x}, \theta) \quad , \tag{17}$$

where the fuzziness is concentrated in the bunch parameter $\tilde{\boldsymbol{s}}$, which contains the three defined fuzzy numbers:

$$\tilde{\boldsymbol{s}} = \{\tilde{\sigma}^2, \tilde{l}_{c,h}, \tilde{l}_{c,u}\} \quad . \tag{18}$$

3.1. *Fuzzification of the input parameters*

For the so-called fuzzification process the fitted results of the correlation parameters $l_{c,h}$, $l_{c,u}$ and the variance $\bar{\sigma}^2$ from Table 2 are presented in a histogram as a first design aid, see Fig. 2. In this paper only convex fuzzy triangular numbers with linear branches are used. It is up to the shell designer to form the fuzzy numbers. They can be modified with the aid of expert knowledge or collecting more data to adopt the membership function for an output with less uncertainty for a more economical design. Here, all samples lie in the support set with a conservative overhanging of the membership function. The result of the fuzzification in form of fuzzy triangular numbers is given by

$$\begin{aligned} \tilde{\sigma}^2 &= \langle 0.002, 0.007, 0.012 \rangle \\ \tilde{l}_{c,h} &= \langle 1000, 9000, 14000 \rangle \\ \tilde{l}_{c,u} &= \langle 175, 225, 400 \rangle \quad . \end{aligned} \tag{19}$$

3.2. *Fuzzy structural analysis with a HDMR-metamodel*

The main task in this paper is the following mapping:

$$w(\tilde{\boldsymbol{s}}, \boldsymbol{x}, \theta) \;\rightarrow\; \tilde{p}_{cr}(\theta) \quad , \tag{20}$$

taking into account the advantageous fuzzy bunch parameter representation, where the fuzzy and stochastic parts can be decoupled. The numerical treatment is therefore a combination of α-Level-Optimization (ALO) and Monte Carlo simulation (MCS), respectively. The MCS estimates for a given configuration of bunch parameters $\boldsymbol{s}_0$ a desired stochastic output, e.g., the sample mean value of the critical buckling load $\bar{p}_{cr}(\boldsymbol{s} = \boldsymbol{s}_0)$. Considering a specific α-level α_k, the fuzzy bunch parameters are then limited by the α-level boundaries

$$\boldsymbol{s} \in [\boldsymbol{s}_{\alpha_k,l}, \boldsymbol{s}_{\alpha_k,r}] = \{\boldsymbol{s}_{\alpha_k}\} \quad . \tag{21}$$

The corresponding α-level boundaries of the exemplary fuzzy output variable $\tilde{p}_{cr}$ are given by the extreme value problems

$$\bar{p}_{cr,\alpha_k,l} = \min_{\boldsymbol{s} \in \{\boldsymbol{s}_{\alpha_k}\}} [\bar{p}_{cr}(\boldsymbol{s})] \quad , \tag{22}$$

$$\bar{p}_{cr,\alpha_k,r} = \max_{\boldsymbol{s} \in \{\boldsymbol{s}_{\alpha_k}\}} [\bar{p}_{cr}(\boldsymbol{s})] \quad , \tag{23}$$

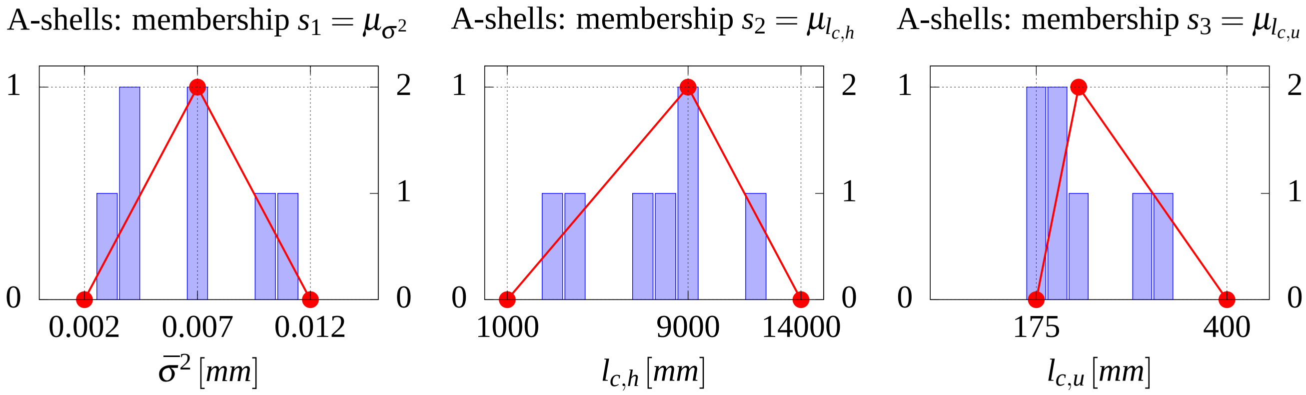

Figure 2: Fuzzification of the correlation structure parameters from Table 2: variance (left), correlation parameter in axial direction (middle) and circumferential direction (right)

which are here not performed on the original function $\bar{p}_{cr}(\mathbf{s})$, but on a previously generated metamodel

$$\bar{p}_{cr}^{MM}(\mathbf{s}) \approx \bar{p}_{cr}(\mathbf{s}), \quad \mathbf{s} \in S(\tilde{\mathbf{s}}) \tag{24}$$

on the support $S(\tilde{\mathbf{s}})$ of the fuzzy bunch parameters. A combined approach of a high-dimensional model representation (HDMR) of second order and Least-Square polynomial approximation is used. Basics of HDMR can be found in Rabitz and Aliş (1999). For the metamodel 61 exact MCS for the desired stochastic output are performed. Therefore, a direct Monte Carlo simulation approach is used with 500 simulations per evaluation point. The complete analysis builds with the metamodel a three-loop computational model. The characteristic of this model is that the outer loop forms the fuzzy analysis and the MCS forms the stochastic analysis (middle loop). The fundamental solution represented here by the numerical buckling analysis (inner loop) is based on a nonlinear FE-model. Besides, the accuracy of the stochastic model depending mostly on the sample size, the quality of the FE-model has a great influence on the quality of the fuzzy output, which is here the fuzzy sample mean $\tilde{p}_{cr}$ of the critical buckling load. $61 \times 500 = 30500$ buckling problems are solved to get the fuzzy mean value of the critical buckling load. Due to the fact that the FE-calculation must be repeated several times it is also worthwhile to optimize the FE-model for speed, e.g., keeping the number of load/displacement steps low, using as many elements as necessary, selecting a fast solver and efficient storage technique.

4. Numerical results

4.1. *Buckling analysis with a nonlinear finite element model*

The FE-model of the cylinder as used in this study is depicted in Fig. 3. The dimensions and mate-

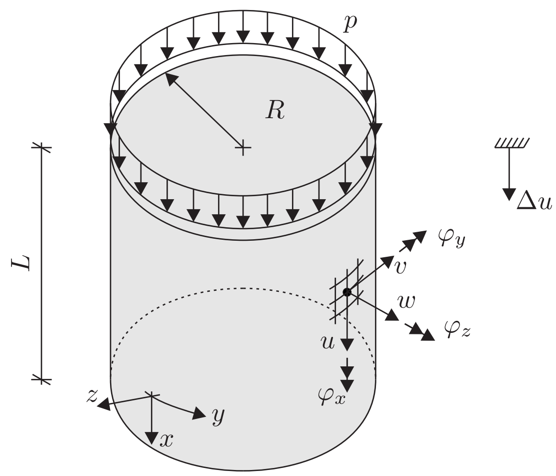

Figure 3: FE-model of the axially loaded cylinder

rial properties of the isotropic shells are the averaged values from Table 1. Furthermore, a geometric nonlinear quadrilateral shell element with moderate rotations from Wagner and Gruttmann (2005) is used. It is based on linear shape functions using the isoparametric concept. Additionally, to avoid shear locking, the assumed natural strain method (ANS) is implemented. The geometric imperfection are modeled as nodal deviation in the $w-$direction.

Since the imperfections alter the coordinates of the FE nodes warping has to be taken into account in terms of axial loading. The cylindrical shell is simply supported on both edges, where the so-called SS-3 boundary condition holds at the lower edge: $u = v = w = 0,\ \varphi_x \neq 0, \varphi_y \neq 0$ and at the upper edge: $u = \Delta u,\ v = w = 0,\ \varphi_x \neq 0, \varphi_y \neq 0$. The cylinder is loaded by displacement control until the stability point is reached, where the reaction forces at the bottom are summed to calculate the critical buckling load. In this paper, if one sign of diagonal elements of the tangent stiffness matrix will become negative the load state is saved and then the calculation is aborted by 'task killing' within the parallelized Monte Carlo loop. All stability loads $P_{cr,imp}$ are then normalized by the critical load $P_{cr,perf} = 5073\,N$ of the perfect shell by

$$\alpha_{cr} = \frac{P_{cr,imp}}{P_{cr,perf}} \quad , \tag{25}$$

where α_{cr} is denoted as the critical load factor. The FE-mesh consists of 200 finite elements in circumferential direction and 100 elements in axial direction. Beside the FE-mesh, another important point is the definition of a random field mesh. Here, the number of random field points $N_C \times N_R$ is given by the number of measuring points from Delft's imperfection data bank. Care must be taken to ensure that the random field properties can be represented by the selected 'stochastic mesh'.

4.2. *Fuzzy result and decision making*

The results of the fuzzy stochastic analysis are presented as fuzzy numbers of the critical buckling load factor, particularly the fuzzy result mean value $\bar{\alpha}_{cr}$ is evaluated, see Fig. 4. In addition to the fuzzy results, critical loads of the original shells, where the measured imperfections are applied as Fourier series on the FE-model are also presented in histogram form, labeled as 'Fourier impf.'. The trend value of the membership function correlates very well with the critical loads referred to Fourier imperfections. The fuzzy result shows a value greater than one for the normalized buckling load factor, which means there are buckling loads greater than the critical perfect load. This is not realistic and is due to a little extrapolation error of the HDMR-metamodel. But in fact, there are no fundamental solutions larger than the perfect buckling load $P_{cr,perf}$. Furthermore, the figure also includes a histogram plot of the normalized experimental results from Delft's imperfection data bank. However, it should be considered that the experimental shells contain not only geometrical imperfections as deviation of the shell surface, but also boundary, load or material imperfections. The fuzzy output shows a large support. This is a hint for large uncertainties and it can be worthwhile to reduce the uncertainties of the input. In other words, it is perhaps possible for the shell designer to get a smaller support of the output with a little effort by narrowing the input, e.g., testing just a few more shells to identify outliers. For demonstration how it can work in practice fictional shells are added for a modified fuzzification process, see Fig. 5. This leads to a narrowed fuzzy input:

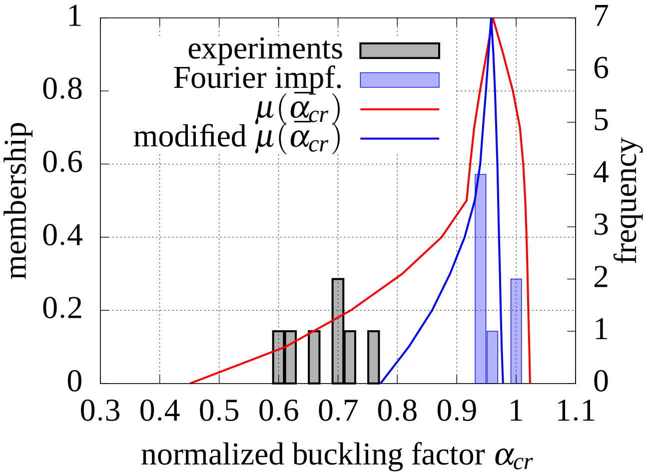

Figure 4: Fuzzy results of the mean value of the critical buckling load factor

$$\tilde{\sigma}^2 = \langle 0.005, 0.007, 0.011 \rangle \tag{26}$$
$$\tilde{l}_{c,h} = \langle 3000, 9000, 13000 \rangle$$
$$\tilde{l}_{c,u} = \langle 175, 225, 325 \rangle \quad .$$

The result is that the modified fuzzy output $\mu(\bar{\alpha}_{cr})$ in Fig. 4 shows a smaller support.

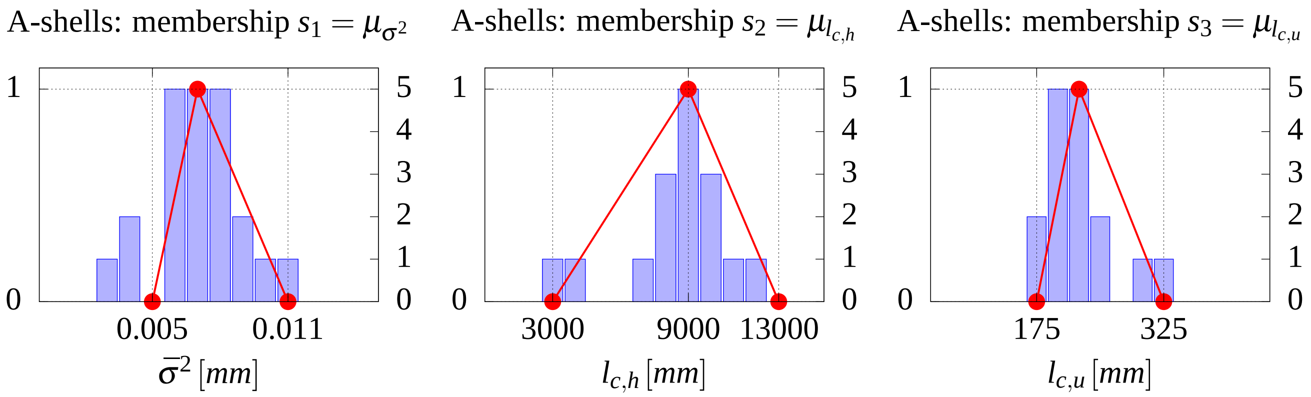

Figure 5: Modified fuzzification of the correlation structure parameters: variance (left), correlation parameter in axial direction (middle) and circumferential direction (right)

5. CONCLUSIONS

The presented approach aims to consider imprecise data to the decision making process in cylindrical shell design, where worst case studies still result in too conservative designs. Therefore, a fuzzy stochastic analysis is introduced for the simulation of the geometric imperfections. More exactly, the polymorphic uncertainty model of fuzzy probability based randomness (fp-r) from Graf et al. (2015) is used to take into account the natural variability and incompleteness. The procedure can be summarized in a possible fuzzy design concept for cylindrical shells:

- Representation of measured geometric imperfections as Fourier series
- Evaluation of correlation functions for a random field representation
- Fitting of the estimated correlation function
- Fuzzification of the correlation parameters: $\tilde{\mathbf{s}} = \{\tilde{\sigma}^2,\ \tilde{l}_{c,h},\ \tilde{l}_{c,u}\}$
- Fuzzy analysis with the aid of a metamodel
- Decision making based on fuzzy output variables, e.g., the sample mean $\tilde{p}_{cr}$

This approach still needs improvement by taking into account all imperfection types, e.g., boundary, load or material imperfections. Furthermore, the dependencies of the different input parameters have to be investigated. For example, the correlation parameter with the greatest influence on the buckling load should be identified, which cannot be seen in the presented fuzzy output.

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