Contests with Linear Externality in Prizes

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This study examines contests in which prizes are affected linearly by aggregate effort. In particular, this research analyzes a contest among individuals as a benchmark to scrutinize the effects of prize externality and sharing-rule information on rent-dissipation rate and social welfare. Thereafter, the current study investigates two types of group contest with linear prize externality: one with private information on intra-group sharing rules and the other with public information on intra-group sharing rules. Results indicate as follows. (1) An increase in prize externality increases rent-dissipation rate but has no effect on social welfare. (2) The group contest with private information on sharing rules yields higher social welfare and lower rent-dissipation rate than the one with public information on sharing rules.

Keywords: Group contest, Linear externality in prize, Intra-group sharing rule, Private information on sharing rules, Public information on sharing rules

JEL Classification: D72

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We would like to thank an anonymous reviewer for constructive comments and suggestions. Any remaining errors are our own responsibility.

[**Seoul Journal of Economics** 2019, Vol. 32, No. 3]

I. Introduction

Many economic, political, and social phenomena can be modeled as contests, in which players expend effort to win prizes. Examples of these contests include elections, environmental conflicts, fund raising competitions among charity organizations, labor tournaments, litigations, lottery competitions, R&D competitions, sports events, and war of attrition. Given the importance of such contests, many scholars have analyzed them under various contexts. In particular, the "lottery" model of Tullock (1980) has been extended in various directions by Dixit (1987), Ellingsen (1991), Konrad (2004), Baik and Lee (2007), Kolmar and Rommeswinkel (2013), and Topolyan (2014).

The majority of the related studies have assumed that the size of a prize is given exogenously. However, the prizes of many contests are affected by aggregate effort. Several studies have reflected on this idea and examined contests in which prizes or costs are affected by aggregate effort. Examples of these studies are Chung (1996), Lee and Kang (1998), Eggert and Kolmar (2006), Shaffer (2006), Lee (2000, 2007), Cohen *et al*. (2008), and Chowdhury and Sheremeta (2011). The contests that they have examined can be classified into two types. In one set of contests, only prize winners are affected by externalities. By contrast, all players in the other set of contests are affected by externalities regardless of whether they are prize winners (Hereafter, the former and latter are called contests with prize and cost externalities, respectively).

Contests with prize externality are ubiquitous. For example, the prize money of a lottery winner depends on the number of people buying lottery tickets and their total expenditure. The more people buying tickets, the larger the money for the winner (Chung 1996, p. 57). Another example is a labor tournament. A typical labor tournament rewards behavior that increases an employer's productivity, thereby augmenting the total surplus available to all members of the organization (Shaffer 2006, p. 251). Chung (1996), Eggert and Kolmar (2006), Shaffer (2006), and Cohen *et al*. (2008) examine contests with prize externality.

Lee and Kang (1998) and Lee (2000, 2007) investigate contests with cost externality. Examples of such contests are Olympic sporting events. Lee and Kang (1998) noted that participants in the Olympics may enjoy a feeling of pride or accomplishment regardless of whether they win a medal. The pride of the participants in such contests is likely to be positively correlated to the aggregate effort expended in the contests (Lee and Kang 1998, p. 728).

The current study focuses on contests in which prizes are affected linearly by the aggregate effort. Chung (1996) analyzes the effect of prize externality and shows that rent-dissipation rate, which is the ratio of the aggregate resources expended relative to the prize, increases when the prize increases with effort. While Chung (1996) analyzes a contest with a strictly positive non-linear spillover, the present research scrutinizes a contest with a linear prize externality that can be either positive or negative. The current study is related to Chowdhury and Sheremeta (2011), which examine contests in which players' payoffs are linear functions of prizes, own effort, and effort of rivals. Their model can represent various contests through parameter modifications.

The present research is also related to Lee and Kang (1998) because individual and group contests are examined. Lee and Kang (1998) introduce a group contest with public information sharing rules and strictly linear externalities. However, their analysis is confined to group contests with public information on intra-group sharing rules. The current research examines group contests with private information on intra-group sharing rules and with public information.

The remainder of this paper is organized as follows. Section II examines the individual contest with linear prize externality. We find that an increase in prize externality increases rent-dissipation rate but has no effect on social welfare in the individual contest. Section III analyzes two types of group contest with prize externality: one with private information on intra-group sharing rules and the other with public information on intra-group sharing rules. We obtain the equilibrium sharing rules, equilibrium outlays, rent dissipation rate, and social welfare under private and public information on intra-group sharing rules. Section IV compares the results derived from the analysis and presents the following outcomes. First, the equilibrium sharing rules and total outlays are less in the group contest with private information on sharing rules than in the group contest with public information. Second, the group contest with private information on sharing rules yields lower rent dissipation and higher social welfare than the one with public information on sharing rules. Section V presents the concluding remarks.

II. Individual contest with linear prize externality

Consider a contest in which N risk-neutral players compete to win a prize the size of which is endogenously determined. Let v be a basic (or initial) prize in the contest. Each player's outlay is denoted by x_h (h = 1, 2, 3, ..., N). We assume that aggregate outlays generate externalities in the form of a change in the size of a prize. That is, the endogenously determined prize is given by $v + \gamma\sum_h x_h$. The parameter γ denotes the degree to which the prize is affected linearly by the aggregate effort and $-1 < \gamma < 1$. As in Tullock (1980), player h's probability of winning the prize p_h is given by his or her outlay relative to the aggregate outlays as follows:

$$\begin{aligned} p_h &= x_h/X, \quad \text{if } X > 0 \text{ and} \\ &= 0 \qquad\quad \text{if } X = 0, \end{aligned} \tag{1}$$

where X ($= \sum_j x_j$) denotes the aggregate outlays.

Thereafter, player h's expected payoff G_h is given as follows:

$$\begin{aligned} G_h &= p_h(v + \gamma X - x_h) + (1 - p_h)(-x_h) \\ &= p_h v - (1 - \gamma)x_h. \end{aligned} \tag{2}$$

When $0 < \gamma < 1$, positive externality is present in the contest. However, if $-1 < \gamma < 0$, then the contest is associated with negative externality. Shaffer (2006) notes that military conflicts would fit into this type of contest. In military conflicts, contenders may bombard enemy facilities, thereby reducing the size of the prize they could claim after winning the conflict. The fiercer the military conflict, the smaller the prize for the winner. When γ is zero, v is the prize with no externality in the contest. This interpretation appears valid in such contests as the previously mentioned labor tournament.

Each player is assumed to behave in a Cournot–Nash manner. That is, each player decides the level of one's own outlay, taking all the other players' outlays as given. The first-order conditions are as follows:

$$(X - x_h)v/X^2 - (1 - \gamma) = 0, \quad \text{for } h = 1, 2, 3, \ldots, N. \tag{3}$$

The second-order conditions are as follows:

$$- 2(X - x_h)\upsilon/X^3 < 0, \text{ for } h = 1, 2, 3, \ldots, N. \tag{4}$$

We focus on the symmetric equilibrium actions. Thus, we obtain the following by summing (3) over h and simplifying:

$$x^E = (N - 1)\upsilon/\{N^2(1 - \gamma)\} \text{ and } X^E = (N - 1)\upsilon/\{N(1 - \gamma)\}, \tag{5}$$

thereby implying that $\partial x^E/\partial\gamma = (N - 1)\upsilon/\{N^2(1 - \gamma)^2\} > 0$, $\partial X^E/\partial\gamma = (N - 1)\upsilon/\{N(1 - \gamma)^2\} > 0$; and $\partial x^E/\partial N = -(N - 2)\upsilon/\{N^3(1 - \gamma)\} < 0$ and $\partial X^E/\partial N = \upsilon/\{N^2(1 - \gamma)\} > 0$. An increase in positive externality augments the individual outlay and aggregate outlays. As the number of players increases, the individual outlay decreases and the aggregate outlays increase.

We now consider the rent-dissipation rate $RD^E = X^E/(\upsilon + \gamma X^E)$. The rent-dissipation rate is given as follows:

$$RD^E = (N - 1)/(N - \gamma), \tag{6}$$

thereby implying that $\partial RD^E/\partial\gamma = (N - 1)/(N - \gamma)^2 > 0$ and $\partial RD^E/\partial N = (1 - \gamma)/(N - \gamma)^2 > 0$.

Lastly, we analyze the expected payoffs for the players and social welfare in equilibrium. We denote social welfare by $W^E = \sum_h G_h$. The respective payoffs for player h and social welfare are as follows:

$$G^E = \upsilon/N^2 \text{ and } W^E = \upsilon/N. \tag{7}$$

The expected payoff does not depend on externality parameter γ but on the number of players N. That is, $\partial G^E/\partial\gamma = 0$ and $\partial G^E/\partial N = -2\upsilon/N^3 < 0$. Evidently, the comparative static effects of externality and number of players on social welfare can be easily derived as follows: $\partial W^E/\partial\gamma = 0$ and $\partial W^E/\partial N = -\upsilon/N^2 < 0$.

III. Group contest with linear externality

This section examines group contests in which players compete by expending outlay and choosing sharing-rule parameter to win endogenous prizes. Each group consists of n risk-neutral members, where $n \geq 2$ and the total population is denoted by $N = 2n$. Member k of group i (= 1, 2) contributes x_{ki} to his own group. Group i expends in the aggregate X_i ($= \sum_k x_{ki}$). The total outlay of the two groups is denoted by

X ($= X_1 + X_2$). The probability of group i winning the prize p_i is given as follows:

$$p_i = X_i / X, \tag{8}$$

where $X_i \geq 0$, for $i = 1, 2$, and $X > 0$. If $X_1 = X_2 = 0$, then we assume that $p_1 = p_2 = 1/2$.

The members of the winning group i share the prize $v + \gamma(X_i + X_j)$ among themselves, for $i = 1, 2$. The fractional share of member k of winning group i is denoted as follows:

$$\sigma_{ki} = \delta_i x_{ki} / X_i + (1 - \delta_i)/n, \tag{9}$$

where δ_i may exceed 1, which is similar to Baik and Lee (1997) and Lee and Kang (1998). The sharing rule of group i is represented by the parameter δ_i, which is chosen by the group members at the beginning of the contest. Moreover, $\delta_i = 0$ implies that players of the winning group share the prize equally regardless of their individual outlays expended. A low value of δ_i implies minimal emphasis on the relative outlay in determining individual share. By contrast, a high value of δ_i implies substantial emphasis on the relative outlay. In addition, $\delta_i = 1$ implies that each player's share of the prize depends only on his outlay relative to his group's total outlays.

The contest is associated with externality because each member's real prize is affected by the aggregate outlays. The expected payoff to member k of group i(*i.e.*, G_{ki}) is given as follows:

$$\begin{aligned} G_{ki} &= p_i[\sigma_{ki}\{v + \gamma(X_i + X_j)\} - x_{ki}] + (1 - p_i)[- x_{ki}] \\ &= p_i\sigma_{ki}\{v + \gamma(X_i + X_j)\} - x_{ki}, \text{ for } i \neq j. \end{aligned} \tag{10}$$

The payoff function in (10) can be viewed similar to that in the group contest, in which each member of the winning group i obtains a percentage of the aggregate effort as externality effect and receives a percentage of the basic prize on the basis of the sharing rule. This interpretation appears valid in such situations as groups' competition to obtain the aforementioned grant. Each group consisting of the fixed number of institutions expends effort to increase its likelihood of winning the grant. Each group distributes the basic grant and matching funds from the government among the institutions in the group on the basis of the pre-specified intra-group sharing rule.

The sharing rules in group contests are either private or public information. Section III, *A* examines the contest when the sharing rules are private information. Section III, *B* analyzes the contest with public information sharing rules.

A. Contest with private information sharing rules

Consider the following game. First, the members of each group decide how to share the prize if they win. That is, they make a binding agreement on the value of their sharing-rule parameter δ. Given that the members of each group are identical, their decision on δ is unanimous. Thereafter, all members of both groups expend their outlays simultaneously and independently. When expending their outlays, the members of each group know their own sharing rule but do not know the rival group's sharing rule. Lastly, the winning group is chosen and the members of this group share the prize on the basis of their previously agreed-upon sharing rule. We assume that negotiating an agreement and sharing the prize do not incur any transaction cost. We also assume that the preceding aspects are common knowledge among the members.

Working backward, we first consider the members' decisions on their outlays. After observing his group's sharing rule or equivalently δ_i, member k of group i maximizes G_{ki} by choosing the level of x_{ki}. The first-order condition to maximize G_{ki} is as follows:

$$\{X_j\sigma_{ki}/X^2 + \delta_i(X_i - x_{ki})/X_iX\}\{\upsilon + \gamma X\} + \gamma\sigma_{ki}X_i/X - 1 = 0,\quad \text{for } k = 1, 2, 3, \ldots, n. \tag{11}$$

The second-order condition is as follows:

$$-2X_j\sigma_{ki}\upsilon/X^3 - 2\delta_i(X_i - x_{ki})\upsilon/\{X_iX^2\} < 0,\quad \text{for } k = 1, 2, 3, \ldots, n, \text{ and for } i \neq j. \tag{12}$$

We focus on the symmetric equilibrium actions. Thus, $x_{ki} = x_i$ is denoted for all k. Thereafter, the first-order condition is reduced as follows:

$$\{\upsilon + \gamma n(x_i + x_j)\}\{x_j + \delta_i(n-1)(x_i + x_j)\} /\{n^2(x_i + x_j)^2\} + \gamma x_i/\{n(x_i + x_j)\} - 1 = 0,\quad \text{for } i \neq j. \tag{13}$$

From (13), we obtain the following best response function:

$$\begin{aligned} x_i(\delta_i, x_j) = \Big[& \delta_i(n-1)(v + 2\gamma n x_j) - 2n(n-\gamma)x_j \\ & + \sqrt{\{\delta_i^2(n-1)^2 v + 4nx_j(n - r - \delta_i\gamma(n-1))\}v}\,\Big] \\ & /[2n\{(n-\gamma) - \delta_i\gamma(n-1)\}], \text{ for } i \neq j \end{aligned} \tag{14}$$

Thereafter, we consider the members' decision on their sharing rule. Given that the members expend the same outlay, they have the same expected payoff: $G_{ki} = G_i$ for all k. The members seek to maximize as follows:

$$\begin{aligned} G_i(\delta_i, x_j) = & [\{v + \gamma n\{x_i(\delta_i, x_j) + x_j\}\}/\{n\{x_i(\delta_i, x_j) + x_j\}\} - 1] \\ & x_i(\delta_i, x_j), \text{ for } i \neq j \end{aligned} \tag{15}$$

with respect to δ_i, taking group j's total outlay X_j (or x_j) as given. Note that we obtain (15) by substituting (14) into (10). From the first-order condition for maximizing $G_i(\delta_i, x_j)$, we obtain the following best response function of group i:

$$\delta_i(x_j) = \{-\gamma n x_j + \sqrt{(1-\gamma)n x_j v}\} \,/\, \{(1-\gamma)v - \gamma^2 n x_j\}. \tag{16}$$

The second-order condition is satisfied.[1] Lastly, we obtain the symmetric equilibrium actions, which is the $2(n+1)$-tuple vector of actions $(\delta_1^*, x_1^*, \cdots, x_1^*, \delta_2^*, x_2^*, \cdots, x_2^*)$, by simultaneously solving the system of four equations: $x_1(\delta_1, x_2)$, $\delta_1(x_2)$, $x_2(\delta_2, x_1)$, and $\delta_2(x_1)$. By substituting (16) into (14), we obtain $x_1(x_2)$ and $x_2(x_1)$.[2] Given that we focus on the symmetric equilibrium actions, the players expend the same outlay and choose the same sharing-rule parameters. Thus, let $x_1^* = x_2^* = x^*$ and $\delta_1^* = \delta_2^* = \delta^*$. We obtain the following by solving $x_1(x_2)$ and $x_2(x_1)$:

$$x^* = v/\{4n(1-\gamma)\} \text{ and } X^* = v/\{2(1-\gamma)\}, \tag{17}$$

thereby implying that $\partial x^*/\partial\gamma = v/\{4n(1-\gamma)^2\} > 0$, $\partial X^*/\partial\gamma = v/\{2(1-\gamma)^2\} > 0$;

[1] We use the computer program Maple to solve for the second-order condition. For concise exposition, we do not provide the expression for the second-order condition. This information is available from the authors upon request.

[2] We use Maple to solve for $x_1(x_2)$ and $x_2(x_1)$.

and $\partial x^*/\partial N = \upsilon/\{2N^2(1-\gamma)\} > 0$ and $\partial X^*/\partial N = 0$. We substitute x^* into (16) and obtain as follows:

$$\delta^* = 1/(2-\gamma), \tag{18}$$

thereby implying $\partial\delta^*/\partial\gamma = 1/(2-\gamma)^2 > 0$ and $\partial\delta^*/\partial N = 0$. The sharing-rule parameter is less than the unity for $-1 < \gamma < 1$. For negative externalities $\gamma < 0$, we have $\delta^* < 1/2$. That is, the relative outlay is minimally emphasized in determining individual share. For positive externalities $\gamma > 0$, we have $1/2 < \delta^* < 1$, which indicates substantial emphasis on the relative outlay.

We analyze the rent-dissipation rate $RD^* = X^*/(\upsilon + \gamma X^*)$. The rent-dissipation rate is given as follows:

$$RD^* = 1/\{2(1-\gamma)\}, \tag{19}$$

thereby implying that $\partial RD^*/\partial\gamma = 1/\{2(1-\gamma)^2\} > 0$ and $\partial RD^*/\partial n = 0$.

Lastly, we derive the expected payoffs for the players and social welfare. We denote social welfare in equilibrium by $W^* = \sum_k G_{k1}{}^* + \sum_k G_{k2}{}^*$. The respective payoffs for a member of group i and the social welfare in equilibrium are as follows:

$$G^* = \upsilon/(2N) \text{ and } W^* = \upsilon/2. \tag{20}$$

The expected payoff does not depend on externality parameter γ but on the number of players N. That is, $\partial G^*/\partial\gamma = 0$ and $\partial G^*/\partial N = -\upsilon/(2N^2) < 0$. Hence, the comparative statics on social welfare, such as $\partial W^*/\partial\gamma = 0$ and $\partial W^*/\partial N = 0$, can be easily derived.

B. Contest with public information sharing rules

Consider the contest that is nearly the same as the one in Section III, *A* with the only difference being that the sharing rule of the rival group is public information. In particular, we consider the following two-stage game. In the first stage, the members of each group decide how to share the prize if they win. Thereafter, both groups simultaneously announce their sharing-rule parameter values that were chosen independently. That is, group i publicly announces the value of δ_i, for $i = 1, 2$. In the second stage, all the players in both groups expend their outlays simultaneously and independently after determining the parameter values. At the end of the second stage, the winning group is chosen and its members share the augmented prize on the basis of the sharing rule chosen in the first stage. We employ subgame-perfect equilibrium as the solution concept.

First, consider the second stage. By using (14), we obtain:

$$x_1(\delta_1, \delta_2) = \{(\delta_1 + \delta_2)(n-1) + 1\}[\{(\delta_1 - \delta_2)n - \gamma\delta_1\}(n-1) + n - \gamma]\upsilon \\ /[n\{(\delta_1 + \delta_2)\gamma(n-1) - 2(n-\gamma)\}^2] \tag{21}$$

and

$$x_2(\delta_1, \delta_2) = \{(\delta_1 + \delta_2)(n-1) + 1\}[\{(\delta_2 - \delta_1)n - \gamma\delta_2\}(n-1) + n - \gamma]\upsilon \\ /[n\{(\delta_1 + \delta_2)\gamma(n-1) - 2(n-\gamma)\}^2]. \tag{22}$$

Let $G_i(\delta_i, \delta_j)$ be the expected payoff of each member in group i at the equilibrium of the second stage. Consider the first stage of the game. By substituting $x_1(\delta_1, \delta_2)$ and $x_1(\delta_1, \delta_2)$ into (10), we obtain:

$$G_i(\delta_i, x_j) = [\{\upsilon + \gamma n\{x_i(\delta_i, \delta_j) + x_j(\delta_i, \delta_j)\}\} \\ /n\{x_i(\delta_i, \delta_j) + x_j(\delta_i, \delta_j)\} - 1]\, x_i(\delta_i, \delta_j), \text{ for } i \neq j. \tag{23}$$

The members of group i seek to maximize $G_i(\delta_i, \delta_j)$, taking δ_j as given. From the first-order conditions for the maximization, we can derive $\delta_i(\delta_j)$ as follows:

$$\delta_i(\delta_j) = [\delta_j^2\gamma^2 + \{(n-1)\delta_j^2 - (3n-2)\delta_j - 2\}\gamma + 2n] \\ /[\gamma^2 - \{(n-1)\delta_j + n + 2\}\gamma + 2n]. \tag{24}$$

The best response function of group i slopes upwards if $\gamma < 0$ and downwards if $\gamma > 0$. The implication is that the groups' interactions exhibit characteristics of strategic complementarities for negative externality. Moreover, they exhibit characteristics of strategic substitutes for positive externality. We also find that the best response function of group i is $2/(2-\gamma) \geq 2/3$ if $\delta_j = 0$ and 1 if $\delta_j = 1$. The players expend the same outlay and choose the same sharing-rule parameters on the symmetric equilibrium sharing-rule parameters. Thus, let $\delta_1^{**} = \delta_2^{**} = \delta^{**}$. From the first-order condition for maximizing $G_i(\delta_i, \delta_j)$, we obtain:

$$\delta^{**} = 1, \tag{25}$$

thereby implying that player k's share of the prize depends on his outlay relative to his group's total outlays. Let $x_1^{**} = x_2^{**} = x^{**}$ for the symmetric equilibrium outlays. By substituting (25) into (21) and (22), we obtain:

$$x^{**} = (N-1)\upsilon/\{N^2(1-\gamma)\} \text{ and } X^{**} = (N-1)\upsilon/\{N(1-\gamma)\}, \tag{26}$$

thereby implying that $x^{**} = x^E$ and $X^{**} = X^E$. Moreover, $\partial x^{**}/\partial\gamma > 0$, $\partial X^{**}/\partial\gamma > 0$; and $\partial x^{**}/\partial N < 0$ and $\partial X^{**}/\partial N > 0$.

We now analyze the real rent-dissipation rate $RD^{**} = X^{**}/(v + \gamma X^{**})$. The rent-dissipation rate is given as follows:

$$RD^{**} = (N-1)/(N-\gamma), \tag{27}$$

where $RD^{**} = RD^E$, and $\partial RD^{**}/\partial\gamma = (N-1)/(N-\gamma)^2 > 0$ and $\partial RD^{**}/\partial N = (1-\gamma)/(N-\gamma)^2 > 0$.

Lastly, we analyze the expected payoffs for the players and social welfare. We denote social welfare in equilibrium by $W^{**} = \sum_k G_{k1}{}^{**} + \sum_k G_{k2}{}^{**}$. Thereafter, the respective payoffs for a member of group i and social welfare in equilibrium are as follows:

$$G^{**} = \upsilon/N^2 \text{ and } W^{**} = \upsilon/N. \tag{28}$$

Evidently, $G^{**} = G^E$ and $W^{**} = W^E$. The expected payoff does not depend on externality parameter γ but on the number of players N. That is, $\partial G^{**}/\partial\gamma = 0$ and $\partial G^{**}/\partial N = -2\upsilon/N^3 < 0$. The comparative statics on social welfare is similar to the payoff for each player. That is, $\partial W^{**}/\partial\gamma = 0$ and $\partial W^{**}/\partial N = -\upsilon/N^2 < 0$.

IV. Comparison of the two group contests

We compare the equilibrium sharing-rule parameters derived from the analysis of the two group contests. Lee and Kang (1998) examine a collective contest with public information on sharing rules and find that the equilibrium sharing-rule parameters are unity when the two groups are of the same size. That is, both groups choose a completely outlay-based incentive scheme. We also show that the equilibrium sharing-rule parameter is unity in the group contest with public information sharing rule. However, with private information on sharing rules, the parameter is constantly less than unity, which implies that $\delta^* < \delta^{**} =$

1. An interesting aspect is comparing the two equilibrium sharing-rule parameters δ^* and δ^{**}. Intuitively, the sharing-rule parameter is naturally less than one half (*i.e.*, $\delta^* < 1/2$) when negative externality is present and is larger than one half (*i.e.*, $\delta^* > 1/2$) with positive externality. Such a situation is established in the private information contest, although it does not occur in the public information contest. Nevertheless, the equilibrium sharing-rule parameter is constantly unity regardless of the degree of the externality. This result can be explained as follows. We find that the groups' interactions exhibit the characteristics of strategic complementarities and strategic substitutes for the negative and positive externalities, respectively, in public information contest. In addition, we have found that the best response function of group i is $2/(2-\gamma)$ if $\delta_j = 0$ and 1 if $\delta_j = 1$.

The rent-dissipation rates derived from individual contest and two group contests are as follows: $RD^* < RD^{**} = RD^E$. The expected payoffs of the players and social welfare derived from individual contest and two group contests are as follows. First, comparing the expected payoffs of the players, we find that $G^E = G^{**} < G^*$. Second, comparing social welfares, we find that $W^E = W^{**} < W^*$.

V. Concluding remarks

This study examined contests in which the contest process generates externality given by a linear function of aggregate effort. Such an externality is added to the prize. This research also compared rent dissipation and payoffs in equilibrium with the prize externality versus those without externality. Moreover, the current study used three models that incorporate the positive and negative externalities of effort.

This study showed that an increase in prize externality increases the rent-dissipation rate but has no effect on social welfare. The rent-dissipation rate and social welfare obtained from the individual contest are identical to those obtained from the group contest with public information on sharing rules. The equilibrium sharing-rule parameter with private information on sharing rules is found to be smaller than the sharing-rule parameter with public information on sharing rules. This research also demonstrated that rent dissipation in the collective contest with private information sharing rules is less than that in the collective contest with public information sharing rules. Lastly, this study showed that social welfare in the collective contest with private information sharing rules is greater than that in the collective contest with public information sharing rules.

The present analysis can be extended in several directions. In particular, an important extension is to consider the possibility of strategic revelation of information on sharing rules. This research assumes that the regime of private or public information on sharing rules is exogenously given. One can consider an extended model in which each group has the option of strategically revealing the sharing rules. Moreover, one may find "social dilemma" in which each group reveals its sharing-rule information. Accordingly, we leave this extension for future research.

(*Received 21 January 2019; Revised 16 May 2019; Accepted 9 June 2019*)

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