경제학석사학위논문

Cooperative game theory approach to airline alliance revenue sharing

협조적 게임 이론을 통한 항공사 연합 수익 분배

2019년 8월

서울대학교 대학원
경제학부 경제학 전공
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이 논문을 경제학석사학위논문으로 제출함

2019년 4월

서울대학교 대학원

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이호정의 석사학위논문을 인준함

2019년 6월

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Abstract

Cooperative game theory approach to airline alliance revenue sharing

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In an airline alliance, airlines combine their resources to serve more customers and reach larger networks. As they jointly operate flights, the need for fare revenue sharing mechanism arises. In the real world, the revenue sharing of the airline alliance follows proportional allocation rule. The current proportional rule is simple but does not guarantee revenue maximization of the alliance. In this paper, we show that the airline alliance game is convex and apply one of the cooperative game theory concept, weighted Shapley value, to compute the allocation for participating airlines. Setting this allocation result as reference point, we evaluate how well proportional allocation rule works.

keywords: Cooperative game theoretic approach, Airline alliance, Revenue sharing, Shapley value
Student Number: 2017-22617

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1. Introduction

In many fields, businesses cooperate strategically to increase their profit. The airline industry is one of them. After the emergence of low-cost airlines in the market, traditional network airlines in the US underwent a decrease in their market share. For example, in 2000, American Airlines operated 9.8% of US domestic passenger miles alone, but it has dropped to 5.6% by 2018.

To deal with increasing competition throughout the industry, traditional airlines make partnerships in several forms. One of the major forms of partnerships is the airline alliance. In an airline alliance, member airlines offer flights service to their passengers by combining their resources, such as flight networks, aircraft, and crew members. By combining their networks, airlines can reach a larger pool of customers and also increase the efficiency of their resource usage. As airline alliances have become popular, needs for fair revenue sharing mechanism arises. In the real world, the revenue sharing of alliance follows proportional allocation rules.

The most common types of proportional allocation rules are flat amounts, mileage-based and fare-based. Flat amounts rule requires to give a fixed amount to the operating carrier of each leg. Under mileage-based and fare-based rules, operating carriers will take the proportional allocation determined by traveled mileage or full Y fare(economic class fare) on each leg. Those proportional rules are simple but not able to achieve efficiency because proportional rules give airlines incentive to maximize their own revenue, not total revenue of the alliance.

1.1. Proportional allocation rule

Consider the following network that airline a and b jointly made. Airline a and b know their own leg demand and demand for the itinerary using both legs, but don't have any idea about the demand of their partner's leg.

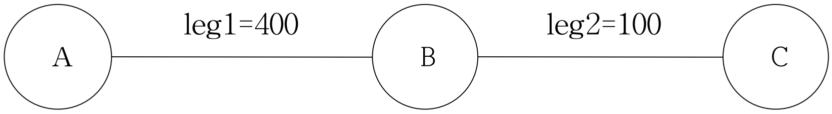

Figure 1. Jointly operated flight by airline a and b

	Operator	Distance	Capacity
leg1	a	400	8
leg2	b	100	8

Table 1. Leg Information

	Demand	Fare(USD)
leg1	4	400
leg2	1	120
leg1+leg2	7	450

Table 2. Ticket Information

Each airline wants to maximize its own profit. Under mileage-based rule, airline a know that it can earn only 360 dollars when it sells 2-leg tickets but can have 400 dollars by selling 1-leg tickets. Therefore, airline a sell 1-leg tickets to 4 passengers who want to go from A to B. Airline b is also aware of the fact that it can earn 120 dollars by selling 1-leg ticket, which goes from B to C, while it could only have 90 dollars from 2-leg ticket.

	Airline a	Airline b	Fare(USD)
Capacity	8	8	
leg1	4		400
leg2		1	120
leg1+leg2	4	4	450
Total Revenue			3,520

Table 3. Sales Results

As a result, there are 4 passengers using both legs, 4 passengers using only leg 1, and 1 passenger for leg 2. The total revenue of this network will be 3,520. Airline a will earn 3,040 dollars, while airline b will have only 480 dollars.

However, to maximize revenue from their cooperation, airline a and b should sell 7 tickets to the passengers who want to go from A to C, another ticket for passenger A to B and the last ticket for a passenger B to C. The maximized total revenue is 3,670 dollars, which is bigger than the result of selfish behavior. We can find a similar example for fare-based allocation rule.

As shown in the example, proportional rules don't guarantee revenue maximization of alliance because the resources of the cooperation cannot be used fully. This problem cannot be easily solved since an airline which can fill its capacity alone always experiences a decrease in its revenue when the alliance maximizes network profit. Instead, we apply cooperative game theory to the alliance revenue sharing and see how well proportional allocation rule works by setting allocation based on cooperative game theory as a reference point.

1.2. Literature Review

There are many studies on the revenue management of an airline alliance, but few of them formally analyze the revenue sharing of the airline alliance. Vinod(2005) discusses the revenue maximization of alliance considering alliance pricing and capacity management. However, this paper gives an only basic idea for the revenue sharing of the alliance. The first paper that takes revenue sharing as a major part of the analysis is Kimms and Çetiner(2012). This paper is divided into two part: first, they make a linear programming model for revenue maximization and use cooperative game theory to make a model for revenue sharing of an airline alliance. They also show that the game has a non-empty core and propose a revenue allocation rule based on the concept of the nucleolus. Çetiner and Kimms(2013) expand their previous revenue maximization model to selfish behavior situation where individual airlines pursue their own profit maximization, not alliance revenue maximization. The paper also computes revenue allocations under selfish revenue sharing mechanism and compares the results with nucleolus-based allocation. Hu et al. (2013) propose a two-stage allocation scheme combining cooperative-game and non-cooperative game. The rule that this paper proposes is basically proportional allocation rule, but the determination of the proration rate considers the stability of the coalition, one of the important issues in cooperative game theory.

1.3. Structure

The organization of this paper is as follows. In section 2, we model the alliance revenue sharing problem and introduce the concept of Shapley value. Also, we show that our game is convex. Section 3 describes the data we used in the analysis. Section 4 gives the analysis of data based on the model presented in Section 2. Finally, section 5 concludes the discussion.

2. The Model

2.1. The Model

Let $N = \{1, 2, \ldots, n\}$ be the set of airline in alliance. And let $S \subseteq N$ be the subset of N. Here, S represents a coalition of alliance N. The set of all flights in alliance are denoted by F, and F_S means the set of all possible flights that can be made by coalition S. Individual flight can be represented as f_k, where $k = 1, 2, \ldots, |F|$. Typical element of the set F is denoted by f. In alliance revenue sharing game, we define te worth of coalition as follows.

Definition 2.1 The worth of coalition S is defined as follow:

$$v(S) = \sum_{f_k \in F_S} Rev(f_k)$$

where $Rev(f_k)$ is the revenue from the flight f_k.

An airline joins in an alliance only when it can increase its revenue by participating in the alliance. Our game is convex so no single airline or coalition has the incentive to leave the alliance.

Theorem 2.2 The airline alliance game is convex.

Proof) To show the airline alliance game is convex, we need to show the value function v should satisfy

$$v(S\bigcup\{i\})-v(S)\leq v(T\bigcup\{i\})-v(T),\ when\ S\subset T\subset N\setminus\{i\}.$$

From the data, we know that individual flight has at most three operators. Then the sets of flights which is operated by 2, 3 airlines in alliance S are represented as F_{ij} and F_{ijk} and typical elements of each set is denoted as f_{ij} and f_{ijk}, where $i,j,k\in S$. So, we can rewrite value function v as

$$\sum_{f_k\in F_S} Rev(f_k)=\sum_{i,j\in S} Rev(f_{ij})+\sum_{i,j,k\in S} Rev(f_{ijk})$$

for each $i,j,k\in S$.

Without loss of generality, let $|S|=s$ and $|T|=t$, and $s<t$. By definition,

$$v(S)=\sum_{f_k\in F_S} Rev(f_k)=\sum_{i,j\in S} Rev(f_{ij})+\sum_{i,j,k\in S} Rev(f_{ijk}).$$

Then,

$$\begin{aligned}
&v(S\bigcup\{i\})-v(S)\\
&=\sum_{j,k\in S\bigcup\{i\}} Rev(f_{jk})+\sum_{j,k,l\in S\bigcup\{i\}} Rev(f_{jkl})\\
&\quad-\sum_{j,k\in S} Rev(f_{jk})-\sum_{j,k,l\in S} Rev(f_{jkl})\\
&=Rev(f_{i1})+Rev(f_{i2})+\cdots+Rev(f_{is})\\
&\quad+Rev(f_{i12})+Rev(f_{i13})+\cdots+Rev(f_{i,s-1,s})\\
&=\sum_{j\in S} Rev(f_{ij})+\sum_{jk\in S} Rev(f_{ijk}).
\end{aligned}$$

Likewise,

$$v\left(T\bigcup\{i\}\right)-v(T)$$

$$= \sum_{j,k\ \in\ T\bigcup\{i\}} Rev(f_{jk}) + \sum_{j,k,l\ \in\ ST\bigcup\{i\}} Rev(f_{jkl})$$

$$- \sum_{j,k\ \in\ T} Rev(f_{jk}) - \sum_{j,k,l\ \in\ T} Rev(f_{jkl})$$

$$= Rev(f_{i1}) + Rev(f_{i2}) + \ \cdots\ + Rev(f)$$

$$+ Rev(f_{i12}) + Rev(f_{i13}) + \ \cdots\ + Rev(f_{i,t-1,t})$$

$$= \sum_{j\ \in\ T} Rev(f_{ij}) + \sum_{jk\ \in\ T} Rev(f_{ijk}).$$

Since the revenue of each flight is always non-negative, and $s < t$,

$\therefore v(S\bigcup\{i\}) - v(S) \leq v(T\bigcup\{i\}) - v(T)$

■

The flight can be consist of a single leg, but many long-distance flights should be consist of 2 or more legs. All flights are consist of up to 3 legs, and each airline contributes to the alliance by operating flights legs or publishing tickets.

For simplicity, we will assume that individual airlines generate value only by operating flights legs. It is clear that the revenue from a single leg flight will go to the airlines who operate it. Similarly, when 2- or 3-leg flights were operated by only one airline, the revenue also goes to that operating airline.

The problem is complicated when 2 or more airlines are operating one flight together. Here, fare revenues sharing is an important issue for the airlines participated. Based on the value function defined above, we will compute a well-known cooperative game theory concept, Shapley value and compare its allocation with the allocation of proportional rule.

2.2. Shapley value

Cooperative game theory is widely used to analyze how to allocate coalitional profit to its participants. One of the famous solution concept is Shapley value, presented by Shaplye(1953), which is based on marginal contribution of each agent.

Definition 2.3 Given a game (N, v), its Shapley value ϕ allocates the surplus $v(N)$ of the grand coalition as follows:
for each agent i,

$$\phi_i = \sum_{S: i \in S} \frac{(|S|-1)!(|N \setminus S|)!}{|N|!} \{v(S) - v(S \setminus i\}.$$

Shaypley(1953) proved that the only value operator satisfying efficiency, symmetry, additivity, and dummy is Shapley value. However, in our game, the value must not be symmetric since the contribution of each agent to a flight is not the same. For instance, in the example presented in section 1, in revenue maximizing situation, 2 airlines jointly earned 3,670 dollars but the traveled mileage and fare of each leg are different. To reflect these differences, we need to consider weighted Shapley value and define reasonable weight system. It is well known that the same flight can be sold in different fare depending on publishing airline. Therefore, we use traveled mileage to define weight system.

We know the fact that the same itinerary can be sold by different fare depending on publishing airline, therefore, we use traveled mileage information to define weight system. One way to construct a weight system based on traveled mileage is by using total traveled mileage of each airline. However, it can induce serious error. Since traveled mileage depends on not only the distance traveled but also the number of flight legs that the airline operates, this system can exaggerate the weight of an airline which operates simply a large number of legs. In this paper, we use average traveled mileage per leg, because it offsets the effects comes from the number of legs, while remaining leg-distance information.

3. Data

Based on the model we presented, we will compute the Shapley value and compare it to the proportional rule(mileage-based rule). The data we used are the ticketing database of February 2017, from OAG(Official Aviation Guide of Airways) and T100 database of BTS(Bureau of Transportation Statistics) to check the length of traveled mileage of each leg. OAG ticketing data gives publisher and operator information of itinerary and its consisting legs, origin-destination record, ticket fare of each cabin class, total revenue and length of the whole flight. BTS T100-segment(leg) database contains non-stop segment data by aircraft type and service class.

Leg composition	Number of observations
1-leg flights	3,624
2-leg flights	194,036
3-leg flights	233,134

Table 4. OAG data observations

We use origin-destination record and total revenue from OAG ticketing data and segment(leg) distance from BTS T100-segment data. In OAG ticketing data, we have 3,624 observation for single leg flights, 194,036 for 2-leg flights and 233,134 observations for 3-leg flights. To make computation simple, we zero-normalized the data and consider only 13 airlines in Oneworld alliance for our computation.

After zero-normalization, we have 4,696 observations for 2-leg flights and 2,597 observations of 3-leg flights. From the T-100 segment data, we extract the distance of 36,047 segments. By matching T-100 segment data to OAG ticketing data of Oneworld alliance, we get 2,884 observation of 2-leg flights and 201 observations of 3-leg flights with full leg length information. An example of the matching is shown in **Table 5.** In processed data, there are only 10 airlines, since the observations without full leg information are deleted.

	leg1(op.al)	leg2(op.al)	Total Revenue
OAG	EZE-MIA(AA)	MIA-LAS(LA)	12,295
BTS(T-100)	4,406 mile	2,714 mile	

Table 5. OAG-BTS(T-100) matching

4. Computation

The numerical study is conducted with the data described in Section 3. We first compute the total traveled mileage and number of legs of each airline in **Table 6.**

Airline	Total traveled mileage(mile)	Leg count
AA	3,241,867	3,282
BA	3,016,773	762
CX	1,858,187	247
AY	135,691	39
IB	374,242	94
JL	2,468,325	401
LA	1,791,948	475
QF	5,199,313	733
QR	849,936	122
RJ	1,234,784	216

Table 6. Total traveled mileage of individual airlines

In our data, Qantas Airways(QF) has the largest amount of traveled mileage, followed by American Airlines(AA) and British Airways(BA). Note that American Airlines has overwhelming number of legs in the data, which results in the second largest amount of traveled mileage. From this fact, considering only total traveled mileage in weight system can distort the contribution of each airline. We will compute the weighted Shapley value modeled in section 2 and see how well the proportional allocation rule works comparing two allocation results. First, we will sho w the average traveled mileage in Table 7 and give the allocation results from proportional allocation rule and Shapley vale, and the differences between two allocations in Table 8.

Airline	Average traveled mileage
AA	908
BA	4,029
CX	7,618
AY	4,117
IB	3,981
JL	6,214
LA	3,785
QF	7,897
QR	7,133
RJ	6,097

Table 7. Average traveled mileage of individual airlines

Airline	Proportional Allocation rule(PA)	Weighted Shapley value(WSH)	Difference (WSH-PA)
AA	6,569,090	6,633,166	64,076
BA	5,641,786	5,292,742	-349,044
CX	5,287,189	5,332,896	45,707
AY	111,319	105,660	-5,659
IB	294,717	274,926	-19,791
JL	7,399,682	7,112,852	-286,830
LA	3,814,261	3,561,422	-252,839
QF	20,531,501	21,374,286	842,785
QR	702,952	675,495	-27,457
RJ	851,315	807,372	-43,943

Table 8. Revenue allocations

In Table 8, except American Airlines, Cathay Pacific Airways, and Qantas Airways, all other airlines undergo decrease in their allocation. This implies that weighted Shapley value gives more amount to airlines which have longer average leg distance. Here, Qantas Airways has the longest average traveled mileage per leg and Cathay Pacific Airways has the second longest. The allocation increase in American Airlines comes from the fact that

American Airlines has the biggest number of operating legs in the data.

Another interesting point is that the difference between the allocations from Shapley value and the proportional rule is not very big. The difference takes only 3 percent of the total revenue of the participating airlines, which means the current proportional allocation rule is easy to apply and guarantee fairness as well.

5. Conclusion

Fair revenue sharing in an alliance is an important problem that member airlines face. Current revenue allocation follows proportional allocation rules, but these rules do not guarantee the maximization of alliance revenue and efficient use of network resources.

In this paper, we model airline alliance revenue sharing game and evaluate current revenue allocation scheme based on the cooperative game theory. As a reference point, we computed the weighted Shapley value. The weighted Shapley value can be a reasonable reference point since it considers both revenue contribution and flight mileage contribution of airlines. To do the computation, we used OAG ticketing database and BTS T-100 segment database. Also, we show that our game is convex so airlines enjoy benefits by forming an alliance.

Although it has some shortcomings, proportional allocation is useful in the sense that it is really simple and gives fair allocation to member airlines. In the example in Section 1, airlines can earn more by selfish behavior. However, if the demand that an airline faces fluctuates, the revenue they get from each allocation rule will be differ. Future study can model changes in demand that airlines face and see how each allocation scheme works.

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Appendix

A. Airline code

Airline code	Airline name
AA	American Airlines
BA	British Airways
CX	Cathy Pacific Airways
AY	Finnair
IB	Iberia
JL	Japan Airlines
LA	LATAM Airlines Group
QF	Qantas Airways
QR	Qatar Airways
RJ	Royal Jordanian

국문 초록

항공 산업에서 항공사들은 그들의 자원을 결합해 더 많은 고객에게 서비스를 제공하고 더 큰 비행 네트워크를 확보하기 위해 연합을 형성한다. 항공사들이 연합을 통해 비행을 함께 운항함에 따라 그 수익을 어떻게 공평하게 분배할 것인지에 대한 문제가 발생한다. 현재 사용 중인 비례 배분 규칙은 간단하지만, 연합의 수익 극대화를 보장하지 못한다는 단점이 있다. 본 논문에서는 항공 연합 게임이 볼록 게임이라는 것을 확인하고, 협조적 게임 이론의 해인 가중적 샤플리 밸류를 이용해 항공 연합의 수익을 어떻게 나눌 것인지 계산하였다. 또한, 샤플리 밸류의 분배 결과를 기준으로 삼아 비례 배분 규칙의 공평성을 평가하였다.

주제어: 협조적 게임이론, 항공 연합, 수익 분배, 가중적 샤플리 밸류

학번: 2017-22617