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A Note on the Optimality of the Debt–Equity Mix*

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Dewatripont and Tirole showed (for the first time) that a mix of debt and equity is optimal and thus provided justification of the most commonly observed capital structure. Using Diamond framework, this note simplifies their model and shows that the main reason why debt–equity mix is optimal is not the divergence of attitude towards risk between shareholders and debtholders as claimed in their papers. It is rather the flexibility debt contract affords in allocating control rights and incentives to liquidate the firm.

I. Introduction

During the 1980s traditional capital structure theories gave rise to a more general question of security design. Studies of security design since the 1980s have produced several interesting results and have deepened our understanding. However, these studies were most successful in the area of hybrid securities such as convertible bonds (see for example, Brennan and Schwartz (1987) and Stein (1992)). They were not equally successful at explaining the most commonly observed securities and capital structure, which are a mix of straight debt and common stock. In short, literature on security design so far has failed to produce a convincing model where a mix of debt and equity emerges as the optimal solution to the security design problem.

The dominant model for the optimality of debt contract is the costly state verification model of Townsend (1978). Yet, this model assumes that debtholders have no information about firm's

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cash flows without conducting costly state verification. Because of that assumption, the same model cannot be used to prove the existence of (outside) equity along with the debt contracts. If we assume that investors have some information about the firm cash flow, so that equity contract can be used, then we will see the optimality of debt contract disappear.

In this regard, papers by Dewatripont and Tirole (1993, 1994a), which later expanded to be a monography on regulation of banks (Dewatripont and Tirole (1994b)), are remarkable because they identified an environment where a mix of debt and equity turns out to be optimal even though it is not uniquely optimal. As is well-known, debtholders become more averse to risk and equity holders develop love for risk once borrowing and lending relationship is established. In their model, it is this difference in attitudes toward risk that the two contract should be used together. This divergence of risk attitude sometimes seems puzzling because it makes the two group of stakeholders have different perspectives on their firm. Dewatripont and Tirole showed that contrary to this general perception, the divergence of attitudes toward risk is exactly what brings them together to support the firm. Different situations require different actions, and different actions come from different attitudes towards risk. When things are going well, the firm needs a risk-loving stakeholders to make decisions, and when things are going badly, the opposite is true. This is why the firm needs both the equityholders and debtholders.

In this note, we are trying to show that while the divergence of attitudes toward risk is important, it is not the main reason why a mix of debt and equity is optimal in their model. It is rather the adjustability of the size of debt contract that can induce the optimal behavior of the manager. Furthermore, we also show that it is not too much risk-aversion of the debtholders that makes them liquidate the project excessively. It is rather because debt is small relative to the total size of the firm. To make our point, we use their model and combine it with a simplified model of cash flows used by Diamond (1989, 1991a, 1991b, 1993a, 1993b) in a series of papers on the structure of debt contracts.

II. Model

A risk-neutral manager[1)] needs outside finance for his project. He is allowed to use any financial contracts he sees fit. There are three dates. At date 0, financial contracts are drawn, and at date 1, more information becomes available, and liquidation/continuation decision is made based on new information. At date 2, cash flows are realized and distributed according to the financial contracts signed at date 0. There is no interim cash flow at date 1.

At date 0, after the contract is signed, the manager chooses his effort level e. Following Dewatripont and Tirole, we assume that there are only two levels of effort, a lower level of effort $\underline{e}$ and a higher level of effort $\overline{e}$ with $\underline{e} < \overline{e}$. We assume that the higher level of effort costs the manager $C(\overline{e}) = K > 0$ while the lower level of effort is costless: $C(\underline{e}) = 0$. So, if other things are equal, the manager prefers not to work hard.

The effort level is important because it determines the probability distribution of the final cash flows at date 2. To emphasize the role of divergent attitude towards risk, Dewatripont and Tirole had to assume a rather complicated form for the probability distribution of the final cash flow. Here, we adopt the Diamond framework and use a very simple form of cash flows. We assume that the date 2 final cash flows can take one of the following two forms.

$$\begin{cases} \text{Safe cash flow: yields } X \text{ for sure} \\ \\ \text{Risky cash flow: yields } X \text{ with probability } \pi \in (0,\ 1) \text{ and } 0 \text{ with probability } (1-\pi) \end{cases}$$

The probability of the safe cash flow is a random variable $\tilde{z} \in [0,1]$. The managerial effort e determines the probability distribution of this random variable $\tilde{z}$. Specifically, we assume:

Assumption 1: Under $\underline{e}$, the density of $\tilde{z}$ is $g(z)$ while under $\overline{e}$, the density is $f(z)$. High

1) Dewatripont and Tirole call this person a manager instead of an entrepreneur mainly because the person has no equity stake. This assumption of zero equity stake plays a crucial role in their model.

effort $\overline{e}$ is better than low effort $\underline{e}$ in the following sense of the *Monotone Likelihood Ratio Property*:

$$\ell(z) \equiv \frac{f(z)}{g(z)} \text{ is increasing in } z$$

The monotone likelihood ratio property is similar to the comparative advantage in economic theory. A low level of effort is more likely to induce a lower cash flow while a high level of effort is more likely to generate a higher cash flow at date 2.

At date 1, the random variable $\tilde{z}$ is realized and becomes *public* knowledge, but the effort level e is not verifiable. Thus contracts contingent on the realization of $\tilde{z}$ are feasible while contracts contingent on e are not.[2] After observing the realized value z, investors may renegotiate with the manager on terms of the contract. Depending on the outcome of the renegotiation process, the project will either be liquidated for the liquidation value L, or will be continued. It is assumed that a safe cash flow makes the project worth continuing while the project had better be liquidated if it is to generate a risky cash flow:

Assumption 2: $\pi X < L < X$

A big difference between our exposition and Dewatripont and Tirole's is that they needed two random variables that affect the probability distribution of the final cash flow. To make attitude toward risk a relevant variable, the probability distribution of the final cash flow needs to be subject to another level of uncertainty. Yet, in our model, that is not necessary.

III. Determination of the Optimal Incentive Scheme

Original contracts may specify actions contingent on the realization of z. Since there are only two possible actions (liquidation or continuation), the probability of liquidation, denoted by

2) Dewatripont and Tirole assumed that there are two random variables u and v, and one of the two is verifiable. We abstract from this complication.

$\lambda(z)$, is all we need to characterize the optimal contract.

Dewatripont and Tirole made some assumptions regarding the renegotiation process. We will maintain their assumptions. First, they assumed that renegotiation involves no extra cost. This assumption is adopted by almost all papers on renegotiation. Second, the manager enjoys control rent $C>0$ only if the project is not liquidated at date 1. The control rent C is assumed to be *transferable*. This assumption is made because if it is not transferable, then the manager has no way to influence the renegotiation outcome. If the renegotiation outcome calls for changes in the probability of liquidation function $\lambda(z)$, outsiders must get the manager's approval as well. If they fail to get his approval, the original plan (i.e., $\lambda(z)$) will be executed. Third, the manager is assumed to have no bargaining power. This implies that if the manager wants to renegotiate with outside financiers, he will have to give up all his control rent. This is a simplifying assumption, but it is not an unrealistic one to make when renegotiation is over liquidation of the project.

Let $q(z)=\pi+z(1-\pi)$ be the probability of success of the project when the realized probability of a safe cash flow is z. Then, it is obvious that no matter what the original contract says, the project will be liquidated if and only if the realized z satisfies

$$q(z)X+C<L$$

Given the contract $\lambda(z)$, the total value of the firm to outsiders is

$$\max\left[q(z)X+C,L\right]-\left[1-\lambda(z)\right]C$$

To understand this, note first that the total value of the project is $\max\left[q(z)X+C,L\right]$ because the control rent C is transferrable and renegotiation is costless. Yet, outside financiers cannot appropriate the full project value because they have to give the control rent to the manager when there is no liquidation. Under the stipulations of the initial contract, the manager is supposed to enjoy $[1-\lambda(z)]C$ of control rent. Therefore, to get the approval of the manager, outsiders have to give him that much of the control rent. In this sense, the term $[1-\lambda(z)]C$ captures the agency costs.

Since the distribution of $\tilde{z}$ depends on the managerial effort e, outsiders' objective is to find the best incentive contract $\lambda(z)$ that will induce best action by the manager. For example, if $\lambda(z)=1$ for all z, the manager will choose the lower level of effort $\underline{e}$. In that case, outsiders will get $\int \max\left[q(z)X+C,L\right]g(z)dz$.

To make the problem worth pursuing, we make the following technical assumption.

Assumption 3: $K<C<L$

To induce the manager to exert proper effort, the benefit of the control right should be greater than the cost of high effort. Furthermore, if $C \geq L$, then the manager can prevent liquidation of the project simply by offering part of the control rent. For liquidation to happen, we need to assume $C<L$ so that there is limit to what the manager can do to block liquidation.

Under the assumptions, the outsiders' problem becomes;

$$\min_{\lambda(\cdot)} AC \equiv C\int_0^1 \left[1-\lambda(z)\right] f(z)dz$$
$$\text{s.t.} \quad AC \geq C\int_0^1 \left[1-\lambda(z)\right] g(z)dz + K$$

The problem is to minimize the agency cost AC by the choice of the function $\lambda(z)$ given the incentive compatibility constraint. Let $\mu \geq 0$ be the Lagrangian multiplier for the constraint. Then, the Lagrangian becomes

$$\mathcal{L} = [1-\lambda(z)]\left[(\mu-1)f(z)-\mu g(z)\right]$$

And we have to maximize this expression (we changed the problem into maximizing $-AC$).

Since the objective function is linear in $\lambda(z)$, $\lambda(z)$ will be either 1 or 0 with probability 1; there will be no randomization. First, note that at optimum, $\mu>1$ holds. If $\mu \leq 1$, then the term in $[\cdot]$ is negative and thus $\lambda(z)=1$ is optimal for all $z \in [0,1]$. However, that cannot be the solution since it will violate the incentive compatibility constraint. So, we have $\mu>1$. The optimal solution would be:

$$\lambda^*(z) = \begin{cases} 1 & \text{for } z < z^* \\ 0 & \text{for } z < z^* \end{cases}$$

where z^* is defined by

$$\ell(z^*) \equiv \frac{f(z^*)}{g(z^*)} = \frac{\mu}{\mu - 1}$$

For the problem to have a solution there must exist such z^*; otherwise, $\lambda(z)$ is either 0, or 1 for all $z \in [0,1]$, thereby violating the incentive compatibility constraint. As Dewatripont and Tirole, we assume that there exists a unique interior solution z^* that satisfies this equation. Note also that the value $\lambda^*(z^*)$ is indeterminate. As long as distributions do not have any atom at z^*, this would not cause any problem.

The most significant thing about the optimal solution is that the success cash flow X and the liquidation value L enter neither the objective function nor the constraint. Thus the critical value z^* is completely independent of X and L. It is determined solely by the functional form of the two density functions $f(\cdot)$ and $g(\cdot)$, the size of the control rent C, and the cost of efforts K. Furthermore, the optimal solution specifies only the allocation of the control rent through the liquidation decision, and it is silent about the distribution of the final cash flow X.

The optimal solution is such that for high values of z, liquidation will not occur, and the manager will enjoy the full control rent. On the other hand, if z turns out to be low, the project will be liquidated, and the manager will lose all his control rent. Since high values of z are more likely under $\overline{e}$, the manager is induced to choose $\overline{e}$ under the optimal contract.

IV. Implementation by a Mix of Debt and Equity

We now show that the above optimal incentive scheme can be implemented by a mix of debt and equity.

We first show that 100% equity cannot implement the optimal solution. An equity contract is characterized by the fact that its holders will get the full final cash flow and has control rights all the time. Suppose that only equity is initially issued to finance the project. Shareholders have both the right and incentives to liquidate the firm at z if and only if $q(z)X < L$.[3] Suppose $q(z^*)X < L$ holds. Then, for $z < z^*$, there will be renegotiation of the contract and the project will be liquidated if and only if $q(z)X + C < L$. Note that when $z < z^*$ and $q(z)X + C \geq L$, the project will not be liquidated, but the manager will have to give up all his control rent C. Thus the optimality condition will not be violated. The trouble rises when $z > z^*$ and $q(z)X \leq L$. For these values of z, the manager will also have to give up all his control rent, but the optimality condition requires $\lambda(z)=0$. Therefore, when $q(z^*)X < L$, equity contract alone cannot implement the optimal solution.

Now, suppose that $q(z^*)X > L$ holds. Then for $z > z^*$ and $q(z)X \geq L$, there will be no renegotiation, and the manager will retain all of his control rent. Hence the optimal solution cannot be implemented. The only case where the equity contract alone can implement the optimal solution is when $q(z^*)X = L$ holds. Note here that equity alone cannot implement the optimal solution even if the corporate charter has a clause that the firm be liquidated if and only if $z < z^*$ because the corporate charter will also be subject to renegotiation.

We now discuss the case of a mix of debt and equity is used to finance the project. A debt contract is characterized by its face amount D and its covenants. In this model, debt covenants can be summarized by a critical value z^c such that debtholders get the control rights if and only if $z \leq z^c$. If $z > z^c$, then shareholders have the control rights. Suppose that debt with face value D is issued to finance the project along with equity. Assume that debt has priority over equity. Then, debtholders have incentives to liquidate the project if and only if $q(z)D < L$. Here, whether there will be liquidation or not depends on whether debtholders have the right to enforce their decisions and whether the manager could prevent liquidation by offering the control rent.

Consider first the case where $q(z^*)X > L$ holds. In this case, chose the amount of debt so that the inequality $q(z^*)D = L$ holds, and set the covenant in such a way that the debtholders

3) We apply a tie-breaking rule that liquidation will not occur at z if $q(z)X = L$.

get the control right whenever $z \leq z^*$. Then, since $q(z)D < L$ holds true for all $z < z^*$, there will be renegotiation, and the manager will have to surrender all his control rent. If $z > z^*$, then shareholders have control rights, but they have no incentive to liquidate the project since it will be clear below that debtholders will get the full liquidation value, leaving nothing for shareholders. So, renegotiation will occur only when $z < z^*$ and only with debtholders. Thus, the debt contract constructed in such a way can implement the optimal solution. In the case of $q(z^*)X > L$, the amount of the debt to be issued contract should satisfy the following condition:

$$D = q\left(z^*\right)^{-1} L > L.$$

As remarked above, the condition $D > L$ ensures that shareholder have no incentive to liquidate the firm when $z > z^*$.

Now, consider the case where $q(z^*)X \leq L$. In this case, $q(z^*)D < L$ for all $D < X$. Therefore, the covenant of the debt contract should be set up in such a way that debt holders have control right only when $z \leq z^*$. Then, there will be renegotiation only when $z \leq z^*$, and the optimal solution can be implemented. In this case, the amount of debt to be issued is indeterminate as long as $D \leq X$.

We have shown that a mix of debt and equity can implement the optimal solution. Note that the optimal solution is implemented not by direct liquidation of the project, but through renegotiation with the manager. Actual liquidation of the project will occur if and only if $q(z)X+C < L$.

V. Conclusion

This note has a very limited objective. That is to simplify the important result of Dewatripont and Tirole (1993, 1994a) and clarify the role of debt. Dewatripont and Tirole made the divergence of attitude toward risk between equityholders and debtholders the central issue in the formulation. Yet, simplification of their model clearly shows that it is the easiness

of controlling amount of debt to be issued and the role of debt covenants in allocating control rights which made their result possible. Our simplification can also explain why debt contract has priority over equity contract, but we did not pursue that line of inquiry.

Dewatripont and Tirole's result depends critically on the assumptions they made on the renegotiation process. Since the optimal liquidation decision is deterministic, and not randomized, they had to assume a rather extreme form of renegotiation where the manager has no bargaining power. Moreover, it is also assumed that the manager gets no pecuniary compensation from the firm's cash flow. It is true that several papers in this literature made similar assumption, that does not make the assumption any more realistic. Notwithstanding these shortcomings, their papers are the first ones that derived a mix of debt and equity as the optimal securities. Obviously, more work needs to be done in this important area.

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