

Market Liquidity and Corporate Control: A Comment on Maug (1998)*

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This note points out a small mistake made in Maug's 1998 paper in the Journal of Finance. It is shown that a correct analysis of the problem yields results different from Maug's. In short, even when the market is very liquid, a large block holder with sufficient incentives to monitor the management does not emerge.

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I. Introduction

When shareholders think that their company is badly managed, they have two options. They can either sell their shares, or voice their discontent, for example, trying to persuade the management to initiate some changes or trying to change the management. Hirschman (1970) called the first the exit option and the second, the voice option. The exit option can be costly if the stock market is not so liquid that shareholders can sell their shares only at a very depressed price. The voice option is also costly because simply talking to the management is not without personal cost (time and energy), and a proxy fight, which is the main instrument for the voice option, is very costly. Shareholders will optimally choose their option only after an elaborate cost-benefit analysis.

Coffee (1991) and Bhidé (1993) argue that when the stock market is very liquid, the cost

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of the exit option is smaller, so that most shareholders will choose the exit option. Consider especially the case where some shareholders get some bad information about the company before others. By selling their shares early before others find out, these shareholders can benefit from their information. If they choose the voice option, they had better have a remedy, and they also have to convince other shareholders that their information is indeed accurate, they do have a remedy. So, with a very liquid stock market, they argue, shareholders tend to choose the exit option, and corrective measures will not be taken. In other words, market liquidity worsens the quality of corporate governance.

In a thought-provoking paper (Maug (1998)), Maug provides a counter-argument. He notes that not all shareholders have incentives to monitor the management, and only shareholders large enough will have such incentives. A liquid stock market makes it easier for someone to emerge as a large shareholder who has strong incentive to monitor. When the market is liquid, an investor can accumulate large shareholdings without affecting the share price much and before he makes the mandatory disclosure of his large shareholding. To show this analytically, Maug builds a simple model of corporate restructuring that incorporates the Kyle model of informed trading.

We believe that the basic idea of Maug is sound. Moreover, his model is one of few examples where a corporate finance issue is embedded in a model of stock trading. However, the execution of the model has one problem. In his analysis, Maug makes a simplifying assumption to make computations easy and clean. The assumption looks innocuous, but we will show that it is not innocuous, and, once we get rid of the assumption, his conclusion does not follow.

This paper is structured as follows. We will first explain the model, and then we will get the equilibrium of the model without the Maug's assumption. At the end, we will show how different the results will be without Maug's simplifying assumption.

II. The Model

Everyone in the model is risk-neutral, and the risk-free rate is zero. This is a one company economy. The future value of the company, denoted by \tilde{v} , depends on whether the company

will be restructured or not:

$$\tilde{v} = \begin{cases} L & \text{if no restructuring} \\ H & \text{if restructuring} \end{cases}$$

In this model, monitoring is identified with restructuring. If a shareholder implements restructuring of the company, it will cost him personally $c_M < \Delta \equiv (H-L)$. Maug assumes $2c_M < \Delta$.

There are two types of shareholders. There is one large shareholder F who owns $\alpha \in (0,1)$ fraction of the firm. The large shareholder is not subject to any liquidity shock. There are many small shareholders, called households and denoted HH . Small shareholders are subject to liquidity shock. Thus with probability 0.5, a $\phi \in (0,1)$ fraction of HH sells their shares, and with probability 0.5, no HH sell their shares.

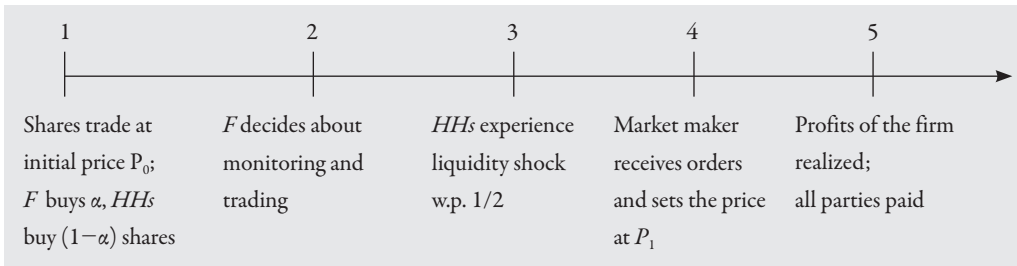
As in the Kyle model, there is a market maker, who observes the total order and sets the price at the expected value of the company value:

$$P = E[\tilde{v} \mid \text{total order}]$$

The following figure gives us the exact time line of the model:

III. Market Equilibrium

We will consider the second round of trading. The first round of trading will be discussed



later. In the second round of trading, there is no pure strategy equilibrium. To see this, note that small shareholders will sell $\phi(1-\alpha)$ or sells nothing with equal probability of 0.5. Without F 's trade, the total order will be either $-\phi(1-\alpha)$, or 0. If F always buys $x_B > 0$ and monitors, the market maker can infer that F is buying and will price the order at H . In this case, F will lose because of the monitoring cost. Therefore, this cannot be an equilibrium. If F always sells and does not monitor, he will not get any profit, either. So, there is no pure strategy equilibrium.

Now, consider a mixed strategy equilibrium where with probability $q \in [0,1]$, F buys and monitors, and with probability $(1-q)$, F sells and does not monitor. Let x_B and x_S be the amount F is buying and selling, respectively, with the convention $x_B \geq 0$ and $x_S \leq 0$. The observable total order will be: x_B , x_S , $x_B - \phi(1-\alpha)$, or $x_S - \phi(1-\alpha)$. If all these four numbers are different, then the market maker can tell whether F is buying or selling. Thus, to garble the information, F will set his orders so that

$$x_B - \phi(1-\alpha) = x_S \leftrightarrow x_B - x_S = \phi(1-\alpha)$$

The following table summarizes what happens in the market.

Order flow	Transactions	Probability	Value	Price
x_B	F buys; HH sell 0	$q/2$	H	H
$x_B - \phi(1-\alpha)$	F buys; HH sell	$q/2$	H	$qH + (1-q)L$
x_S	F sells; HH sell 0	$(1-q)/2$	L	$qH + (1-q)L$
$x_S - \phi(1-\alpha)$	F sells; HH sell	$(1-q)/2$	L	L

Given this pricing strategy of the market maker, buying and monitoring gives F :

$$\frac{1}{2}x_B[H - qH - (1-q)L] + \alpha H - c_M = \frac{1}{2}x_B(1-q)\Delta + \alpha H - c_M$$

Selling and not monitoring gives F :

$$-\frac{1}{2}x_s[qH + (1-q)L - L] + \alpha L = -\frac{1}{2}x_s q\Delta + \alpha L$$

Since F has to be indifferent between the two options, we should get

$$q = \frac{x_B\Delta - 2(c_M - \alpha\Delta)}{(x_B - x_s)\Delta} = \frac{1}{\phi(1-\alpha)} \left[x_B - 2\left(\frac{c_M}{\Delta} - \alpha\right) \right]$$

Maug deals only with the case where $x_B = -x_s = u \equiv \phi(1-\alpha)/2$ and $q = \frac{1}{2} - \frac{2(c_M - \alpha\Delta)}{\phi(1-\alpha)\Delta}$, but we are going to see that this is problematic.

VI. Determination of Optimal (x_B, x_s)

We are now trying to determine the optimal choice of (x_B, x_s) subject to the constraints specified above. First, note that to have $q \in [0, 1]$, the following inequality must hold.

$$2\left(\frac{c_M}{\Delta} - \alpha\right) \leq x_B \leq \phi(1-\alpha) + 2\left(\frac{c_M}{\Delta} - \alpha\right) \equiv \bar{x} \quad (*)$$

If $x_B \leq 2(c_M/\Delta - \alpha)$, then $q \leq 0$ and it is better not to restructure. If $x_B \geq \phi(1-\alpha) + 2(c_M/\Delta - \alpha)$, then $q \geq 1$ and it is better to restructure.

The large shareholder F will choose x_B to maximize the expected profit

$$\frac{x_B\Delta}{2\phi(1-\alpha)} \left[\phi(1-\alpha) + 2\left(\frac{c_M}{\Delta} - \alpha\right) - x_B \right] + \alpha H - c_M$$

subject to the constraint (*). The equation $f(x) = x[\phi(1-\alpha) + 2(c_M/\Delta - \alpha) - x] = 0$ has two roots, 0 and \bar{x} , and the unconstrained maximum occurs at $x^* = \frac{1}{2}\bar{x}$.

Note that $\bar{x} \geq 0$ if and only if

$$\alpha \leq \bar{\alpha} \equiv \frac{2(c_M / \Delta) + \phi}{2 + \phi}$$

For the constraint set not to be empty, we should have $\alpha \leq \bar{\alpha}$. If $\alpha > \bar{\alpha}$, then no non-negative x_B can make q a probability because $q > 1$, and the large shareholder will simply restructure the firm with probability 1 and get $\alpha H - c_M$.

Now, the question is whether $x^* = \frac{1}{2}\bar{x}$ belongs to the constraint set when $\bar{x} \geq 0$. This is determined by whether or not $2[(c_M / \Delta) - \alpha] \leq x^*$. The inequality will hold if and only if

$$\alpha \geq \bar{\alpha} \equiv \frac{2(c_M / \Delta) - \phi}{2 - \phi}$$

Note $\underline{\alpha} < (c_M / \Delta) < \bar{\alpha}$. If $2c_M \leq \phi\Delta$, this condition is automatically satisfied.

We first discuss the case where $2c_M \leq \phi\Delta$ holds

- (a) If $0 \leq \alpha \leq \bar{\alpha}$, the constraint set is $\max[2(c_M / \Delta - \alpha), 0] \leq x_B \leq \bar{x}$ and includes x^* . Thus, the optimal choice is $x_B = x^*$, and

$$q = \frac{1}{2} - \frac{\frac{c_M}{\Delta} - \alpha}{\phi(1 - \alpha)}$$

The maximized value is

$$\pi(\alpha) = \frac{\Delta}{\phi(1 - \alpha)} \left[\frac{1}{2}\phi(1 - \alpha) + \left(\frac{c_M}{\Delta} - \alpha \right) \right]^2 + \alpha H - c_M$$

- (b) If $\bar{\alpha} < \alpha \leq 1$, $q=1$ and the large shareholder will simply restructure the firm and get $\pi(\alpha) = \alpha H - c_M$.

If $2c_M > \phi\Delta$ holds, the only difference is the possibility of $0 \leq \alpha \leq \underline{\alpha}$. In this case, the constraint set is $x^* \leq 2(c_M / \Delta - \alpha) \leq x_B \leq \bar{x}$, so that the optimum occurs at $x_B = 2(c_M / \Delta - \alpha)$. Therefore, $q=0$, and no restructuring will occur. The large shareholder gets $\pi(\alpha) = \alpha L$. The two remaining cases of $\underline{\alpha} < \alpha < \bar{\alpha}$, and $\bar{\alpha} < \alpha \leq 1$ are identical to the previous cases.

To sum up, the optimal value for $x_B = x^*$ as long as q can be made a probability. Otherwise, there will be no mixed strategy equilibrium and no trading profit can be made. The expected profit for the larger shareholder will be: if the monitoring cost is not so high, so that $2c_M \leq \phi\Delta$ holds, we have:

$$\pi(\alpha) = \begin{cases} \frac{\Delta}{\phi(1-\alpha)} \left[\frac{1}{2} \phi(1-\alpha) + \left(\frac{c_M}{\Delta} - \alpha \right) \right]^2 + \alpha H - c_M & \text{if } 0 \leq \alpha \leq \bar{\alpha} \\ \alpha H - c_M & \text{if } \bar{\alpha} < \alpha \leq 1 \end{cases}$$

If the monitoring cost is high so that $2c_M > \phi\Delta$ holds, the expected profit is given by

$$\pi(\alpha) = \begin{cases} \alpha L & \text{if } 0 \leq \alpha < \underline{\alpha} \\ \frac{\Delta}{\phi(1-\alpha)} \left[\frac{1}{2} \phi(1-\alpha) + \left(\frac{c_M}{\Delta} - \alpha \right) \right]^2 + \alpha H - c_M & \text{if } \underline{\alpha} \leq \alpha \leq \bar{\alpha} \\ \alpha H - c_M & \text{if } \bar{\alpha} < \alpha \leq 1 \end{cases}$$

This leads us to Maug's Proposition 1 and 2:

Proposition 1 & 2: The probability of monitoring q is not decreasing in F 's initial stake α . It unambiguously decreases in the monitoring costs c_M . The impact of the market liquidity ϕ on the probability of monitoring is not monotonic.

(*Proof*) If $2c_M \leq \phi\Delta$, the probability of monitoring is given by

$$q = \begin{cases} \frac{1}{2} - \frac{(c_M / \Delta) - \alpha}{\phi(1-\alpha)} & \text{if } 0 \leq \alpha \leq \bar{\alpha} \\ 1 & \text{if } \bar{\alpha} < \alpha \leq 1 \end{cases}$$

Since $q = \frac{1}{2} - \frac{1}{\phi} + \frac{1 - (c_M / \Delta)}{\phi(1-\alpha)}$, q is strictly increasing in α up to the point $\alpha = \bar{\alpha}$, after which α has no impact on q . When $2c_M > \phi\Delta$, α strictly increases q only when $\underline{\alpha} \leq \alpha \leq \bar{\alpha}$; otherwise, it has no impact on q . If the monitoring cost c_M increases, q decreases if $2c_M \leq \phi\Delta$ and $0 \leq \alpha \leq \bar{\alpha}$ or $2c_M > \phi\Delta$ and $\underline{\alpha} \leq \alpha \leq \bar{\alpha}$. Yet, at the same time, an increase in

c_M raises both $\underline{\alpha}$ and $\bar{\alpha}$. Increases in $\underline{\alpha}$ and $\bar{\alpha}$ also have negative effect on q .

If $2c_M \leq \phi\Delta$ and $0 \leq \alpha \leq \bar{\alpha}$ or $2c_M > \phi\Delta$ and $\underline{\alpha} \leq \alpha \leq \bar{\alpha}$, the direct impact of an increase in the market liquidity ϕ on q is to increase q if and only if $c_M > \alpha\Delta$. Yet, $\bar{\alpha}$ is increasing in ϕ while $\underline{\alpha}$ is decreasing in ϕ . Therefore, if $2c_M \leq \phi\Delta$, an increase in ϕ reduces monitoring probability in the sense that the area where perfect monitoring $q=1$ occurs. When $2c_M > \phi\Delta$, an increase in the market liquidity reduces both the non-monitoring area and perfect monitoring area. Therefore, the relationship is not monotonic.

We restate Maug's Proposition 3 using our calculations.

Proposition 3 (Social Optimum) The social optimum $q=1$ is achieved if and only if $\bar{\alpha} < \alpha \leq 1$ (compare this with Maug's result). $\bar{\alpha}$ is increasing in ϕ ; F chooses $q=1$ less frequently if the market is more liquid.

Now, we move to the first round of trading. First, calculate the expected trading profit separately when $0 < q < 1$. Then:

$$\frac{1}{2}x_B q(1-q)\Delta - \frac{1}{2}x_S q(1-q)\Delta = (1-\alpha)\frac{1}{2}\phi q(1-q)\Delta \equiv (1-\alpha)G$$

The total expected profit is given by

$$(1-\alpha)G + q(\alpha\Delta - c_M) + \alpha L$$

Assume the relevant range. Then, at time 0, the market price will be set according to the valuation by the households. Thus, the initial price P_0 will be

$$\begin{aligned} P_0 &= \frac{1}{2} \left[qH + q(1-\phi)H + q\phi(qH + (1-q)L) + (1-q)L \right] \\ &= qH + (1-q)L - \frac{\phi}{2} q(1-q)\Delta \\ &= qH + (1-q)L - G \end{aligned}$$

Thus, by buying α fraction of shares, F expects to earn

$$\begin{aligned} & (1-\alpha)G + q(\alpha\Delta - c_M) + \alpha L - \alpha P_0 \\ &= G - qc_M \\ &= \frac{\phi}{2}q(1-q)\Delta - qc_M \equiv h(q) \end{aligned}$$

Compare the following two different versions of Proposition 4. If we throw away Maug's ad hoc assumption, a large shareholder does not emerge at all after the first round of trading:

Proposition 4 (Maug's calculation: Commitment Effect) At optimum, $\alpha^* = \frac{c_M}{2\Delta - c_M} < \frac{c_M}{\Delta}$, and $q^* = \frac{1}{2} - \frac{c_M}{\phi\Delta}$. Thus, the probability of monitoring is strictly increasing in the market liquidity ϕ .

Proposition 4' (Our calculation) At optimum, $\alpha^*=0$, or the large shareholder does not accumulate any share at date 0. If $2c_M \leq \phi\Delta$, he monitors with probability $q^* = \frac{1}{2} - \frac{c_M}{\phi\Delta}$. If $2c_M > \phi\Delta$, he does not monitor at all. The probability of monitoring is independent of the market liquidity ϕ .

(Proof) First consider the case with $2c_M \leq \phi\Delta$ and $0 \leq \alpha \leq \bar{\alpha}$. In this case, $\frac{1}{2} - \frac{c_M}{\Delta} \leq q \leq 1$.

The expression $h(q)$ attains unconstrained maximum at $q = \frac{1}{2} - \frac{c_M}{\phi\Delta} < \frac{1}{2} - \frac{c_M}{\Delta}$. Therefore, the maximum under the constraint $\frac{1}{2} - \frac{c_M}{\Delta} \leq q \leq 1$ occurs at $q = \frac{1}{2} - \frac{c_M}{\Delta}$ or at $\alpha^*=0$.

If $2c_M > \phi\Delta$, the relevant range is $\underline{\alpha} \leq \alpha \leq \bar{\alpha}$ and $0 \leq q \leq 1$. In this case, the non-zero root of the equation $h(q)=0$ is negative ($q = \frac{\phi\Delta - 2c_M}{\phi\Delta} < 0$). Therefore, the function $h(q)$ is strictly decreasing for $0 \leq q \leq 1$. The maximum is thus achieved by $q^*=0$ or $\alpha^* = \underline{\alpha}$. Since the large shareholder does not monitor and gets no trading profits, he can also choose $\alpha^*=0$. Thus, either way, the large shareholder does not buy any share at date 0.

We now come to the ultimate difference between Maug's and our calculations:

Proposition 7(Shareholder Value) Initial shareholders' wealth P_0 is either decreasing in the market liquidity ϕ or independent of it.

(*Proof*) In the first case of $2c_M \leq \phi\Delta$, $P_0 = qH + (1-q)L - \frac{1}{2}\phi q(1-q)\Delta$. Since q is independent of the market liquidity, and since $q(1-q)\Delta > 0$, we know that P_0 is strictly decreasing in the market liquidity ϕ . In the second case of $2c_M > \phi\Delta$, $P_0 = L$. So market liquidity has no impact.

These Propositions clearly show the importance of Maug's assumption. If we throw away the ad-hoc assumption Maug made, no large shareholder emerges in the first round of trading no matter how liquid the market is, and the market liquidity does not affect the initial stock price at all.

V. Conclusion

This paper has a very narrow objective of showing that different conclusion results if we correctly calculate the equilibrium of the game without making unnecessary ad-hoc assumption. This difference seems to be caused by the setup that market liquidity at the second round trading is negatively affected by the large shareholder's initial shareholding α . In other words, if he acquires a very large block of shares in the first round, the market liquidity in the second round of trading is reduced. Thus he chooses not to buy any shares in the first round of trading.

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