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The user-pays principle in the river pollution problem

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Abstract

The user-pays principle in the river pollution problem

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In this paper, we are going to think about the user-pays principle in the river pollution problem. each region enjoys benefit doing economic activity, in other words, discharging emissions. However, the emission causes damage to the downstream regions. In this situation, we consider methods of increasing social welfare using transfer scheme based on the user-pays principle. Futhermore, we study the properties of this transfer scheme and the induced welfare distribution in equilibrium. The water-use charge in Korea is a regulation scheme based on the user-pays principle.

Keywords: The user-pays principle, river pollution problem, negative externality problem, The water use charge, Axiomatic characterization.
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1. Introduction

1.1 The problem

In real life, there are many conflicts which comes from negative externality problems. In a nation, various pollution problems and noise problems occurs. Internationally, there are issues related with greenhouse gas problem, acid rain problem, fine dust problem, and river pollution problem. especially, there has been many conflicts and researches concerning the river pollution problem. The river pollution problem and sharing problem can be considered as an important example of negative externality problem.

Usually, river flows across nations, regions, or municipalities. Around the world, about 200 rivers flow across national borders(Ambec & Sprumont(2002)), and much more rivers flow across regions. The property right over the flowing river is hard to define. In a river, the upstream can influnence the downstream in many ways. For example, if the upstream uses water so much, the downstream can experience a drought. On the other hand, if the upstream uses water less, there can be a flood in the downstream. Not only that, but river also carries the pollution. If the upstream discharges pollution a lot, the downstream water will be contaminated. There has been a lot of water conflicts for these properties of the river. And the importance of fair and efficient economic instruments (taxes, regulation, finantial assistance, cooperation) has been increased. Here, we are going to think about the river pollution problems.

1.2 Literature review

A river, which is polluted at some degree but usable have both benefitial and harmful aspects. Ambec and Sprumont(2002) proposed a method how agents along the water share the water resource using cooperative game theory. According to the two doctrines of international dispute called absolute terriotrial integrity and unlimited territorial integrity, they proposed two axioms called core lower bound and aspiration upper bound. Using above two axioms, they characterized the downstream incremental distribution which specifies the allocation of welfare of each agent. Ambec and Ehler(2008) extended the model to the agent who have satiation point of water use. Van der Brink et al(2012) extended the previous model by allowing multiple springs of water.

Concerning the harmful aspects, Ni and Wang(2007) proposed methods to split the cost of cleaning the whole river among the agents located along it when the cost is exogenously given. Along these lines, they characterized two solutions, upstream equal sharing method and local responsibility sharing method. They also showed these methods correpond to the Shapley value of appropriately defined TU game and as solutions belonging to the core of this game. Gomez-Rua(2013), under basically same model of Ni and Wang (2007), changed an axiom of upstream equal sharing method. Then, they characterized another methods using new axioms. Alcalde-Unzu et al (2015) also used similar model of Ni and Wang (2007). They introduced the model the transfer rate of discharged polution from upstream to downstream. Using transfer rate, they proposed the upstream responsibility rule.

These three papers all assume that cost of cleaning the river is exogenously given, and try to find methods to split the cost. Van der Laan and Moes (2012) doesn't take the cost exogenously given. It basically depends on quantity of emission amount. According to 2 doctrines of international law, called absoulte territorial sovereignty, and unlimited territorial integrity, they proposed two axioms called upstream autonomy and downstream autonomy. They characterized ATS value and UTI value using two axioms through cooperative game theoretic approach.

Ambec and Ehler (2014) proposed transfer scheme which is based on the polluter-pays principle. They showed that the efficient emission amount is implemented under the transfer scheme in non-cooperative Nash equilibrium and welfare distribution in the equilibrium satisfies appropriate properties. Their results include not only the river pollution problem, but also other general negative externality problems such as the green house gas problem, and acid rain problem.

There are basically two regulation principle dealing with negative externality problem. One is the polluter-pays principle studied in the Ambec & Ehler (2014). The other is the user-pays principle which is not yet studied theoretically as far as I know. In this paper, our basic model is special case of the Ambec and Ehler's (2014) model. In our model, there are upstream agnets and downstream agents and downstream agents cannot pollute the upstream agent. However, The Ambec and Ehler' s model have no upstream or downstream agents. So their model is more general. Then, we propose a different transfer scheme based on the user-pays principle. Coase theorem (1960) says if the property right is well established, the social optimum emission amount can be obtained by negotiation. The polluter-pays principle render the property right of the water to the downstream agents, while the user-pays principle to the upstream agents. Both rule accomplish efficiency but another property of user-pays principle is not studied detailed. So we study the properties of this transfer scheme and the welfare distribution based on the user-pays principle.

2. A model

Basic model is similar with the Ambec & Ehler[2014]. Consider a set $N=\{1,2,\cdots,n\}$ of agents and that the agents are labeled from upstream to downstream, i.e., agent 1 is the most upstream agent, followed by agent 2 and so on until the most downstream agent n. Agent i enjoys a benefit $b_i(e_i)$ from production where $e_i \geq 0$ denotes i' s level of economic activity hereafter we call 'emissions'.

The benefit function b_i is assumed to be both strictly concave and strictly increasing from 0 to a maximum $\hat{e_i}$ with $b'(\hat{e_i})=0$ for every $i \in N$ and twice continuoulsy differentiable. We normalise $b_i(0)=0$ and assume that the marginal benefit at $e_i=0$ is high enough (say infinite) so it is optimal for all agents to produce and/or to consume.

Pollution from agent i causes marginal damage $a_{ij} \geq 0$ to agent j. The parameter a_{ij} measures the magnitude of the pollution impact of i' s emission on j. In this paper, we consider only constant marginal damage. A river pollution problem (N,b,a) is defined by a set of agents N, a profile of benefit functions $b=(b_i)_{i \in N}$, and a matrix of marginal impacts $a=[a_{ij}]_{ij \in N \times N}$. When there is no confusion, we write a instead of (N,b,a). Throughout, we assume that $a_{ii}>0$ for any $i \in N$ with $\sum_{j \in N \setminus \{i\}} a_{ij} > 0$. i.e. if i is polluting other agents, then his pollution causes some damage at his location.

$$A=\left\{a=[a_{ij}]_{ij \in N \times N} : a_{ij} \geq 0 \ \forall\, ij \in N \times N \text{ and } a_{ii}>0 \ \forall\, i \in N \text{ with } \sum_{j \in N \setminus \{i\}} a_{ij} > 0\right\}$$

denotes the set of problems.

For each agent $i \in N$, write $Pi = \{1, \cdots, i\}$ as the subset of N containing agent i and all its upstream agents. Write $Fi = \{i, \cdots, n\}$ as the subset of N containing i and all its downstream agents. $F^0 i$ denotes $Fi \backslash \{i\}$, and $P^0 i$ denotes $Pi \backslash \{i\}$.

The damage suffered by i in the emission vector $e = (e_i)_{i \in N}$ is, $d_i = \sum_{j \in Pi} a_{ji} e_j$ The welfare of agent i with emissions $e = (e_i)_{i \in N}$ is $b_i(e_i) - d_i = b_i(e_i) - \sum_{j \in Pi} a_{ji} e_j$. The first term is i's benefit from his own emissions, whereas the second term is i's welfare loss due to pollution.

Stand alone emission amount $\overline{e} = (\overline{e_i})_{i \in N}$ maximises stand alone welfare $b_i(e_i) - d_i = b_i(e_i) - \sum_{j \in Fi} a_{ji} e_j$ which satisfies $b'(\overline{e_i}) = a_{ii}$. An efficient emission amount $e^* = (e_i^*)_{i \in N}$ maximises total welfare $\sum_{i \in N} (b_i(e_i) - d_i) = \sum_{i \in N} b_i(e_i) - \sum_{i \in N} \sum_{j \in Pi} a_{ji} e_j$. It satisfies the following first order conditions for every $i \in N$, $b_i'(e_i^*) = \sum_{j \in Fi} a_{ij}$.

Note that our assumptions on the benefit function b_i guarantee that e_i^* is unique because $b_i'(\widehat{e_i}) = 0$ and b_i is strictly concave and strictly increasing between 0 and $\widehat{e_i}$.

3. The user-pays principle

The user-pays principle is the variation of the polluter-pays principle that calls upon the user of a natural resource to bear the cost of running down natural capital. This principal can be implemented in this model through proper transfer scheme.

3.1 Transfer scheme

A transfer scheme $t(a,e)$ is a function $t: A \times R^N \rightarrow R^N$ specifies for any problem and emissions a vector of payments $t(a,e) = [t_i(a,e)]_{i \in N}$. Given the transfer t and the emission amount e, agent i' s welfare under the vector $t(a,e)$ is given by $b_i(e_i) - d_i(a,e) + t_i(a,e) = b_i(e_i) - \sum_{j \in Pi} a_{ji} e_j + t_i(a,e)$

The downstream agents want to use clean water. To use the clean water, downstream agents pays to upstream the cost of running down the clean water. So, downstream agent j, should subsidize to his upstream agent $i \in P^0 i$ the amount $a_{ij}(\overline{e_i} - e_i)$ for the reduction of pollution. Under this transfer, if i discharge emission at stand alone level, he receives nothing from his downstream agents. But the less he discharges emission, the more he receives from his dowsntream agents.

Summing up all these side-payments, the user-pays principle leads to the transfer $t^U(a,e)$ defined as follows for any agent

$$t_i^U(a,e)=-\sum_{j\in P^0 i} a_{ji}(\overline{e_j}-e_j)+\sum_{j\in F^0 i} a_{ij}(\overline{e_i}-e_i)$$
$$=\sum_{j\in P^0 i} a_{ji}e_j-\sum_{j\in F^0 i} a_{ij}e_i+\sum_{j\in F^0 i} a_{ij}\overline{e_i}-\sum_{j\in P^0 i} a_{ji}\overline{e_j}$$
$$=d_i-\sum_{j\in F^0 i} a_{ij}e_i+\sum_{j\in F^0 i} a_{ij}\overline{e_i}-\sum_{j\in P^0 i} a_{ji}\overline{e_j}$$

Since the user-pays principle involves side-payments among agents, the transfers sum up to zero. It is, therefore, budget balanced.

Agent i' s welfare under the payments $t^U(a,e)$ with emission amount e is

$$u_i(a,e)=b_i(e_i)-d_i(a,e)+t_i^U(a,e)$$
$$=b_i(e_i)-d_i(a,e)+d_i(a,e)-\sum_{j\in Fi} a_{ij}e_i+\sum_{j\in F^0 i} a_{ij}\overline{e_i}-\sum_{j\in P^0 i} a_{ji}\overline{e_j}$$
$$=b_i(e_i)-\sum_{j\in Fi} a_{ij}\overline{e_i}+(\sum_{j\in F^0 i} a_{ij}\overline{e_i}-\sum_{j\in P^0 i} a_{ji}\overline{e_j})$$

Therefore, agent i has incentive to emit the efficient level e_i^* for any given emissions emitted by other agents in nash equilibrium.

The equilibrium concept of emission e in this paper is (pure) non-cooperative Nash equilibrium given problem a, and transfer scheme t. We denote $N(t,a)$ the set of (pure) non-cooperative Nash equilibria in the emission problem under the regulation scheme t and the problem a. In the non-cooperative Nash equilibrium of the river pollution problem with transfer t, each player i maximises welfare with respect to e_i given $e_{-i}=(e_j)_{j\in N\backslash\{i\}}$. Let $e^t\in N(t,a)$ be a Nash equilibrium emission amount. Therefore, e_i^t satisfies $e_i^t=\arg\max_{e_i}(b_i(e_i)-d_i(a,e)+t_i(a,e))$

One of important transfer scheme is the laissez-faire scheme t^{lf} defined by $t_i^{lf}(a,e)=0$ for all $i\in N$ and all $a\in A, e\in R_+^N$. The laissez-faire scheme represents a situation without regulation. It implements the emission amount $e^{lf}=(e_i^{lf})_{i\in N}$ satisfying the following first-order conditions, $b_i'(e_i^{lf}) = a_{ii}$ for every $i \in N$. i.e, $e^{lf} = \overline{e}$. At this e^{lf}, $t(a, e^{lf}) = 0$.

From now on, we denote $t(a)$ the transfer in equilibrium emission amount. i.e $t(a) = t(a, e^t)$. While $t(a, e)$ means the transfer which can change according to e. A rule ϕ associates with any problem (N, b, a) a pair $(e^t, t(a)) \in (R_+^N, R^N)$, i.e $\phi(a) = (e^t, t(a))$. For any problem $a \in A$, the equilibrium welfare under transfer scheme t is given by $z_i^t(a) = b_i(e_i^t) - \sum_{j \in P i} a_{ji} e_j^t + t_i(a)$ $\forall i \in N$.

3.2 The polluter-pays principle

There is another well-known principle called the polluter-pays principle. It basically renders the polluter responsible for the damage it causes to the environment. Actually most of regulation schemes based on the polluter-pays principle. In our model, an arbitrary agent i who pollutes should compensate every agent $j \in F^0 i$ for the damage he caused, $a_{ij} e_i$.

Therefore, as a victim of pollution, agent i receives the compensation $a_{ji} e_j$ from each agent $j \in P^0 i$ who pollutes him. Summing up all these side-payments, the PP principle leads to the regulation scheme $t^{PP}(a, e)$ defined as follows for any agent $i \in N$: $t_i^{PP}(a, e) = \sum_{j \in P^0 i} a_{ji} e_j - \sum_{j \in F^0 i} a_{ij} e_i = d_i - a_{ii} e_i - \sum_{j \in F^0 i} a_{ij} e_i = d_i - \sum_{j \in Fi} a_{ij} e_i$.

Agent i receives the net transfer from the cost of pollution he suffers minus the cost of pollution he causes to society.

The polluter-pays distribution rule is the only rule satisfying Non-negativity and Responsibility for pollution impact.(Ambec &

Ehler(2014)) Non-negativity simply requires that nobody should be worse off under pollution than without pollution (in the situation all agents utility is zero) Responsibility for pollution impact renders the polluter responsible for any change of his pollution impact on the economy.

To see the properties of the user-pays principle, lets think of an example. Setting of the problem is below. The set of agents is $N=\{1,2,3,4\}$. If $i<j$, i is upstream of j. Benefit functions of each agent are $b_1=-(e_1-2)^2+4$, $b_2=-(e_2-2)^2+4$,$b_3=-(e_3-1)^2+1$ $b_4=-(e_4-1)^2+1$. The matrix of marginal impacts is $\begin{pmatrix} 0.2 & 0.2 & 0.1 & 0.1 \\ 0 & 0.2 & 0.1 & 0.1 \\ 0 & 0 & 0.1 & 0.1 \\ 0 & 0 & 0 & 0.1 \end{pmatrix}$.

In this problem, socially optimal emission amount is $e_1^*=1.7$, $e_2^*=1.8$, $e_3^*=0.9$, $e_4^*=0.95$. The eqilibrium welfare according to the polluter-pays principle is below. $z_1^{PP}(a)=2.89$, $z_2^{PP}(a)=3.24$, $z_3^{PP}(a)=0.81$, $z_4^{PP}(a)=0.9025$ In the same example, equilibrium welfare by the user-pays principle is below. $z_1^U(a)=3.65$, $z_2^U=3.24$, $z_3^U(a)=0.525$, $z_4^U(a)=0.4275$. And the below table compares the equilibrium welfare distribution of polluter-pays principle and the user-pays principle. Again, note that the lassiez-faire transfer scheme means the situation when there is no regulation.

	lassiez-faire	Polluter-pays	User-pays
agent1	3.61	2.89	3.65
agent2	3.23	3.24	3.24
agent3	0.1425	0.81	0.525
agent4	0.0475	0.9025	0.4275
Total	7.03	7.8425	7.8425

Table 1. Polluter-pays versus User-pays

As one can see, the polluter-pays rule fails to acheive individual rationality constraint for the agent 1(when the status quo is no-regulation situation). If a regulation scheme fails to achieve individual rationality for some agents, they will refuse the regulation scheme to be adopted. However, the user-pays rule satisfies individual rationality. So all the agents can increase their equilibrium welfare compared with no-regulation situation.

4. Properties of the user-pays principle

The user-pays rule $\phi_i^U(a)$ satisfies following properties.

Efficiency : For any problem $a \in A$, when $\phi^U(a) = (e^t, t^U(a))$, $e^t \in N(t^U, a)$ is unique and $e^t = e^*$

Efficiency requires that the non-cooperative Nash equilibrium emission amount under the transfer scheme t is unique, and equal to the social optimum emission amount e^*.

Budget balance : For any problem $a \in A$, when $\phi^U(a) = (e^t, t^U(a))$, $\sum_{i \in N} t_i^U(a) = 0$.

Budget balance means that there is no budget deficit or surplus.

Individual rationality : For any problem $a \in A$, when $\phi^U(a) = (e^t, t^U(a))$, $z_i^U(a) \geq z_i^{lf}(a)$.

Individual rationality requires the equilibrium welfare under transfer scheme t is greater than the welfare when there is no regulation.

Independence of irrelevant pollution impacts: For any problem $a \in A$, when $\phi^U(a) = (e^t, t^U(a))$, If the marginal impact matrix changes from a to a' and $a_{ij} \neq a'_{ij}$ for some $i, j \neq k$, $i \neq j$ (multiple changes are allowed) Then $z_k^U(a) = z_k^U(a')$.

This axiom means that the change of irrelevent pollution impacts doesn't change the equilibrium welfare of an agent.

Downstream responsibility for pollution impact : For any problem $a \in A$, when $\phi^U(a) = (e^t, t^U(a))$, Then, for $i \in P^0 j$, $\frac{\partial}{\partial a_{ij}} z_j^U = -\overline{e_i}$.

This axiom means that when there is an increase of marginal pollution impact of i to j, then j's equilibrium welfare decrease by $-\overline{e_i}$.

Under the polluter-pays transfer scheme, the equilibrium welfare of j doesn't change even if a_{ij} change. It means that agent $i \in P^0 j$ takes the full-responsibility of changes in marginal impact a_{ij}. However, under the user-pays transfer scheme, agent $j \in F^0 i$ are charged for the increase of a_{ij}.

Theorem 1. *A rule* $\phi(a) = (e^t, t(a))$ *satisfies efficiency, budget balance, individual rationality, downstream responsibility for pollution impact, and independence of irrelevent pollution impact if and only if it is the user-pays rule.*

Proof is given in the appendix.

5. Relation with the VCG mechanism

The VCG mechaism is popular in mechanism design literatures and it is truthful mechanisim. Let us assume that the set of social altenatives is given as $X=\{x=(k,t_1,\cdots,t_n):k\in K \text{ and } t_i\in R,\ \forall i\}$ where k denotes a project choice and belongs to a set K t_i denotes the transfer to agent i. the type of each agent is θ_i, $i\in N$, and each i's utility is $u_i(x,\theta_i)=v_i(k,\theta_i)+t_i$. let k^* be an efficienct project choice. So k^* satisfies $\sum_{i\in N} v_i(k^*(\theta),\theta_i)\geq \sum_{i\in N} v_i(k'(\theta),\theta_i)$ $\forall\theta\in\Theta$. Transfer scheme based on the VCG mechanism is given by $t_i^V(\theta)=\sum_{j\neq i} v_j(k^*(\theta),\theta_j)+h_i(\theta_{-i})$.

In this model, there can be two kind of type. First, the type of each agent can be benefit function. And second is information of marginal impact to others. In this two case of type, the efficient project $k^*(\theta)$ is the efficient emission amount vector and $v_i(k,\theta_i)=b_i(e_i)-d_i(a,e)$, $v_i(k^*(\theta),\theta_i)=b_i(e_i^*)-d_i(a,e^*)$.

The transfer scheme based on the user-pays principle t_i^U is

$$t_i^U=-\sum_{j\in P^0i} a_{ji}(\overline{e_j}-e_j)+\sum_{j\in F^0i} a_{ij}(\overline{e_i}-e_i)=\sum_{j\neq i}(b_j(e_j^*)-d_j(a,e^*))+h_i(\theta_{-i})$$

So in this equation, $h_i(\theta_{-i})$ is given by

$$\begin{aligned} h_i(\theta_{-i}) &= -\sum_{j\neq i}(b_j(e_j^*)-d_j(a,e^*))+t_i^U \\ &= -\sum_{j\neq i}(b_j(e_j^*)-d_j(a,e^*))-\sum_{j\in P^0i} a_{ji}(\overline{e_j}-e_j)+\sum_{j\in F^0i} a_{ij}(\overline{e_i}-e_i) \end{aligned}$$

First, when we think the type as benefit function, $\overline{e_i}$ depends on the function b_i. So, $h_i(\theta_{-i})$ is not independent of b_i. Second, if we think the type as information of marginal impact a_{ij}, $j\in F^0i$ $h_i(\theta_{-i})$ is also not independent of a_{ij}, $j\in F^0i$. Therefore, the transfer scheme based on the user-pays principle is not included in VCG mechanism.

6. Conclusion

In this paper, I proposed a regulation scheme based on the user-pays principle and characterized the induced rule. this rule has may properties. this rule satisfies efficiency, budget balance, and individual rationality. In Korea, most of taxes related with the negative externality problem are based on the polluter-pays problem. Only few taxes are based on the user-pays principle. One of the taxes are water-use charge. there are many disputes on the taxes based on the user-pays principle. This paper could be a theoretical background of the user-pays principle.

In many literature, usually the marginal damage is considered to be increasing on the amount of pollution. However, I only thought constant marginal damage case in this paper. So the user pay principle with the convex damage function should be considered to make it more natural. And the independence of axioms is needed to be checked.

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Appendix

proof of theorem 1.

First, we show if part.

The user-pays rule satisfies individual rationality.

By definition, For each $i \in N$, $z_i^U(a)$ and $z_i^{lf}(a)$ is given by

$z_i^U(a) = b_i(e_i^*) - \sum_{j \in Fi} a_{ij} e_i^* + \sum_{j \in F^0 i} a_{ij} \overline{e_i} - \sum_{j \in P^0 i} a_{ji} \overline{e_j}.$

$z_i^{lf}(a) = b_i(\overline{e_i}) - a_{ii} \overline{e_i} - \sum_{j \in P^0 i} a_{ji} \overline{e_j}$ Then,

$z_i^U(a) - z_i^{lf}(a) = (b_i(e_i^*) - \sum_{j \in Fi} a_{ij} e_i^*) - (b_i(\overline{e_i}) - \sum_{j \in Fi} a_{ij} \overline{e_i}).$

Since $e_i^* = \arg\max_{e_i} \ (b_i(e_i) - \sum_{j \in Fi} a_{ij} e_i)$, So, $z_i^U(a) - z_i^{lf}(a) \geq 0.$

I.e, individually rational.

The user-pays rule satisfies budget balance.

By definition of t_i^U,

$t_i^U = d_i(e) - \sum_{j \in Fi} a_{ij} e_i + (\sum_{j \in F^0 i} a_{ij} \overline{e_i} - \sum_{j \in P^0 i} a_{ji} \overline{e_j})$. Since

$\sum_{i \in N} d_i(e) = \sum_{i \in N} \sum_{j \in Pi} a_{ji} e_j = \sum_{(i,j) \in (N \times N), j \in Pi} a_{ji} e_j = \sum_{(i,j) \in (N \times N), i \in Fj} a_{ji} e_j$ =

$\sum_{i \in N} \sum_{j \in Fi} a_{ij} e_i.$

Using a similar argument, we can prove that $\sum_{i \in N} \sum_{j \in F^0 i} a_{ij} \overline{e_i} = \sum_{i \in N} \sum_{j \in P^0 i} a_{ji} \overline{e_j}.$

Therefore, $\sum_{i \in N} t_i^U = 0.$ I.e, budget-balanced.

The user-pays rule satisfies efficiency.

By definition, for each $i \in N$,

$e_i^t = \arg\max_{e_i} (b_i(e_i) - d_i(e) + t_i^U(a,e))$. Since

$t_i^U(a,e) = d_i(e) - \sum_{j \in Fi} a_{ij} e_i + \sum_{j \in F^0 i} a_{ij} \overline{e_i} - \sum_{j \in P^0 i} a_{ji} \overline{e_j}$ we have,

$b_i(e_i) - d_i(e) + t_i^U(a,e) = b_i(e_i) - \sum_{j \in Fi} a_{ij} e_i + (\sum_{j \in F^0 i} a_{ij} \overline{e_i} - \sum_{j \in P^0 i} a_{ji} \overline{e_j})$. Therefore,

$\frac{\partial}{\partial e_i}(b_i(e_i) - d_i(a,e) + t_i^U(a,e)) = b_i{}'(e_i) - \sum_{j \in Fi} a_{ij}$.

e_i^* satisfies

1. $b_i{}'(e_i^*) - \sum_{j \in Fi} a_{ij} = 0$
2. $b_i{}''(e_i^*) \leq 0$

And since b_i is concave function, $e_i^t = e_i^*$. I.e, it satisfies efficiency.

The user-pays rule satisfies independence of irrelevant polltution impacts.

By definition of z^U, for each $i \in N$,

$$\begin{aligned} z_i^U &= b_i(e_i^*) - d_i(e^*) + t_i^U(a) \\ &= b_i(e_i^*) - \sum_{j \in Fi} a_{ij} e_i^* + (\sum_{j \in F^0 i} a_{ij} \overline{e_i} - \sum_{j \in P^0 i} a_{ji} \overline{e_j}) \end{aligned}$$

As one can see, for any a_{kl} such that $k, l \neq i$, $k \neq l$, $\frac{\partial}{\partial a_{kl}} z_i^U = 0$.

and for $k \in P^0 i$, $\frac{\partial}{\partial a_{ki}} z_i^U = -\overline{e_k}$.

Therefore, ϕ^U satisfies independence of irrelevant polltuion impacts and downstream responsibility for pollution impact.

Now we show only if part.

By definition of z_j^U, $z_j^U(a) = b_j(a) - d_j(a) + t_j(a)$.

By Efficiency, e^t is exist and $e^t = e^*$.

So, $b_j(a) = b_j(e_j^*)$, $d_j(a) = \sum_{i \in Pj} a_{ij} e_i^*$.

By Downstream responsibility for pollution impact,

$\frac{\partial}{\partial a_{ij}} z_j = \frac{\partial}{\partial a_{ij}} b_j(e_j^*) + \frac{\partial}{\partial a_{ij}} d_j(a) + \frac{\partial}{\partial a_{ij}} t_j(a) = -\overline{e_i}$. Also, since e_j^* is independent of a_{ij}, $\frac{\partial}{\partial a_{ij}}(b_j(e_j^*)) = 0$.

By definition of $d_j(a)$, $d_j(a) = \sum_{i \in Pj} a_{ij} e_i^*$. So, $\frac{\partial}{\partial a_{ij}} d_j(a) = e_i^* + a_{ij} \frac{\partial}{\partial a_{ij}} e_i^*$.

$\frac{\partial}{\partial a_{ij}} t_j(e^*) = -(\overline{e_i} - e_i^*) + a_{ij} \frac{\partial e_i^*}{\partial a_{ij}} = \frac{\partial}{\partial a_{ij}}(a_{ij} e_i^* - \overline{e_i} a_{ij})$.

By Integrating each side by a_{ij}, $t_j(a) = -a_{ij}(\overline{e_i} - e_i^*) + t_j'(a)$.

And $t_j'(a)$ satisfies $\frac{\partial}{\partial a_{ij}} t_j'(a) = 0$.

Since we can use same argument for arbitrary $i \in P^0 j$, we can say that $t_j(e^*) = -\sum_{i \in P^0 j} a_{ij}(\overline{e_i} - e_i^*) + t_j'(a)$

And $t_j'(a)$ satisfies $\frac{\partial}{\partial a_{ij}} t_j'(a) = 0 \;\; \forall\, i \in P^0 j$.

With the same logic, $\forall k \in F^0 j$, $t_k(e^*) = -\sum_{i \in P^0 k} a_{ik}(\overline{e_i} - e_i^*) + t_k'(a)$.

Since $j \in P^0 k$, $t_k(e^*)$ have $-a_{jk}(\overline{e_j} - e_j^*)$ term.

Now by independence of irrelevent pollution impacts,

$\frac{\partial}{\partial a_{jk}} z_t = 0$ if $t \neq j,k$, $j \neq k$. now for the t s.t $t \neq j,k$,

1) when $t < j$

$0 = \frac{\partial}{\partial a_{jk}} z_t = \frac{\partial}{\partial a_{jk}} t_t(e^*)$

2) when $t > j$

$0 = \frac{\partial}{\partial a_{jk}} z_t = -a_{jt} \frac{\partial}{\partial a_{jk}} e_j^* + \frac{\partial}{\partial a_{jk}} t_t(e^*)$

By budget-balance, $\sum_{i \in N} t_i(a) = 0 \;\; \forall\, a \in A$. So, $\frac{\partial}{\partial a_{jk}} \sum_{i \in N} t_i(a) = 0$.

$$0=\frac{\partial}{\partial a_{jk}}\sum_{i\in N}t_i(a)=\frac{\partial}{\partial a_{jk}}(\sum_{i<j}t_i(a))+\frac{\partial}{\partial a_{jk}}(\sum_{i>j,i\neq k}t_i(a))+\frac{\partial}{\partial a_{jk}}(t_j(a)+t_k(a))$$
$$=\sum_{i>j,i\neq k}a_{ji}\times\frac{\partial e_j^*}{\partial a_{jk}}+(-(\overline{e_j}-e_j^*)+a_{jk}\frac{\partial e_j^*}{\partial a_{jk}})+\frac{\partial}{\partial a_{jk}}t_j(a)$$

Therefore,

$$\frac{\partial}{\partial a_{jk}}t_j(a)=(\overline{e_j}-e_j^*)-a_{jk}\frac{\partial e_j^*}{\partial a_{jk}}-\sum_{i>j,i\neq k}a_{ji}\frac{\partial e_j^*}{\partial a_{jk}}.$$

Since $t_j(a)=-\sum_{i\in P^0j}a_{ij}(\overline{e_i}-e_i^*)+t_j'(a)$, if we plug in this equation, it leads to

$$\frac{\partial}{\partial a_{jk}}(-\sum_{i\in P^0j}a_{ij}(\overline{e_i}-e_i^*)+t_j'(a))=(\overline{e_j}-e_j^*)-a_{jk}\frac{\partial e_j^*}{\partial a_{jk}}-\sum_{i>j,i\neq k}a_{ji}\frac{\partial e_j^*}{\partial a_{jk}}.$$

By integrating both equations by a_{jk}, we can get $t_j'(a)=a_{jk}(\overline{e_j}-e_j^*)-\sum_{i>j,i\neq k}a_{ji}e_j^*+t_j''(a)$ and $t_j''(a)$ is independent of a_{jk}.

So for arbitrary $k\in F^0j$, $t_j(a)$ satisfies $t_j'(a)=a_{jk}(\overline{e_j}-e_j^*)-\sum_{i>j,i\neq k}a_{ji}e_j^*+t_j''(a)$ and $t_j''(a)$ is independent of a_{jk}.

Therefore, for arbitrary $k\in F^0j$, $t_j'(a)$ has a term $a_{jk}(\overline{e_j}-e_j^*)$.

So, combining the two results, $t_j(a)$ becomes $t_j(a)=\sum_{k\in F^0i}a_{jk}(\overline{e_j}-e_j^*)-\sum_{t\in P^0j}a_{tj}(\overline{e_t}-e_t^*)+t_j''(a)$ and $t_j''(a)$ is independent of a_{kj}, $(\forall k\in P^0j)$ and a_{jk}, $(\forall k\in F^0j)$.

Since $t_j^U(a)=\sum_{k\in F^0i}a_{jk}(\overline{e_j}-e_j^*)-\sum_{t\in P^0j}a_{tj}(\overline{e_t}-e_t^*)$, $t_j(a)=t_j^U(a)+t_j''(a)$.

So, it is enough if we show that $t_j''(a)=0\quad\forall a\in\mathcal{A}$.

By individual rationality, $z_j(a)\geq z_j^{lf}(a)$. using definitions of $z_j^{lf}(a)$, we can get

$$b_j(e_j^*) - \sum_{i\in Pj} a_{ij}e_i^* + \sum_{i\in P^0j} - a_{ij}(\overline{e_i} - e_i^*) + \sum_{i\in F^0j} a_{ji}(\overline{e_j} - e_j^*) + t_j''(a) \geq b_j(\overline{e_j}) - \sum_{i\in Pj} a_{ij}\overline{e_i}$$

subtracting $\sum_{i\in P^0j} - a_{ij}\overline{e_i}$ both sides, then this inequality becomes

$$b_j(e_j^*) - \sum_{i\in Fj} a_{ji}e_j^* + t_j''(a) \geq b_j(\overline{e_j}) - \sum_{i\in Fj} a_{ji}\overline{e_j}.$$

Now suppose that $t_i''(a) \neq 0$ for an $i\in N$. Then, $t_i''(a)$ can be positive or negative.

By budget balance, $\sum_{i\in N} t_i(a) = \sum_{i\in N} t_i^U(a) + \sum_{i\in N} t_i''(a) = \sum_{i\in N} t_i''(a) = 0$.

Therefore, if $t_i''(a) > 0$ for an $i\in N$, then there exist $j\in N$ s.t $t_j''(a) < 0$.

Now without loss of generality, suppose that $t_j''(a) < 0$. We already know that $t_j''(a)$ is independet of $a_{ij}, (i\in P^0j)$ and $a_{ji}, (i\in F^0j)$.(in other words, $\frac{\partial}{\partial a_{ij}} t_j''(a) = 0 = \frac{\partial}{\partial a_{ji}} t_j''(a)$).

We know that if $a_{ji} \to 0 \ \forall i\in F^0j$, then $e_j^* \to \overline{e_j}$. Also, b_j is continuous because we already assumed that b_j is twice continuously differentiable, and trivially, $\sum_{i\in Fj} a_{ji}e_j^*$ is continuous on e_j^*. So, if we send e_j^* to $\overline{e_j}$, then $b_j(e_j^*) - \sum_{i\in Fj} a_{ji}e_j^* \to b_j(\overline{e_j}) - \sum_{i\in Fj} a_{ji}\overline{e_j}$.

So, if $e_j^* \to \overline{e_j}$, the inequality $b_j(e_j^*) - \sum_{i\in Fj} a_{ji}e_j^* + t_j''(a) \geq b_j(\overline{e_j}) - \sum_{i\in Fj} a_{ji}\overline{e_j}$ leads to $t_j''(a) \geq 0$. Therefore, $t_j''(a)$ satisfies $0 \leq t_j''(a) < 0$. it is contradiction.

So, $t_j''(a) = 0 \ \forall a\in A, j\in N$. It concludes that $z_j(a) = z_j^U(a)$.

국문초록

본 논문은 강물의 오염 상황에서 사용자 부담원칙에 기초해서 문제를 해결하는 방법을 생각 해 본다. 각 지역은 생산, 곧 오염을 배출하는 행위를 통해서 이익을 얻게 되고, 이는 동시에 강의 하류에 있는 지역에 피해를 주게 된다. 이때 피해를 받는 지역이 피해를 주는 지역에게 돈을 이전하여 줌으로 피해를 줄이는 방법에 관해 생각 해 보고, 그러한 이전 방법(transfer scheme)이 가지는 특성과 그러한 방법에 의해 도출되는 균형에서의 효용이 어떠한 특성을 만족하는지 알아본다. 한국에서의 물이용 부담금 제도는 사용자 부담원칙에 기초한 한가지 세금방식이다.

주요어 : 사용자 부담원칙, 강 오염 문제, 부정적 외부성 문제, 물 이용 부담금, 공리적 특성화.
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