Ph.D. DISSERTATION

Analysis of Maximal Topologies and Their DoFs in Topological Interference Management

토폴로지 간섭관리에서 최대 토폴로지와 자유도에 관한 분석

BY

JONG-YOON YOON

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이 논문을 공학박사 학위논문으로 제출함

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전기 컴퓨터 공학부

윤 종 윤

윤종윤의 공학박사 학위 논문을 인준함

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Abstract

In this dissertation, four main contributions are given as i) design of maximal topology in topological interference management (TIM), ii) design of maximal topology matrix and generalized alliance construction, iii) topological interference management-treating interference as noise (TIM-TIN) decomposition, and iv) inter-cell interference coordination (ICIC) based on cell zooming are considered.

First, we propose a method of alliance construction, which derives maximal topology by stipulating several conditions for message relationship in the alignment graph and conflict graph. Maximal topologies are the topologies of K-user interference channel, where any interference link cannot be added without degenerating current degrees of freedom (DoF). It is proved that a topology is maximal if and only if it is derived from the alliance construction. Through alliance construction, any maximal topologies achieving symmetric DoF $1/2$ can be designed. Properties of alliance construction are derived such as the maximum number of alliances to be constructed for the given number of messages K and a method to partition messages into sub-alliances.

Second, message relationship based on alliance construction is translated into topology matrix in TIM. Permutation of the topology matrix is used to demonstrate the characteristics of the alliances easily in the topology matrix. The conditions for maximal topology matrix (MTM) are characterized and the discriminant of topology matrix for maximality and transformation of non-MTM into MTM are proposed. Alliance construction is generalized by introducing generalized sub-alliances, which extends the range of topologies derived from alliance construction in the achievable DoFs. The analysis of generalized alliance construction in the topology matrix is also proposed.

Third, TIM-TIN decomposition is proposed in order to handle with intermediate links in interference channel. The criterion how to separate interference links into TIM and TIN is proposed for generalized degrees of freedom (GDoF) performance. Since

GDoF in TIN depends on the Hamiltonian path in graph of interference channel, it is NP-hard problem and the optimal solution is hard to be proposed for GDoF. Instead of the optimal solution, a method to derive sub-optimal solution is proposed using modified channel matrix (MCM) and simulation result will be followed to show the performance of the proposed decomposition.

Lastly, ICIC for self organizing cellular network is proposed, where each base station (BS) is not able to share information through backhaul to perform conventional ICIC schemes.The proposed ICIC scheme is based on distributed cell zooming, where non-cooperative game theory is used. Further, it is shown that proposed scheme can efficiently handle inter-cell interference and coverage hole problem in self organizing network by simulation result.

keywords: Degrees-of-freedom (DoF), interference channel, topological interference management (TIM), treating interference as noise (TIN), interference channel, maximal topology, alliance, alliance construction, maximal topology matrix (MTM), TIM-TIN decomposition, modified channel matrix (MCM), inter-cell interference coordination (ICIC), cell zooming, self-organizing network (SON)

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Chapter 1

INTRODUCTION

1.1 Background

Recently, there have been many advances in the wireless networks with interference, where the most remarkable achievement is the idea of interference alignment (IA) [1]. IA is a scheme to design signals in such a way that interference signals can be overlapped and separated from desired signal at each receiver so that each receiver can recover its desired message with gains of degrees of freedom (DoF). IA greatly enhances research on interference channels and a number of IA-based related studies have been performed. The initial researches on the IA mainly depend on the perfect and instantaneous channel state information at the transmitters (CSIT) [9],[13]. However, perfect CSIT assumption is not practical and challenging, because perfect CSIT is rarely available to transmitter. Also, when the number of users is large or the channel rapidly changes, the burden of CSIT becomes large. Considering the difficulty of perfect CSIT, researchers begin to explore settings with relaxed CSIT assumptions. It is shown that the setting with delayed CSIT can achieve the optimal symmetric DoF using the benefit of reconfigurable antenna, which is the same as perfect CSIT assumption model [10]. Also blind IA could improve DoF with some structured patterns of fading channels of different users beyond the absolutely no CSIT case [12].

Nevertheless, most of the studies are based on the theoretical insights, which remain fragile so far to be applied to practice directly. Also, these traditional interference management schemes based on IA always consider all interference links regardless of their strength, which results in unnecessary waste on resources such as time and antennas. As the strength of interference rapidly decays with distance due to shadowing, blocking, and path loss, interference from some sources is necessarily weaker than others, which is enough to be ignored. There are more opportunities in terms of DoF and resources by utilizing the characteristic of partial connectivity in actual interference channel.

With the more practical assumptions of interference channel and relaxation for heavy CSIT assumptions, interference management with no channel state information except the knowledge of the connectivity at the transmitters has been suggested under the name of the ”topological interference management (TIM)”[16]. Jafar suggested that index coding problem could be applied to TIM problem only with linear solutions and translated the index coding problem into TIM problem in a way of analyzing DoF gains [16]. It has been shown that under the topology satisfying certain conditions, TIM can obtain gains in terms of DoF and further achieve one half DoF per user, which is optimal for an interference channel with perfect CSIT. And it can be achieved with only topological information.

Inspired by the new framework of topological interference management that has a merit of tremendous reduction of CSIT, there have been a lot of follow-up researches in line with various assumptions such as channel, antenna, cellular network, transmit cooperation, and message passing. Fast fading channel [18] and alternating connectivity [15] were also considered and fundamental limits on multiple antennas in the TIM setting was derived [17]. Furthermore, TIM was studied in the downlink cellular network with hexagonal structure [20] and more gains of DoF is achievable with the help of message passing in uplink cellular network [21]. TIM was also studied in the interference broadcast channels [22], [26] and device-to-device communications [25].

Most of the above existing works on TIM try to establish the conditions of topology for symmetric DoF based on graph theory. In contrast, Shi et al. [23] present algorithmic method to find the achievable DoFs by interpreting DoF problem in TIM as an LRMC (low-rank matrix completion) problem. Riemannian pursuit framework is proposed to detect the rank of matrix to be recovered by iteratively increasing the rank. Shi et al. [24] deliver user admission control that maximizes the number of admitted users for achieving the feasibility of TIM compared to traditional TIM, where all the users are assumed to be admitted and target is to maximize the achievable symmetric DoFs for all the users. In order to handle the problem, sparse and low-rank optimization framework is proposed and the Riemannian trust-region algorithm is developed.

Unlike above follow-up studies of TIM, we further develop the research in [16] in more practical sense rather than changing assumptions or putting some schemes which help to enhance DoF. Theorem 4 in [16] suggests condition for topology achieving symmetric DoF $1/2$. However, since the study on TIM in [16] is based on index coding problems which mainly focus on each message, it is hard to design a specific topology achieving symmetric DoF $1/2$ directly from Theorem 4 in [16] , which is not suitable for dealing with actual network topologies. Even except for the topology achieving symmetric DoF $1/2$, topologies achieving symmetric DoF less than $1/2$ have not well studied, where most of them have only the upper bound of their DoFs.

For these problems, we raise a question, ”Is it possible to derive and determine all topologies achieving symmetric DoF $1/2$ in TIM easily?” This is the motivation of the researches on TIM. In order to avoid finding unnecessary topologies, which are sub-topologies of other topologies, we focus on finding only maximal topologies, where any interference link cannot be added without degenerating current DoF. In this dissertation, we reinterpret the previous condition of topology for the DoF $1/2$ into more understandable conditions of topology by introducing the subset of messages with constraints, called *alliance* and propose how to construct a maximal topology.

Maximality conditions for topology are not derived in the previous studies including [7]. The challenges of finding maximal topologies are routed for difficulty of finding alignment sets and internal conflict in alignment and conflict graph. For these reasons, it is meaningful that maximal topology for DoF $1/2$ is identified by using alliance. Even though conditions for maximal topologies for DoF $1/2$ are derived, it is still difficult to apply TIM into practical wireless communication network without design method of maximal topologies. Here, design of maximal topology matrix is also proposed using conditions for maximal topologies. Through design of maximal topology matrix, TIM can be applied to practical wireless network.

Meanwhile, there is a still unsolved question, "Is it possible to derive a topology achieving symmetric DoF less than $1/2$? We derive topologies achieving symmetric DoF $1/n$ for any $n \geq 3$ by generalizing alliance construction. In general, it is not guaranteed that actual interference network contains weak interference links enough to achieve symmetric DoF $1/2$ in TIM. Nevertheless, not many studies for topologies that cannot achieve symmetric DoF $1/2$ have been done, because these cases are more difficult to be analyzed compared to the case for symmetric DoF $1/2$. Thus, some of topologies and their design methods are proposed. It seems that our work and results in [23] and [24] have a common point, that is, matrix completion. However, our work is basically different from the results in [23], and [24]. Compared to dealing with LRMC problem in [23], design of topology matrix in our work is based on index coding problem (i.e, graph theory). For this reason, our topology design covers only some cases, not the whole, of topology achieving symmetric DoF less than $1/2$. However, contrast to design of [23], designs of matrix in this dissertation do not require NP-hard complexity. Also, design objects, matrices in [23] and [24] are not topology matrix, but related to precoding and decoding vectors. On the other hand, topology matrices related to network topology are considered in this dissertation.

TIM reduces CSIT extremely by considering only topology of interference network while managing interference problem. However, the symmetric DoF performance in TIM hardly depends on the property of topology, that is, how many weak interference links exist and how they are distributed. In other words, TIM has weakness to handle interference links of intermediate strength. To compensate TIM for dealing with intermediate links, treating interference as noise (TIN) with power allocation could be a solution. TIN with power allocation is proved to be optimal in the sense of generalized degrees of freedom (GDoF) in general K-user interference channel [19]. Also, TIM and TIN manage different parts of signal, that is, signal space and signal power level. The combination of TIM and TIN, named TIM-TIN decomposition is proposed [14]. However it is difficult to separate interference links into TIM and TIN for GDoF optimization and only baseline of separation is suggested in [14]. In this dissertation, sub-optimal criterion for TIM-TIN decomposition is proposed in the sense of GDoF.

In conventional cellular network, the topology and resource allocation of network is preset based on the estimation of traffic load and cell planning to avoid heavy inter-cell interferences. And many interference avoidance schemes are based on exchange of information between transmitters through backhaul. But for a self-organizing cellular network (SON), this assumptions may not be possible and thus, conventional inter-cell interference coordination (ICIC) schemes are not suitable. Thus, it is needed to propose an ICIC based on idea of cell zooming, where there is no base station (BS) cooperation and cell planning.

A cell zooming was originally proposed for energy saving purpose in cellular network [7]. Cell zooming is managing BS transmission power, which adaptively adjusts the cell size according to traffic load, user requirements, and channel conditions. BS switching and cell zooming is used to achieve QoS condition for each user equipment (UE). In [7], a usage case of cell zooming for energy saving is investigated and centralized and distributed cell zooming algorithms are developed. Also, simulation results show that the cell zooming can reduce the energy consumption.

In this dissertation, inter-cell interference coordination (ICIC) for SON is proposed, where each BS is not able to share information through backhaul to perform conventional ICIC schemes. The proposed ICIC scheme is based on distributed cell zooming, where non-cooperative game theory is used. Further, we show that proposed scheme can efficiently handle inter-cell interference and coverage hole problem and thus improve the minimum signal to interference and noise ratio (SINR) of UE in self organizing network by numerical analysis.

1.2 Overview of Dissertation

This dissertation is organized as follows.

In Chapter 2, basic concepts of topological interference management, treating interference as noise, and cell zooming are presented as preliminaries for understanding the whole of this dissertation. The definition of K-user interference channel and its degrees of freedom are introduced. Then, brief concept of topological interference management is described. Lastly, for cell zooming, basic concept of cell zooming and its implementation by physical adjustments are given.

In Chapter 3, one of our main contributions is suggested, that is, alliance construction, which determines relationship of messages so that maximal topology can be derived. In order to derive maximal topology, we develop the definition of alignment set with constraints and propose new definition, named alliance. Further, the relationship of alliances is also proposed as mutual partial hostility (MPH), where its topology of interference network is maximal. It is proved that topology is maximal for symmetric DoF $1/2$ if and only if it is derived from alliance construction. Thus, all maximal topology for DoF $1/2$ can be designed through alliance construction. Moreover, properties of alliance construction are described such as the maximum number of alliances and partition of messages into alliance.

In Chapter 4, alliance and alliance construction is delivered in matrix perspective. Permutation of the topology matrix is used to demonstrate the characteristics of the alliances easily in the topology matrix. The conditions for maximal topology matrix

(MTM) are characterized and the discriminant of topology matrix for maximality and transformation of non-MTM into MTM are proposed. Further, alliance construction is generalized by introducing generalized sub-alliances, which extends the range of topologies derived from alliance construction in the achievable DoFs. The analysis of generalized alliance construction in the topology matrix is also proposed.

In Chapter 5, a method for TIM-TIN decomposition is proposed using channel matrix. Instead of the optimal separation for sum of GDoF, a method for sub-optimal solution is proposed using alliance construction and modified channel matrix (MCM). Alliance construction is utilized to design TIM-TIN decomposition, where TIM can achieve symmetric DoF $1/2$. General channel matrix is modified as normalizing non-diagonal components (i.e., interference links) to measure each interference link's relative influence on each transceiver.

In Chapter 6, ICIC design for SON, where each BS is not able to share information through backhaul to perform conventional ICIC schemes is proposed based on distributed cell zooming. The proposed scheme is designed based on distributed cell zooming, where non-cooperative game theory is used. Further, we show that the proposed scheme can efficiently handle inter-cell interference and coverage hole problem in SON by simulation result.

Finally, the concluding remarks are given in Chapter 7.

1.3 Notations

Throughout the dissertation, some notations are defined as follows. A, $\boldsymbol{A}$, and $\mathcal{A}$ represent a variable, a matrix, and a set, respectively. $|\mathcal{A}|$ denotes the cardinality of the set $\mathcal{A}$ and a_{ij} is the (i,j)-th entry of the matrix $\boldsymbol{A}$. Let $\mathcal{K} = \{1, 2, \cdots, K\}$ be a set of users, $\mathcal{N} = \{1, 2, \cdots, N\}$, and $\mathcal{M} = \{1, 2, \cdots, M\}$. A set of all messages is denoted as $\mathcal{W}$ with $K = |\mathcal{W}|$. Let $\mathcal{V}$ and $\mathcal{E}$ be sets of vertices and edges. A graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a set of vertices $\mathcal{V}$ and a set of edges $\mathcal{E}$ between two vertices. A directed graph $\mathcal{D} = (\mathcal{V}, \mathcal{E})$ consists of a set of vertices $\mathcal{V}$ and a set of directed edges $\mathcal{E}$ between two vertices.

Chapter 2

Preliminaries

2.1 Degrees of Freedom

The MIMO communication systems use the multiple antennas at both the transmitter and receiver as in Fig. 2.1 to improve the performance of the communication systems. MIMO communications can be used for obtaining higher data transmission rate measured by DoF. MIMO technique can achieve higher DoF without any further communication resources such as transmit power and frequency spectrum.

Since MIMO can offer significant enhancement in data throughput without additional bandwidth or increased transmit power, it has drawn great attention in wireless communications. In [1], the capacity of the Gaussian MIMO channels was derived and the advantage of MIMO communications was proved theoretically.

Consider an $M \times N$ MIMO communication system, where M antennas and N antennas are equipped at the transmitter and receiver, respectively. Then, the received signal in one time slot is represented as

$$\mathbf{y} = \sqrt{\rho}\mathbf{H}\mathbf{x} + \mathbf{n}, \tag{2.1}$$

where $\mathbf{H}$, $\mathbf{x}$, and $\mathbf{n}$ are the $N \times M$ channel matrix, the $M \times 1$ transmitted signal vector, and the $N \times 1$ noise vector, respectively and ρ is the signal-to-noise ratio (SNR)

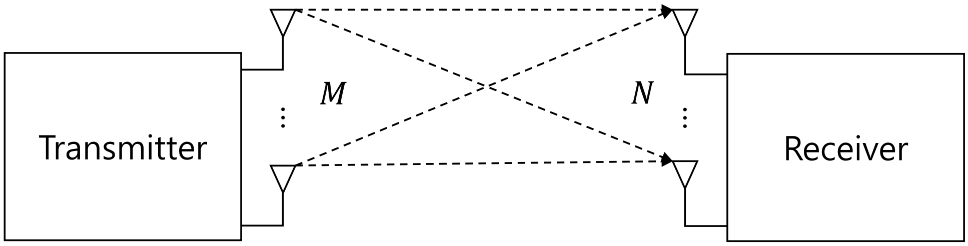


Figure 2.1: $M \times N$ wireless MIMO channel.

divided by number of receive antennas N. It is assumed that elements of $\mathbf{n}$ are i.i.d. with $\mathcal{CN}(0,1)$, where $\mathcal{CN}(0,1)$ is complex normal distribution with zero mean and unit variance.

If the CSI is known at both transmitter and receiver, the channel capacity in (2.1) is derived as [1]

$$C = \log_2 \det\Big(\mathbf{I}_N + \rho \mathbf{H}\mathcal{K}_{\mathbf{x}}\mathbf{H}^\dagger\Big),$$

where $\mathcal{K}_{\mathbf{x}}$ is covariance matrix of $\mathbf{x}$.

If the channel state is not known to the transmitter, the transmitter transmits the data according to $\mathcal{K}_{\mathbf{x}} = \frac{\mathbf{I}_M}{M}$, which implies that the uniform power allocations and independent codes are used for each antenna. Then the channel capacity can be represented as

$$C = \sum_{m=1}^{\min(M,N)} \log_2\Big(1 + \frac{\rho}{M}\lambda_m\Big),$$

where λ_m's are the eigenvalues of $\mathbf{H}\mathbf{H}^\dagger$.

Definition 2.1 (Degrees of freedom)**:** The DoF or multiplexing gain d of the channel is defined as

$$d = \lim_{SNR\to\infty} \frac{R(SNR)}{\log(SNR)},$$

where $R(SNR)$ is an achievable rate at the SNR.

Note that DoF of the $M \times N$ MIMO channel is $\min(M, N)$.

2.2 Interference Management

As a number of transceivers increase in communications, the influence of interference management becomes larger. As the cell size becomes smaller, the cell interference region becomes wider. This implies that the interference increases and causes the bad effect on the reliable communications specially in the cell interference region. Therefore, the interference management becomes a hot issue in the wireless communications. To deal with interference in communications, there are some basic ways to handle interference in interference channel such as:

- Decode interference

  The interference signal can be decoded and then subtracted from the desired signal when interference is strong.

- Treat interference as noise

  The interference signal is treated as noise and single user encoding/decoding suffices as in the conventional communications systems when interference is weak.

- Orthogonalization

  This approach is to orthogonalize interferences and desired signals in time, frequency, or code when the strength of interference is comparable to the desired signal.

However, such basis approaches have weakness that when signal power of interference is comparable to the desired signal, 'decode interference' and 'treat interference as noise' do not guarantee high reliability. Also, such orthogonal schemes have fundamental limit for DoF, which results in low throughput in high SNR region. To overcome these problems, Jafar suggests optimal DoF region of K-user interference channel and interference alignment (IA) as a scheme to achieve the optimal DoF. The key idea of IA is to separate the signal spaces of interference signal from the desired signal in signal space, where all interference signal spaces overlap maximally at each receiver so that optimal DoF can be achieved.

In this situation, IA scheme is drawing great attention as a solution for the above conventional approaches. Thus, by projecting signal to **u**, the desired signal x_1 can be recovered without interference signal. In more details, **u** is orthogonal to x_2, x_3, x_4, x_5 and thus

$$\mathbf{u}^\dagger \mathbf{y} = \mathbf{u}^\dagger \mathbf{v}_1 x_1 + \mathbf{u}^\dagger \mathbf{v}_2 x_2 + \mathbf{u}^\dagger \mathbf{v}_3 x_3 + \mathbf{u}^\dagger \mathbf{v}_4 x_4 + \mathbf{u}^\dagger \mathbf{v}_5 x_5 = \mathbf{u}^\dagger \mathbf{v}_1 x_1 \, .$$

In this case, orthogonalization can provide DoF of $\frac{1}{5}$, but IA achieves DoF of $\frac{1}{3}$.

Usually, IA is implemented by using multiple antennas, carriers, or time extension. Cadambe and Jafar proposed IA scheme for K-user interference channel and X channel by using time extension [2] [6].

There were many follow-up studies on IA with different settings. Nevertheless, most of the studies are based on the theoretical insights, which remain fragile so far to be applied to practice directly. However, it is assumed that the channel is time-varying and the global channel state information (CSI) is available at the transmitters, which implies that each transmitter knows the CSIs of all links between transmitters and receivers. These assumptions are infeasible because for global CSI knowledge, each transmitter requires lots of instantaneous feedback and the channel state may vary during feedback of global CSI.

In order to relax heavy CSIT assumptions for IA, interference management with no channel state information except the knowledge of the connectivity at the transmitters has been suggested under the name of the "topological interference management

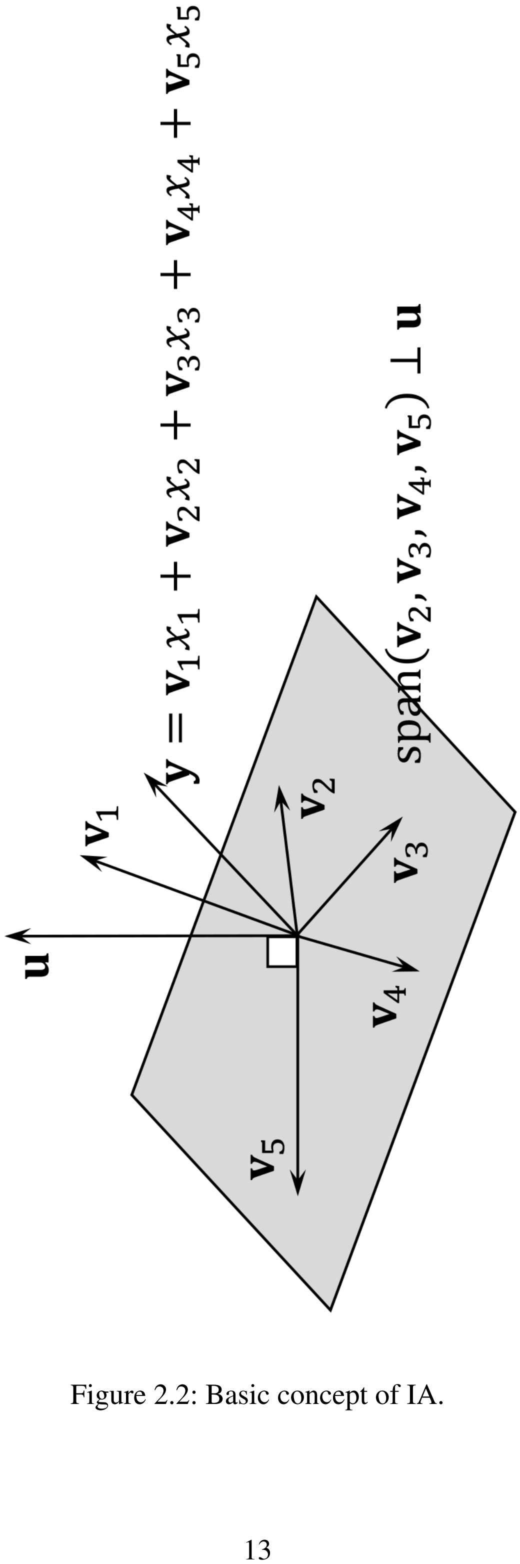


Figure 2.2: Basic concept of IA.

(TIM)"[16]. Jafar suggested that index coding problem could be applied to TIM problem only with linear solutions and translated the index coding problem into TIM problem in a way of analyzing DoF gains [16]. It has been shown that under the topology satisfying certain conditions, TIM can obtain gains in terms of DoF and further achieve one half DoF per user, which is optimal for an interference channel with perfect CSIT.

In fact the actual K-user interference channel is not partially connected. However, in TIM framework, each interference link is considered as "weak" or "strong" using one bit CSIT and only strong interference links are considered as "connected". It is necessary to approximate original fully-connected K-user interference channel to the topology in TIM, where only information about connection exists. The weak (disconnected) interference channels are identified as following condition:

$$\sum_{i:t_{ij}=0} |h_{ij}|^2 P \leq N_0. \tag{2.2}$$

That is, the average received signal power at receiver j from all weak interferers is less than or equal to the noise floor.

2.3 Graph Theory

In this section, we introduce several definitions in index coding and TIM. Since one of our contributions is mainly focused on results in [16], some of notations in [16] are needed.

Definition 2.2 (Conflict graph [16])**:** For a network topology, its conflict graph is a directed graph $\mathcal{D}_c = (\mathcal{V}, \mathcal{E}_c)$ such that $i \in \mathcal{V}$ represents the message W_i from the transmitter i to the receiver i and conflict edge $e_{ij} \in \mathcal{E}_c$ represents the interfering link from the transmitter i to receiver j in the interference network.

We simply say that message W_i conflicts with W_j if there is a conflict edge from message W_i to message W_j in the conflict graph.

Definition 2.3 (Alignment graph [16])**:** For a network topology, its alignment graph is a graph $\mathcal{G}_a = (\mathcal{V}, \mathcal{E}_a)$ such that $i \in \mathcal{V}$ represents the message W_i from transmitter i to receiver i and an alignment edge $e_{ij} \in \mathcal{E}_a$ exists if the transmitters i and j interfere with receiver k that wants to receive message $W_k, k \neq i$ and $k \neq j$.

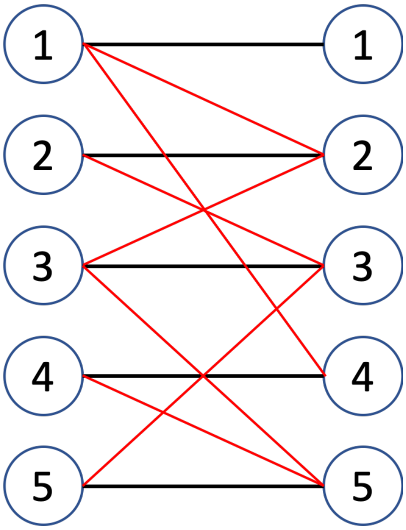
Definition 2.4 (Alignment set [16])**:** Each set of connected vertices in an alignment graph is called an alignment set.

Definition 2.5 (Internal conflict [16])**:** If any two messages that belong to the same alignment set have a conflict edge between them in the conflict graph, it is called an internal conflict.

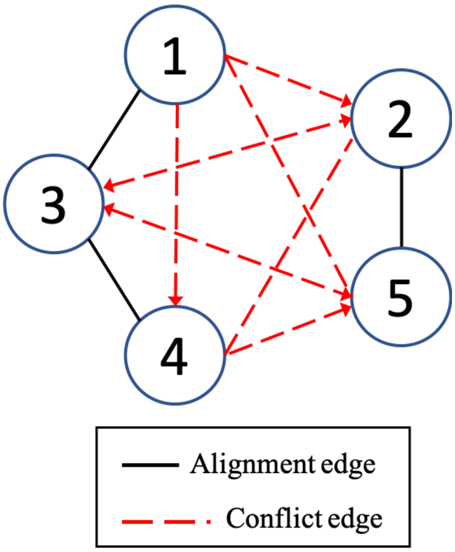
Example 2.1: In Fig. 2.3 (a), there is a topology for 5-user interference channel in TIM. The alignment graph and conflict graph for Fig. 2.3 (a) are shown in Fig. 2.3 (b). Since the alignment graph and conflict graph have the same set of vertices, they can be plotted in Fig. 2.3 (b) together and we briefly call it alignment-conflict graph. There is an alignment edge between message nodes 1 and 3 because both transmitters 1 and 3 interfere with receiver 2 that wants to message W_2. Likewise, message vertices 2 and 5 are connected with an alignment edge because they are interference signals for receiver 3 and message vertices 3 and 4 are connected with an alignment edge because they are interference signals for receiver 5. In Fig. 2.3 (b), there are two alignment sets, $\{W_1, W_3, W_4\}$ and $\{W_2, W_5\}$ and there is an internal conflict in the alignment set $\{W_1, W_3, W_4\}$ due to the conflict edge between message vertices 1 and 4.

2.4 Treating Interference as Noise with Power Allocation

TIN is a classical technique to handle interference in the signal power level and it is quite effective when the strength of maximal interference signal is quite small enough to be ignored. TIN is practical method in that it requires low complexity and has ro-



(a) A topology for 5-user interference channel



(b) Alignment-conflict graph

Figure 2.3: Topology and alignment-conflict graph.

bustness to channel. Not only practical issue, but TIN is also attractive scheme in theoretical perspective. Despite the simplicity of TIN, the rate region of TIN is nontrivial, because it is also required to optimize power of each transmitter.

Recently, there were works on the optimality of TIN and its conditions assuming various channel model such as 2-user symmetric Gaussian interference channel, K-user fully-connected Gaussian channel, and cyclic asymmetric Gaussian interference channel. Finally, the optimality of TIN in GDoF and its conditions are presented in K-user fully-connected, fully-asymmetric Gaussian interference channel. To help to understand, we first introduce optimal conditions of TIN and optimal GDoF region [19].

Remark 2.1 (Optimal condition of TIN and optimal GDoF region [19])**:** In a K-user interference channel, where the channel strength level from transmitter i to receiver j is $a_{ji}, \forall i, j \in \mathcal{K}$, if the following condition is satisfied

$$\alpha_{ii} \geq \max_{j:j \neq i} \alpha_{ji} + \max_{k:k \neq i} \alpha_{ik}, \forall i, j, k \in \mathcal{K} \tag{2.3}$$

then treating interference as noise with power allocation can achieve the whole GDoF region. Moreover, the GDoF region is the set of all K-tuples $(d_1, d_2, \ldots, d_K)$ satisfying

$$0 \leq d_i \leq \alpha_{ii}, \forall i \in \mathcal{K} \tag{2.4}$$

$$\sum_{j=1}^{m} d_{ij} \leq \sum_{j=1}^{m} (\alpha_{i_j} \alpha_{i_j} - \alpha_{i_{j-1} i_j}), \forall (i_1, i_2, \ldots, i_m) \in \Pi_K, \tag{2.5}$$

$$\forall m \in \{2, 3 \ldots, K\} \tag{2.6}$$

where Π_K is the set of all possible cyclic sequences of all subsets of $\mathcal{K}$ with cardinality no less than 2, and the modulo-m arithmetic is implicitly used on the user indices, e.g., $i_m = i_0$.

The power allocation for the optimal GDoF sum is not our interest and omitted in this dissertation.

2.5 Cell Zooming

Cell size in cellular networks is in general fixed based on the estimated traffic load. However, the traffic load can have significant spatial and temporal fluctuations, which bring both challenges and opportunities to the planning and operating of cellular networks. In this subsection, we introduce a concept of cell zooming, which adaptively adjusts the cell size according to traffic load, user requirements, and channel conditions.

In each cell, BS transmits common control signals and data signals to mobile users (MUs), and the cell size is defined as the area in which MUs can receive control signals from the BS. At the stage of network planning, cell size and capacity are usually fixed based on the estimation of peak traffic load. Since the traffic load in cellular networks can have significant spatial and temporal fluctuations due to user mobility and bursty nature of many data applications, static network planning degrades the network performance. Therefore, cell zooming was proposed [7], which adaptively adjusts the cell size according to traffic conditions. Cell zooming has the potential to balance the traffic load. An example of cell zooming is illustrated in Fig. 2.4.

The earlier version of cell zooming is centralized cell zooming, where a cell zooming server (CS) exists and controls the procedure of cell zooming. CS is a virtual entity in the network, which can be either implemented in the gateway or distributed in the BSs. The CS will first sense the network state information for cell zooming, such as traffic load, channel conditions, user requirements, and so on. The sensing process can be realized by specific control messages. After collecting the information, the CS will analyze whether there are opportunities for cell zooming and make decisions. If a cell needs to zoom in or zoom out, it will coordinate with its neighbor cells with the help

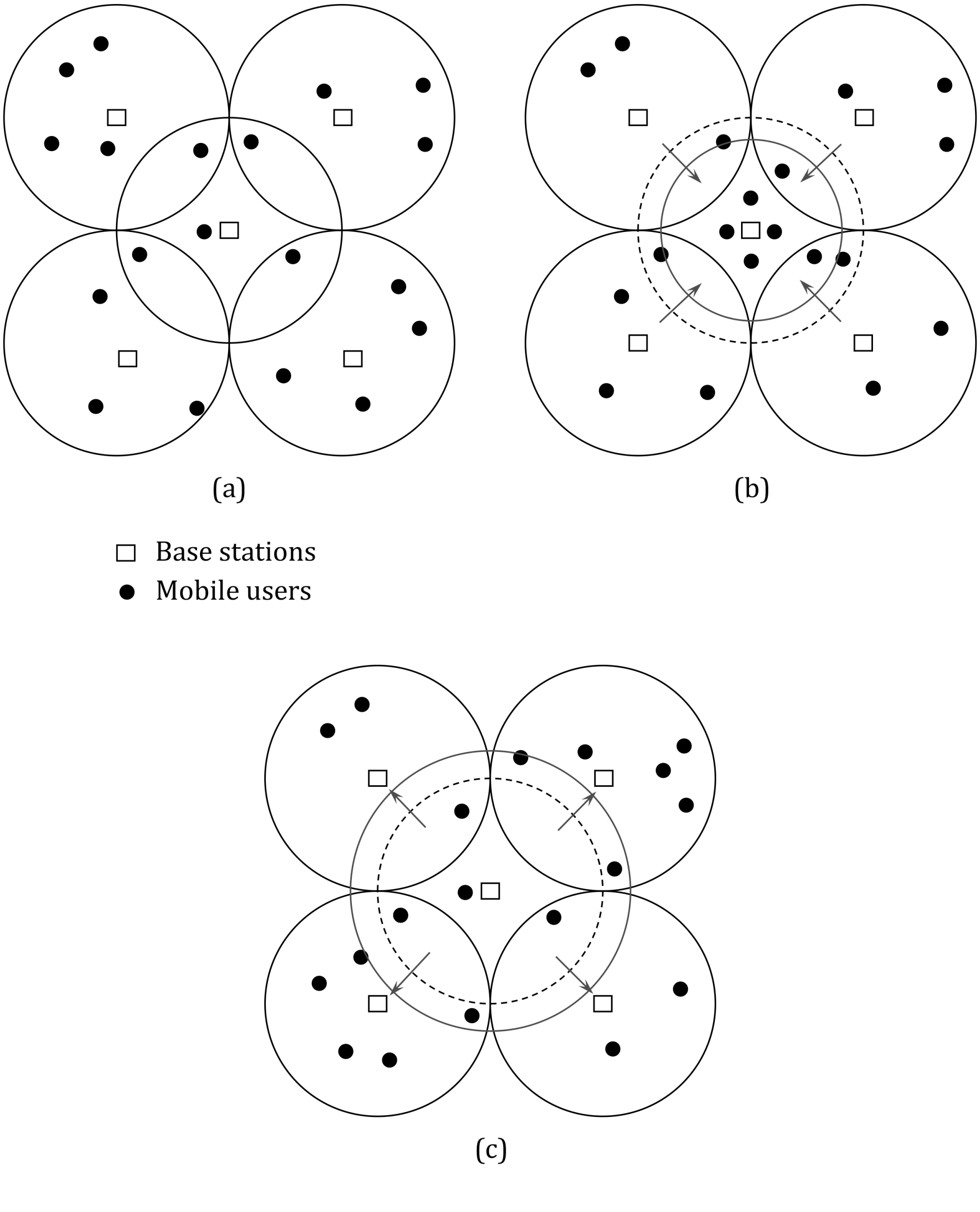


Figure 2.4: Cell zooming in cellular networks: (a) Cells with original size; (b) Central cell zooms in when load increases; (c) Central cell zooms out when load decreases.

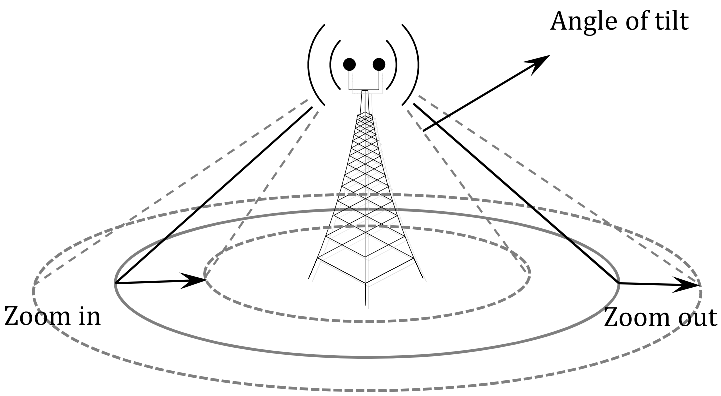


Figure 2.5: Cell zooming by adjusting antenna tilt.

of CS. Then these cells will either zoom in or zoom out.

Cell zooming can be implemented by adjusting physical parameters of network deployment. Cells can zoom out by increasing the transmit power of BS, and vice versa. Furthermore, in Fig. 2.5, antenna height and antenna tilt of BSs can also be adjusted for cells to zoom in or zoom out.

User experience, such as throughput can be improved by cell zooming. But, cell zooming may also cause some problems, such as inter-cell interference and coverage holes. When some neighboring cells zoom out together, there will be more inter-cell interference among them. If BS cooperation is infeasible, additional interference management schemes are needed to reduce the interference. Cell zooming may also produce coverage holes. When cells zoom in or zoom out, some areas in the network are possible to have no coverage. Above mentioned problems should be taken into account on designing cell zooming.

Since it is not possible to make cells to be distributed evenly in SON network, adaptive scheme that enlarges cell area like cell-zooming is needed in order to handle the coverage hole problem. Especially, SON is generally used in military network rather than commercial cellular network and the most important issue in military network is to ensure coverage region. Thus, it is suitable to apply cell-zooming to SON for reliability.

Chapter 3

Analysis of Maximal Topologies and Their DoFs in TIM

3.1 Introduction

In this chapter, alliance construction is proposed and maximal topology is derived from alliance construction in TIM. The condition of topology for symmetric DoF $1/2$ per user in TIM is already proposed in Theorem 4 in [16] that a topology can achieve symmetric DoF $1/2$ per user in TIM if and only if its alignment sets do not have any internal conflict.

However, since alignment graph and conflict graph take into account each message, not the message set, it is not easy to check whether a given topology achieves symmetric DoF $1/2$ or not without drawing alignment-conflict graph for whole messages and investigating the existence of internal conflict, which requires lots of works. In other words, Theorem 4 in [16] does not directly produce a topology achieving symmetric DoF $1/2$. This is the beginning of our study and one of our main contributions is to derive all topologies achieving symmetric DoF $1/2$ in TIM by combining and reinterpreting the alignment set and the internal conflict into a single concept, referred to as *alliance*. To this end, we defined a maximal topology in the previous section. Only maximal topology is considered because any non-maximal topology is sub-topology of maximal topology.

But, it is not enough to derive maximal topology for symmetric DoF $1/2$ only with the concept of alliance. After defining the alliance, we propose how to associate alliances with each other. We name it *alliance construction* to define the relationship among alliances in order to derive maximal topology. Using alliance construction, the relationship among messages together with alliances is determined naturally, which makes it possible to derive all maximal topologies achieving symmetric DoF $1/2$.

3.2 Alliance Construction for Maximal Topologies

3.2.1 System Model: K-User Interference Channel

We consider the TIM setting [16] in a partially connected K-user interference channel, where K transmitters want to send K independent messages to K receivers equipped with a single antenna. Then, the received signal at the receiver j through partially connected channel over the nth time slot is represented as

$$y_j(n) = \sum_{i \in \mathcal{S}_j} h_{ij} x_i(n) + z_j(n),\ j \in \mathcal{K},\ n \in \mathcal{N}, \tag{3.1}$$

where $x_i(n)$ is the transmitted signal with the average power constraint $\mathbb{E}[x_i^2(n)] \leq P$, $z_j(n)$ is the Gaussian noise with zero-mean and noise power spectral density N_0, h_{ij} is the channel coefficient between transmitter i and receiver j, and $\mathcal{S}_j$ represents a set of the indices of transmitters that are heard by receiver j.

The network topology is denoted by $\mathcal{T}$, which is directed bipartite graph with transmitters and receivers at each side, and with edges from transmitters to receivers only when they are connected. We also define sub-topology as follows. If a topology $\mathcal{T}_s$ is a sub-graph of $\mathcal{T}$, it is briefly called a sub-topology of $\mathcal{T}$.

Similar to TIM researches in [16]-[21], the following CSI is assumed:

(i) The channel coefficients are assumed to be fixed throughout the duration of communication such that $h_{ij}(n) = h_{ij}$ and thus the network topology $\mathcal{T}$ is also assumed to be fixed.

(ii) The channel coefficients h_{ij} for all i, j are unavailable at the transmitters, but the network topology $\mathcal{T}$ is known to all transmitters and receivers.

(iii) The channel state information at the receiver (CSIR) includes only the information of the desired channel coefficient h_{ii} at each receiver.

3.2.2 Definitions

A topology matrix from $\mathcal{T}$ is defined as follows.

Definition 3.1 (Topology matrix [16])**:** A topology matrix for K-user interference channel, $\boldsymbol{T}_K = [t_{ij}]_{K\times K}$ is defined as $t_{ij} = 1$, if there is a link between transmitter i and receiver j, and $t_{ij} = 0$, otherwise.

For the topology analysis, we define maximal topology as follows.

Definition 3.2 (Maximal topology)**:** A topology is maximal if any interference link cannot be added without reducing symmetric DoF that it can currently achieve. A maximal topology for K-user interference channel is denoted by $\mathcal{T}_{\mathcal{M}}$.

The achievable DoF of interference channel in TIM is determined by the topology of interference channel. In this chapter, we mainly focus on the topologies achieving symmetric DoF $1/2$ and the condition for symmetric DoF $1/2$ will be described at the following subsection.

3.2.3 Alliance

In the alignment-conflict graph, each message $W_i \in \mathcal{W}$ implicitly represents a pair of transmitter i and receiver i which sends and wants it, where DoF is analyzed based on each message rather than set of messages. However, it is better to consider topology in terms of the alignment set with no internal conflict rather than messages themselves for symmetric DoF $1/2$, because internal conflict is not just relationship of two messages but group of messages. Moreover, it is necessary to make each alignment set to have maximality of interference links with maintaining current symmetric DoF that its topology can achieve. Thus, the alignment set with constraints is needed, that is, no conflict edge between messages within the alignment sets and satisfying maximality of topology. In order to propose how to design a maximal topology in TIM, we introduce alliance of messages as follows.

Definition 3.3 (Partition of set)**:** A family of sets $\mathcal{P}_{\mathcal{W}}$ is a called partition of $\mathcal{W}$ if and only if the following conditions hold:

(i) The union of the sets in $\mathcal{P}_{\mathcal{W}}$ is equal to $\mathcal{W}$.

(ii) The intersection of any two distinct sets in $P_{\mathcal{W}}$ is empty.

Also, each set in $\mathcal{P}_{\mathcal{W}}$ is referred to as a block.

Definition 3.4 (Alliance)**:** Let $\mathcal{P}_{\mathcal{W}} = \{\mathcal{A}_1, \cdots, \mathcal{A}_M\}$ be a partition of $\mathcal{W}$. For each block $\mathcal{A} \in \mathcal{P}_{\mathcal{W}}$ satisfying the following two conditions, $\mathcal{A}$ is called an alliance.

(i) (No conflict) There is no conflict edge between any two messages in $\mathcal{A}$.

(ii) (Set conflict) All messages in $\mathcal{A}$ conflict with a subset of messages that are not in $\mathcal{A}$.

According to the above definition, there are two conditions for messages in alliance. The first condition is no conflict among messages in an alliance, which prevents internal conflict in each alignment set. The second condition means that if single message W_i in $\mathcal{A}$ conflicts with message $W_k \notin \mathcal{A}$, then the other messages in $\mathcal{A}$ should also conflict with W_k, which results that all messages in an alliance are fully connected with alignment edges. The set conflict condition makes the topology to be able to contain as many interference links as possible without changing message relationship, that is, it is the necessary condition for a maximal topology.

Lemma 3.1 (Set conflict)**:** Set conflict is necessary if a topology of K-user interference network is maximal in TIM.

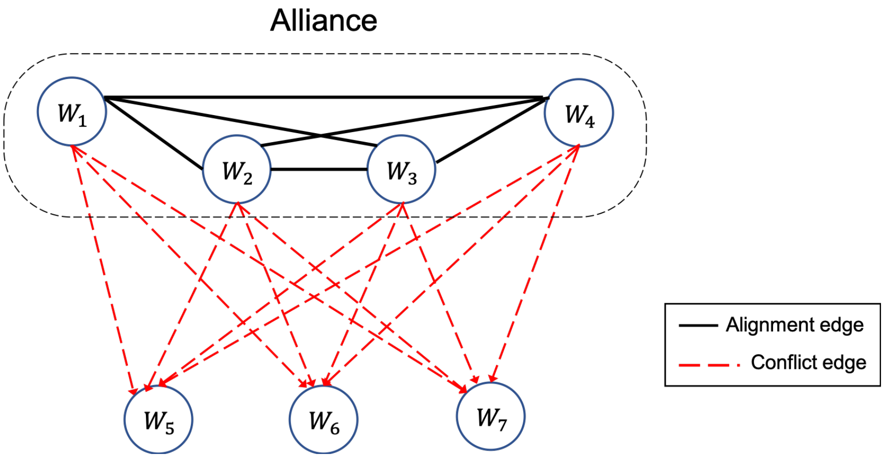
Proof. Suppose that messages in an alignment set do not satisfy the set conflict, that is, messages in a subset of the alignment set conflicts with W_k. Consider the possible relationship of the remaining messages in the alignment set and W_k without incurring internal conflict. Due to internal conflict, the remaining messages and W_k can be connected not with alignment edges but with conflict edges. Since the messages in the subset and the remaining messages are already in the same alignment set, connecting the remaining messages and W_k with conflict edges does not change the message relationship. Thus, the topology is not maximal and we prove it. □

In order to help to understand the above proof more clearly, we will explain the proof by Fig. 3.2. There is an alignment set, $\{W_1, W_2, W_3, W_4, W_k\}$ in Fig. 3.2 (a) and (b). Fig. 3.2 (a) does not satisfy the set conflict in the alignment set of the alignment-conflict graph. Then, we can add a conflict edge from W_4 to W_k as in Fig. 3.2 (b).

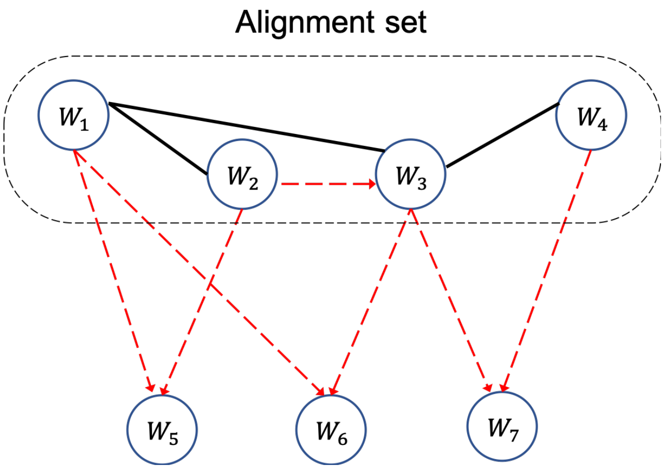
Further, we also describe differences between an alliance and an alignment set in Fig. 3.1. The alignment set $\{W_1, W_2, W_3, W_4\}$ can have internal conflict in the set and does not always follow the set conflict. As a result, it is not a clique in the alignment graph as in Fig. 3.1 (b). On the contrary, messages in an alliance have no conflict among them, which prevents internal conflict and follows the set conflict as in Fig. 3.1 (a).

3.2.4 Alliance Construction

The alliance itself is not enough to derive topology of K-user interference channel, because it is just a subset of messages, which has the relationship of messages in the

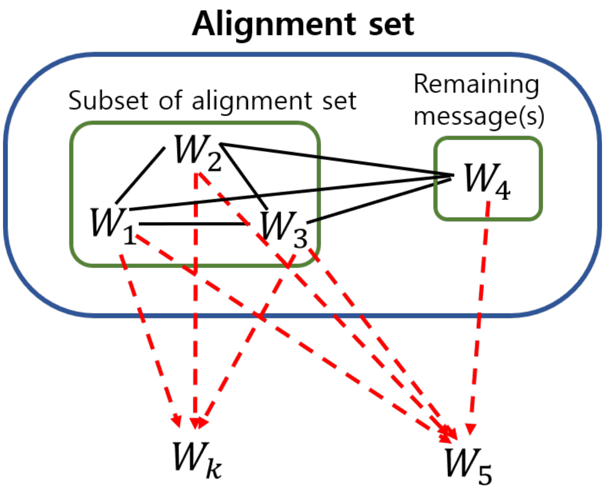


(a) Alliance

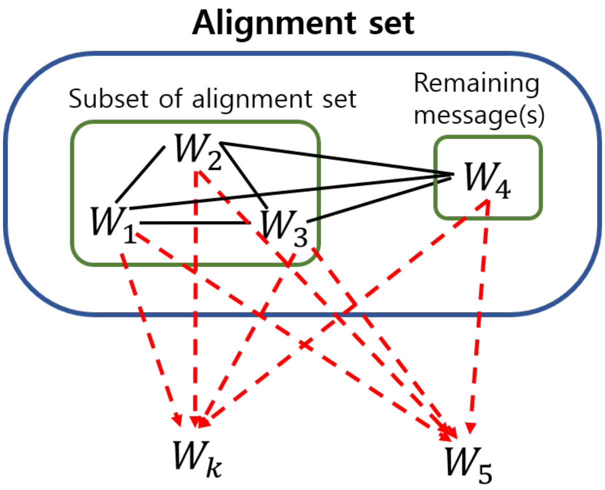


(b) Alignment set

Figure 3.1: The differences between alliance and alignment set.



(a) Alignment set that does not satisfy the set conflict



(b) Alignment set that satisfies the set conflict

Figure 3.2: Set conflict.

alliance and its conflicting messages as in Definition 3.4. The definition of alliance does not require relationship among alliances. Now, it is needed to establish inter-alliance relationship as relationship of whole messages in the alignment-conflict graph. Here, we need some definitions.

Definition 3.5 (Hostility and mutual hostility)**:** Alliance $\mathcal{A}_m$ is said to be hostile to alliance $\mathcal{A}_l$ if all messages in $\mathcal{A}_m$ conflict with all messages in $\mathcal{A}_l$, denoted by

$$\mathcal{A}_m \to \mathcal{A}_l. \tag{3.2}$$

Also, alliances $\mathcal{A}_m$ and $\mathcal{A}_l$ are said to be mutually hostile if and only if all messages in $\mathcal{A}_m$ conflict with all messages in $\mathcal{A}_l$ and vice versa, denoted by

$$\mathcal{A}_m \Longleftrightarrow \mathcal{A}_l. \tag{3.3}$$

The possible number of alliances is limited to two if we assume mutual hostility of all alliances as in the following lemma.

Lemma 3.2: If all alliances are mutually hostile, there exist only two alliances.

Proof. Suppose that there are three alliances $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$ with mutual hostility as in Fig. 4. Due to the mutual hostility of all alliances, both $\mathcal{A}_1$ and $\mathcal{A}_2$ are hostile to $\mathcal{A}_3$. This is contradiction that $\mathcal{A}_1$ and $\mathcal{A}_2$ should be combined into a single alliance because all messages in $\mathcal{A}_1$ and $\mathcal{A}_2$ have conflict edges with all messages in $\mathcal{A}_3$, but they cannot be combined due to hostility between them called internal conflict. Similarly, more than three alliances cannot exist with the mutual hostility of all alliances. Therefore, there exist only two alliances if all alliances are mutually hostile. □

The mutual hostility can be related to maximality of topology as in the following theorem.

Theorem 3.1 (2-alliance with mutual hostility)**:** For $\mathcal{W}$ in K-user interference channel, there are two alliances $\mathcal{P}_{\mathcal{W}} = \{\mathcal{A}_1, \mathcal{A}_2\}$. A topology of K-user interference chan-

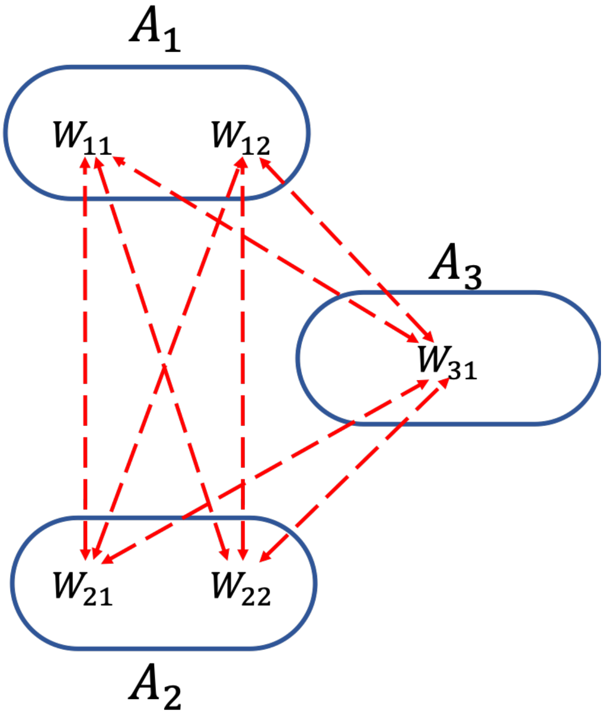


Figure 3.3: Three alliances with mutual hostility.

nel is maximal if and only if $\mathcal{A}_1$ and $\mathcal{A}_2$ are mutually hostile

$$\mathcal{A}_1 \Longleftrightarrow \mathcal{A}_2. \tag{3.4}$$

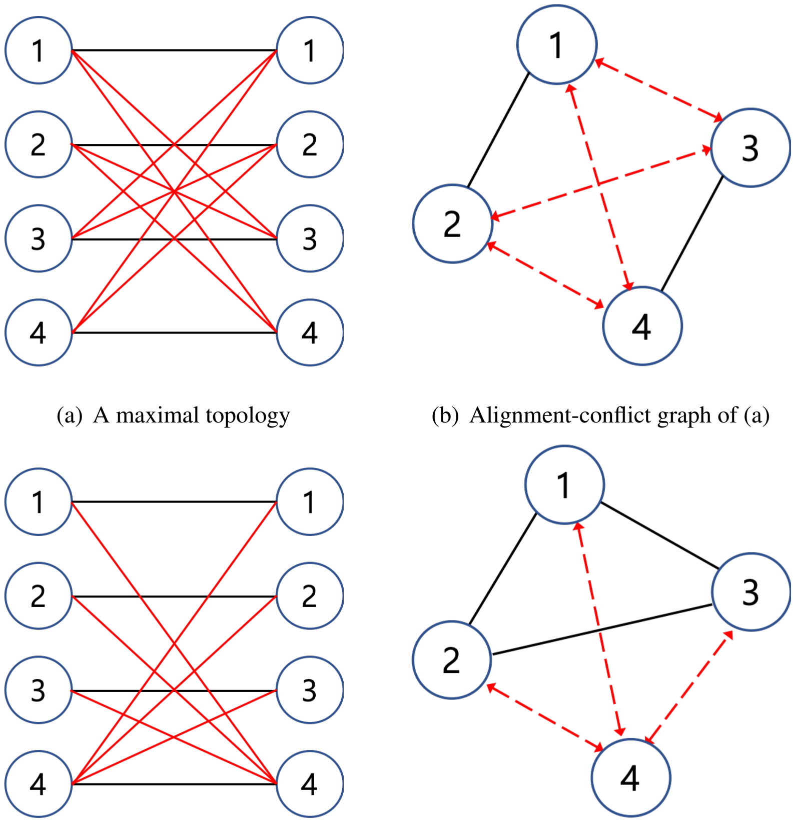
The achievable symmetric DoF of the topology with 2-alliance with mutual hostility is optimal,

$$d_{sym} = \frac{1}{2}. \tag{3.5}$$

Proof. (Necessity) Assume that $\mathcal{A}_1$ and $\mathcal{A}_2$ are not mutually hostile, that is, all messages in $\mathcal{A}_1$ conflict with some messages in $\mathcal{A}_2$ or all messages in $\mathcal{A}_2$ conflict with some messages in $\mathcal{A}_1$. Then, we can add conflict edges between messages in $\mathcal{A}_1$ and $\mathcal{A}_2$ without occurring the internal conflict and thus it is not maximal.
(Sufficiency) From Lemma 2, there are only two alliances if all alliances are mutually hostile. Then all messages in $\mathcal{A}_1$ fully conflict with all messages in $\mathcal{A}_2$ and vice versa. Also, it is not possible to add any conflict edges among messages in the same alliance due to the no conflict of messages in alliance. Thus the topology is maximal. □

We omit the proof of DoF optimality because it has already been shown in [16] for the alignment-conflict graph. The achievable scheme is proposed in the following Section.

Example 3.1: In Fig. 3.4, there are two maximal topologies for 4-user interference channel. They are maximal topologies derived from 2-alliance with mutual hostility. Each solid edge indicates that two messages are connected as alignment edge and belong to the same alliance (alignment set). The dashed edges indicate the conflict edge between them. In Fig. 3.4 (a), there are two alliances, $\mathcal{A}_1 = \{W_1, W_2\}$ and $\mathcal{A}_2 = \{W_3, W_4\}$ whose alignment-conflict graph is given in Fig. 3.4 (b). The interference channel in Fig. 3.4 (c) is another maximal topology, where $\mathcal{A}_1 = \{W_1, W_2, W_3\}$ and $\mathcal{A}_2 = \{W_4\}$ whose alignment-conflict graph is given in Fig. 3.4 (d).



(a) A maximal topology (b) Alignment-conflict graph of (a)

(c) Another maximal topology (d) Alignment-conflict graph of (c)

Figure 3.4: Two maximal topologies for 4-user interference channel and related graphs.

The two topologies in Example 2 are all possible maximal topologies derived from 2-alliance with mutual hostility for 4-user interference channel if we do not take into account the indices of messages. Note that if there is no hostility between any two alliances, two alliances (alignment sets) are combined by adding conflict edges without internal conflict and thus its topology is not maximal.

However, there are other maximal topologies which are generated from other than design with two alliances. The natural question is, ”Is there other way to satisfy maximality with giving up mutual hostility of all alliances?” The alliance construction can be generalized by setting hostility of alliances in a more general way. The key idea is to construct alliances, where each subset of messages in an alliance is interfered separately from all messages of each alliance. To this end, some definitions are needed as follows.

Definition 3.6 (Sub-alliance)**:** For alliance $\mathcal{A}_m \in \mathcal{P}_{\mathcal{W}}$, $\mathcal{A}_{m,k}$ and $\mathcal{A}_{m,k'}$ are blocks of partition of $\mathcal{A}_m$, $\mathcal{A}_{m,k} \cap \mathcal{A}_{m,k'} = \emptyset$ for distinct m, k, and k' and $\bigcup_{k \neq m} \mathcal{A}_{m,k} = \mathcal{A}_m$. Then $\mathcal{A}_{m,k}$ is called a sub-alliance of $\mathcal{A}_m$, where all messages in $\mathcal{A}_k$ conflict with each message in $\mathcal{A}_{m,k}$.

The structure of alliance and its sub-alliances is described in Figs. 3.5 and 3.6. There are alliance $\mathcal{A}_m$ and its sub-alliances as $\mathcal{A}_{m,1}, \mathcal{A}_{m,2}, \cdots, \mathcal{A}_{m,m-1}, \mathcal{A}_{m,m+1}, \cdots,$ $\mathcal{A}_{m,M}$. Sub-alliance $\mathcal{A}_{m,k}$ is the subset of messages in $\mathcal{A}_m$, where there exist conflict edges from all messages in $\mathcal{A}_k$ to all messages in $\mathcal{A}_{m,k}$ in Fig. 3.5. That is, the second index n of sub-alliance $\mathcal{A}_{m,n}$ is the index of alliance $\mathcal{A}_n$, whose messages are connected to messages in sub-alliance $\mathcal{A}_{m,n}$ with conflict edges. The directed line in Fig. 3.6 (a) indicates that all messages in $\mathcal{A}_k$ are connected to all messages in $\mathcal{A}_{m,k}$ with conflict edges. Fig. 3.6 (a) is equivalent to Fig. 3.6 (b), which is the alignment-conflict graph for messages in $\mathcal{A}_1$ and $\mathcal{A}_{m,1}$. In Fig. 3.6 (b), there exist an alliance $\mathcal{A}_1 = \{W_1, W_2\}$ and a sub-alliance $\mathcal{A}_{m,1} = \{W_3, W_4, W_5\}$. There exist conflict edges from $\{W_1, W_2\}$ in $\mathcal{A}_1$ to $\{W_3, W_4, W_5\}$ in $\mathcal{A}_{m,1}$.

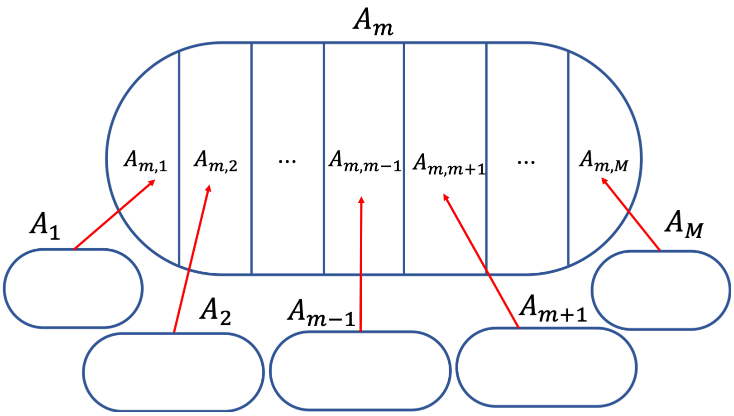
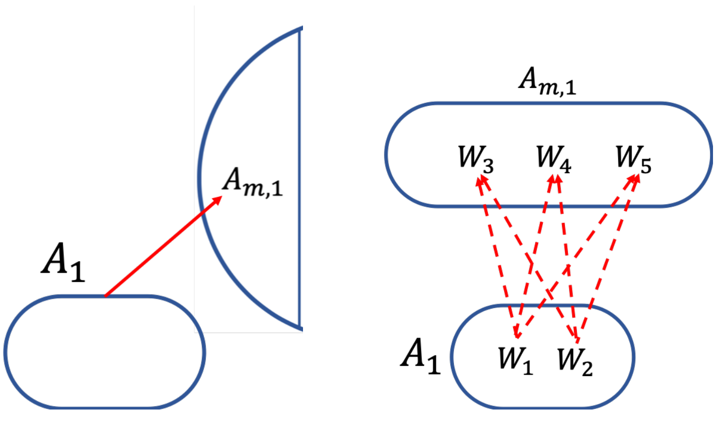


Figure 3.5: Alliance and its sub-alliances.



(a) Sub-alliance $\mathcal{A}_{m,1}$ (b) Conflict graph

Figure 3.6: Sub-alliance and conflict graph.

Definition 3.7 (Partial hostility)**:** $\mathcal{A}_l$ is said to be partially hostile to $\mathcal{A}_m$ if all messages in $\mathcal{A}_l$ only conflict with all messages in $\mathcal{A}_{m,l}$ of $\mathcal{A}_m$, denoted as

$$\mathcal{A}_l \rightharpoonup \mathcal{A}_m, \text{identically}\, \mathcal{A}_l \rightarrow \mathcal{A}_{m,l}. \tag{3.6}$$

Definition 3.8 (Mutually partial hostility (MPH))**:** Alliances $\mathcal{A}_m$ and $\mathcal{A}_l$ are said to be mutually partial hostile if all messages in $\mathcal{A}_m$ only conflict with all message in $\mathcal{A}_{l,m}$ of $\mathcal{A}_l$ and all messages in $\mathcal{A}_l$ only conflict with all messages in $\mathcal{A}_{m,l}$ of $\mathcal{A}_m$, where at least one of $\mathcal{A}_{m,l}$ and $\mathcal{A}_{l,m}$ are non empty sets, denoted by

$$\mathcal{A}_m \rightleftharpoons \mathcal{A}_l \tag{3.7}$$

$$\text{identically}\, \mathcal{A}_m \rightarrow \mathcal{A}_{l,m} \text{ and/or } \mathcal{A}_l \rightarrow \mathcal{A}_{m,l}. \tag{3.8}$$

Even though one of sub-alliances $\mathcal{A}_{m,l}$ and $\mathcal{A}_{l,m}$ is an empty set, $\mathcal{A}_m$ and $\mathcal{A}_l$ are also said to be mutually partial hostile in this paper. That is, $\mathcal{A}_m \rightleftharpoons \mathcal{A}_l$ includes three cases that only $\mathcal{A}_m$ is partially hostile to $\mathcal{A}_l$ or only $\mathcal{A}_l$ is partially hostile to $\mathcal{A}_m$ or both $\mathcal{A}_m$ and $\mathcal{A}_l$ are partially hostile to each other.

Lemma 3.3: If sub-alliances $\mathcal{A}_{m,l}$ and $\mathcal{A}_{l,m}$ are all empty sets, the topology is not maximal.

Proof. Since there is no conflict edge between messages in $\mathcal{A}_m$ and $\mathcal{A}_l$ in the conflict graph, two alliances can be merged into an alliance. If we merge them, the merged alliance should follow the set conflict of alliance. However, conflict edges between messages in $\mathcal{A}_l$ and $\mathcal{A}_{k,m}$ and conflict edges between messages in $\mathcal{A}_m$ and $\mathcal{A}_{k,l}$ in the conflict graph can be added for all distinct m, l, and k, which means that the topology is not maximal. □

Using the sub-alliances and MPH, Theorem 3.1 can be modified into the following theorem.

Theorem 3.2 (M-alliances with MPH)**:** For $\mathcal{W}$ in K-user interference channel, there are M-alliances given as a partition $\mathcal{P}_{\mathcal{W}} = \{\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_M\}$. A topology of K-user interference channel is maximal if and only if any distinct $\mathcal{A}_m$ and $\mathcal{A}_l$ are mutually partial hostile, that is,

$$\mathcal{A}_m \rightleftharpoons \mathcal{A}_l, \text{ for any } m, l \in \mathcal{M}. \tag{3.9}$$

Further, its achievable symmetric DoF is optimal, that is,

$$d_{sym} = \frac{1}{2}. \tag{3.10}$$

Proof. (Necessity) Assume that for some m and l, $\mathcal{A}_m$ and $\mathcal{A}_l$ are not mutually partial hostile, where there are three cases:
i) $\mathcal{A}_{m,l} = \mathcal{A}_{l,m} = \emptyset$,
ii) $\mathcal{A}_{m,l} \cap \mathcal{A}_{m,k} \neq \emptyset$,
iii) $\cup_i \mathcal{A}_{m,i} \neq \mathcal{A}_m$.

From Lemma 3, we have already proved the case i). For the second case, since messages in $\mathcal{A}_l$ and $\mathcal{A}_k$ have common messages to conflict with messages in $\mathcal{A}_m$, messages in $\mathcal{A}_l$ and $\mathcal{A}_k$ should be combined into an alignment set. But there exist conflict edges between messages in $\mathcal{A}_{l,k}$ and $\mathcal{A}_k$ or messages in $\mathcal{A}_{k,l}$ and $\mathcal{A}_l$ in the conflict graph, which means that internal conflict exists and its topology is not maximal. For the third case, there are at least one messages in $\mathcal{A}_m$, which is not interfered with. Then, some interference links can be added and thus the topology is not maximal.

(Sufficiency) Assume that M alliances are MPH. Due to MPH for all pairs of alliances, there exist conflict edges between all messages in $\mathcal{A}_m$ and $\mathcal{A}_{l,m}$ and between all messages in $\mathcal{A}_l$ and $\mathcal{A}_{m,l}$ for any m and l in the conflict graph. Let us add a conflict edge from a message W_e in $\mathcal{A}_m$ to W_p in $\mathcal{A}_{l,k}$ for any distinct m, l, and k. Then all messages in $\mathcal{A}_{k,m}$ and W_e are connected with alignment edges, which results that all messages in $\mathcal{A}_m$ and $\mathcal{A}_k$ are tied as an alignment set. Since $\mathcal{A}_m$ and $\mathcal{A}_k$ are hostile to each other, there exists internal conflict in the alignment set $\mathcal{A}_m \cup \mathcal{A}_k$. Thus, internal conflict always occurs by adding a conflict edge between any two messages and thus its topology is maximal. Similarly, we omit the proof of DoF optimality. □

Then, the following corollary can be stated without proof.

Corollary 3.1: A topology of K-user interference channel is maximal if and only if all distinct alignment sets are alliances with MPH.

The above corollary tells that it is possible to derive all maximal topologies for symmetric DoF $1/2$ by designing $\mathcal{P}_{\mathcal{W}}$ and its alliances with MPH. A design method of maximal topology is proposed with the definition of sub-alliance graph as follows. For $\mathcal{W}$, let $\mathcal{P}_{\mathcal{W}} = \{\mathcal{A}_1, \mathcal{A}_2, \cdots, \mathcal{A}_M\}$. Here, alliance $\mathcal{A}_m$ is partitioned into sub-alliances $\mathcal{A}_{m,l}$, $l \in \mathcal{M}$, $l \neq m$. Let $\mathcal{A}_{sub}$ be a set of all sub-alliances given as $\mathcal{A}_{sub} = \{\mathcal{A}_{m,l} \mid m, l \in \mathcal{M}, m \neq l\}$. Then, the sub-alliance graph is defined as follows.

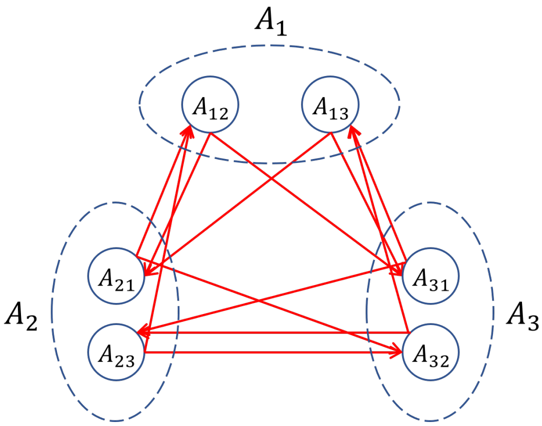
Definition 3.9 (Sub-alliance graph)**:** For $\mathcal{W}$, let $\mathcal{P}_{\mathcal{W}} = \{\mathcal{A}_1, \mathcal{A}_2, \cdots, \mathcal{A}_M\}$ be a partition of $\mathcal{W}$. A directed graph $\mathcal{D} = (\mathcal{A}_{sub}, \mathcal{E}_{sub})$ is called a sub-alliance graph, if there exist directed edges from all sub-alliances in $\mathcal{A}_m$ to sub-alliance $\mathcal{A}_{l,m}$ in $\mathcal{A}_l$ for all $m, l \in \mathcal{M}, m \neq l$.

Proposition 3.1 (Design of maximal topology)**:** A maximal topology $\mathcal{T}_M$ is derived from sub-alliance graph as follows:

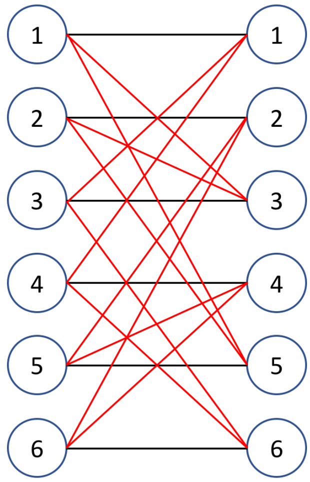
(i) There exists a direct link from transmitter i to receiver i for each message $W_i \in \mathcal{W}$.

(ii) There exists an interference link from transmitter i, whose message belongs to alliance $\mathcal{A}_m$ to receiver j, whose message belongs to $\mathcal{A}_{l,m}$ for all distinct m and l.

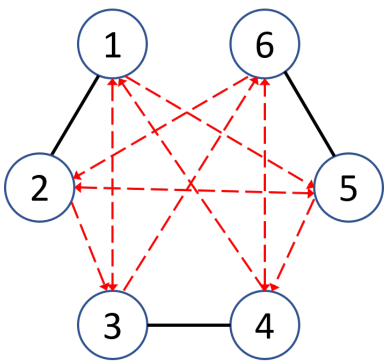
Example 3.2: A sub-alliance graph is given in Fig. 3.7 (a). A maximal topology from Fig. 3.7 (a) is given in Fig. 3.7 (b) and its alignment-conflict graph is given in Fig. 3.7 (c).



(a) Sub-alliance graph



(b) Maximal topology



(c) Alignment-conflict graph

Figure 3.7: Maximal topology derived from 3-alliance construction for 6-user interference channel.

Let $\mathcal{A}_1 = \mathcal{A}_{1,2} \cup \mathcal{A}_{1,3}$, whose sub-alliances are $\mathcal{A}_{1,2} = \{W_1\}$ and $\mathcal{A}_{1,3} = \{W_2\}$, $\mathcal{A}_2 = \mathcal{A}_{2,1} \cup \mathcal{A}_{2,3}$, whose sub-alliances are $\mathcal{A}_{2,1} = \{W_3\}$ and $\mathcal{A}_{2,3} = \{W_4\}$, and $\mathcal{A}_3 = \mathcal{A}_{3,1} \cup \mathcal{A}_{3,2}$, whose sub-alliances are $\mathcal{A}_{3,1} = \{W_5\}$ and $\mathcal{A}_{3,2} = \{W_6\}$. Contrary to 2-alliance with mutual hostility, receivers for messages in each alliance are partially interfered by transmitters of all messages in each alliance, where messages in each alliance are partitioned into sub-alliances indicating interferers. There are many ways to construct alliances by changing the number of alliances and partitioning messages into sub-alliances differently.

3.3 Properties of Alliance Constructions

In this section, properties of alliance construction are derived such as the maximum number of alliances to be constructed for the given number of messages K and a method to partition messages into sub-alliances.

3.3.1 Beamforming Vector Design for Alliance Construction

In this subsection, we design a beamforming vector assigned for messages in each alliance and show that the linear beamforming scheme for alliance construction achieves symmetric DoF $1/2$ in TIM. Suppose that there are M alliances with MPH for K-user interference channel and we use two time extensions for beamforming vectors. The beamforming vectors split each received signal space into two subspaces with desired message and one directional aligned interference signals for all receivers. M pairwise linearly independent beamforming vectors can be constructed and each of them is allotted to each alliance. Let $\boldsymbol{V}_m$ be a 2×1 beamforming vector for messages in alliance $\mathcal{A}_m$, $m \in \mathcal{M}$. There is no conflict edge among messages in an alliance and the messages in $\mathcal{A}_{m,l}$ are only interfered by all messages in $\mathcal{A}_l$. Consider the receiver j that wants message W_j, which belongs to sub-alliance $\mathcal{A}_{m,l}$ after the alliance construction. Then 2×1 received signal vector at receiver j for two time slots is given as

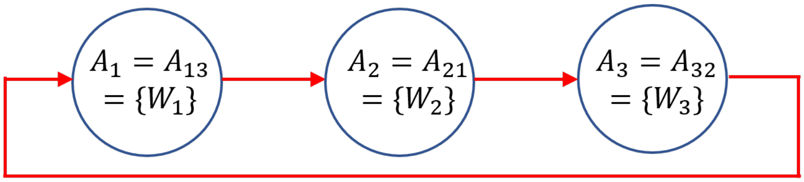


Figure 3.8: 3-alliance construction requiring the minimum number of messages.

$$\boldsymbol{Y}_j = h_{jj}\boldsymbol{V_m}W_j + \sum_{W_k \in \mathcal{A}_l} h_{kj}\boldsymbol{V_l}W_k + \boldsymbol{Z_j}. \tag{3.11}$$

Since $\boldsymbol{V}_m$ and $\boldsymbol{V}_l$ are linearly independent, receiver j can null the aligned interference signals corresponding to messages in $\mathcal{A}_l$ and recover W_i. In the same way, every receiver can decode its desired message by only two time extensions, which means that the network achieves symmetric DoF $1/2$ in TIM.

3.3.2 Maximum Number of Alliances and Partition of Messages into Alliances

In this subsection, we derive the maximum number of alliances for a given number of messages K for the alliance construction. In fact, we derive the minimum number of messages required to construct M alliances with MPH rather than the maximum number of alliances that can be made with K messages. Let a_M be the minimum number of messages which can construct M alliances with MPH. It is trivial that $a_1 = 1$ and $a_2 = 2$. When $M = 3$, the alliance construction requires the minimum number of messages, $K = 3$, where there is only a message in each alliance and hostility between alliances is a tail-bite as in Fig. 3.8.

It is clear that when all alliances are related with only one-way hostility, where any non-empty sub-alliance has only one message, the alliance construction has the minimum number of messages. That is, for any $m, l \in \mathcal{M}$, one of two sub-alliances

$\mathcal{A}_{m,l}$ and $\mathcal{A}_{l,m}$ is empty and the other has only one message.

Every non-empty sub-alliance requires at least one message. And it is enough to make mutually partial hostility for each pair of alliances with only one non-empty sub-alliance of them. Assume that M alliances have already been constructed using the minimum number of messages a_N and we want to add a new alliance $\mathcal{A}_{M+1}$. In this situation, $\mathcal{A}_{M+1}$ should relate hostility with all existing M alliances, which requires non-empty sub-alliance $\mathcal{A}_{m,M+1}$ or $\mathcal{A}_{M+1,m}$ for each $m \in \mathcal{M}$ and this requires at least M additional messages. Thus, the recurrence relation is formulated as

$$a_{M+1} = a_M + M, \quad M \geq 3 \tag{3.12}$$

and thus a_M is computed as

$$a_M = \binom{M}{2}, \quad M \geq 3. \tag{3.13}$$

In fact, alliance construction with the minimum number of messages is equivalent to the handshake problem. Using (3.13), the maximum number of alliances for given K users can be derived as follows. Let $M_{\max}$ be the maximum number of alliances with MPH for a given number of messages K. The range of K which can construct alliances up to $M_{\max}$ is given as

$$\binom{M_{\max}}{2} \leq K < \binom{M_{\max}+1}{2}. \tag{3.14}$$

It is clear that different alliance constructions are possible for the same numbers of alliances and messages because there are many ways to partition messages into sub-alliances. The partition of messages for a given number of alliances M is summarized as follows.

Corollary 3.2 (Partition of messages)**:** There exist M alliances with MPH for K user interference channel, if the number of messages in sub-alliance satisfies the following conditions for $K \geq \binom{M}{2}$:

(i) $\sum_{l=1,l\neq m}^{M} |\mathcal{A}_{l,m}| \geq 1, m \in \mathcal{M}$

(ii) $\sum_{l=1,l\neq m}^{M} |\mathcal{A}_{m,l}| \geq 1, m \in \mathcal{M}$

(iii) $|\mathcal{A}_{m,l}| + |\mathcal{A}_{l,m}| \geq 1, m, l \in \mathcal{M}$

(iv) $\sum_{m=1}^{M} \sum_{l=1,l\neq m}^{M} |\mathcal{A}_{m,l}| = K.$

The first inequality constraints that every alliance has at least one common message to conflict with. The second one constraints that every alliance has at least one message. The third inequality is necessary and sufficient conditions of MPH between two alliances. The last one means that every message should belong to a sub-alliance. The condition for $K \geq \binom{M}{2}$ is required to ensure enough messages for constructing M alliances. We omit the proof of above corollary.

3.4 Discriminant and Transformation of Maximal Topologies

In this section, a method to determine the maximality of topology is proposed using alliance construction with MPH and the transformation of non-maximal topology into maximal one is also proposed.

3.4.1 Discriminant of Maximal Topologies

Proposition 3.2 (Discriminant of maximal topology)**:** The maximality of topology is determined as follows:

(i) Construct all alignment sets (i.e., tentative alliances) for a given topology.

(ii) Investigate all messages in each alignment set whether they follow the conditions of no conflict and set conflict or not. If yes, alignment sets become alliances and otherwise, it is not maximal.

(iii) Investigate whether all alliances are pairwise mutually hostile or not, that is, there is no message which is not interfered and there is no pair of alliances $\mathcal{A}_m$ and $\mathcal{A}_l$ for $m, l \in \mathcal{M}$, whose sub-alliances $\mathcal{A}_{m,l}$ and $\mathcal{A}_{l,m}$ are both empty sets. If yes, the topology is maximal and otherwise, it is not maximal.

3.4.2 Transformation of Maximal Topology

Proposition 3.3 (Transformation of non-MTM into MTM)**:** First, check whether each principal submatrix is an identity matrix or not. If yes, the transformation procedure can be stated as:

(i) Insert element 1 to the topology matrix in a such way that incomplete interference blocks do not exist.

(ii) If two alliance blocks $\mathcal{A}_m$ and $\mathcal{A}_l$ do not have any corresponding interference block, there are two ways to transform the topology matrix as;

  (a) Merge them by permuting matrix indices in a such way that all indices in $\mathcal{A}_m$ and $\mathcal{A}_l$ are rearranged in a consecutive order.

  (b) If there exists the ith column with no interference block for message $W_i \in \mathcal{A}_m$ or $W_i \in \mathcal{A}_l$, add corresponding interference block to the ith column of $\mathcal{A}_m$ or $\mathcal{A}_l$.

(iii) If there still exists a column with no interference block, add an arbitrary interference block to the column.

Proposition 3.3 shows that transformation is not unique for a given topology matrix. There are many ways to merge provisional alliance blocks into single alliance block. Also for the column with no interference block, there are many ways to put an arbitrary interference block into the column.

Chapter 4

Maximal Topology Matrix and Generalized Alliance Construction

4.1 Introduction

In this chapter, we propose generalized alliance construction for topologies achieving DoF less than $1/2$ by modifying the definition of sub-alliance and derive its topology.

4.2 Conditions for Maximal Topology Matrix

In this subsection, the sufficient and necessary conditions for MTM are derived based on alliance construction. First, some of definitions related to topology matrix are givne as follows.

Definition 4.1 (Alliance block and interference block)**:** Suppose that the indices of messages in each alliance are ordered consecutively as $\mathcal{A}_m = \{W_i, W_{i+1}, \cdots, W_{i+|\mathcal{A}_m|-1}\}$. A principal submatrix of the topology matrix $\boldsymbol{T}$ is called an alliance block of $\mathcal{A}_m$ if $\boldsymbol{T}$ satisfies the following conditions:

(i) The principal submatrix corresponding to $\mathcal{A}_m$ is an identity matrix of size $|\mathcal{A}_m| \times |\mathcal{A}_m|$.

(ii) There exist at least one j in $\boldsymbol{T}$ such that $t_{kj} = 1$ for all $k \in \{i, i+1, \cdots, i + |\mathcal{A}_m| - 1\}$ and $W_j \notin \mathcal{A}_m$.

(iii) There does not exist j in $\boldsymbol{T}$ such that $t_{kj} = 1$ and $t_{lj} = 0$ for some $k, l \in \{i, i+1, \cdots, i + |\mathcal{A}_m| - 1\}$ and $W_j \notin \mathcal{A}_m$.

The above $|\mathcal{A}_m| \times 1$ submatrix $[t_{kj}]$ is called an interference block from $\mathcal{A}_m$.

The first condition corresponds to no conflict of messages in alliance preventing internal conflict and the second and third ones are the set conflict.

Example 4.1: Fig. 4.1 shows topology and its topology matrix for 6-user interference channel. There are three principal submatrices whose sizes are 3×3, 2×2, and 1×1. The 3×3 and 1×1 square matrices are alliance blocks. But 2×2 one is not due to $t_{5,4} = 1$, which represents internal conflict in alignment set $\{W_4, W_5\}$. There is an interference block from the alliance block $\mathcal{A}_1 = \{W_1, W_2, W_3\}$ given as $\{t_{i,5} | i \in \{1, 2, 3\}\}$, which represents the set conflict to W_5. Thus the above matrix is not MTM.

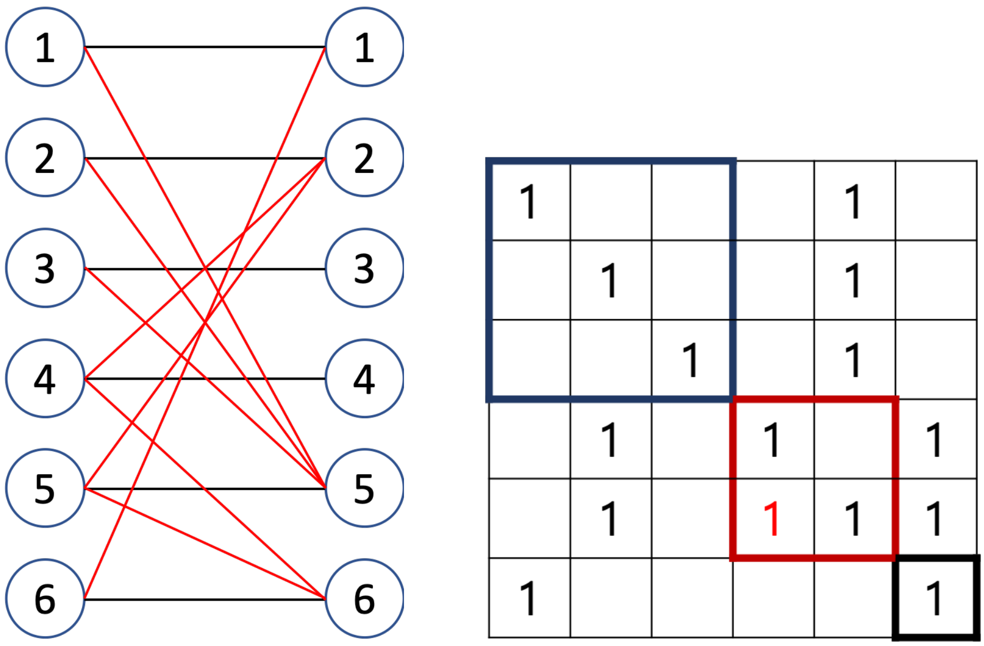
It is also required to translate the mutually partial hostility in alliance construction into topology matrix for MTM.

Theorem 4.1 (MTM)**:** Suppose that there are M alliance blocks in a topology matrix and messages in each alliance are ordered consecutively in indices. A topology matrix is MTM if and only if all alliance blocks satisfy the following conditions:

(i) Each column of the alliance blocks has a single interference block.

(ii) There exists at least one interference block between any two alliance blocks.

The first condition ensures that there is no message whose receiver is not interfered and each receiver for every message is interfered from all messages in a single alliance. The second condition ensures that at least one of sub-alliances $\mathcal{A}_{m,l}$ and $\mathcal{A}_{l,m}$ are not



(a) Topology (b) Topology matrix

Figure 4.1: Topology and its topology matrix for 6-user interference channel.

empty-sets for any $m, l \in \mathcal{M}$. In fact, the above two conditions correspond to MPH for M alliances. Thus we omit the proof of above Theorem.

Example 4.2: In Fig. 4.2, there are two topology matrices for 9-user interference channel. The topology matrix in Fig. 4.2 (a) satisfies all conditions for MTM. On the other hand, the topology matrix in Fig. 4.2 (b) is not MTM because the column of message W_6 in $\mathcal{A}_2 = \{W_5, W_6, W_7\}$ has two interference blocks from $\mathcal{A}_1 = \{W_1, W_2, W_3, W_4\}$ and $\mathcal{A}_3 = \{W_8, W_9\}$.

4.3 Discriminant and Transformation of MTM

In this section, we propose the discriminant of MTM in matrix perspective. The interpretation of the discriminant of maximal topology into the topology matrix is needed because the characteristics of maximal topology are more easily analyzed in topology matrix than sub-alliance graph. It is desirable that the messages that belong to the same alliance are ordered consecutively in indices. However, the indices of messages in a given topology matrix are always not well sorted and the alliance and interference blocks are not easily discerned. Thus, the permutation of messages for consecutive ordering of indices for each alliance should precede the analysis of maximality of topology in matrix perspective.

Proposition 4.1 (Discriminant of MTM)**:** The maximality of topology matrix is determined as follows:

(i) Construct all tentative alliance blocks by permuting matrix indices in a such way that any ith and jth rows and ith and jth columns are simultaneously permuted into consecutive order if $t_{i,k} = t_{j,k} = 1$.

(ii) Investigate whether all tentative alliance blocks are alliance blocks or not. If not, it is not MTM.

1				1		1		1
	1			1		1		1
		1		1		1		1
			1	1		1		1
1	1			1			1	
1	1				1		1	
1	1					1	1	
		1	1		1		1	
		1	1		1			1

(a) Maximal topology

1				1	1	1		1
	1			1	1	1		1
		1		1	1	1		1
			1	1	1	1		1
1	1			1			1	
1	1				1		1	
1	1					1	1	
		1	1		1		1	
		1	1		1			1

(b) Non-maximal topology

Figure 4.2: Topology matrices for 9-user interference channel.

(iii) If yes, investigate whether all alliance blocks follow two conditions in the Theorem or not.

(iv) If yes, it is an MTM and otherwise, it is not an MTM.

Note that the above (i) corresponds to reordering of message indices in order to make interfence blocks.

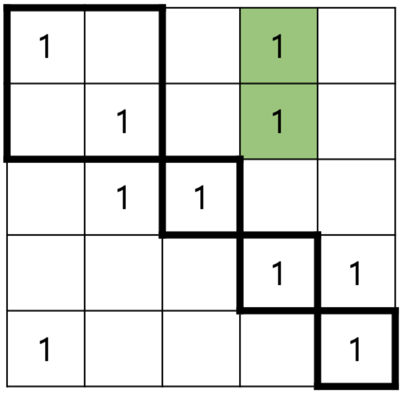
Example 4.3: The topology matrix in Fig. 4.3 represents a 5-user interference channel. However, if we swap message indices 2 and 3 in both rows and columns simultaneously in Fig. 4.3 (a), $\mathcal{A}_1 = \{W_1\}$ and $\mathcal{A}_3 = \{W_3\}$ can be combined into a single alliance block because $t_{1,4} = t_{3,4} = 1$ as in Fig. 4.3 (b). Even though message indices are reordered, this topology matrix is still not an MTM, because every column in the matrix does not have one interference block. Thus it is possible to add more interference links while maintaining the current DoF. It is not trivial to determine which empty spaces should be filled with element 1 (interference link) in the topology matrix. We propose how to transform non-MTM into MTM by filling some empty spaces with element 1 as in the following proposition.

4.4 Generalized Alliance Construction

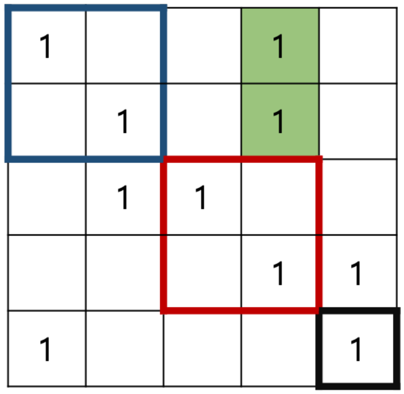
In this section, we propose generalized alliance construction for topologies achieving DoF less than $1/2$ by modifying the definition of sub-alliance and derive its topology.

4.4.1 Generalized Sub-Alliance

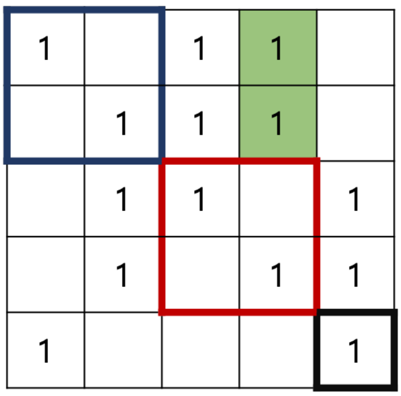
Until now, we focus on alliance construction for maximal topologies achieving symmetric DoF 1/2 and analyze characteristics of alliance construction and its topology matrix. In TIM, there exist other topologies whose achievable DoFs are less than $1/2$. But it is more difficult to show achievability and optimality of DoF less than $1/2$. In



(a) Original matrix



(b) Matrix with proper tentative alliance blocks



(c) MTM after transformation

Figure 4.3: Topology matrices for 5-user interference channel.

this subsection, we propose generalized alliance construction for topologies achieving symmetric DoFs less than $1/2$. In order to do so, some definitions in the previous section are modified for the generalized alliance construction as follows.

4.4.2 Topology Matrix for Generalized Allaince Construction

Definition 4.2 (Generalized sub-alliance)**:** The alliance $\mathcal{A}_m$ is partitioned into n_m generalized sub-alliances $\mathcal{A}_{m,\mathcal{E}_m^k}$, where $\mathcal{E}_m^k$ is the set of indices of alliances whose messages give interference to all messages in $\mathcal{A}_{m,\mathcal{E}_m^k}$ with $\cup_{k=1}^{n_m}\mathcal{A}_{m,\mathcal{E}_m^k} = \mathcal{A}_m$ but $\mathcal{E}_m^{k_1}$ and $\mathcal{E}_m^{k_2}$ can have a common subset for any distinct k_1 and k_2. Then $\mathcal{A}_{m,\mathcal{E}_m^k}$ is called a generalized sub-alliance of $\mathcal{A}_m$.

Definition 4.3 (Mutually multiple partial hostility(MMPH))**:** For alliances $\mathcal{A}_m$ and $\mathcal{A}_l$, it is called mutually multiple partial hostile if $l \in \cup_{k=1}^{n_m}\mathcal{E}_m^k$ and $m \in \cup_{k=1}^{n_l}\mathcal{E}_l^k$.

Theorem 4.2 (Generalized symmetric DoF)**:** For $\mathcal{W}$, there is a partition $\mathcal{P}_{\mathcal{W}} = \{\mathcal{A}_1, \mathcal{A}_2, \cdots, \mathcal{A}_M\}$ for M alliances with generalized sub-alliances and MMPH. Let E_M be $\max_{m,k} \left|\mathcal{E}_m^k\right|$ for all $\mathcal{A}_m \in \mathcal{P}_{\mathcal{W}}$. Then, the achievable symmetric DoF using the proposed linear beamforming scheme is

$$d_{sym} = \frac{1}{E_M + 1}. \tag{4.1}$$

Proof. Suppose that there are M alliances with generalized sub-alliances and MMPH for K-user interference channel and we use $(E_M + 1)$ time extensions for beamforming vectors. It is possible to construct M beamforming vectors allotted to each alliance, where any $E_M + 1$ vectors in M vectors are linearly independent. Let $\boldsymbol{V}_m$ be an $(E_M + 1) \times 1$ beamforming vector for messages in $\mathcal{A}_m$, $m \in \mathcal{M}$. There is no conflict edge among messages in each $\mathcal{A}_m$ in the conflict graph. Also each receiver of message in $\mathcal{A}_{m,\mathcal{E}_m^k}$ is interfered by all messages in all alliances $\mathcal{A}_l$, $l \in \mathcal{E}_m^k$. Consider the receiver j that wants message W_j, which belongs to $\mathcal{A}_{m,\mathcal{E}_m^k}$ after the generalized alliance construction. Then the $(E_M + 1) \times 1$ received signal vector at receiver j for

$(E_M + 1)$ time slots is given as

$$\boldsymbol{Y}_j = h_{jj}\boldsymbol{V_m}W_j + \sum_{l \in \mathcal{E}_m^k} \sum_{W_i \in \mathcal{A}_l} h_{ij}\boldsymbol{V_l}W_i + \boldsymbol{Z_j}. \quad (4.2)$$

Since there are at most E_M alliances with indices in $\mathcal{E}_m^k$ and any $E_M + 1$ beamforming vectors are linearly independent, receiver j can null the aligned interference signals and recover W_j. In the same way, every receiver can decode its desired message by only $E_M + 1$ time extensions, which means that the interference channel achieves DoF 1/$(E_M + 1)$ in TIM.

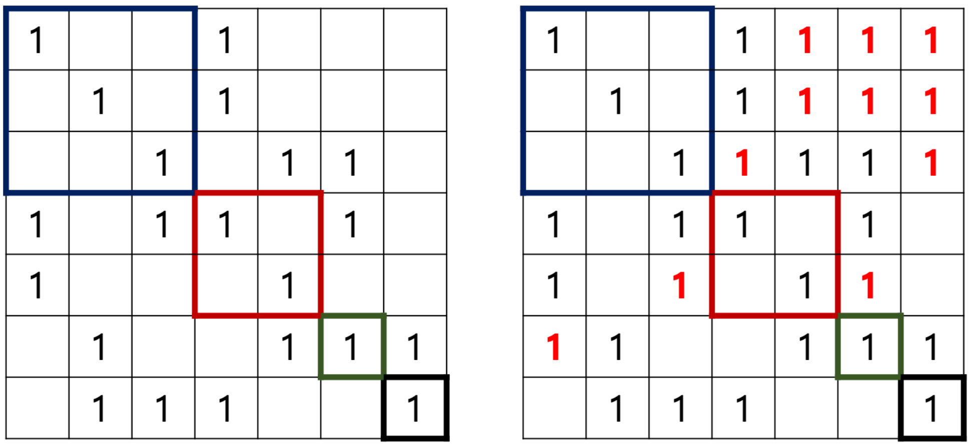
The symmetric DoF achieved by linear beamforming scheme is bounded by the maximum number of interfering alliances for all generalized sub-alliances. Note that the interference channel can achieve symmetric DoF 1/2 when each receiver of message in each sub-alliance in the interference channel is interfered by all messages from a single alliance, that is, $E_M = 1$, which results in $d_{sym} = 1/2$. □

4.5 Topology Matrix for Generalized Alliance Construction

Generalized alliance covers not only maximal topologies for DoF 1/2 but also topologies for DoFs less than 1/2 by generalizing sub-alliances. Theorem 4.2 tells that the achievable DoF does not change even if $\mathcal{E}_m^k = 1$ for all m and its k, $1 \leq k \leq n_m$. Thus, it is possible to consider maximality of topology matrix with the proposed linear beamforming scheme. The following corollary is the matrix version of Theorem 4.2 and suggests conditions for MTM with DoF $1/n$ for $n \geq 3$.

Corollary 4.1 (MTM with DoF $1/(E_M + 1)$ with the proposed scheme)**:** Suppose that there are M alliance blocks in a topology matrix and messages in each alliance are ordered consecutively in indices. A topology matrix is MTM with DoF $1/(1 + E_M)$ with the proposed scheme, if topology matrix satisfies following conditions:

(i) Every column has E_M interference blocks.



(a) Non-MTM (b) MTM

Figure 4.4: Topology matrices for 7-user interference channel achieving DoF $\frac{1}{3}$.

(ii) There is at least one interference block between any two alliance blocks.

Example 4.4: In Fig. 4.4, there are two topology matrices for 7-user interference channel, which have been already well permutated. Two toplogy matrices have four tentative alliance blocks, respectively and both matrices can achieve symmetric DoF 1/3 in TIM, because E_M is equal to two. However, the topology matrix in Fig. 4.4 (a) is not MTM with the proposed scheme because there are lots of rooms for additional interference links. The topology matrix in Fig. 4.4 (b) is designed as an example of MTM from the topology matrix in Fig. 4.4 (a). The bold elements are inserted properly to satisfy the maximality of topology in Fig. 4.4 (b). After transformation, it can be seen that the topology matrix in Fig. 4.4 (b) satisfies two conditions in Corollary 4.1 and thus, it is an MTM with DoF 1/3.

Note that Corollary 4.1 restricts definition of MTM with the proposed beamforming scheme. This is because it may be possible to achieve higher DoF for the same topology in a way other than the one we propose. The maximality means that topology matrix contains as many as possible with the proposed scheme because all messages in each alliance is connected to conflict edges with messages in exact E_M alliances. For the reason, the discrimination of MTM for DoFs less than $1/2$ is not proposed in the paper. It can be a future work to prove the optimality condition for topology DoFs less than $1/2$ and suggest achievable schemes for DoFs.

Chapter 5

Multi-level Topological Interference Management

5.1 Introduction

Unlike traditional IA, TIM can manage interference in more practical view by reducing the required CSI and implementation complexity. However, DoFs achieved by TIM heavily depends on the traits of topology, that is, the connectivity of interference links in the interference channel. For weak but not too weak interference links, the criterion is vague as to whether they are disconnected. For these reasons, the achievable symmetric DoF by TIM can be greatly degraded by ambiguous interference links. Thus, it is necessary to handle intermediate interference links effectively and treating interference as noise (TIN) with power allocation could be a solution .

5.2 Topological Interference Management

A various of symmetric DoFs including $1/2$ can be achieved according to topology in TIM and the optimal TIM-TIN decomposition should consider all the cases. However, it is NP-hard problem and hard to be analyzed. Similar to baseline in [19], we consider TIM-TIN decomposition only when the symmetric DoF is $1/2$ in TIM. This restriction can be beneficial to reduce complexity of decomposition significantly and connects

TIM-TIN decomposition to alliance construction.

In the previous chapters, alliance construction and maximal topologies are proposed and using alliance construction, it is possible to design any MTM and its maximal topology achieving symmetric DoF $1/2$. A maximal topology from alliance construction is easily derived at topology matrix. Thus, if channel matrix including channel gains is considered instead of topology matrix, it is possible to analyze whole GDoF in the matrix perspective. Similar to Chapter 4, alliance block and interference block will be discussed at the followings.

5.3 Treating Interference as Noise with Power Allocation

TIN with power allocation is proved to be optimal in the sense of generalized degrees of freedom (GDoF) in K-user interference channels [19]. In this dissertation, we utilize Theorem in [19] to analyze GDoF sum achieved by TIN. According to Theorem 1 in [19], the GDoF region is the set of all K-tuples $d_1, d_2, \ldots, d_K$ satisfying as follows:

$$0 \leq d_i \leq \alpha_{ii}, \forall i \in \mathcal{K} \tag{5.1}$$

$$\sum_{j=1}^{m} d_{ij} \leq \sum_{j=1}^{m} (\alpha_{i_j} \alpha_{i_j} - \alpha_{i_{j-1} i_j}), \forall (i_1, i_2, \ldots, i_m) \in \Pi_K. \tag{5.2}$$

Thus GDoF sum is bounded by the sum of all gains of desired links minus sum of all gains of cross links visiting all indices exactly one time.

5.3.1 System Model

In order to analyze GDoF performance in TIN, consider the general K-user interference channel presented in [19]. Suppose that each user $k \in \mathcal{K}$ sends b_k independent scalar data streams, each of which carries one symbol $s_{k,l}$ and it is transmitted along the $n \times 1$ beamforming vector $v_{k,l}$ over n channel uses

5.4 TIM-TIN Decomposition

5.4.1 Baseline

A baseline of TIM-TIN decomposition is proposed in [19]. The purpose of the TIM-TIN decomposition is to simplify the problem by solving the TIM and TIN components separately because joint optimization for signal power levels and signal vector spaces is far more challenging. First, all the non-zero interference links

Suppose that proper power allocation is performed with TIN, then the achievable GDoF tuple is $(d_{1,\text{TIN}}, d_{2,\text{TIN}}, \ldots, d_{K,\text{TIN}})$, which means the achievable signal power levels. The whole GDoF tuple is the product of the two fractions for each user, that is, $(d_{1,\text{TIN}} \times d_{1,\text{TIM}}, d_{2,\text{TIN}} \times d_{2,\text{TIM}}, \ldots, d_{K,\text{TIN}} \times d_{K,\text{TIM}})$.

5.4.2 Separation Criterion

The main point of separation criterion is how to distribute intermediate links to TIM and TIN. In this dissertation, the TIM-TIN decomposition is considered only in cases $1/2$ symmetric DoF is achievable in TIM. Thus purpose of the decomposition is to maximize GDoF in TIN as much as possible while achieving $1/2$ DoF in TIM. According to Theorem in [19], the sum GDoF of TIN is determined by the Hamiltonian path in K-user interference channel. Moreover, the optimal separation way is not restricted when TIM part can achieve symmetric DoF $1/2$. Thus, the optimal decomposition method is NP-hard problem. In this dissertation, we escape from finding the optimal solution rather propose a method to find sub-optimal solution based on topology matrix. Our solution is also restricted on the case $d_{i,\text{TIM}} = 1/2, \forall i \in \mathcal{K}$. However, this restriction is valid because the degeneration of DoF in TIM is much more critical than the degeneration of GDoF in TIN.

In Chapter 4, we already applied topology matrix to design maximal topology matrix in TIM. We also utilize channel matrix for TIM-TIN decomposition because matrix can show whole K-user interference channel compactly. However, TIN should consider gain of each channel between transceiver, modified channel matrix is defined as,

Definition 5.1 (Channel matrix)**:** Let define α_{ij} is channel gain between transmitter j and receiver i. Then, the channel matrix $\boldsymbol{M}$ for K-user interference channel is defined as

$$\boldsymbol{M}_K = [\alpha_{ij}]_{K\times K}, \forall i, j \in \mathcal{K} \tag{5.3}$$

In TIN, $d_{1,\text{TIM}}$ for each transceiver pair is affected by the maximum gain of ingoing channel and the maximum gain of outgoing channel. The ingoing channel affects transmit power P_i because other receiver receives larger interference from transmitter i as P_i increases. On the other hand, outgoing channel degenerates $d_{i,\text{TIM}}$ itself. Thus, in the view of TIM-TIN decomposition, it is necessary to consider both ingoing and outgoing channels simultaneously for each transceiver. For these reasons we define modified channel matrix as follows:

Definition 5.2 (Modified channel matrix)**:** Let define α_{ij} is channel gain between transmitter j and receiver i. Then, the channel matrix $\boldsymbol{M}$ for K-user interference channel is defined as

$$\boldsymbol{M}_K = [\frac{\alpha_{ij}}{\min(\alpha_{ii}, \alpha_{jj})}]_{K\times K}, \forall \text{different} i, j \in \mathcal{K} \tag{5.4}$$

The modified channel matrix reflects that every desired link is considered equally in TIM and every cross link α_{ij} is weighted with the maximum desired link between α_{ii} and α_{jj} whose transmitter and receiver are related to α_{ij}. The cross channel α_{ij} is normalized by the maximum desired link of the ith and jth transceivers. This normalization is required to make the cross link to reflect relative gain. The maximum desired link is used to consider worst case in the ith and jth transceivers.

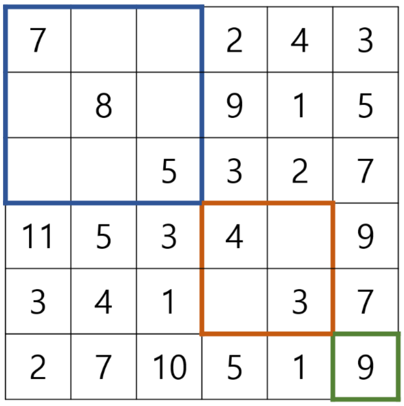
Proposition 5.1 (TIM-TIN decomposition)**:** Suppose that there are arbitrary N alliance blocks in MCM. The interference links TIM handles are determined as follows:

(i) For each column, only one interference block can be selected from existing other alliance blocks.

(ii) The chosen column has the maximum component (i.e., the maximum relative interference) in it.

(iii) The interference links for TIM are interference links corresponding to selected interference block with the maximum component.

(iv) The interference links corresponding to the other unselected interference blocks are not considered in TIM. In other words, TIN handles them.



(a) Original channel matrix

(b) Modified channel matrix

(c) MCM with maximum component for each column

(d) MCM with selected interference blocks

Figure 5.1: TIM-TIN decomposition using MCM.

Chapter 6

Inter-Cell Interference Coordination Based on Game Theory by Cell Zooming for Self-Organizing Cellular Network

6.1 Introduction

In this chapter, we consider a cellular network with limited backhaul such as SON with wireless backhaul. In the conventional cellular network, the topology and resource allocation of network is preset based on the estimation of traffic load and cell planning to avoid heavy inter-cell interferences.

When the backhaul is rich enough to share control messages among the cellular network, inter-cell interferences due to the activation of new cells can be managed efficiently through cooperation between cells. But in the cellular network consisting of self-organizing cells, ICIC without cell planning and BS cooperation should be considered.

Thus, we propose an ICIC scheme based on game theory by cell zooming for the SON, which improves SINR of UEs in the SON. In fact, cell zooming was originally proposed for energy saving in the cellular network. In [11], QoS-aware BS switching and cell zooming problem for green cellular networks was investigated. The proposed scheme in [11] considers user QoS requirements, which correspond to the inter-cell interference in our environment.

6.2 Non-Cooperative Game Theory

Many cell zooming algorithms are designed in the centralized network, which are not suitable for our backhaul-limited cellular network. Because control message among cells cannot be shared and also no cell can play role of CS. Therefore, we use the idea of non-cooperative game theory to design distributed ICIC cell zooming algorithm. First, we define a cell zooming factor as follows.

Definition 6.1 (Cell zooming factor)**:** The cell zooming factor r_j of cell j is defined as

$$r_j = \left(\frac{P_{\text{CZ}}}{P}\right)^a,$$

where P_{CZ}, P, and a are BS transmission power after cell zooming, current BS transmission power, and path-loss factor, respectively.

Cell zooming factor r_j can be interpreted as the measure of increase or decrease of cell radius after cell zooming.

Game theory is a theory of mathematical models of conflict and cooperation among intelligent rational decision-makers. A game is called non-cooperative if players cannot form alliances or if all agreements need to be self-enforcing. Basic components of a game are given as;

- Player: decision-maker
- Action: behavior of player based on its strategy
- Strategy: decision rule of player

- Utility: payoff for player based on its action

Then utility consists of two elements as:

- Revenue; profit (positive term) of a player, which should be maximized based on its own strategy.
- Cost; cost (negative term) of a player, which should be paid for its revenue.

If each player has chosen a strategy and no player can benefit by changing strategies while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding utilities constitutes a Nash equilibrium.

Definition 6.2 (Nash equilibrium)**:** Nash equilibrium is a solution concept of a non-cooperative game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy.

6.3 Design of Utility Function Based on Neighboring Signal Power Estimation

6.3.1 Design of Revenue Function

We propose the revenue function of cell k to be minimum received signal power among UEs in cell k such that

(i) Player k of non-cooperative game is BS of cell k.

(ii) Action of player k is the choice of cell zooming factor (CZF) r_k.

(iii) Strategy of player k is to choose an action that maximizes its utility.

Revenue function of player k which is minimum received signal power of UEs in cell k, is given as

$$\text{revenue}_k = P_k(\frac{x_{k_{\min,k}}}{r_k})^{-a}, \tag{6.1}$$

where $k_{\min}$ denotes the UE with minimum received signal power in cell k, and P_k, $x_{k_{\min,k}}$, and a are transmit power of BS k, distance between BS k and UE $k_{\min}$, and channel attenuation factor, respectively.

6.3.2 Design of Cost Function

Cost function should be designed considering that cooperation among BSs is not allowed. If each BS increases its cell zooming factor to obtain high revenue, inter-cell interference over cellular network becomes worse.

We propose how to design cost function based on the interference signal strength to the neighboring cells. Since the geographic information of neighboring cells is not allowed to be shared among BSs, the cell cannot know its most interfering neighbor cell.

Assume that the cell m is the strongest interferer to the cell k, which provides the second strongest reference signal received power (RSRP) to UE $k_{\min}$, i.e., $m = \arg\max_{i,i\neq k}\{P_i(\frac{x_{k_{\min,i}}}{r_i})^{-a}\}$. Then, in Fig. 6.1, it can be assumed that UEs of cell m near to the UE $k_{\min}$ will have its strongest interfering signals from BS k. Thus, UE $m_{\min}$ which receives the second strongest RSRP from BS k is assumed to be located near to UE $k_{\min}$, i.e., $x_{m_{\min,k}} \simeq x_{k_{\min,k}}$.

The cost function of cell k, cost_k which is based on inter-cell interference from cell k to UE $m_{\min}$ can be derived as

$$\text{cost}_k = -b\delta P_k^{\alpha}(\frac{x_{m_{\min,k}}}{r_k})^{-\alpha a} \simeq -b\delta P_k^{\alpha}(\frac{x_{k_{\min,k}}}{r_k})^{-\alpha a}, \tag{6.2}$$

where δ and b are average number of adjacent cells per cell and weight for the cost, respectively. In general, $\alpha = 1$ is used in the cellular network. In game theory, the sum

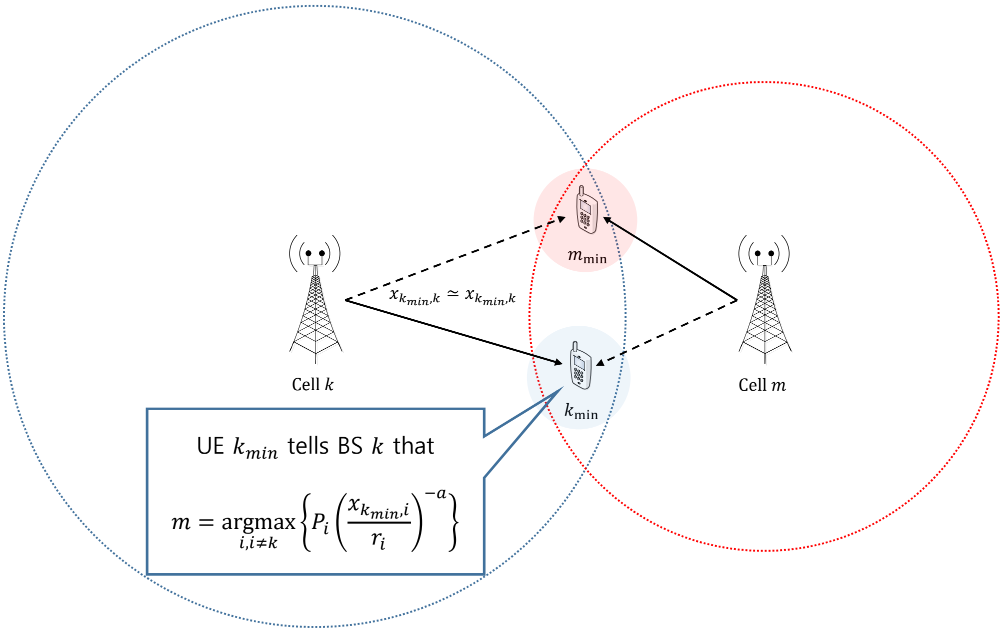


Figure 6.1: Estimation of $x_{m_{\min,k}}$.

of revenue function and cost function, that is, revenue$_k$ + cost$_k$ should be converged to Nash equilibrium. Thus, α should be larger than 1 and we heuristically decide $\alpha = 2$.

6.3.3 Utility Function and Nash Equilibrium

From (6.1) and (6.2), the utility function U_k of cell k is given as

$$U_k = \text{revenue}_k + \text{cost}_k = P_k(\frac{x_{k_{\min,k}}}{r_k})^{-a} - b\delta P_k^2(\frac{x_{k_{\min,k}}}{r_k})^{-2a}.$$

Nash equilibrium of player k should satisfy the following equation

$$\frac{\partial U_k}{\partial r_k} = aP_k(x_{k_{\min,k}})^{-a}{r_k}^{a-1} - 2ab\delta P_k^2(x_{k_{\min,k}})^{-2a}{r_k}^{2a-1} = 0. \quad (6.3)$$

Nash equilibrium r_k^* can be calculated from (6.3) as

$$r_k^* = \sqrt[a]{\frac{(x_{k_{\min},k})^a}{2b\delta P_k}}. \quad (6.4)$$

To ensure that U_k becomes maximum at Nash equilibrium in (6.4), the following inequality should be satisfied as

$$\frac{\partial^2 U_k}{\partial {r_k}^2} = a(a-1)P_k(x_{k_{\min,k}})^{-a}{r_k}^{a-2} - 2a(2a-1)b\delta P_k^2(x_{k_{\min,k}})^{-2a}{r_k}^{2a-2} \leq 0. \quad (6.5)$$

Equation (6.5) can be rewritten as

$$r_k \geq \sqrt[a]{\frac{(a-1)(x_{k_{\min},k})^a}{2(2a-1)b\delta P_k}}.$$

Note that $r_k^* \geq \sqrt[a]{\frac{(a-1)(x_{k_{\min},k})^a}{2(2a-1)b\delta P_k}}$. Therefore, each CZF r_k should be chosen from the following range

$$\sqrt[a]{\frac{(a-1)(x_{k_{\min},k})^a}{2(2a-1)b\delta P_k}} \leq r_k \leq r_{\max},$$

where $r_{\max}$ is decided by physical limitation of transmit power in BS.

Each UE is served by the BS which provides maximum RSRP to itself. If a cell is overloaded, BS chooses its serving UEs in the highest RSRP order and rest of UEs are handovered to other cells which provide the second highest RSRP to itself. Once the UE loads among cells are properly distributed, cell zooming is performed at the cell, where its minimum SINR is lower than threshold as

$$r_k = \min\left(r_k^*, r_{\max}\right).$$

The overall cell zooming process is summarized as in Algorithm 1.

Algorithm 1 Cell zooming for ICIC

INITIALIZATION

1. To be served, each UE requests to the BS which provides the highest RSRP.
2. BS k chooses its serving UEs in order of the highest RSRP upto cell capacity.
3. If cell k is overloaded, the rest of UEs in cell k are handovered to other cells which provides the second highest RSRP to themselves.

CELL ZOOMING

1. Each BS k finds UE k_{min} which suffers the highest interference in cell k.
2. BS k calculates its minimum SINR among UEs as $\text{SINR}_{k,\text{min}}$.
3. $\text{SINR}_{k,\text{min}}$ is compared to SINR threshold value, $\text{SINR}_{\text{threshold}}$.
   (a) $\text{SINR}_{k,\text{min}} \leq \text{SINR}_{\text{threshold}}$
   BS k updates its CZF r_k as $r_k = \min\left(r_k^*, r_{\text{max}}\right)$ as in (6.4).
   (b) $\text{SINR}_{k,\text{min}} \geq \text{SINR}_{\text{threshold}}$
   BS k does not change its CZF r_k.

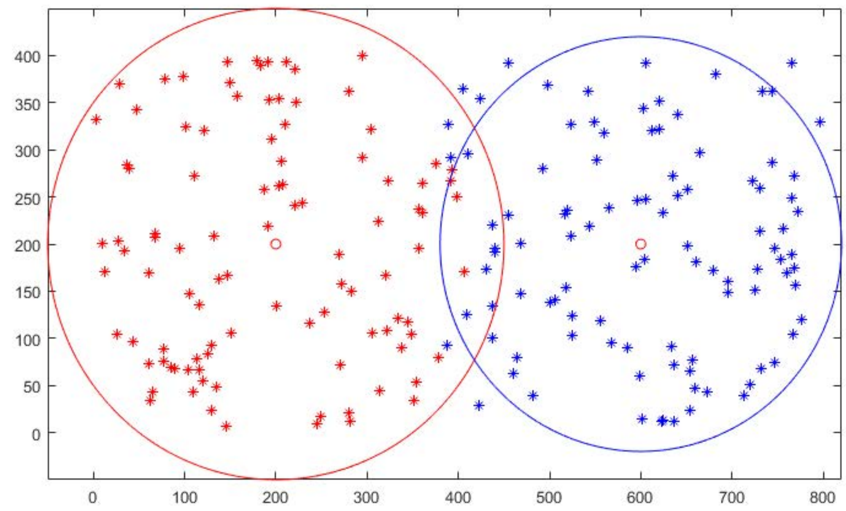
6.3.4 Simulation Result

Simulation environment for the proposed ICIC scheme is given as;

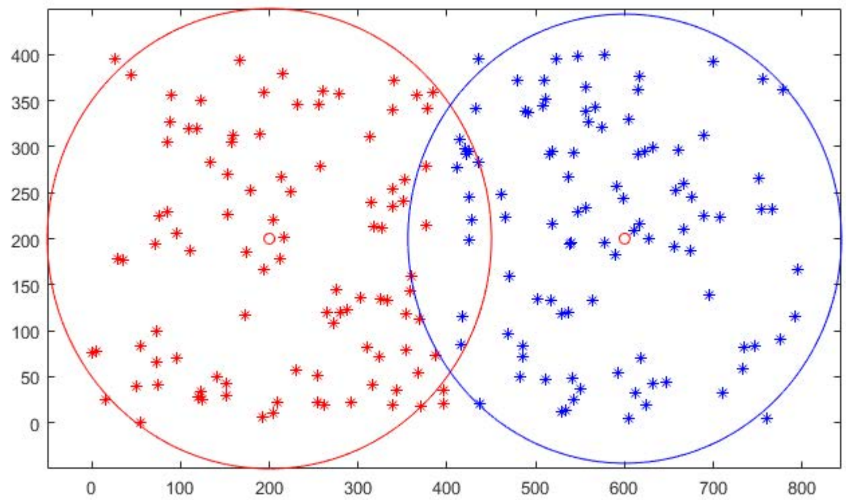
- Number of cells: 2
- $\alpha = 2$
- Channel attenuation factor a: 3
- Intensity of UEs: 100
- Initial radius of cell: 200 m
- Transmit power at BS: 50 dBm
- Noise power: -50 dBm
- $\text{SINR}_{\text{threshold}}$: 0.5
- $r_{\text{max}} : \sqrt[a]{2} = \sqrt[3]{2}$
- Value of $b \cdot \delta$: 6.2×10^4

Simulation results for the proposed scheme are shown in Fig. 6.2. In Fig. 6.2 (a), the proposed cell zooming algorithm does not converge and each cell switches its CZF among certain values, because UE distribution in Fig. 6.2 (a) is bad to be served by the network. In Fig. 6.2 (b), the proposed cell zooming algorithm converges, where UE distribution in Fig. 6.2 (b) is good to be served by the network.

We also extend the environment for simulation, where 5 cells exist and each BS is randomly distributed and maintains the minimum distance as 100m among BSs in order to show that the proposed scheme improves the minimum SINR of UE in the whole network in SON.



(a) Minimum SINRs in left and right cells become 1.2010 and 0.5189 after cell zooming.



(b) Minimum SINRs in left and right cells become 1.1456 and 1.2563 after cell zooming.

Figure 6.2: Simulation of ICIC based on cell zooming.

- Number of cells: 5
- $\alpha = 3$
- Channel attenuation factor a: 3
- Intensity of UEs: 80
- Initial radius of cell: 200 m
- Transmit power at BS: 50 dBm
- Noise power: = 40,-35,-30,-25 dBm
- $r_{\text{max}} : \sqrt[a]{2} = \sqrt[3]{2}$.

In the second simulation, each BS is randomly located in a square area of 1km each and maintains at least 100m distance among them. There are 80 UEs on average in each cell and thus the total number of UEs is 400. Also it is assumed that each cell can contain UE up to 100 as in Fig. 6.3. In Table 6.1, we assume that the transmit power of each BS is equal to the average transmit power of BSs after cell zooming in order to compare performances fairly.

The SINRs in Table 6.1 are the average value of the minimum SINRs in the whole network by randomly generated simulation environment 10,000 times. It is shown that the improvement ratio for the minimum SINR of UE is increased as the power of noise increases.

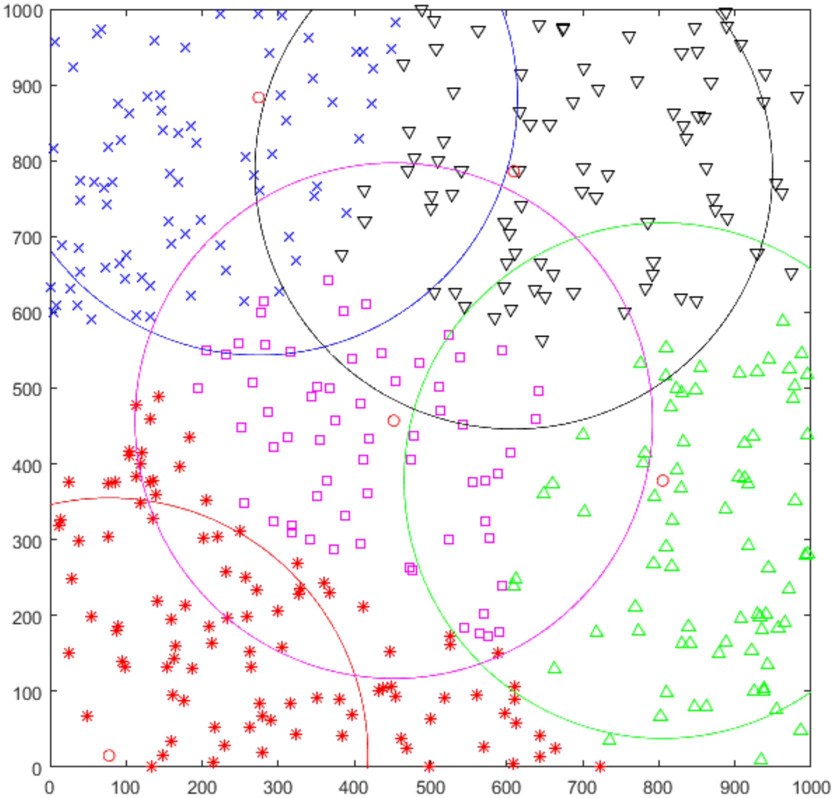


Figure 6.3: Cells and UE distributions after cell zooming.

Table 6.1: The minimum SINR of UE before and after cell-zooming.

Noise Power (dBm)	The minimum SINR of UE before CZ	The minimum SINR of UE after CZ	Improvement ratio
-40	0.4676	0.4753	1.6%
-35	0.4490	0.4693	4.5%
-30	0.3954	0.4436	12%
-25	0.3212	0.3785	18%

Chapter 7

Conclusion

In this dissertation, alliance construction which derives maximal topology in TIM, MTM and generalized alliance construction, TIM-TIN decomposition using MCM, and ICIC design for SON are studied.

First, we extended alignment set and introduced the alliance as a set of messages that follows no internal conflict and set conflict in the alignment-conflict graph. Based on alliance, we proposed the alliance construction with MPH, which results in generating maximal topology. We proved that any maximal topology achieving symmetric DoF $1/2$ can be derived from alliance construction. Using alliance construction, some properties of maximal topologies were given such as the maximum number of alliances and partition of messages into alliance. Using alliance construction, the discriminant and transformation for maximal topology were also proposed.

Second, we convert alliance construction in the alignment-conflict graph into topology matrix in order to analyze the maximality of topology easily. The sufficient and necessary conditions for MTM were derived and the discriminant of MTM and the transformation of non-MTM into MTM were also proposed. Furthermore, we generalized the alliance construction with generalized sub-alliances dealing with topologies for DoF $1/n$. The generalized alliance construction was represented in matrix form and the conditions of MTM with DoF $1/n$ with the proposed scheme were described.

Third, a method of TIM-TIN decomposition is proposed for sub-optimal GDoF sum. From the baseline of TIM-TIN decomposition, we proposed a concrete way to distribute interference links to TIM and TIN by utilizing alliance construction and MCM. It is possible to consider relative influence on GDoF for each transceiver using MCM.

Lastly, a new ICIC scheme for self-organizing cellular network is proposed to improve the SINR. The proposed scheme consists of distributed cell zooming based on non-cooperative game theory, where information exchange is not allowed among BSs. It is shown that the proposed scheme can efficiently adjust transmit power of each cell and its coverage to manage inter-cell interferences through simulation.

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초 록

본 논문에서는, i) 동맹 건설을 이용한 토폴로지 간섭관리에서 최대 토폴로지 설계, ii) 최대 토폴로지 행렬 설계 및 일반화된 동맹 건설과 이를 이용한 자유도 $1/2$ 미만의 토폴로지 설계, iii) TIM-TIN 분리 기법 iv) 셀 간 간섭 조정 (ICIC)이 연구되었다.

먼저, 기존의 정렬 집합을 확장시켜 내적 갈등이 없고 집합 갈등을 만족하는 메세지들의 집합인 동맹 (alliance)을 정의한다. 동맹을 기반으로 상호 부분 적대를 만족하는 동맹 건설을 제안하고 이를 통해 최대 토폴로지를 생성한다. 대칭 자유도가 $1/2$인 모든 최대 토폴로지는 동맹 건설을 통해 설계가 된다는 것을 증명한다. 또한 동맹 건설을 이용하여 동맹의 최대 수, 동맹으로 메세지 할당 등 최대 토폴로지의 특성에 관한 내용을 제시한다. 동맹 건설을 활용하여, 최대 토폴로지 판별과 변형을 제안한다.

두 번째로, 토폴로지의 최대성을 보다 쉽게 분석하기 위해, 정렬-갈등 그래프와 관련된 동맹 건설을 토폴로지 행렬로 변형시킨다. 최대 토폴로지 행렬 (maximal topology matrix; MTM)의 필요 충분 조건을 유도하고 MTM의 판별과 변형 역시 제안한다. 나아가, 일반화된 부분동맹을 통해 동맹 건설을 일반화하고 $1/n$ 자유도를 얻는 토폴로지를 설계한다. 일반화된 동맹 건설도 행렬 형태로 표현되고 제안하는 기법에서 자유도 $1/n$을 얻는 최대 토폴로지의 조건을 제시한다.

세 번째로 일반화 자유도 합의 차선해를 위한 TIM-TIN 분리 기법을 제안한다. TIM-TIN 분리의 기초에서 시작하여, TIM과 TIN에 간섭 링크들을 분배하는 구체적인 방법을 동맹 건설과 변형 채널 행렬 (modified channel matrix; MCM)을 활용하여

제안한다. MCM을 이용하여 각각 간섭 링크들이 각 송수신 쌍의 일반화 자유도에 대한 상대적 영향을 측정할 수 있다.

마지막으로, 자가 조직화 셀룰러 네트워크를 위한 셀 간 간섭 조정 기법이 제안되었는데, 각 기지국은 종래의 셀 간 간섭 조정 방식을 수행하기 위한 정보를 백홀을 통해 공유할 수 없는 상황에서 간섭 조정을 수행한다. 제안된 셀 간 간섭 조정 기법은 비협조적 게임 이론이 사용되는 분산 셀 확대 기법에 기반을 두고 있다. 또한, 제안된 기법이 자가 조직화 셀룰러 네트워크에서 셀 간 간섭 및 커버리지 공동 문제를 효율적으로 처리 할 수 있음을 모의 실험을 통하여 보인다.

주요어: 전송 자유도, 간섭 채널, 토폴로지 간섭 관리, 최대 토폴로지, 동맹, 동맹 건설, 최대 토폴로지 행렬, TIM-TIN 분해, 변형 채널 행렬, 셀간 간섭 조정, 셀 확대

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