Trade of Vertically Differentiated Products, Quality Improvement, and Welfare

Le Duc Niem and Taegi Kim

This paper constructs a simple model to explain how trade in vertically differentiated products affects the welfare of the trading countries when product quality can be selected by firms. We specifically devise a model showing that when trade expands a market, firms will intensively invest in R&D activities to build higher quality into their goods and thus benefit the entire region. We find that trade does not increase the variety of goods but increases their cost due to their higher quality. We conclude that quality improvement is the main mechanism that helps countries gain from trade in vertically differentiated goods.

Keywords: Product quality, Variety of goods, Vertical intra-industry trade, Welfare

JEL Classification: F12, L13

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I. Introduction

Recent studies have shown that international trade is concentrated among industrialized nations which have similar factor endowments and that this trade is mostly for the same goods (Bergoeing and Kehoe 2003; Gabrisch and Segnana 2003). Generally accepted explanations for this phenomenon have stressed economies of scale and variety of goods as fundamental causes (*e.g.*, see Krugman *1979* and Lancaster 1980). The predictive power of these models thus relies on the horizontal differentiation of products and an assumption that the quality of trading products is identical. However, the observation confirms that a large part of trade among developed countries is vertical intra-industry trade (VIIT)—that is, trade of the same goods at different quality levels (Gabrisch and Segnana 2003). To address this situation, Falvey and Kierzkowski (1987), Falvey (1981), and Flam and Helpman (1987) show how the trade of quality differentiated goods can occur between countries with different per capita incomes. They assumed that quality is an output of an increasing function of capital intensity so that capital-abundant countries will have a comparative advantage in trading higher quality goods, whereas labor-abundant countries will have a comparative advantage in trading lower quality ones. Bhagwati and Davis (2012) and Davis (1995) show that VIIT can occur in traditional trade models in the presence of technology differences within an industry. These models claim that VIIT operates according to an H-O model based on comparative advantages arising from factors intrinsic to each country. Accordingly, it has been suggested that this kind of trade be called *inter-industry trade* instead of VIIT.

Gruber *et al.* (1967) report that, in the case of US industries, a strong correlation exists between the intensity of R&D effort and international trade activities. Brito *et al.* (2012) analyze the UN Comtrade data from 19 OECD countries and show that high-tech industries focus their R&D efforts on improving product quality rather than reducing the cost of their products. According to Fabrizio *et al.* (1997), the principal countries benefiting from higher product qualities are those that trade internationally. This finding implies that companies that trade internationally are motivated to improve their products by investing in R&D. Hence, consumers who live in countries where many industries trade on international markets benefit from higher quality products. Bloom, Draca, and Reenen (2016) investigated the impact of Chinese import competition on technical change using panel data across twelve European countries from 1996 to 2007 and pointed out that trade strongly and significantly induced technological upgrading within firms and between firms in Europe.

In this paper, we construct a theoretical model of VIIT involving similar countries trading for the same goods and show that trade between similar countries can also induce technology upgrading *within* and *between firms*. We show that market expansion when countries are in trade likely triggers firms to invest more in R&D to produce goods of *a higher quality level*, thereby increasing both consumers and producers' surpluses. Consequently, the entire region is better off. Thus, two similar countries may be eager to trade with each other because they may find a win-win outcome, with both countries gaining from the trade. When we allow this process of selecting product quality, quality improvement becomes extremely dominant that internal increasing returns to scale and varieties of goods no longer serve as the reasons for trading quality differentiated goods.

The paper is organized as follows. Section II provides the basic model. Section III derives the impact of VIIT on trading countries' welfare. Section IV discusses the finding implications. Our conclusion is provided in the final section.

II. Model

We assume a region with only two similar countries, namely, Home and Foreign. Many industries exist in each country. However, we *hereinafter* focus on the trade of goods in a single industry which has identical goods but can be differentiated by quality.

Numerous consumers purchase the goods: S in Home and T in Foreign (T, S > 0). Reasonably, S and T are assumed proxies for Home and Foreign sizes, respectively. In each country, consumers are uniformly distributed along the interval [1, b] according to their marginal willingness to pay for quality, denoted by θ_j for consumer *j*.[1] A consumer's marginal willingness to pay for quality is dependent on her income (as conceptualized by Gabszewicz and Thisse [1979]), or it

[1] This assumption is widely used in vertical production differentiation studies, such as those of Sutton (1986), Wauthy (1996), and Beloqui and Usategui (2005).

is the reciprocal of the marginal utility of income. The more income a consumer has, the more she is willing to pay for goods at any quality level. Thus, b can be considered a proxy of per capita income in a country.[2] Given that Home and Foreign are identical in terms of this income, we assume that both countries have the same distribution range of consumers' marginal willingness to pay for quality (from 1 to b).

By following Motta (1993), Wauthy (1996), Liao (2008), and Nguyen (2015), the utility function of consumer j identified by θ_j is given by

$$U_j = \begin{cases} \theta_j q - p & \text{if buying a unit of goods} \\ 0 & \text{if not buying} \end{cases}, \tag{1}$$

where the quality level of the purchased good is q, the price she must pay for the good is p. A consumer's utility is zero if she does not purchase the good. Considering the many goods available, she will elect to purchase the one that generates the highest and non-negative utility.

In each country, we assume an infinite number of free-entry/exit firms that are willing to produce one type of goods. Production is associated with quality development costs, which are assumed to increase as the goods reach higher quality levels. Nguyen (2015) assumed that the unit cost is quadratically proportional to the quality of a product and constant for a given level of quality.[3] Alternatively, Mussa and Rosen (1978), Motta (1993), Liao (2008), and Schubert (2017) considered a quadratic fixed quality cost.[4] By combining these assumptions, we assume that firm i's total cost function is as follows:

$$TC_i = D_i a q_i + \frac{1}{z} q_i^z \qquad i = 1, 2, \ldots, n;\ n \to +\infty, \tag{2}$$

where D_i is the demand for firm i's goods, a is a constant, and q_i is

[2] Consumer's marginal willingness to pay for quality is dependent on personal income. Additionally, we assume that consumers are uniformly distributed from 1 to b with regard to their marginal willingness. Given that per-capita income is the average income of all consumers, it is directly proportional to the average of the consumer's marginal willingness, (b−1)/2 or simply b.

[3] Nguyen (2015) assumed $TC_i = D_i q_i^2/2$, where D_i is the demand for firm i's good.

[4] Mussa and Rosen (1978), Motta (1993), and Liao (2008) used quality cost functions with a quadratic form $TC = q_i^2/2$.

the quality level of the product (see Motta 1993 and Nguyen 2015 for a similar set up). Thus, aq_i, unit cost of goods of firm i, is constant for a given level of quality; however, it is proportional to quality level. An increase in the quality of the goods also increases its unit cost. The fixed cost for product quality improvement (R&D cost) is $f(q_i) = q_i^z/z$ ($z \in N$, $z \geq 2$). It is the same for all firms and is seen as quality-dependent fixed cost with $f(0) = 0$. Notably, $f'(q) > 0$ or the marginal quality cost is increasing. In addition, $f(q)$ is strictly convex in q for all feasible quality levels or $f''(q) > 0$. The total cost function employed in this model incorporates the assumptions made by Mussa and Rosen (1978), Motta (1993), Liao (2008), and Nguyen (2015).

All firms are indexed and named according to the quality rank of their goods: the firm with a quality rank i is called *firm i* . We note that the total cost function in (2) has a property of *increasing returns to scale (IRS)*, for a given level of quality because the average cost decreases when the output (or the demand D_i) increases for a fixed level of quality.

The timing of the two-stage game is as follows: firms simultaneously select the quality of their goods in the first stage and then engage in simultaneous price competition during the second stage. This assumption is very common in the literature on vertical product differentiation. Specifically, firm i selects quality q_i ($i = 1,2,\ldots,n$) during the first stage and price p_i during the second stage for the maximization of its profit. Given that firms are named by their product quality ranks, we have $q_1 > q_2 > \ldots > q_{n-1} > q_n$. Remarkably, firms can trade in products with a quality rating of zero.

We assume that no trade barriers exist between Home and Foreign and that transportation cost is zero. In addition, complete and perfect information is also assumed. We solve this problem through backwards induction.

III. Impact of vertical intra-industry trade on the welfares

A. Unitary Country

This section considers a country called *a similar country,* that is identical to Home and Foreign in all aspects, but its size can be different. When a consumer size is 1, we call this country the *Unitary Country.*

Lemma 1 *A number N exists such that, at any Nash equilibrium, at most, N firms with positive market shares and positive prices exist in the Unitary Country.*

Proof. Lemma 1 is a corollary of *Condition (F) for the Finiteness Property* defined by Shaked and Sutton (1983). If Condition (F) is satisfied, a finite number of firms with positive market shares are expected to co-exist at equilibrium. This condition is violated only under the assumption of zero marginal costs, combined with the lower bound on the marginal willingness being zero. In this model, the lower bound of the willingness is equal to 1 and the marginal cost aq_i is greater than zero for a positive quality. Put differently, Condition (F) is either satisfied under the conditions of our assumptions, or the finiteness property is satisfied. Thus, a finite number of firms with positive market shares and prices exist during the price selection stage.

Lemma 2 *The Unitary Country is covered by N firms with positive market shares, where N is unique.*

Proof. Now, we suppose that an uncovered market configuration is set up with a number of firms. Some consumers exist in the interval $[1, 1 + \varepsilon)$ who do not buy any goods (where ε is a very small increment). Considering that firms can freely enter/ exit the market, an entry firm (firm *e*) with a strategy offers goods with sufficiently low quality such that $q_e - p_e \geq 0$ will attract these consumers and make a positive profit. Thus, an uncovered market is unstable in this setting. The entry of firms covers the market at equilibrium.

An infinite number of firms are free to enter or exit the market. Thus, N is an *ex-ante* assumption of the maximum number of firms co-existing at equilibrium. Then, we will prove that any arrangement of M firms (M ≠ N) cannot construct any equilibrium.

As each consumer can buy, at most, one unit of goods, the demand for a firm is also the number of consumers who decide to buy its goods. We follow Sutton (1986), Wauthy (1996), and Beloqui and Usategui (2005) to compute profit functions of *N* firms in the country with market size 1.

$$\begin{cases} \Pi_1 = \dfrac{1}{b-1}\left[b - \dfrac{p_1 - p_2}{q_1 - q_2}\right] p_1 - \dfrac{1}{b-1}\left[b - \dfrac{p_1 - p_2}{q_1 - q_2}\right] aq_1 - \dfrac{1}{z} q_1^z \\ \Pi_i = \dfrac{1}{b-1}\left[\dfrac{p_{i-1} - p_i}{q_{i-1} - q_i} - \dfrac{p_i - p_{i+1}}{q_i - q_{i+1}}\right] p_i - \dfrac{1}{b-1}\left[\dfrac{p_{i-1} - p_i}{q_{i-1} - q_i} - \dfrac{p_i - p_{i+1}}{q_i - q_{i+1}}\right] aq_i - \dfrac{1}{z} q_i^z \quad for \;\; i \neq 1, N \\ \Pi_N = \dfrac{1}{b-1}\left[\dfrac{p_{N-1} - p_N}{q_{N-1} - q_N} - 1\right] p_N - \dfrac{1}{b-1}\left[\dfrac{p_{N-1} - p_N}{q_{N-1} - q_N} - 1\right] aq_N - \dfrac{1}{z} q_N^z \end{cases} . \quad (3)$$

The term $1/(b-1)$ denotes the consumer density. With an *ex-ante* assumption of two firms, Liao (2008) proves that a covered market with an interior solution in the price stage is not an equilibrium when fixed costs of quality improvement are considered. In this model, the threat of firms' entry prevents a covered market with an interior solution from being an equilibrium.[5] Thus, the market must be covered with a corner solution at equilibrium (or $q_N - p_N = 0$). To obtain the optimal prices in system (3), we solve the following system of linear equations:

$$\begin{cases} \partial \Pi_1 / \partial p_1 = b(q_1 - q_2) - 2p_1 + p_2 + aq_1 = 0 \\ \partial \Pi_i / \partial p_i = (q_i - q_{i+1})p_{i-1} - 2(q_{i-1} - q_{i+1})p_i + (q_{i-1} - q_i)p_{1+1} + aq_i(q_{i-1} - q_{i+1}) = 0 \quad for \quad i \neq 1, N \\ q_N - p_N = 0 \end{cases} \quad (4)$$

Notably, the set of qualities $\{q_1, q_2, \ldots, q_N\}$ is given in this stage. Given that (4) consists of N variables $\{p_1, p_2, \ldots, p_N\}$ and N linear equations, it only has one solution.

Now, we prove that a unique Nash Equilibrium with N firms exists with positive market shares and positive prices. We suppose the existence of an arrangement of M firms with positive market shares and positive prices such that $M \neq N$. We will prove that this case is not a Nash Equilibrium.

For any M such that $M > N$, Lemma 1 prevents such arrangement from forming an equilibrium.

For any M such that $M < N$, a firm ranked i will offer $q_i > 0$, $i = 1, \ldots, M$. However, many other firms can freely enter the market by deciding to offer goods with positive qualities if they find opportunities to make a positive profit. In this case, at least $(N - M)$ other firms expect positive profits by entering the market. Thus, the arrangement of M firms is no longer stable. Consequently, the market is eventually settled down at the equilibrium of N firms as stated in Lemma 1. Put differently, the arrangement of M firms such that $M < N$ is not an equilibrium. Thus, Nash Equilibrium with N firms having positive market shares and positive prices is unique. Lemma 2 is proven.

Lemma 3 *If* (q_i^*, p_i^*) *is the optimal combination of quality and price selected by firm i (ranked i) in the Unitary Country, then, the optimal combination of quality and price selected by firm i in a similar country*

[5] Detailed analysis will be provided by the authors upon request.

with the market size Ψ *is* $(\sqrt[z-1]{\Psi} q_i^*, \sqrt[z-1]{\Psi} p_i^*)$. *The number of firms co-existing at equilibrium N is independent from the market size.*

Lemma 3 is proven in Appendix A1.

B. Autarkic and trading situations

The result that a finite number of firms co-exist and that this number is independent of the market size suggests that we can investigate the trade between countries as follows:

i) We consider a case in which Home and Foreign do not engage in trade (Autarky Case).
ii) We compare the outcomes of trading with the ones of the Autarky Case.

When the number of firms is large, the process of obtaining specific values for optimal prices and qualities using the profit maximizations in system (3) is complex. Fortunately, we can compare the optimal qualities and prices when a country maintains autarky or when countries trade with each other, with those of the Unitary Country without calculating explicit solutions. This fact explains why this paper includes the Unitary Country.

Now, imagine that we have applied the maximization of profit functions in system (3) and obtained the optimal qualities and prices of the N firms in the Unitary Country during the two stages. The optimal quality and optimal price determined by *firm i* is denoted as (q_i^*, p_i^*), where i = 1,2,...,N. We calculated the average cost of goods, total consumers' surplus, and total producers' surplus as presented in Table

TABLE 1
COSTS AND SURPLUSES IN THE UNITARY COUNTRY

Terms	Formula
Average cost (AC_i^u)	$AC_i^U = aq_i^* + \frac{(b-1)(q_i^*)^Z}{z(\theta_{i-1} - \theta_i)}$
Consumers' surplus (CS^U)	$CS^U = \frac{1}{b-1} \sum_{i=1}^{N} \{ \int_{\theta_i}^{\theta_{i-1}} (q_i^* \theta_j - p_i^*) d\theta_j \}$
Producers' surplus (PS^U)	$PS^U = \sum_{i=1}^{N} \{ \frac{[\theta_{i-1} - \theta_i]}{b-1} (p_i^* - aq_i^*) - \frac{1}{z} (q_i^*)^z \}$

Note: $\theta_0 = b, \theta_i = \frac{p_i^* - p_{i+1}^*}{q_i^* - q_{i+1}^*}, \theta_N = 1$

1 (See Appendix A2 for calculations).

Proposition 1 *Trade between Home and Foreign does not increase the variety of goods.*

Proof. Lemma 2 carries an implication that an increase in the market size does not change the number of firms at equilibrium. Without trade, Home's market size is S and Foreign's market size is T. The trade between Home and Foreign expands the market size to S + T, but the same number of firms N coexist at equilibrium with positive market shares. In addition, proving that the market share of *firm i* (the firm ranked *i* according to its product quality) is the same in Home, Foreign, and the region is easy.

Proposition 2 *When Home and Foreign are engaged in trade of vertically differentiated goods,*

i) a firm ranked i will produce its goods at a higher level of quality than those of the firm with the same rank in an autarkic country (Home or Foreign);

ii) goods become more costly to produce as a consequence of quality improvement;

iii) a win-win outcome generally results for both countries because the trade makes the region better off. However, the welfare of a trading country might be harmed when its firms lose from international competition and when the relative size of its trading partner is not sufficiently large.

Proof. Notably, $z \geq 2$. From Lemma 3, deriving the optimal quality of firm *i is* straightforward: $\sqrt[z-1]{S}q_i^*$ in Home, $\sqrt[z-1]{T}q_i^*$ in Foreign, and $\sqrt[z-1]{S+T}q_i^*$ in the region (Home and Foreign with trade). Thus, the quality increases when countries are engaged in trade. Referring to Table 2, showing that $\sqrt[z-1]{(S+T)^z}AC_i^U > \sqrt[z-1]{S^z}AC_i^U$ and $\sqrt[z-1]{(S+T)^z}AC_i^U > \sqrt[z-1]{S^z}AC_i^U$ or goods become more costly is easy.

From the data in Table 2, proving that trade enhances the welfare of the region as a whole is straightforward. The extent of the regional gains from trading is proportional to the sizes of the trading countries because $\sqrt[z-1]{(S+T)^z} - \sqrt[z-1]{S^z} - \sqrt[z-1]{T^z}$ increases when S or T increases. Moreover, for a given regional size (S+T), the gain from trade is highest when trading countries are similar in size (or S = T). This finding supports *Country Similarity Theory* proposed by Linder (1961).

We note that when Home and Foreign are engaged in trade, all firms from both countries will compete with each other. Consequently, only N firms exist and offer goods with qualities greater than zero. We cannot tell where these firms come from, that is, whether they are Home or Foreign. The competition for vertically differentiated goods is much stronger than that for horizontally differentiated ones (as analyzed by Krugman [1979]). Thus, although the market size is extended, the number of firms does not increase.

Now, we consider a case of Home as an example. Let $\omega \in [0,1]$ be the share of the regional total producers' surplus gained by firms located within Home. The welfare of Home with trade is

$$W^H = S \sqrt[z-1]{(S+T)} CS^U + \omega \sqrt[z-1]{(S+T)^z} PS^U \tag{5}$$

The welfare of Home without trade is $\sqrt[z-1]{S^z} CS^U + \sqrt[z-1]{S^z} PS^U$. Thus, the welfare of Home will be hurt by trade if

$$S \sqrt[z-1]{(S+T)} CS^U + \omega \sqrt[z-1]{(S+T)^z} PS^U < \sqrt[z-1]{S^z} CS^U + \sqrt[z-1]{S^z} PS^U$$

$$\Leftrightarrow S[\sqrt[z-1]{(S+T)} - \sqrt[z-1]{S}] CS^U + [\omega \sqrt[z-1]{(S+T)^z} - \sqrt[z-1]{S^z}] PS^U < 0. \tag{6}$$

We let $\frac{T}{S}$ be λ(the relative country size of Foreign). Dividing both sides of (6) by $S \sqrt[z-1]{S}$, we can rewrite it as follows:

$$[\sqrt[z-1]{(1+\lambda)} - 1] CS^U + [\omega \sqrt[z-1]{(1+\lambda)^z} - 1] PS^U < 0. \tag{7}$$

Showing that inequality (7) is more likely to be satisfied when λ and ω are small is easy.

Proposition 3 is proven.

Table 2 summarizes the impact of trade on quality, average cost, and consumers and producers' surpluses in Home, Foreign, and the region (See Appendix A3 for details).

TABLE 2

IMPACT OF VERTICAL INTRA-INDUSTRY TRADE

Impact	Autarky		Trading
	Home	Foreign	Region
Quality (q_i)	$\sqrt[z-1]{S}q_i^*$	$\sqrt[z-1]{T}q_i^*$	$\sqrt[z-1]{S+T}q_i^*$
Average cost (AC_i)	$\sqrt[z-1]{S^z}AC_i^U$	$\sqrt[z-1]{T^z}AC_i^U$	$\sqrt[z-1]{(S+T)^z}AC_i^U$
Consumers' Surplus (CS)	$\sqrt[z-1]{S^z}CS^U$	$\sqrt[z-1]{T^z}CS^U$	Home: $S\sqrt[z-1]{(S+T)}CS^U$ Foreign: $T\sqrt[z-1]{(S+T)}CS^U$
	Home and Foreign: $(\sqrt[z-1]{S^z}+\sqrt[z-1]{T^z})CS^U$		Region: $\sqrt[z-1]{(S+T)^z}CS^U$
Producers' Surplus (PS)	$\sqrt[z-1]{S^z}PS^U$	$\sqrt[z-1]{T^z}PS^U$	Firms' locations are not defined
	Home and Foreign: $(\sqrt[z-1]{S^z}+\sqrt[z-1]{T^z})PS^U$		Region: $\sqrt[z-1]{(S+T)^z}PS^U$
Regional Welfare	$(\sqrt[z-1]{S^z}+\sqrt[z-1]{T^z})(CS^U+PS^U)$		$\sqrt[z-1]{(S+T)^z}(CS^U+PS^U)$
Regional Gain	$(\sqrt[z-1]{(S+T)^z}-\sqrt[z-1]{S^z}-\sqrt[z-1]{T^z})(CS^U+PS^U)>0$		

C. Numerical analysis

For this numerical analysis, we consider a case with $S = 10$ (Home) and T = 2 (Foreign). In addition, we let $a = 0.1$, $b = 4$, and $z = 2$. Liao (2008) proves that the market is covered by only two firms because $2 < b < 4.7125$. The high and low quality firms are named *firms 1* and *2,* respectively. Using Maple Software (Version 17.0), we obtain values of optimal qualities, prices, average costs, welfares, and gains from trade in Home, Foreign, and the Region (Home trades with Foreign). In addition, when both firms locate in Foreign, we show that Home is worse off when it engages in trade. Table 3 presents the numerical analysis.

TABLE 3
NUMERICAL ANALYSIS OF THE IMPACT OF TRADE

Equilibrium values	Home ($S = 10$)		Foreign ($T = 10$)		Region ($S + T = 12$)	
	Firm 2	Firm 1	Firm 2	Firm 1	Firm 2	Firm 1
Optimal prices	2.430	22.260	0.486	4.452	2.917	26.711
Optimal qualities	2.430	12.637	0.486	2.527	2.917	15.164
Averaged cost	1.183	12.907	0.237	2.581	1.419	15.489
Consumers' surplus (generated by goods)	10.803	314.528	0.432	12.581	15.556	452.921
Producers' surplus	3.921	64.130	0.157	2.565	5.646	92.347
Welfares (autarky)	393.382		15.735		409.117	
Welfares (trade)	-		-		566.470	
Gains from trade	-		-		157.353	
Gains/losses from trade when both firms locate in Foreign	-2.984		160.337		157.353	

As shown in Table 3, when Home and Foreign trade with each other, quality levels as well as average costs of both goods increase. In addition, the regional gain from trade is positive. However, when both firms locate in Foreign, Home becomes worse off. Thus, the numeric analysis totally supports the findings derived from the theoretical part of this paper.

D. Discussion

We have considered a model of VIIT in which firms can conduct R&D to adjust the quality of their goods with zero entry costs. Implications are as follows:

First, we have shown that trade increases R&D investment (fixed quality improvement cost) for quality improvement. Thus, the increase in the quality of goods is the main mechanism that makes a region better off. Bhagwati and Davis (2012) and Davis (1995) claim that VIIT actually operates on the basis of comparative advantages and suggested that this kind of trade should be called *inter-industry trade* instead of VIIT. This finding supports the idea that the reason for VIIT derives from firms' level or from a vertical differentiation of products rather than countries' level. Thus, this kind of trade should be classified as *a kind of intra-industry trade* (VIIT).

R&D activities are duplicated (by two firms ranked i) across countries in the autarkic case, and trade can avoid this duplication to save unnecessary R&D cost. This scenario is another source for gains from trade. The intensive R&D investment when countries engaging in trade may provide an explanation for a movement of employment from production to R&D sectors as evidenced by Gruber *et al.* (1967); Gabrisch and Segnana (2003); Brito *et al.* (2012); and Bloom, Draca, and Reenen (2016).

Second, we found that trading vertically differentiated goods does not improve the variety of goods and that the possibility for the effect of internal increasing returns to scale is destroyed by the rise in quality. Owing to the strong competition for quality, companies are forced to invest in R&D to improve the quality of their products for their own survival. Thus, the average cost of their products increases. Consequently, the causes of international trade based on good quality differentiation (or VIIT) may be different from those of intra-industry trade of horizontally differentiated goods as concluded by Krugman (1979).

Third, similar countries exchange the same goods with each other because this trade generally benefits them both. However, a country may be worse off when it engages in trade. The welfare from trade added to a country is a consequence of two factors: the success of its firms in the international market and the scale of quality improvement possible (resulting from its trading partner size). Thus, a larger country often attracts trading partners more effectively because of the opportunity of quality improvement, whereas a country whose firms are already strong exerts a weaker (or even an opposite) effect.

V. Conclusion

By using a basic model, we have identified that similar countries engage in vertical intra-industry trade because it can make them better off. Specifically, the intensive investment in R&D for the quality improvement of goods as a result of trade is the mechanism for achieving gains from VIIT. In addition, we have shown that internal increasing returns to scale as well as varieties of goods, may not play a role in explaining VIIT when firms can vary the quality of their products. Thus, we concluded that the causes of VIIT may be quite different from those suggested by Krugman (1979).

We formulated a model that reached conclusions using assumptions that are commonly made concerning vertical product differentiation. However, these findings were derived from a purely theoretical model and thus require validation through empirical studies. In addition, this model employs an identical range of consumers' willingness to pay across countries. Future studies may relax this assumption to address a more realistic situation.

Appendix

A1. Proof of Lemma 3

We start proving Lemma 3 with an *ex-ante* assumption that N firms enter the market with market size of Ψ. Next, we prove that if N firms co-exist at equilibrium in the Unitary Country, the equilibrium number of firms is also N in any similar country.

In the second stage:

In a country with a market size of Ψ, firm i derives its optimal price from a system of linear equations similar to (4). This can be written by (A3.1):

$$\begin{cases} \partial \Pi_1 / \partial p_1 = b(q_1 - q_2) - 2p_1 + p_2 + aq_1 = 0 \\ \partial \Pi_i / \partial p_i = (q_i - q_{i+1})p_{i-1} - 2(q_{i-1} - q_{i+1})p_i \\ \qquad + (q_{i-1} - q_i)p_{1+1} + aq_i(q_{i-1} - q_{i+1}) = 0 \quad for \quad i \neq 1, N. \\ q_N - p_N = 0 \end{cases} \tag{A3.1}$$

Notably, the last equation in (A3.1), $q_N - p_N = 0$, ensures that the market is covered with a corner solution. Only one solution to the system (A3.1) is available. Now, let $p_i^* = g_i(q_1, q_2, \ldots, q_N)$ be the optimal price chosen by firm i, obtained by solving the system (A3.1). The proof that $g_i(.)$ is the first-degree homogeneous function is trivial because if (q_i^*, p_i^*) $i = 1, 2, \ldots, N$ satisfies (A3.1), then, $(\mathrm{k}q_i^*, \mathrm{k}p_i^*)$ $i = 1, 2, \ldots, N$ also satisfies (A3.1). Hence, $g_i(\mathrm{k}q_1, \mathrm{k}q_2, \ldots, \mathrm{k}q_N) = \mathrm{k}g_i(q_1, q_2, \ldots, q_N)$.

First stage:

The following system can be used to derive the optimal qualities, which are based on the maximization of profit function:

$$\begin{cases} firm\,1: \max_{q_1}\{\frac{\Psi}{b-1}\left[b-\frac{p_1^*-p_2^*}{q_1-q_2}\right]p_1^*-\frac{\Psi}{b-1}\left[b-\frac{p_1^*-p_2^*}{q_1-q_2}\right]aq_1-\frac{1}{z}q_1^z\} \\ firm\,i: \max_{q_i}\{\frac{\Psi}{b-1}\left[\frac{p_{i-1}^*-p_i^*}{q_{i-1}-q_i}-\frac{p_i^*-p_{i+1}^*}{q_i-q_{i+1}}\right]p_i^* \\ \qquad -\frac{\Psi}{b-1}\left[\frac{p_{i-1}^*-p_i^*}{q_{i-1}-q_i}-\frac{p_i^*-p_{i+1}^*}{q_i-q_{i+1}}\right]aq_i-\frac{1}{z}q_i^z\}\, for\;\; i\neq 1,N. \\ firm\,N: \max_{q_N}\{\frac{\Psi}{b-1}\left[\frac{p_{N-1}^*-p_N^*}{q_{N-1}-q_N}-1\right]p_N^*-\frac{\Psi}{b-1}\left[\frac{p_{N-1}^*-p_N^*}{q_{N-1}-q_N}-1\right]aq_N-\frac{1}{z}q_N^z\} \end{cases} \tag{A3.2}$$

We shorten this system by denoting $p_i^* = g_i(q_1,q_2,\ldots,q_N)$ as $g_i(q_j)$, where $j = 1,2,\ldots,N$.

Replacing $p_i^* = g_i(q_j)$ in (A3.2) results in (A3.3),

$$\begin{cases} \max_{q_1}\{\frac{\Psi}{b-1}\left[b-\frac{g_1(q_j)-g_2(q_j)}{q_1-q_2}\right]g_1(q_j)-\frac{\Psi}{b-1}\left[b-\frac{g_1(q_j)-g_2(q_j)}{q_1-q_2}\right]aq_1-\frac{1}{z}q_1^z\} \\ \max_{q_i}\{\frac{\Psi}{b-1}\left[\frac{g_{i-1}(q_j)-g_i(q_j)}{q_{i-1}-q_i}-\frac{g_i(q_j)-g_{i+1}(q_j)}{q_i-q_{i+1}}\right]g_i(q_j) \\ \qquad -\frac{\Psi}{b-1}\left[\frac{g_{i-1}(q_j)-g_i(q_j)}{q_{i-1}-q_i}-\frac{g_i(q_j)-g_{i+1}(q_j)}{q_i-q_{i+1}}\right]aq_i-\frac{1}{z}q_i^z\}\, for\;\; i\neq 1,N. \\ \max_{q_N}\{\frac{\Psi}{b-1}\left[\frac{g_{N-1}(q_j)-g_N(q_j)}{q_{N-1}-q_N}-1\right]g_N(q_j)-\frac{\Psi}{b-1}\left[\frac{g_{N-1}(q_j)-g_N(q_j)}{q_{N-1}-q_N}-1\right]aq_N-\frac{1}{z}q_N^z\} \end{cases} \tag{A3.3}$$

The Unitary Country: We set $\Psi = 1$ in (A3.3), to obtain the following:

$$\begin{cases} \max_{q_1}\{\frac{1}{b-1}\left[b-\frac{g_1(q_j)-g_2(q_j)}{q_1-q_2}\right]g_1(q_j)-\frac{1}{b-1}\left[b-\frac{g_1(q_j)-g_2(q_j)}{q_1-q_2}\right]aq_1-\frac{1}{z}q_1^z\} \\ \max_{q_i}\{\frac{1}{b-1}\left[\frac{g_{i-1}(q_j)-g_i(q_j)}{q_{i-1}-q_i}-\frac{g_i(q_j)-g_{i+1}(q_j)}{q_i-q_{i+1}}\right]g_i(q_j) \\ \qquad -\frac{1}{b-1}\left[\frac{g_{i-1}(q_j)-g_i(q_j)}{q_{i-1}-q_i}-\frac{g_i(q_j)-g_{i+1}(q_j)}{q_i-q_{i+1}}\right]aq_i-\frac{1}{z}q_i^z\}\, for\;\; i\neq 1,N \\ \max_{q_N}\{\frac{1}{b-1}\left[\frac{g_{N-1}(q_j)-g_N(q_j)}{q_{N-1}-q_N}-1\right]g_N(q_j)-\frac{1}{b-1}\left[\frac{g_{N-1}(q_j)-g_N(q_j)}{q_{N-1}-q_N}-1\right]aq_N-\frac{1}{z}q_N^z\} \end{cases} \tag{A3.4}$$

In a country with market size Ψ, we replace q_i with $\sqrt[z-1]{\Psi}u_i$ in (A3.3) to obtain (A3.5). Given that $\sqrt[z-1]{\Psi}$ is a constant, this replacement is equivalent to maximizing the profit functions in (A3.5) with respect to u_i rather than q_i.

$$\begin{cases} \max_{u_1}\{\frac{\Psi}{b-1}\left[b-\frac{g_1(\sqrt[z-1]{\Psi}u_j)-g_2(\sqrt[z-1]{\Psi}u_j)}{\sqrt[z-1]{\Psi}u_1-\sqrt[z-1]{\Psi}u_2}\right]g_1(\sqrt[z-1]{\Psi}u_j) \\ \qquad -\frac{\Psi}{b-1}\left[b-\frac{g_1(\sqrt[z-1]{\Psi}u_j)-g_2(\sqrt[z-1]{\Psi}u_j)}{\sqrt[z-1]{\Psi}u_1-\sqrt[z-1]{\Psi}u_2}\right]a\sqrt[z-1]{\Psi}u_1-\frac{\Psi\sqrt[z-1]{\Psi}}{z}u_1^z\} \\ \max_{u_i}\{\frac{\Psi}{b-1}\left[\frac{g_{i-1}(\sqrt[z-1]{\Psi}u_j)-g_i(\sqrt[z-1]{\Psi}u_j)}{\sqrt[z-1]{\Psi}u_{i-1}-\sqrt[z-1]{\Psi}u_i}-\frac{g_i(\sqrt[z-1]{\Psi}u_j)-g_{i+1}(\sqrt[z-1]{\Psi}u_j)}{\sqrt[z-1]{\Psi}u_i-\sqrt[z-1]{\Psi}u_{i+1}}\right]g_i(\sqrt[z-1]{\Psi}q_j) \\ \quad -\frac{\Psi}{b-1}\left[\frac{g_{i-1}(\sqrt[z-1]{\Psi}u_j)-g_i(\sqrt[z-1]{\Psi}u_j)}{\sqrt[z-1]{\Psi}u_{i-1}-\sqrt[z-1]{\Psi}u_i}-\frac{g_i(\sqrt[z-1]{\Psi}u_j)-g_{i+1}(\sqrt[z-1]{\Psi}u_j)}{\sqrt[z-1]{\Psi}u_i-\sqrt[z-1]{\Psi}u_{i+1}}\right]a\sqrt[z-1]{\Psi}u_i \\ \qquad -\frac{\Psi\sqrt[z-1]{\Psi}}{z}u_i^z\}\ for\ \ i\neq 1,N. \\ \max_{u_N}\{\frac{\Psi}{b-1}\left[\frac{g_{N-1}(\sqrt[z-1]{\Psi}u_j)-g_N(\sqrt[z-1]{\Psi}u_j)}{\sqrt[z-1]{\Psi}u_{N-1}-\sqrt[z-1]{\Psi}u_N}-1\right]g_N(\sqrt[z-1]{\Psi}u_j) \\ \qquad -\frac{\Psi}{b-1}\left[\frac{g_{N-1}(\sqrt[z-1]{\Psi}u_j)-g_N(\sqrt[z-1]{\Psi}u_j)}{\sqrt[z-1]{\Psi}u_{N-1}-\sqrt[z-1]{\Psi}u_N}-1\right]a\sqrt[z-1]{\Psi}u_N-\frac{\Psi\sqrt[z-1]{\Psi}}{z}u_N^z\} \end{cases} \tag{A3.5}$$

Recall that $g_i(.)$ is the first-degree homogenous function or equivalently that $g_i(\sqrt[z-1]{\Psi}u_j)=\sqrt[z-1]{\Psi}g_i(u_j)$. Given that it is constant, we can remove Ψ from the maximizing notation in each problem in (A3.5). Each firm must now solve

$$\begin{cases} \max_{u_1}\{\frac{1}{b-1}\left[b-\frac{g_1(u_j)-g_2(u_j)}{u_1-u_2}\right]g_1(u_j)-\frac{1}{b-1}\left[b-\frac{g_1(u_j)-g_2(u_j)}{u_1-u_2}\right]au_1-\frac{1}{z}u_1^z\} \\ \max_{u_i}\{\frac{1}{b-1}\left[\frac{g_{i-1}(u_j)-g_i(u_j)}{u_{i-1}-u_i}-\frac{g_i(u_j)-g_{i+1}(u_j)}{u_i-u_{i+1}}\right]g_i(u_j) \\ \qquad -\frac{1}{b-1}\left[\frac{g_{i-1}(u_j)-g_i(u_j)}{u_{i-1}-u_i}-\frac{g_i(u_j)-g_{i+1}(u_j)}{u_i-u_{i+1}}\right]au_i-\frac{1}{z}u_i^z\}\ for\ \ i\neq 1,N \\ \max_{u_N}\{\frac{1}{b-1}\left[\frac{g_{N-1}(u_j)-g_N(u_j)}{u_{N-1}-u_N}-1\right]g_N(u_j)-\frac{1}{b-1}\left[\frac{g_{N-1}(u_j)-g_N(u_j)}{u_{N-1}-u_N}-1\right]au_N-\frac{1}{z}u_N^z\} \end{cases} \tag{A3.6}$$

Each of the maximization functions in (A3.4) and (A3.6) is identical. Thus, the optimal value for u_i obtained by solving (A3.6) is equal to the optimal value q_i derived from (A3.4), so $u_i^* = q_i^*$. Thus, the optimal quality chosen by firm i in a country with market size Ψ is $\sqrt[z-1]{\Psi}q_i^*$, the optimal price for firm i is $\sqrt[z-1]{\Psi}p_i^*$.

From Lemma 2, N is unique in the Unitary Country. Now, we prove that N is also unique in other similar countries. If the market outcome in the Unitary Country is (N, q_i^*, p_i^*) (a triple of N firms, optimal quality and optimal price), it is easy to prove that $(N, \sqrt[z-1]{\Psi}q_i^*, \sqrt[z-1]{\Psi}p_i^*)$ is also the market outcome in a similar country with a market size of Ψ and *vice versa*. As N is unique in the Unitary Country, it is also unique in a similar country. This process completes the proof of Lemma 3.

A2. Derivation of Table 1

i) Average cost of firm i:

The average cost of a good can be obtained via dividing its total cost by its demand. The demand of good i is

$$\frac{1}{b-1}[\theta_{i-1} - \theta_i], \text{ where } \theta_0 = b, \ \theta_i = \frac{p_i^* - p_{i+1}^*}{q_i^* - q_{i+1}^*},$$

and $\theta_N = 1$. The total cost incurred by firm i is

$$TC_i = \frac{1}{b-1}[\theta_{i-1} - \theta_i](aq_i^*) + \frac{1}{z}(q_i^*)^Z. \text{ Thus, } AC_i^U = aq_i^* + \frac{(b-1)(q_i^*)^Z}{z(\theta_{i-1} - \theta_i)}.$$

ii) Total consumers' surplus

As firm i sells its goods to consumers from θ_i to θ_{i-1}, these consumers receive a surplus

$$CS_i^U = \frac{1}{b-1}\int_{\theta_i}^{\theta_{i-1}} (\theta_j q_i^* - p_i^*)d\theta_j.$$

Thus, the total consumers' surplus is

$$CS^U = \frac{1}{b-1}\sum_{i=1}^{N}\{\int_{\theta_i}^{\theta_{i-1}} (\theta_j q_i^* - p_i^*)d\theta_j\}.$$

iii) Total Producers' Surplus

The profit of firm i is $PS_i^U = \frac{[\theta_{i-1} - \theta_i]}{b-1}(p_i^* - aq_i^*) - \frac{1}{z}(q_i^*)^Z$.

Thus, the total producers' surplus is

$$PS^U = \sum_{i=1}^{N} \{\frac{[\theta_{i-1} - \theta_i]}{b-1}(p_i^* - aq_i^*) - \frac{1}{z}(q_i^*)^z\}.$$

A3. Derivation of Table 2

i) Average cost of firm i:

A consumer with $\theta_i(i \neq 1,N)$ is indifferent to whether goods come from firm i or from firm $i+1$. Additionally, showing that the set $\Omega = \{\theta_1,\ldots,\theta_N\}$ is the same in the Unitary Country, Home, or Foreign as well as in the region (Home and Foreign in trade) is easy.

In Home, the demand for goods i is

$$\frac{S}{b-1}[\theta_{i-1} - \theta_i],$$

where $\theta_0 = b$, $\theta_N = 1$, and

$$\theta_i = \frac{p_i^* - p_{i+1}^*}{q_i^* - q_{i+1}^*}.$$

The total cost incurred by firm i is

$$\frac{S}{b-1}[\theta_{i-1} - \theta_i]\, a(\sqrt[z-1]{S}q_i^*) + \frac{S^{z-1}\sqrt{S}}{z}(q_i^*)^z = \sqrt[z-1]{S^z}\{\frac{1}{b-1}[\theta_{i-1} - \theta_i]\, aq_i^* + \frac{1}{z}(q_i^*)^z\}.$$

Thus,

$$AC_i^H = (\sqrt[z-1]{S^z})[aq_i^* + \frac{(b-1)(q_i^*)^Z}{z(\theta_{i-1} - \theta_i)}] = \sqrt[z-1]{S^z}AC_i^U.$$

Similarly, the average cost in Foreign is $AC_i^F = \sqrt[z-1]{T^z}AC_i^U$, and the average cost in the region is $AC_i^R = \sqrt[z-1]{(S+T)^z}AC_i^U$.

ii) Total consumer surplus

In Home, in the case where no international trade exists, firm i will sell its good to consumers from θ_i to θ_{i-1}. These consumers will obtain a surplus of

$$\frac{S}{b-1}\int_{\theta_i}^{\theta_{i-1}} (\theta_j \sqrt[z-1]{S}q_i^* - \sqrt[z-1]{S}p_i^*)d\theta_j.$$

Thus, the total surplus is

$$\frac{S}{b-1}\sum_{i=1}^{N}\{\int_{\theta_i}^{\theta_{i-1}}(\theta_j\sqrt[z-1]{S}q_i^* - \sqrt[z-1]{S}p_i^*)d\theta_j\},\text{ or } \sqrt[z-1]{S^z}CS^U$$

is the total consumers' surplus in Home.

With similar calculations, we can derive total consumers' surpluses in Foreign as well as in Home and Foreign when they are engaged in trade.

iii) Total producer surplus

In Home, profit of *firm i*:

$$\frac{S[\theta_{i-1}-\theta_i]}{b-1}(\sqrt[z-1]{S}p_i^* - a\sqrt[z-1]{S}q_i^*) - \frac{1}{z}(\sqrt[z-1]{S}q_i^*)^z = \sqrt[z-1]{S^z}PS_i^U.$$

Thus, the total producers' surplus is

$$\sum_{i=1}^{N}\{\sqrt[z-1]{S^z}PS_i^U\} = \sqrt[z-1]{S^z}PS^U.$$

With similar calculations, we can derive total producers' surpluses in Foreign ($\sqrt[z-1]{T^z}PS^U$) as well as in the region ($\sqrt[z-1]{(S+T)^z}PS^U$) when Home and Foreign trade with each other.

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