Heterogeneous Entrepreneurs and Managerial Delegation in an Open Economy

Yang Seung Lee

This study examines the impact of firm-level managerial delegation on aggregate economy. First, managerial delegation demands skilled labor and hence affects the labor market. Second, managerial delegation also affects the product market by improving production efficiency. Managerial delegation incurs costs but does not equally benefit firms. Higher entrepreneurial-skill entrepreneurs are selected, which dramatizes the firm size distribution. These entrepreneurs run larger and more productive firms, and contribute to the aggregate economy by creating higher-paying jobs. This paper provides new implication of trade gain, which is larger for the country where managerial delegation is easier.

Keywords: Entrepreneurial Skill, Decentralization, Firm Heterogeneity, Management, Monopolistic Competition

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I. Introduction

The insight of selection has highly contributed to various research areas, especially to international trade. Melitz (2003) showed that the external shock of trade openness causes firms' selection within an industry. That is, not all firms can export; only a few can be selected for exporting and allocated with resources. In this paper, I highlight that firm selection is highlighted to occur without the external shock. In Melitz (2003), firm productivity is a lottery while in the present paper, entrepreneurial ability is a lottery. Using Schoar's characterization[1], this study finds two types of enterprises, subsistence and transformational. Subsistence enterprises are smaller and run mostly by family members while transformational enterprises are larger and run by professional managers. The emphasis is that, using managerial delegation, small firms can transition to large firms. That is, a transitional dynamism occurs for entrepreneurs. Within the present framework, all firms use the same production technology, but their output differs with entrepreneurial skill as in Lucas (1978). Then, the productivity of a firm depends mainly on entrepreneurial skill, where productivity is defined as the ratio of output and input. That is, entrepreneurial skill represents the innate ability of firms.

In this paper, heterogeneous entrepreneurs run firms and can delegate their authority to recruited managers. Managerial delegation can improve the production efficiency, because professional managers can help the firm to operate more efficiently.[2] The better the business

[1] Schoar (2010) argued that two types of entrepreneurship exist, namely, subsistence and transformational entrepreneurship. According to her study, transformational entrepreneurship intends to grow the firm while subsistence entrepreneurship does not.

[2] According to the survey of Syverson (2011), the determinants of firm productivity are multifold. To understand the magnitude of the relationship between mangement practices and productivity, Bloom, Brynjolfsson, Foster, Jarmin, Patnaik, Saporta-Esksten, and Van Reenen (2019) compared management practices to R&D, ICT (information and communication technology), and human capital. According to their analysis, the role of management practices account for 44.1 percent of the 90–10 productivity spread. Bloom, Eifert, Mahajan, McKenzie, and Roberts (2013) randomly selected a sample of Indian textile firms for the treatment of consulting service and found that management practices are positively correlated with productivity, profitability, and firm employment. In this paper, firm productivity represents

environment, the greater the effect of managerial delegation. However, managerial delegation incurs costs[3] and causes unequal benefits to firms. With the ability to draw a better synergy from managers, higher-skill entrepreneurs benefit more from managerial delegation than lower-skill ones. Thus, entrepreneurs self-select for delegation only if its return is expected to outweigh its cost.[4] With delegation, the firm's productivity becomes dependent not only on the entrepreneurial skill but also on the average quality of managers. Firm productivity is linear in entrepreneurial skill for centralized firms, but nonlinear in entrepreneurial skill for decentralized firms. The distribution of firm productivity is discontinuous and shifts upward from a threshold level of entrepreneurial skill. This paper highlights that discontinuity dramatizes the Melitz-selection effect, where firms endogenize their own productivity through decentralization. Poschke (2018) showed that technologic change benefits higher-skill entrepreneurs under a perfect competition. In his paper, the occupational choice of either entrepreneur or worker determines masses of firms and jobs. In the present paper, decentralization benefits higher-skill entrepreneurs without any technologic change, where agents choose an occupation of either manager or worker. Within this framework, all firms compete monopolistically and the highest-skill entrepreneurs can be selected for managerial delegation. Eventually, firm productivity is endogenously determined with a combination of entrepreneurial skill and average quality of managers. Melitz (2003) also underlies three main assumptions: 1) firm productivity is a random draw; 2) the economy is

total factor productivity (TFP).

[3] Brynjolfsson and Milgrom (2013) insinuated that the adoption of new management skills incurs a fixed cost. This adoption requires coordination among all the interests within the firm and also consumes time. Examining Indian textile firms, Bloom, Sadun, and Van Reenen (2012) found that the fixed cost arises from management consulting. Approximately, the cost was measured as $250,000.

[4] This statement has empirical evidence. Bloom, Sadun, and Van Reenen (2012) found that many Indian textile firms are self-selecting for poor management of family-oriented centralization. The study emphasized the main reasons as information barrier and overhead cost. That is, several firms do not recognize that managerial delegation can improve firm productivity. Moreover, despite such recognition, certain firms do not adopt managerial innovation because of its high costs.

under full employment; and 3) all jobs are identical. Thus, aggregate income is constant and is determined merely with population size. Relaxing the assumptions shows that aggregate income is endogenous. The main reason is that decentralization of firms generates higher-paying jobs. In this paper, a firm consists of an entrepreneur and workers. Through decentralization, the firm becomes constituted of an entrepreneur, managers, and workers. Decentralization endogenizes firm productivity at managerial inputs. In this paper, the entrepreneur plays the role of drawing synergy from recruited managers. The novelty of this paper shows that decentralization affects not only firm productivity but also the pattern of labor demand. This paper links to a growing literature, which highlights the roles of management practices in productivity differences (see Bloom and Van Reenen 2010, 2011; Bloom, Sadun, and Van Reenen 2017; among others). In these studies, well-managed firms are larger, more productive, and have higher survival rates. In the present paper, one objective is to examine the effect of management practices on intra-industry trade. Recently, Bloom, Manova, Van Reneen, Sun, and Yu (2020) established a model that management competence improves firm performance by dually enhancing production efficiency and quality capacity. Empirical evidence shows that well-managed firms export more and earn larger profits. The study analyzed plant-level data on production and management practices, and transaction-level trade activity for 485 Chinese firms (in 1999–2008) and over 10,000 American firms (in 2010). Management status and exporting performance are found to be correlated. The empirical findings can be summarized as follows.

Dependent Variable	United States		China	
	Exporter Dummy	Export Revenue	Exporter Dummy	Export Revenue
Management	0.042 (13.92)	0.488 (21.72)	0.040 (2.30)	0.260 (2.14)

The table examines the relationship between management status, probability of exporting, and export revenues. For each country, the column of 'Exporter Dummy' shows the estimates when the dependent variable is a binary indicator that is equal to 1 for exporters, and 0 otherwise. The column of 'Export Revenue' shows the estimates when the dependent variable is the log total exports. T-statistics are in parentheses. Apparently, management status significantly affects the exporting performance for both countries. The statistics motivated this study for the relationship between decentralization and exporting performance. Decentralized firms tend to be managed well and to have higher qualities of management. Well-managed firms have advantages not only on exporting performance but also technology upgrading. That is, well-managed firms can upgrade their production technology through R&D investment and produce goods of higher quality. Recently, Niem and Kim (2020) constructed a theoretical model of vertical intra-industry trade, and explained how trade between similar countries can induce technology upgrading within and between firms. In their model, market expansion triggers the R&D investments of firms. In the present paper, technology upgrading or R&D investment is beyond the scope, and rather focuses on the possibility that well-managed firms have competitive advantage over poorly managed firms for exporting varieties when trade is liberated. That is, well-managed firms are already selected for international competence and are prepared to take advantage of trade openness. Thus, firms benefit more from trade. Within this framework, firms have not only large production scales but also different organizational structures. Without decentralization, firms are run traditionally by family members. Mostly, such firms tend to be smaller and to have lower productivity. Within this framework, higher-skill entrepreneurs are self-selecting for decentralization[5] while lower-skill entrepreneurs are for centralization. As a result, decentralized firms have much larger differences of firm profits. Managerial delegation is at the risk of misbehavior; managers might act in their interest (*i.e.,* bribing). When the level of social trust is higher, the risk is lower.

Then, the next question arises. Who would become entrepreneurs? Several studies provide different accounts. Lucas (1978) explained that higher-skill individuals choose to become entrepreneurs, whereas Kihlstrom and Laffont (1979) explained that less risk-averse individuals choose to become entrepreneurs. Meanwhile, Poschke (2013, 2018) found that the relationship between entrepreneurship and ability is U-shaped. That is, either higher-skill or lower-skill individuals choose

[5] Rajan and Wulf (2006) and Guadalupe and Wulf (2010) found that market competition is associated with decentralization. Bloom, Sadun, and Van Reenen (2012) addressed that decentralization might be risky when the level of social trust is low. The managers are more likely to show adverse behave for their own interest when given the authority of the entrepreneur.

to become entrepreneurs, whereas moderate-skill individuals choose to become employees. Finally, this study focuses on inheritance. That is, several individuals never work for others, for the reason of inheritance of either wealth or personal trait. Thus, the assumption is that the set of entrepreneurs is given as exogenous.[6] Each entrepreneur is endowed with a level of skill that is randomly distributed. Entrepreneurs never enter the labor market. Meanwhile, agents who are willing to work for others enter the labor market. In assumption, the total population of agents and their average quality are publicly known. Within this framework, a manager affects firm productivity while a worker does not. Regardless of how many managers are employed, the average manager quality remains identical for firms. The status of social trust and the average quality of agents represent business environment of the economy. Firms differ only in the level of entrepreneurial skill.

Two sectors are considered. One sector (Sector 1) produces homogeneous goods under a perfect competition while another (Sector 2) produces a continuum of varieties under monopolistic competition. Decentralization occurs only within Sector 2. In this paper, firm selection occurs in two ways, by production and decentralization. Thus, the distribution of entrepreneurial skill has two cut-offs, namely, production cut-off and decentralization cut-off. For the production cut-off, entrepreneurs can run firms to produce varieties. In such cases, production scale is determined by entrepreneurial skill. For the decentralization cut-off, entrepreneurs can decentralize their firm. In such cases, production scale is determined endogenously by entrepreneurial skill, average manager quality, and status of social trust. In this regard, two studies can be reviewed. Monte (2011) developed a model in which firms and managers are both heterogeneous but the workers are homogeneous. Wage inequality occurs only for managers. Thus, technical change is skill-biased. Next, Caroli and Van Reenen (2001) addressed skill-biased organizational change, which includes decentralization and delayering. With the data of British and French firms, the authors found empirical evidence that organizational change leads to higher firm productivity if firms are endowed with

[6] Crawford, Aguinis, Lichtenstein, Davisson, and McKelvey (2015) highlighted power law distribution for enterprises. Examining over 12,000 young and hyper-growth firms, they found that many entrepreneurship-related variables exhibit highly skewed power law distributions.

larger initial skills. That is, skill supply is important. In deciding organizational change, this paper emphasizes on skill distribution more than skill supply. As mentioned, skill distribution is a determinant of business environment.

This paper presents a tractable model for North–North or North–South trade. North–North trade can be characterized by decentralization that is available in both countries. By contrast, North–South trade only has decentralization available in the North. In this paper, all countries gain from trade with the preference of 'love for variety'. That is, trade benefits all consumers of all countries because it increases the number of varieties. With trade, price level becomes the same between two countries, although the aggregate income still differs. The difference stems from that of skill distribution. Suppose that high skills are more abundant in Country A than Country B. Then, the prediction is that trade gain is larger for Country A, for the reason that the masses of both their decentralized and exporting firms are larger than in Country B. As an implication, more well-paying jobs are created for Country A. For both countries, price level falls because products are supplied at lower prices, domestically and internationally. Thus, the level of welfare improves in all countries. This paper provides new implications of trade gain, that is, as determined largely by skill distribution and business environment. The remainder of this paper is structured as follows. Section II outlines the model. Section III analyzes the autarky equilibrium. Section IV analyzes the trade equilibrium for symmetric countries while Section V discusses asymmetric countries, which differ only in skill distribution. Section VI provides concluding remarks.

II. Model

This section introduces the model with two sectors, a homogeneous sector (Sector 1) and a differentiated one (Sector 2). A representative consumer is assumed to have a Cobb–Douglas utility function for the homogenous and differentiated goods. A unit of labor is supplied with a level of quality. Average labor quality is given and publicly known. The differentiated good constitutes a continuum of varieties and the consumer has the preference of constant elasticity substitution (CES). Within Sector 2, an entrepreneur runs a firm to produce a variety of products for a given demand. Production technology is given identically for all firms within Sector 2; the technology is linear in worker inputs. In this model, firm heterogeneity arises from heterogeneous entrepreneurs. Firms can be decentralized for managerial efficiency. However, decentralization incurs costs because managers are recruited for delegation. With decentralization, firm productivity is determined by the combination of entrepreneurial skill and average manager quality.

A. Consumption

A representative consumer is endowed with a level of labor quality. He/she can work as either a manager or a worker and can earn wages according to his occupation. His/her preference relation can be represented by the following Cobb–Douglas function.

$$U = X_1^{\beta_1} X_2^{\beta_2}\text{, where } \beta_1 + \beta_2 = 1. \quad (1)$$

The preference is defined over the consumption of the homogenous good (X_1) and differentiated good (X_2). The price of the homogeneous good is taken as numeraire; $P_1 = 1$. A continuum of varieties constitutes the differentiated good. The consumer has a CES preference as follows.

$$X_2 = \left[\int_{z \in Z} q(z)^{\rho} dz \right]^{\frac{1}{\rho}}, \ \rho > 1. \quad (2)$$

The measure of the set Z represents a mass of available products, which are substitutes where $0 < \rho < 1$ is a given parameter. Elasticity of substitution between any two varieties is $1/(1-\rho) = \sigma > 1$. The Cobb–Douglas utility function implies that a share of the income is constantly spent on each good. Let I denote the income. Then, the total expenditure on good 2 is $X_2 = \beta_2 I$. P_2 denotes the aggregate price of Sector 2. Then, the demand for a variety z is obtained as

$$q(z) = (\frac{p(z)}{p_2})^{-\sigma} X_2, \quad (3)$$

where $P_2 = \left[\int_{z \in Z} p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}$. For the economy, price level can be defined as a geometric mean between P_1 and P_2.

B. Production Technology

As mentioned above, the production technology is linear in worker input. Competition mode differs between the sectors. Sector 1 is under perfect competition while Sector 2 is under monopolistic competition. Within Sector 2, firms are run by heterogeneous entrepreneurs. Similar to Melitz (2003), production technology can be represented by a cost function, which comprises a marginal cost and a fixed overhead cost f. Entrepreneurs can observe their own skill and decide whether to enter the differentiated sector or not. To enter, entrepreneur must make an initial sunk cost f_e. The novelty of this study is that, upon entry, an entrepreneur decides whether to decentralize his firm or not. Decentralization improves firm productivity at costs. Without decentralization, firm productivity reflects mainly entrepreneurial skill ψ as in Lucas (1978). Given the linearity of the production technology, the output implies the worker demand; $l(\psi) = f + \frac{q(\psi)}{\psi}$.

i) Centralization

Centralized firms do not need to recruit managers. Thus, the productivity of a centralized firm relies on the entrepreneurial skill. Thus, the price of a product is $p(\psi) = \frac{w_L}{\rho\psi}$, where w_L is the worker wage within Sector 2, and ψ is the entrepreneurial skill. Then, the firm profit is

$$\pi(\psi) = \frac{r(\psi)}{\sigma} - f \tag{4}$$

From Eq. (3), firm revenue $r(\psi)$ is obtained as $r(\psi) = (\rho\psi)^{\sigma-1}P^{\sigma}Y$. Between centralized firms, the output and revenue ratios depend only on the ratio of the entrepreneurial skills.

$$\frac{q_h(\psi_h)}{q_k(\psi_k)} = \left(\frac{\psi_h}{\psi_k}\right)^{\sigma}, \text{ and } \frac{r_h(\psi_h)}{r_k(\psi_k)} = \left(\frac{\psi_h}{\psi_k}\right)^{\sigma-1}. \tag{5}$$

Higher-skill entrepreneurs run larger firms and earn higher profits than lower-skill entrepreneurs. A centralized firm has the optimal price; $P = \frac{w_L}{\rho\psi}$. As explained, an entrepreneur can decide to decentralize the firm upon entry if its return is expected to be larger than its cost.

ii) Decentralization

For simplicity, decentralization is assumed to lead to the same organization and the same control span for firms. At a decentralized firm, the entrepreneur and managers determine the firm productivity together through a synergy. That is, firm productivity becomes endogenous. Notably, decentralization affects the pattern of labor demand by creating higher-paying jobs. In Lucas (1978), the production function was set as $q = \psi f(l)$, where ψ and l denote entrepreneurial skill and the number of worker inputs, respectively. That is, entrepreneurial skill is multiplied to the production of workers. In his model, individuals self-select for entrepreneurs or workers. In the present model, individuals comprise entrepreneurs and agents. Agents are assigned to managers or workers. At a decentralized firm, each manager is assumed to supervise a fixed number α of workers, which represents control span. Entrepreneurs do not enter the labor market. Instead, their firms are selected for successful business. The assumption is that labor qualities follow a random distribution with mean $\bar{w}$. Then, at each decentralized firm, the production function becomes

$$q = \phi(\psi, \bar{w}) f(l, m) = \psi g(\bar{w}) \min\left\{l, \frac{m}{\alpha}\right\}, \tag{6}$$

where I and m denote the number of worker inputs and of manager inputs, respectively. Given the business environment, the output varies according to entrepreneurial skill. Bloom, Sadun, and Van Reenen (2012) emphasized the role of social trust in decentralization. In their model, managers can misbehave with a probability λ. Then, the entrepreneur can obtain the share $(1 - \lambda)$ of the output. In this model, the status of social trust plays a role of translating the manager qualities into firm productivity. Thus, when the level of social trust is higher, firm productivity is higher. Thus, the model is $g(\bar{w}) = \bar{w}^{\gamma}$, where $0 < \gamma < 1$ indicates a level of social trust. Unlike in Lucas (1978), in this paper, the productivity term, $\phi(\psi, \bar{w}) = \psi \bar{w}^{\gamma}$, contains synergy effect.[7] The implication is that decentralization improves firm productivity at additional costs, namely, the fixed cost f_m and wages for managers. Entrepreneurs are heterogeneous in skill, which is indexed by $\psi > 0$. Higher index indicates higher skill. Regardless of productivity, each firm faces a residual demand curve with constant elasticity σ, and the price markup equals $\left(\frac{\sigma}{\sigma - 1}\right) = \frac{1}{\rho}$. Each decentralized firm faces the marginal

[7] The synergy creates more than the simple sum of parts. Implicatively, the productivity is not reducible to the 'atoms' of managers in the firm.

cost $MC_d = w_L + \left(\frac{w_m}{\alpha}\right)$. Then, the optimal price is

$$p = \frac{\left(w_L + \frac{w_m}{\alpha}\right)}{\rho\phi(\psi)} = \frac{MC_d}{\rho\phi(\psi)}, \tag{7}$$

where w_m denotes the manager wage and α is the control span. Then, the firm profit is

$$\pi(\phi(\psi)) = \frac{r(\phi(\psi))}{\sigma} - (f + f_m) . \tag{8}$$

III. Market Equilibrium

This section derives a general equilibrium in which both goods and labor market clear. Within Sector 2, centralized and decentralized firms coexist. Between two centralized firms, h and k, the ratio of the revenues is represented by the ratio of entrepreneurial skills. That is, $r_h(\phi_h) / r_k(\phi_k) = (\psi_{Eh} / \psi_{Ek})^{(\sigma-1)}$. Between a centralized firm and a decentralized firm, the ratio of revenues additionally reflects average manager skills and status of social trust.

Remark 1: Between a centralized firm h and a decentralized firm k, ratio of the revenues is

$$\frac{r_h(\phi_h)}{r_k(\phi_k)} = \left(\frac{\frac{w_L}{\rho\psi_h}}{\frac{MC_d}{\rho\phi(\psi_k)}}\right)^{1-\sigma} . \tag{9}$$

As mentioned, a mass of entrepreneurs is given as E.[8] The assumption is that entrepreneurial skill ψ follows a Pareto distribution, $G(\psi) = 1 - \psi^{-k}$ for $k > 1$. Let M denote the mass of producing firms, $M \subset E$. Then, two cut-offs exist in equilibrium.

[8] Monte (2011) and Samson (2014) analyzed the assortment of managers or workers in firms. In Monte (2011), managers are assigned to firms being different in 'idea' while workers are homogeneous. In Samson (2014), workers are assigned to firms of different technology. In this paper, managers are assigned to entrepreneurs of different skills.

A. Production Cut-off

Without decentralization, firm productivity can be determined mainly by the entrepreneur's skill. Due to conditions of zero profit and of free entry, a cut-off ψ' should exist. Let $r(\psi')$ and $\pi(\psi')$ denote firm revenue and profit at the cut-off ψ', respectively. Then, the firm profit should be zero. That is,

$$\pi(\psi') = \frac{r(\psi')}{\sigma} - f = 0\,. \tag{10}$$

Eq. (10) implies that $r(\psi') = \sigma$f.

Between any two centralized firms, the ratio of revenues is $r_h(\psi_h)$ / $r_k(\psi_k) = (\psi_h / \psi_k)^{(\sigma-1)}$. Let $r(\overline{\psi})$ denote the average firm revenue, where $\overline{\psi}$ is average entrepreneurial within Sector 2. Using the ratio, $r(\overline{\psi})$ can be found as

$$\frac{r(\overline{\psi})}{r(\psi')} = \left(\frac{\overline{\psi}}{\psi'}\right)^{(\sigma-1)} \quad \text{or} \quad r(\overline{\psi}) = \left(\frac{\overline{\psi}}{\psi'}\right)^{(\sigma-1)} r(\psi'), \tag{11}$$

where $r(\psi') = \sigma f$. Then, average firm profit is obtained as

$$\pi(\overline{\psi}) = \frac{r(\overline{\psi})}{\sigma} - f\,. \tag{12}$$

Plugging Eq. (11) into Eq. (12) yields

$$\pi(\overline{\psi}) = \left[\left(\frac{\overline{\psi}}{\psi'}\right)^{(\sigma-1)} - 1\right] f. \tag{13}$$

For each period, the mass of producing firms is determined as M. An exogenous shock hits the economy with probability δ. As a result, a fraction of producing firms should exit and is substituted with new entrants. In assumption, no time is discounted between entry and exit. Thus, the value function of a producing firm can be calculated as

$$\upsilon(\psi) = \sum_{t=0}^{\infty} (1-\delta)^t \pi(\psi) = \frac{\pi(\psi)}{\delta}, \tag{14}$$

where superscript t denotes time period. Given the production cut-off, ex-ante probability of successful entry is $p_s = p(\psi > \psi') = (\psi')^{-k}$. Let $\bar{\upsilon}$ denote the present value of average firm profit flows. Then, $\bar{\upsilon} = \sum_{t=0}^{\infty} (1-\delta)^t \pi(\bar{\psi}) = \pi(\bar{\psi}) / \delta$. When an entrepreneur attempts to enter the differentiated sector, the net value of his entry (υ_e) can be calculated as

$$\upsilon_e = p_s \bar{\upsilon} - f = \frac{\left[(\psi')^{-k} \right] \pi(\bar{\psi})}{\delta} - f_e, \tag{15}$$

where f_e is a sunk fixed cost for entry. In equilibrium, the net value of entry should be zero under the condition of free entry. Eq. (15) implies that

$$\pi(\bar{\psi}) = \frac{\delta f_e}{(\psi')^{-k}}. \tag{16}$$

Eqs. (13) and (16) imply the existence of a stationary equilibrium ($\pi(\bar{\psi})$, ψ'), where $\bar{\psi}$ is defined as

$$\bar{\psi}(\psi') = \left[\int_{\psi'}^{\infty} \psi^{\sigma-1} \mu(\psi) d\psi \right]^{\frac{1}{\sigma-1}}, \tag{17}$$

and $\mu(\psi) = \frac{g(\psi)}{[1 - G(\psi')]} = \frac{k}{\psi} \left(\frac{\psi'}{\psi} \right)^t$ for $\psi \geq \psi'$. The integral Eq. (17) can be calculated as

$$\bar{\psi}(\psi') = \left[\frac{k}{k - (\sigma - 1)} \right]^{\frac{1}{\sigma-1}} \psi'. \tag{18}$$

Plugging Eq. (18) into Eq. (13) yields

$$\pi(\bar{\psi}) = \left[\frac{\sigma - 1}{k - (\sigma - 1)} \right] f. \tag{19}$$

Both Eqs. (19) and (16) yield that

$$\psi' = \left[\frac{\delta f_e (k - \sigma + 1)}{f(\sigma - 1)} \right]^{\frac{-1}{k}}. \tag{20}$$

The production cut-off reflects a distributional property of entrepreneurial skill and substitution elasticity. As a matter of fact, the production cut-off is similar as that in Melitz (2003). Firm productivity is switched with entrepreneurial ability. For the sake of simplicity, ψ' can be normalized as 1. The implication is that all entrepreneurs can enter the sector for the level of subsistence, and a few of them can be selected for decentralization.

B. Decentralization Cut-off

The decentralization cut-off ψ^* can be derived. At ψ^*, firm productivity is endogenously determined as $\phi^* = \psi^* \bar{\omega}^{\gamma}$. The firm profit from decentralization is at least as great as that from centralization. Thus, the firm profit at the cut-off ψ^* is

$$\pi(\phi(\psi^*)) = \frac{r(\phi(\psi^*))}{\sigma} - (f + f_m) = \frac{r(\psi^*)}{\sigma} - f = \pi(\psi^*). \tag{21}$$

From Eq. (5), the revenue ratio can be found as

$$\frac{r(\phi(\psi^*))}{r(\psi^*)} = \left(\frac{\dfrac{MC_d}{\rho\psi^*(\bar{\omega})^{\gamma}}}{\dfrac{w_L}{\rho\psi^*}} \right)^{1-\sigma} = \left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}} \right)^{1-\sigma}.$$

Then, with decentralization, the firm revenue at ψ^* can be found as

$$r(\psi^*) = \left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}} \right)^{\sigma-1} r\left(\phi(\psi^*)\right). \tag{22}$$

Plugging Eq. (22) into Eq. (21) yields

$$r\left(\phi(\psi^*)\right) = \frac{\sigma f_m}{\left[1 - \left(\dfrac{MC_d}{w_L(\bar{\omega})^{\gamma}} \right)^{\sigma-1} \right]}. \tag{23}$$

At the production cut-off ψ', $r(\psi') = \sigma f$.

Thus,

$$\frac{r\left(\phi(\psi^*)\right)}{r(\psi')} = \left(\frac{\dfrac{MC_d}{\rho\psi^*(\varpi)^\gamma}}{\dfrac{w_L}{\rho\psi'}}\right)^{1-\sigma} = \frac{\dfrac{\sigma f_m}{\left[1-\left(\dfrac{MC_d}{w_L(\varpi)^\gamma}\right)^{\sigma-1}\right]}}{f}. \tag{24}$$

From Eq. (24), the cut-off ψ^* can be obtained as

$$\psi^* = \frac{\rho(MC_d)}{w_L(\varpi)^\gamma}\left[\frac{f}{\sigma f_m}\left[1-\left(\frac{MC_d}{w_L(\varpi)^\gamma}\right)^{\sigma-1}\right]\right]^{\frac{1}{1-\sigma}}\psi'. \tag{25}$$

Normalizing ψ' as 1, ψ^* is obtained as

$$\psi^* = \frac{\rho(MC_d)}{w_L(\varpi)^\gamma}\left[\frac{f}{\sigma f_m}\left[1-\left(\frac{MC_d}{w_L(\varpi)^\gamma}\right)^{\sigma-1}\right]\right]^{\frac{1}{1-\sigma}}. \tag{26}$$

Firm profit can be defined as $\pi_c(\psi) = (r(\psi)/\sigma) - f$ without decentralization, and as $\pi_d\left(\phi(\psi)\right) = (r\left(\phi(\psi)\right)/\sigma) - (f + f_m)$ with decentralization.

Remark 2: When $\psi \geq \psi^*$, $\dfrac{d\pi_d\left(\phi(\psi)\right)}{d\psi} > \dfrac{d\pi_c(\psi)}{d\psi}$. (27)

The result is similar as that in Bustos (2011). In her paper, technological upgrading can sharpen the slope of the profit function although the payment of a higher fixed cost is required. In the present paper, managerial delegation can sharpen the slope of the profit function although the payments of higher fixed cost and managerial inputs are required. Thus, marginal cost increases from $MC_c = w_L$ to $MC_d = w_L + (w_m / \alpha)$, where α is the control span.

C. Aggregate Outcome

Given the higher efficiency of production, decentralized firms supply products at lower prices, and therefore contribute to lowering the aggregate price while centralized firms do not.

For Sector 2, the aggregate price is $P_2 = P_C + P_d$, where $P_c = \left[\int_{\psi'}^{\psi^*} P_c(\psi)^{1-\sigma} M_c \mu_c(\psi) d\psi\right]^{\frac{1}{1-\sigma}}$ and $P_d = \left[\int_{\psi^*}^{\infty} P_d(\psi)^{1-\sigma} M_d \mu_d(\psi) d\psi\right]^{\frac{1}{1-\sigma}}$.

P_c and P_d can be calculated as follows.

$$P_c = \left[\int_{\psi'}^{\psi^*} P_c(\psi)^{1-\sigma} M_c \mu_c(\psi) d\psi\right]^{\frac{1}{1-\sigma}} = \left(\frac{w_L}{\rho}\right)^{1-\sigma} \frac{\left[1-\left(\psi^*\right)^{\sigma-k-1}\right]\left[1-\left(\psi^*\right)^{-k}\right]}{k+1-\sigma} E, \quad (28)$$

$$P_d = \left[\int_{\psi^*}^{\infty} P_d(\psi)^{1-\sigma} M_d \mu_d(\psi) d\psi\right]^{\frac{1}{1-\sigma}} = \left(\frac{MC_d}{\rho(\bar{\omega})^{\gamma}}\right)^{1-\sigma} \frac{\left[\left(\psi^*\right)^{\sigma-2k-1}\right]}{k+1-\sigma} E, \quad (29)$$

where $\psi^* = \frac{\rho(MC_d)}{w_L(\bar{\omega})^{\gamma}}\left[\frac{f}{\sigma f_m}\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]\right]^{\frac{1}{1-\sigma}}$.

Price level heavily relies on P_2 given that P_1 is given as 1. Now, aggregate income can be found. Due to linearity of the production technology, the output implies the worker inputs at each firm. Let q' denote the output at the cut-off ψ'. Given that $r(\psi') = (w_L/\rho\psi')q' = \sigma f$, $r(\psi') = (w_L/\rho\psi')q' = \sigma f$, That is, $q' = (\rho\sigma f \,/\, w_L)$.

Let q^* denote the output at the cut-off ψ^*. Eq. (23) implies that

$$r(\phi(\psi^*)) = \frac{\sigma f_m}{\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]} = \left(\frac{MC_d}{\rho\phi(\psi^*)}\right) q^*. \quad (30)$$

Then,

$$q^* = \left(\frac{\rho\psi^*\bar{\omega}^{\gamma}}{MC_d}\right) \frac{\sigma f_m}{\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]}. \quad (31)$$

At ψ, the output of a decentralized firm can be calculated as

$$q_d(\psi) = \left(\frac{\psi}{\psi^*}\right)^{\sigma} q^* = \left(\frac{\psi}{\psi^*}\right)^{\sigma} \frac{\rho\psi^*\bar{\omega}^{\gamma}\left(\sigma f_m\right)}{(MC_d)\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]}. \quad (32)$$

With linearity of the production function, the worker demand is

$$l_d(\psi) = \frac{q_d(\psi)}{\phi(\psi)} + \left(f + f_m\right) = \left(\frac{\psi}{\psi^*}\right)^{\sigma-1} \frac{\rho\sigma f_m}{(MC_d)\left[1 - \left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]} + (f + f_m). \quad (33)$$

For each range of entrepreneurial skills, the demands of worker and manager can be found separately.
For $\psi \geq \psi^*$,

$$l_d(\psi) = \frac{\left(\rho\sigma f_m\right)\left(\frac{\psi}{\psi^*}\right)^{\sigma-1}}{\left(MC_d\right)\left[1 - \left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]} + (f + f_m), \text{ and}$$

$$m(\psi) = \frac{\left(\rho\sigma f_m\right)\left(\frac{\psi}{\psi^*}\right)^{\sigma-1}}{\alpha(MC_d)\left[1 - \left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]} + \frac{\left(f + f_m\right)}{\alpha}. \quad (34)$$

For $\psi' \leq \psi < \psi^*$,

$$l_c(\psi) = \left[\frac{\rho\sigma f}{w_L}\right]\left(\frac{\psi}{\psi'}\right)^{\sigma-1} + f, \text{ and } m(\psi) = 0. \quad (35)$$

In equilibrium, the output increases in ψ. That is, higher-skill entrepreneurs produce greater outputs, hire more workers and managers. For Sector 2, the aggregate labor demand can be represented by the integral as

$$L_2 = \int_{\psi'}^{\psi^*} l_c(\psi)\mu_c(\psi)d\psi + \left(\frac{\alpha+1}{\alpha}\right)\int_{\psi^*}^{\infty} l_d(\psi)\mu_d(\psi)d\psi, \quad (36)$$

where $\mu_c(\psi) = \frac{g(\psi)}{\left[1 - G(\psi')\right]} = \frac{k}{\psi}\left(\frac{\psi'}{\psi}\right)^k$ and $\mu_d(\psi) = \frac{g(\psi)}{\left[1 - G(\psi^*)\right]} = \frac{k}{\psi}\left(\frac{\psi'}{\psi}\right)^k$.

The integral Eq. (36) can be calculated as[9]

$$L_2^* = \left[\frac{\rho\sigma fk\left[\psi^{*\sigma-k-1}-1\right]}{w_L(\sigma-k+1)}\right] + f + \left(\frac{\alpha+1}{\alpha\left(k-\sigma+1\right)}\right)\frac{\rho\sigma f_m}{\left(MC_d\right)\left[1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]} + \left(\frac{\alpha+1}{\alpha}\right)\left(f+f_m\right), \quad (37)$$

where $\psi^* = \frac{\rho\left(MC_d\right)}{w_L(\bar{\omega})^{\gamma}}\left[\frac{f}{\sigma f_m}\left[1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]\right]^{\frac{1}{1-\sigma}}$.

For sector 2, the aggregate labor income is

$$Y_2 = w_L\left[\frac{\rho\sigma fk\left[\psi^{*\sigma-k}-1\right]}{w_L\left(\sigma-k+1\right)}\right] + w_L f + \frac{w_L\rho\sigma f_m}{\left(MC_d\right)(k-\sigma-1)\left[1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]} + w_L(f+f_m) + \frac{\left(\frac{w_m}{\alpha}\right)\rho\sigma f_m}{\left(MC_d\right)\left(k-\sigma-1\right)\left[1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]} + \frac{w_m}{\alpha}\left(f+f_m\right). \quad (38)$$

Eq. (38) can be simplified as

$$Y_2 = \frac{\rho\sigma fk\left[\psi^{*\sigma-k}-1\right]}{\left(\sigma-k+1\right)} + w_L f + \frac{\rho\sigma f_m}{(k-\sigma-1)\left[1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]} + \left(w_L+\frac{w_m}{\alpha}\right)\left(f+f_m\right). \quad (39)$$

The remaining agents work for Sector 1, for which the aggregate labor demand can be determined as

$$L_1 = \bar{L} - L^*_2 = \bar{L} - \left[\frac{\rho\sigma fk\left[\psi^{*\sigma-k+1}-1\right]}{w_L(\sigma-k+1)} + w_L f + \frac{(\alpha+1)\rho\sigma f_m}{\alpha(k-\sigma-1)(MC_d)\left[1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]} + \left(w_L+\frac{w_m}{\alpha}\right)\left(f+f_m\right)\right].$$

[9] The integration is shown in detail in Appendix 1.

For the economy, aggregate income is obtained as

$$Y = \bar{L} + \left(\frac{w_L - 1}{w_L}\right)\frac{\rho\sigma fk\left[\psi^{*\sigma-k+1} - 1\right]}{(\sigma - k + 1)} + \left[\frac{(\alpha+1) - \alpha(MC_d)}{\alpha(MC_d)(k-\sigma-1)}\right]\frac{\rho\sigma f_m}{\left[1 - \left(\frac{MC_d}{w_L(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]}, \quad (40)$$

where $\psi^* = \frac{\rho(MC_d)}{w_L(\bar{\varpi})^{\gamma}}\left[\frac{f}{\sigma f_m}\left[1 - \left(\frac{MC_d}{w_L(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]\right]^{\frac{1}{1-\sigma}}$.

Aggregate income represents the aggregate labor income within the economy. The mass of decentralized firms, M^*, is determined by the cut-off ψ^*. That is, $M^* = (\psi^*)^{-k}E$, where E is the mass of entrepreneurs. This result differs from that of Melitz (2003), who underlies full employment and no occupational choice. Thus, aggregate income is constant and is determined merely by population size. The present result underlies full employment and occupational choice, and thus wage differs between occupations and between sectors. That is, the wage of the manager is higher than that of the worker. Again, the worker wage is higher in Sector 2 than in Sector 1. Thus, aggregate income increases as the mass of decentralized firms expands. as determined by skills distribution. Intuitively, aggregate income is larger for a country where higher-skill entrepreneurs are greater in number given the same business environment.

IV. Trade Equilibrium

This section discusses the trade equilibrium. Only decentralized firms can export, where exporting incurs an additional fixed cost f_x and an 'iceberg' transportation cost τ. Let d and x denote domestic-associated and export-associated variables, respectively. For each decentralized firm, the optimal price is

$$P_d(\psi) = \frac{MC_d}{\rho\phi(\psi)}. \quad (41)$$

$$P_x(\psi) = \frac{\tau MC_d}{\rho\phi(\psi)}, \text{ where } \tau \text{ is the per-unit 'iceberg' trade cost.} \quad (42)$$

An exporting firm earns revenue $r_d(\cdot)$ from the domestic market, while earning revenue $r_x(\cdot)(= \tau^{1-\sigma} r_d(\cdot))$ from the foreign market. For the firm, the total revenue is

$$r(\psi) = r_d(\psi) + r_x(\psi) = (1 + \tau^{1-\sigma}) r_d(\psi). \tag{43}$$

Not all firms can engage in exporting activities due to the additional costs. Among decentralized firms, the highest-skill entrepreneurs can export. Thus, another cut-off exists for exporting. Let ψ_x^{**} denote the exporting cut-off. Let ψ^{**} and ψ'' denote the decentralization and the production cut-offs, respectively, in the open economy. The point is that the production cut-off ψ' shifts to ψ'' as trade is liberated.

Proposition 1: $\psi'' = \left[\dfrac{\delta(k-\sigma+1)}{(1+\tau^{1-\sigma})(\sigma-1)} \right]^{\frac{-1}{k}} > \psi' = \left[\dfrac{\delta(k-\sigma+1)}{\sigma-1} \right]^{\frac{-1}{k}}$.

Proof: in Appendix 2

Proposition 2: $\psi_x^{**} > \psi^{**} > \psi^{*}$.

Proof: in Appendix 3

Proposition 3: $\psi^{*} > \psi''$ if and only if $f_m > \dfrac{f}{\sigma}$.

Proof: in Appendix 4

Proposition 3 implies that the selection for decentralization can occur only when f_m is sufficiently large. Higher-skill entrepreneurs are selected for decentralization, and among those firms, the highest-skill entrepreneurs are selected for exporting. The ex-ante probability of decentralization is $p_d = (\psi^{**})^{-k}$ while that of exporting is $p_x = (\psi_x^{**})^{-k}$. In equilibrium, worker demand differs across firms, and is $l(\psi) = \dfrac{q(\psi)}{\phi}$ each. Let q^* denote the output at the cut-off ψ^*. Eq. (30) can be rewritten as follows.

$$r\left(\phi\left(\psi_x^{**}\right)\right) = \frac{\sigma(f_m + f_x)}{\left[1 - \left(\dfrac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]} = \left(1+\tau^{1-\sigma}\right)\left(\frac{MC_d}{\rho\phi(\psi_x^{**})}\right) q_x^{**}. \tag{44}$$

Then, $q_x^{**} = \dfrac{\rho\bar{\omega}^{\gamma}\sigma\left(f_m + f_x\right)\psi_x^{**}}{\left(1+\tau^{1-\sigma}\right)\left(MC_d\right)\left[1 - \left(\dfrac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]}$.

At ψ, the output of a decentralized firm can be calculated as

$$q_x(\psi) = \left(\frac{\psi}{\psi_x^{**}}\right)^{\sigma} q_x^{**} = \left(\frac{\psi}{\psi_x^{**}}\right)^{\sigma} \frac{\rho\bar{\omega}^{\gamma}\sigma\left(f_m + f_x\right)\psi_x^{**}}{\left(1+\tau^{1-\sigma}\right)\left(MC_d\right)\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]}. \quad (45)$$

With linearity of the production technology, the worker demand is

$$l_x(\psi) = \frac{q_x(\psi)}{\phi(\psi)} + f_x = \frac{\rho\sigma\left(f_m + f_x\right)\left(\frac{\psi}{\psi_x^{**}}\right)^{\sigma-1}}{\left(1+\tau^{1-\sigma}\right)\left(MC_d\right)\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]} + f_x. \quad (46)$$

For each range of entrepreneurial skills, the demands of worker and manager can be obtained.

For $\psi'' \le \psi < \psi^{**}$, $l_c(\psi) = \left(\frac{\psi}{\psi''}\right)^{\sigma-1}\left[\frac{\rho\sigma f}{w_L}\right] + f$, and $m(\psi) = 0$.

For $\psi^{**} \le \psi < \psi_x^{**}$,

$$l_d(\psi) = \frac{\rho\sigma f_m\left(\frac{\psi}{\psi^{**}}\right)^{\sigma-1}}{(MC_d)\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]} + \left(f + f_m\right), \text{ and}$$

$$m(\psi) = \frac{\rho\sigma f_m\left(\frac{\psi}{\psi^{**}}\right)^{\sigma-1}}{\alpha(MC_d)\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]} + \frac{\left(f + f_m\right)}{\alpha}.$$

For $\psi_x^{**} \le \psi < \infty$,

$$l_x(\psi) = \frac{\rho\sigma\left(f_m + f_x\right)\left(\frac{\psi}{\psi_x^{**}}\right)^{\sigma-1}}{\left(1+\tau^{1-\sigma}\right)\left(MC_d\right)\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]} + f_x \text{ and}$$

$$m_x(\psi)=\frac{1}{\alpha}\frac{\rho\sigma\left(f_m+f_x\right)\left(\frac{\psi}{\psi_x^{**}}\right)^{\sigma-1}}{(1+\tau^{1-\sigma})(MC_d)\left[1-\left(\frac{MC_d}{w_L(\varpi)^{\gamma}}\right)^{\sigma-1}\right]}+\frac{f_x}{\alpha}.$$

In equilibrium, higher-skill entrepreneurs can engage in exporting activities, topped by the ones with the highest skills. For Sector 2, the aggregate labor demand can be represented by the integral as

$$\begin{aligned}L_2=&\int_{\psi''}^{\psi^{**}} l_c(\psi)\mu_c(\psi)d\psi+\left(\frac{\alpha+1}{\alpha}\right)\int_{\psi^{**}}^{\psi_x^{**}} l_d(\psi)\mu_d(\psi)d\psi\\&+\left(\frac{\alpha+1}{\alpha}\right)\int_{\psi_x^{**}}^{\infty} l_x(\psi)\mu_x(\psi)d\psi,\end{aligned}\qquad(47)$$

where $\mu_c(\psi)=\frac{g(\psi)}{[1-G(\psi'')]}=\frac{k}{\psi}\left(\frac{\psi''}{\psi}\right)^k$, $\mu_d(\psi)=\frac{g(\psi)}{[1-G(\psi'')]}=\frac{k}{\psi}\left(\frac{\psi^{**}}{\psi}\right)^k$, and $\mu_d(\psi)=\frac{g(\psi)}{\left[1-G\left(\psi_x^{**}\right)\right]}=\frac{k}{\psi}\left(\frac{\psi_x^{**}}{\psi}\right)^k$.

The integral Eq. (47) can be calculated as[10]

$$\begin{aligned}L_2^*=&\frac{\rho\sigma fk\left[\psi^{**\sigma-k-1}\left(1+\tau^{1-\sigma}\right)^{\frac{k-\sigma+1}{k}}-1\right]}{w_L\left(\sigma-k-1\right)}+f\\&+\frac{\left(\frac{(\alpha+1)\rho\sigma}{\alpha\left(k-\sigma+1\right)}\right)\left[\frac{(f_m+f_x)}{(1+\tau^{1-\sigma})}+f_m\left[\tau^{\sigma-k-1}\left(\frac{f_x}{f}\right)^{\frac{\sigma-k-1}{(\sigma-1)}}-1\right]\right]}{\left(MC_d\right)\left[1-\left(\frac{MC_d}{w_L(\varpi)^{\gamma}}\right)^{\sigma-1}\right]}\\&+\left(\frac{\alpha+1}{\alpha}\right)\left(f+f_m+f_x\right)\end{aligned}\qquad(48)$$

where $\psi^{**}=\left(\frac{\left(1+\tau^{1-\sigma}\right)\sigma f_m}{f\left[1-\left(\frac{MC_d}{w_L(\varpi)^{\gamma}}\right)^{\sigma-1}\right]}\right)^{\frac{1}{\sigma-1}}\left(\frac{MC_d\left[1+\tau^{1-\sigma}\right]^{\frac{1}{k}}}{w_L\varpi^{\gamma}}\right).$

[10] The integration is shown in detail in Appendix 5.

For Sector 2, the aggregate labor income is

$$Y_2 = w_L \frac{\rho\sigma fk\left[\psi^{**\sigma-k-1}\left(1+\tau^{1-\sigma}\right)^{\frac{k-\sigma+1}{k}}-1\right]}{w_L\left(\sigma-k-1\right)} + w_L f$$

$$+\frac{\left(\frac{w_L\rho\sigma}{(k-\sigma+1)}\right)\left[\frac{(f_m+f_x)}{\left(1+\tau^{1-\sigma}\right)}+f_m\left[\tau^{\sigma-k-1}\left(\frac{f_x}{f}\right)^{\frac{\sigma-k-1}{(\sigma-1)}}-1\right]\right]}{(MC_d)\left[1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]}$$

$$+\frac{\left(\frac{w_m\rho\sigma}{a(k-\sigma+1)}\right)\left[\frac{(f_m+f_x)}{\left(1+\tau^{1-\sigma}\right)}+f_m\left[\tau^{\sigma-k-1}\left(\frac{f_x}{f}\right)^{\frac{\sigma-k-1}{(\sigma-1)}}-1\right]\right]}{(MC_d)\left[1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]}$$

$$+\left(w_L+\frac{w_m}{\alpha}\right)\left(f+f_m+f_x\right) \tag{49}$$

Eq. (49) can be simplified as

$$Y_2 = \frac{\rho\sigma fk\left[\psi^{**\sigma-k-1}\left(1+\tau^{1-\sigma}\right)^{\frac{k-\sigma+1}{k}}-1\right]}{\left(\sigma-k-1\right)} + w_L f$$

$$+\frac{\left(\frac{\rho\sigma}{k-\sigma+1}\right)\left[\frac{(f_m+f_x)}{\left(1+\tau^{1-\sigma}\right)}+f_m\left[\tau^{\sigma-k-1}\left(\frac{f_x}{f}\right)^{\frac{\sigma-k-1}{(\sigma-1)}}-1\right]\right]}{\left[1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]} \tag{50}$$

$$+\left(w_L+\frac{w_m}{\alpha}\right)\left(f+f_m+f_x\right).$$

The remaining agents work for Sector 1, and thus its aggregate labor demand can be found as

$$Y_1 = L_1 = \bar{L} - L_2^* = \bar{L} - \frac{\rho\sigma fk\left[\psi^{**\sigma-k-1}\left(1+\tau^{1-\sigma}\right)^{\frac{k-\sigma+1}{k}} - 1\right]}{w_L\left(\sigma-k-1\right)}$$
$$- \frac{\frac{(\alpha+1)\rho\sigma}{\alpha(k-\sigma+1)}\left[\frac{(f_m+f_x)}{\left(1+\tau^{1-\sigma}\right)} + f_m\left[\tau^{\sigma-k-1}\left(\frac{f_x}{f}\right)^{\frac{\sigma-k-1}{(\sigma-1)}} - 1\right]\right]}{(MC_d)\left[1-\left(\frac{MC_d}{w_L(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]} \tag{51}$$

For the economy, aggregate income is obtained as
$Y = Y_1 + Y_2$,

$$= \bar{L} + \left(\frac{w_L-1}{w_L}\right)\frac{\rho\sigma fk\left[\psi^{**\sigma-k-1}\left(1+\tau^{1-\sigma}\right)^{\frac{k-\sigma+1}{k}} - 1\right]}{(\sigma-k-1)}$$
$$+ \frac{\rho\sigma\left(1-\left(\frac{\alpha+1}{\alpha w_L + w_m}\right)\right)}{(k-\sigma+1)}\left[\frac{\frac{(f_m+f_x)}{\left(1+\tau^{1-\sigma}\right)} + f_m\left[\tau^{\sigma-k-1}\left(\frac{f_x}{f}\right)^{\frac{\sigma-k-1}{(\sigma-1)}} - 1\right]}{1-\left(\frac{MC_d}{w_L(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}}\right], \tag{52}$$

where $\psi^{**} = \left(\frac{\left(1+\tau^{1-\sigma}\right)\sigma f_m}{f\left[1-\left(\frac{MC_d}{w_L(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]}\right)^{\frac{1}{\sigma-1}}\left(\frac{MC_d\left[1+\tau^{1-\sigma}\right]^{\frac{1}{k}}}{w_L\bar{\varpi}^{\gamma}}\right).$

In this economy, price level diminishes because firm productivity improves for decentralized firms. P_1 is given as 1. Sector 2 comprises a mix of centralized and decentralized firms. The product prices of centralized firms merely reflect entrepreneurial ability, whereas those of decentralized firms reflect a business environment that is represented by the labor market characteristics and status of social trust. Trade openness enhances aggregate industry productivity and the lowers price level. With symmetry, the business environment for entrepreneurship is identical between two countries. In such a case, the masses of centralized and exporting firms are the same between two countries. The assumption that skill distribution differs between two countries is thus rational.

V. Asymmetric Countries

Suppose that skill distribution differs between two countries. In this case, distributional property of entrepreneurial skill and average labor quality play important roles in trade equilibrium. When average labor quality is higher in one country, the decentralization and exporting cut-offs are lowered. This means that the masses of decentralized and exporting firms expand. Hence, the number of higher-paying jobs increases. Suppose that k is lower in Country A than Country B. In Country i (A or B), normalizing ψ' as 1,

$$\psi_i^* = \frac{\rho\left(MC_d\right)}{w_L\left(\bar{\varpi}_i\right)^{\gamma_i}}\left[\frac{f}{\sigma f_m}\left[1-\left(\frac{MC_d}{w_L\left(\bar{\varpi}_i\right)^{\gamma_i}}\right)^{\sigma-1}\right]\right]^{\frac{1}{1-\sigma}}. \tag{53}$$

$$\psi_i'' = \left[1+\tau^{1-\sigma}\right]^{\frac{1}{k_i}}. \tag{54}$$

$$\psi_i^{**} = \left(\frac{\left(1+\tau^{1-\sigma}\right)\sigma f_m}{f\left[1-\left(\frac{MC_d}{w_L\left(\bar{\varpi}_i\right)^{\gamma_i}}\right)^{\sigma-1}\right]}\right)^{\frac{1}{\sigma-1}}\left(\frac{MC_d}{w_L\left(\bar{\varpi}_i\right)^{\gamma_i}}\right)\left[1+\tau^{1-\sigma}\right]^{\frac{1}{k_i}}. \tag{55}$$

$$\psi_{xi}^{**} = \tau\left(\frac{\left(1+\tau^{1-\sigma}\right)\sigma f_x}{f}\right)^{\frac{1}{(\sigma-1)}}\left(\frac{MC_d\left[1+\tau^{1-\sigma}\right]^{\frac{1}{k_i}}}{w_L\left(\bar{\varpi}_i\right)^{\gamma_i}}\right). \tag{56}$$

Thus far, the production cut-off ψ_i' is normalized. Apparently, this value declines as k_i increases (equivalently, as higher entrepreneurial skills are less available). Then, ψ^{**} and ψ_x^{**} increase. Suppose that $k_A < k_B$. Then, the masses of decentralized and exporting firms are smaller in Country B than in Country A. In note, decentralized firms create higher-paying jobs and exporting firms create the highest-paying jobs. Thus, good jobs are more abundant in Country A than in Country B. Aggregate income is larger and price level is lower in Country A than in Country B. Thus, the level of welfare is definitely greater in Country A than in Country B. Trade openness causes firm selection more drastically in Country B than in Country A. In autarky, Country A has the greater mass of firms than Country B. With trade openness, firm mass can shrink in both countries. However, the shrinkage is greater in Country B than in Country A, where the more productive firms substitute away the least productive firms of Country B.

VI. Conclusion

This study examines how distribution of entrepreneurial skill can affect the aggregate economy. Most importantly, decentralization can affect not only firm productivity but also the pattern of labor demand by generating well-paying jobs. Lower-skill entrepreneurs have no incentives to delegate their authority to professional managers. The reason is that the entrepreneurs gain less from managerial delegation while it increases both the fixed and variable costs. Within this framework, the equilibrium can be characterized with self-selecting entrepreneurs. First, they are self-selecting for production or no production. Upon entry, entrepreneurs can recognize that decentralization leads to higher production efficiency. Second, entrepreneurs self-select for centralization or decentralization. Thus, two cut-offs are, namely, the production and decentralization cut-offs. Firm profit shifts upward from the decentralization cut-off because firm productivity is enhanced. Unlike in centralized firms, the production of decentralized firms requires managerial inputs. Managers are more skilled than workers. Thus, decentralization is skill-biased. In this paper, wage inequality occurs between occupations, across firms, and between sectors. When the economy is exposed to trade openness, decentralized firms have competitive advantage for exporting over centralized firms. As a result, resources are reallocated towards decentralized firms of higher productivity. The resource reallocation squeezes centralized and decentralized firms of lower productivity. Therefore, wage inequality widens. Between two countries, difference in skill distribution can generate inter-country income distribution. Suppose that high-skill individuals are more abundant in Country A than in Country B. Then, as a model of 'love for variety' predicts, both countries benefit from trade. However, the benefit is larger for Country A because masses of both decentralized and exporting firms are larger in Country A than in Country B. Therefore, well-paying jobs are more available in Country A than in Country B. In both countries, price level falls because products can be supplied at lower prices, domestically and internationally. Thus, the level of welfare improves for both countries. This paper provides new implications of trade gain, which is larger for the country where decentralization is easier. The study can be extended with the assumption that skills distribution is exogenous. Commonly, individuals invest in education or learning, for they know that manager wages are higher than worker wages. That is, educational investment endogenizes skill distribution, and its consideration promises more interesting results. Such mission is left for future researchers.

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Appendix 1

For Sector 2, aggregate labor demand can be represented by the integral as

$$L_2 = \int_{\psi'}^{\psi^*} l_c(\psi)\,\mu_c(\psi)\,d\psi + \left(\frac{\alpha+1}{\alpha}\right)\int_{\psi^*}^{\infty} l_d(\psi)\,\mu_d(\psi)\,d\psi, \tag{36}$$

where $\mu_c(\psi) = \frac{g(\psi)}{\left[1-G(\psi')\right]} = \frac{k}{\psi}\left(\frac{\psi'}{\psi}\right)^k$ and $\mu_d(\psi) = \frac{g(\psi)}{\left[1-G(\psi^*)\right]} = \frac{k}{\psi}\left(\frac{\psi^*}{\psi}\right)^k$.

The first integral of Eq. (36) can be rewritten as

$$\begin{aligned}\int_{\psi'}^{\psi^*} l_c(\psi)\,\mu_c(\psi)\,d\psi &= \int_{\psi'}^{\psi^*}\left(\frac{\psi}{\psi'}\right)^{\sigma-1}\left[\frac{\rho\sigma f}{w_L}\right]\frac{k}{\psi}\left(\frac{\psi'}{\psi}\right)^k d\psi + f \\ &= \left[\frac{\rho\sigma f k\,(\psi')^{k-\sigma+1}}{w_L}\right]\int_{\psi'}^{\psi^*}\psi^{\sigma-k-2}d\psi + f.\end{aligned} \tag{A1-1}$$

(A1-1) can be calculated as

$$\begin{aligned}&\left[\frac{\rho\sigma f k\,(\psi')^{k-\sigma+1}}{w_L}\right]\left[\frac{1}{\sigma-k-1}\psi^{\sigma-k-1}\right]_{\psi'}^{\psi^*} + f \\ &= \left[\frac{\rho\sigma f k\,(\psi')^{k-\sigma+1}}{w_L(\sigma-k-1)}\right]\left[\psi^{*\sigma-k-1} - \psi'^{\sigma-k-1}\right] + f.\end{aligned} \tag{A1-2}$$

The second integral of Eq. (36) can be rewritten as

$$\begin{aligned}&\left(\frac{\alpha+1}{\alpha}\right)\int_{\psi^*}^{\infty} l_d(\psi)\,\mu_d(\psi)\,d\psi + \left(\frac{\alpha+1}{\alpha}\right)(f+f_m) \\ &= \left(\frac{\alpha+1}{\alpha}\right)\int_{\psi^*}^{\infty}\left[\frac{\rho\sigma f_m\left(\frac{\psi}{\psi^*}\right)^{\sigma-1}}{(MC_d)\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]}\right]\frac{k}{\psi}\left(\frac{\psi^*}{\psi}\right)^k d\psi + \left(\frac{\alpha+1}{\alpha}\right)(f+f_m), \\ &= \frac{(\alpha+1)\,\rho\sigma f_m\,(\psi^*)^{k-\sigma-1}}{\alpha\,(MC_d)\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]}\int_{\psi^*}^{\infty}\psi^{\sigma-k-2}d\psi + \left(\frac{\alpha+1}{\alpha}\right)(f+f_m).\end{aligned} \tag{A1-3}$$

(A1-3) can be calculated as

$$\frac{(\alpha+1)\rho\sigma f_m (\psi^*)^{k-\sigma-1}}{\alpha(MC_d)\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^\gamma}\right)^{\sigma-1}\right]}\left[\frac{1}{\sigma-k-1}\psi^{\sigma-k-1}\right]_{\psi^*}^{\infty} d\psi+\left(\frac{\alpha+1}{\alpha}\right)(f+f_m)$$

$$=\frac{(\alpha+1)\rho\sigma f_m}{\alpha(k-\sigma+1)(MC_d)\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^\gamma}\right)^{\sigma-1}\right]}+\left(\frac{\alpha+1}{\alpha}\right)(f+f_m). \quad \text{(A1-4)}$$

Thus, Eq. (36) can be written as

$$L_2^* = \frac{\rho\sigma f k\left[\psi^{*\sigma-k-1}-1\right]}{w_L(\sigma-k+1)}+f$$
$$+\frac{(\alpha+1)\rho\sigma f_m}{\alpha(k-\sigma+1)(MC_d)\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^\gamma}\right)^{\sigma-1}\right]}+\left(\frac{\alpha+1}{\alpha}\right)(f+f_m), \quad \text{(A1-5)}$$

where $\psi^* = \frac{\rho(MC_d)}{w_L(\bar{\omega})^\gamma}\left[\frac{f}{\sigma f_m}\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^\gamma}\right)^{\sigma-1}\right]\right]^{\frac{1}{1-\sigma}}$.

Appendix 2

Proposition 1: $\psi'' = \left[\frac{\delta(k-\sigma+1)}{(1+\tau^{1-\sigma})(\sigma-1)}\right]^{\frac{-1}{k}} > \psi' = \left[\frac{\delta(k-\sigma+\ )}{\sigma-1}\right]^{\frac{-1}{k}}$.

Proof: Given the production cut-off, ex-ante probability of successful entry is $p_s = p(\psi > \psi'') = (\psi'')^{-k}$. Let $\bar{\upsilon}$ denote the present value of average firm profit flows; $\bar{\upsilon} = \sum_{t=0}^{\infty}(1-\delta)^t \pi(\bar{\psi}) = \frac{(1+\tau^{1-\sigma})\pi(\bar{\psi})}{\delta}$.

Then, when an entrepreneur attempts to enter the sector, the net value of his entry (υ_e) is calculated as

$$v_e = p_s\bar{v} - f_e = \frac{\left[(\psi'')^{-k}\right]\left(1+\tau^{1-\sigma}\right)\pi(\bar{\psi})}{\delta} - f_e. \tag{A2-1}$$

In equilibrium, the net value of entry should be zero under the condition of free entry.

$$\text{Thus, } \pi(\bar{\psi}) = \frac{\delta f_e}{\left(1+\tau^{1-\sigma}\right)(\psi'')^{-k}}. \tag{A2-2}$$

Eqs. (44) and (45) imply a stationary equilibrium, $\left(\pi(\bar{\psi}), \psi''\right)$. $\bar{\psi}$ is defined as

$$\bar{\psi}(\psi'') = \left[\int_{\psi''}^{\infty} \psi^{\sigma-1}\mu(\psi)\,d\psi\right]^{\frac{1}{\sigma-1}}, \tag{A2-3}$$

where $\mu(\psi) = \frac{g(\psi)}{\left[1-G(\psi'')\right]} = \frac{k}{\psi}\left(\frac{\psi''}{\psi}\right)^{t}$ for $\psi \geq \psi''$. The integral (A2-3) can be calculated as

$$\bar{\psi}(\psi'') = \left[\frac{k}{k-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \psi''. \tag{A2-4}$$

Average firm profit is

$$\left(1+\tau^{1-\sigma}\right)\pi(\bar{\psi}) = \left[\left(\frac{\bar{\psi}}{\psi''}\right)^{(\sigma-1)} - 1\right] f. \tag{A2-5}$$

Plugging Eq. (A2-4) into Eq. (A2-5) yields

$$\left(1+\tau^{1-\sigma}\right)\pi(\bar{\psi}) = \left[\frac{\sigma-1}{k-(\sigma-1)}\right] f. \tag{A2-6}$$

Eqs. (A2-2) and (A2-6) yield that

$$\psi'' = \left[\frac{\delta f_e (k-\sigma+1)}{\left(1+\tau^{1-\sigma}\right) f (\sigma-1)}\right]^{\frac{-1}{k}}. \tag{A2-7}$$

That is, $\psi'' = \left[\dfrac{\delta f_e (k - \sigma + 1)}{\left(1 + \tau^{1-\sigma}\right) f(\sigma - 1)}\right]^{\frac{-1}{k}} > \psi' = \left[\dfrac{\delta f_e (k - \sigma + 1)}{f(\sigma - 1)}\right]^{\frac{-1}{k}}$. (A2-8)

Appendix 3

Proposition 2: $\psi_x^{**} > \psi^{**} > \psi^{*}$.

Proof: By definition, the cut-off value ψ^{**} should satisfy the condition that $\pi_d\left(\varphi\left(\psi^{**}\right)\right) = \pi_d\left(\psi^{**}\right)$. Similarly, the cut-off value ψ_x^{**} should satisfy the condition that $\pi_x\left(\varphi\left(\psi_x^{**}\right)\right) = \pi_x\left(\psi_x^{**}\right)$. From the former condition, $r_d\left(\varphi\left(\psi^{**}\right)\right)$ can be calculated as $\dfrac{\sigma f_m}{\left[1 - \left(\dfrac{MC_d}{w_L(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]}$. From the latter condition, $r_x\left(\varphi\left(\psi_x^{**}\right)\right)$ can be calculated as $\dfrac{\sigma f_x}{\left[1 - \left(\dfrac{MC_d}{w_L(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]}$. Between an exporting firm and a non-exporting firm, the revenue ratio is

$$\frac{r_x\left(\phi_x^{**}\right)}{r_d\left(\phi^{**}\right)} = \tau^{1-\sigma}\left(\frac{\psi_x^{**}}{\psi^{**}}\right)^{(\sigma-1)} = \frac{f_x\left[1 - \left(\dfrac{MC_d}{w_L(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]}{f_m}. \tag{A3-1}$$

$$\text{Thus, } \psi_x^{**} = \tau\left(\frac{f_x\left[1 - \left(\dfrac{MC_d}{w_L(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]}{f_m}\right)^{\frac{1}{(\sigma-1)}}\psi^{**}. \tag{A3-2}$$

The partitioning of firms by export status can occur if and only if .

$$\tau\left(\frac{f_x\left[1 - \left(\dfrac{MC_d}{w_L(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]}{f_m}\right)^{\frac{1}{(\sigma-1)}} > 1.$$

Thus, $\psi_x^{**} > \psi^{**}$. (A3-3)

In detail, ψ^{**} can be obtained as follows.

As derived, $r\left(\phi\left(\psi^*\right)\right) = \dfrac{\sigma f_m}{\left[1 - \left(\dfrac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]}.$ (A3-4)

$$r\left(\phi\left(\psi^{**}\right)\right) = \left(1+\tau^{1-\sigma}\right) r\left(\phi\left(\psi^*\right)\right) = \frac{\left(1+\tau^{1-\sigma}\right)\sigma f_m}{\left[1 - \left(\dfrac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]}. \quad \text{(A3-5)}$$

Then, $\dfrac{r\left(\phi\left(\psi^{**}\right)\right)}{r\left(\phi\left(\psi^*\right)\right)} = \left(\dfrac{\dfrac{MC_d}{\rho\psi^{**}\left(\bar{\omega}\right)^{\gamma}}}{\dfrac{MC_d}{\rho\psi^{*}\left(\bar{\omega}\right)^{\gamma}}}\right)^{1-\sigma} = \left(\dfrac{\psi^*}{\psi^{**}}\right)^{1-\sigma} = \left(1+\tau^{1-\sigma}\right).$ (A3-6)

Eq. (A3-6) implies that

$$\psi^{**} = \left(1+\tau^{1-\sigma}\right)^{\frac{1}{\sigma-1}}\psi^*. \quad \text{(A3-7)}$$

Thus, $\psi^{**} > \psi^*$.

Appendix 4

From (A2-7), $\psi'' = \left[\dfrac{\delta f_e\left(k-\sigma+1\right)}{\left(1+\tau^{1-\sigma}\right) f\left(\sigma-1\right)}\right]^{\frac{-1}{k}}.$

$$\psi^* = \frac{\rho\left(MC_d\right)}{w_L\left(\bar{\omega}\right)^{\gamma}}\left[\frac{f}{\sigma f_m}\left[1 - \left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]\right]^{\frac{1}{1-\sigma}}\psi'. \quad \text{(A4-1)}$$

Normalizing ψ' as 1, $\psi^* = \dfrac{\rho\left(MC_d\right)}{w_L\left(\bar{\omega}\right)^{\gamma}}\left[\dfrac{f}{\sigma f_m}\left[1 - \left(\dfrac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]\right]^{\frac{1}{1-\sigma}}$. (A4-1-1)

Given that $\psi^* > 1$, then $MC_d = w_L + \frac{w_m}{\alpha} < w_L\left(\bar{\omega}\right)^{\gamma}$. (A4-2)

From (A2-7), $\psi'' = \left[1+\tau^{1-\sigma}\right]^{\frac{1}{k}} < 2$. (A4-3)

(A4-1-1) can be rewritten as

$$\psi^* = \frac{\rho\left(MC_d\right)}{w_L\left(\bar{\omega}\right)^{\gamma}}\left[1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]^{\frac{1}{1-\sigma}}\left[\frac{f}{\sigma f_m}\right]^{\frac{1}{1-\sigma}}. \tag{A4-1-2}$$

Given that $\frac{\rho\left(MC_d\right)}{w_L\left(\bar{\omega}\right)^{\gamma}} < 1$, and $1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1} < 1$, the sign of ($\psi^*$ - ψ'') depends on $\frac{f}{\sigma f_m}$.

That is, $\psi^* > \psi''$ only if $f_m > \frac{f}{\sigma}$.

Appendix 5

For Sector 2, the aggregate labor demand can be represented by the integral as

$$\begin{aligned} L_2 = &\int_{\psi''}^{\psi^{**}} l_c\left(\psi\right)\mu_c\left(\psi\right)d\psi + \left(\frac{\alpha+1}{\alpha}\right)\int_{\psi^{**}}^{\psi_x^{**}} l_d\left(\psi\right)\mu_d\left(\psi\right)d\psi \\ &+\left(\frac{\alpha+1}{\alpha}\right)\int_{\psi_x^{**}}^{\infty} l_x\left(\psi\right)\mu_x\left(\psi\right)d\psi, \end{aligned} \tag{47}$$

where $\mu_c\left(\psi\right) = \frac{g\left(\psi\right)}{\left[1-G\left(\psi''\right)\right]} = \frac{k}{\psi}\left(\frac{\psi''}{\psi}\right)^k$, $\mu_d\left(\psi\right) = \frac{g\left(\psi\right)}{\left[1-G\left(\psi^{**}\right)\right]} = \frac{k}{\psi}\left(\frac{\psi^{**}}{\psi}\right)^k$,
and $\mu_d\left(\psi\right) = \frac{g\left(\psi\right)}{\left[1-G\left(\psi_x^{**}\right)\right]} = \frac{k}{\psi}\left(\frac{\psi_x^{**}}{\psi}\right)^k$.

The first integral of Eq. (47) can be rewritten as

$$\begin{aligned} \int_{\psi''}^{\psi^{**}} l_c\left(\psi\right)\mu_c\left(\psi\right)d\psi + f &= \int_{\psi''}^{\psi^{**}}\left(\frac{\psi}{\psi''}\right)^{\sigma-1}\left[\frac{\rho\sigma f}{w_L}\right]\frac{k}{\psi}\left(\frac{\psi''}{\psi}\right)^k d\psi + f \\ &= \left[\frac{\rho\sigma fk\left(\psi''\right)^{k-\sigma+1}}{w_L}\right]\int_{\psi''}^{\psi^{**}}\psi^{\sigma-k-2}d\psi + f. \end{aligned} \tag{A5-1}$$

(A5-1) can be calculated as

$$\left[\frac{\rho\sigma fk\left(\psi''\right)^{k-\sigma+1}}{w_L}\right]\left[\frac{1}{\sigma-k-1}\psi^{\sigma-k-1}\right]_{\psi''}^{\psi^{**}}+f$$
$$=\left[\frac{\rho\sigma fk\left(\psi''\right)^{k-\sigma+1}}{w_L\left(\sigma-k-1\right)}\right]\left[\psi^{**\sigma-k-1}-\psi''^{\sigma-k-1}\right]+f. \qquad \text{(A5-2)}$$

The second integral of Eq. (47) can be rewritten as

$$\left(\frac{\alpha+1}{\alpha}\right)\int_{\psi^{**}}^{\psi_x^{**}} l_d\left(\psi\right)\mu_d\left(\psi\right)d\psi+\left(\frac{\alpha+1}{\alpha}\right)\left(f+f_m\right)$$
$$=\left(\frac{\alpha+1}{\alpha}\right)\int_{\psi^{**}}^{\psi_x^{**}}\left(\frac{\psi}{\psi^{**}}\right)^{\sigma-1}\left[\frac{\rho\sigma f_m}{\left(MC_d\right)\left[1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]}\right]\frac{k}{\psi}\left(\frac{\psi^{**}}{\psi}\right)^{k}d\psi \qquad \text{(A5-3)}$$
$$+\left(\frac{\alpha+1}{\alpha}\right)\left(f+f_m\right),$$
$$=\frac{\left(\alpha+1\right)\rho\sigma f_m\left(\psi^{**}\right)^{k-\sigma+1}}{\alpha\left(MC_d\right)\left[1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]}\int_{\psi^{**}}^{\psi_x^{**}}\psi^{\sigma-k-2}d\psi+\left(\frac{\alpha+1}{\alpha}\right)\left(f+f_m\right)$$

(A5-3) can be calculated as

$$=\frac{\left(\alpha+1\right)\rho\sigma f_m\left(\psi^{**}\right)^{k-\sigma+1}}{\alpha\left(MC_d\right)\left[1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]}\left[\frac{1}{\sigma-k-1}\psi^{\sigma-k-1}\right]_{\psi^{**}}^{\psi_x^{**}}+\left(\frac{\alpha+1}{\alpha}\right)\left(f+f_m\right),$$

$$=\frac{\left(\alpha+1\right)\rho\sigma f_m}{\alpha\left(MC_d\right)\left[1-\left(\frac{MC_d}{w_L\left(\bar{\omega}\right)^{\gamma}}\right)^{\sigma-1}\right]}\left[\tau^{\sigma-k-1}\left(\frac{f_x}{f}\right)^{\frac{\sigma-k-1}{(\sigma-1)}}-1\right]+\left(\frac{\alpha+1}{\alpha}\right)\left(f+f_m\right). \qquad \text{(A5-4)}$$

The third integral of Eq. (47) can be rewritten as

$$\left(\frac{\alpha+1}{\alpha}\right)\int_{\psi_x^{**}}^{\infty} l_x(\psi)\,\mu_x(\psi)\,d\psi + \left(\frac{\alpha+1}{\alpha}\right) f_x$$

$$= \left(\frac{\alpha+1}{\alpha}\right)\int_{\psi_x^{**}}^{\infty}\left(\frac{\psi}{\psi_x^{**}}\right)^{\sigma-1}\left[\frac{\rho\sigma(f_m+f_x)}{\left(1+\tau^{1-\sigma}\right)(MC_d)\left[1-\left(\frac{MC_d}{w_L\,(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]}\right]\frac{k}{\psi}\left(\frac{\psi_x^{**}}{\psi}\right)^{k} d\psi$$

$$+\left(\frac{\alpha+1}{\alpha}\right) f_x, \tag{A5-5}$$

$$= \frac{(\alpha+1)\,\rho\sigma(f_m+f_x)\left(\psi_x^{**}\right)^{k-\sigma+1}}{\alpha\left(1+\tau^{1-\sigma}\right)(MC_d)\left[1-\left(\frac{MC_d}{w_L\,(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]}\int_{\psi_x^{**}}^{\infty}\psi^{\sigma-k-2}d\psi + \left(\frac{\alpha+1}{\alpha}\right) f_x.$$

(A5-5) can be calculated as

$$= \frac{(\alpha+1)\,\rho\sigma(f_m+f_x)\left(\psi_x^{**}\right)^{k-\sigma+1}}{\alpha\left(1+\tau^{1-\sigma}\right)(MC_d)\left[1-\left(\frac{MC_d}{w_L\,(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]}\left[\frac{1}{\sigma-k-1}\,\psi^{\sigma-k-1}\right]_{\psi_x^{**}}^{\infty} + \left(\frac{\alpha+1}{\alpha}\right) f_x$$

$$= \frac{(\alpha+1)\,\rho\sigma(f_m+f_x)}{\alpha\left(1+\tau^{1-\sigma}\right)(k-\sigma+1)(MC_d)\left[1-\left(\frac{MC_d}{w_L\,(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]} + \left(\frac{\alpha+1}{\alpha}\right) f_x. \tag{A5-6}$$

Thus, the integral Eq. (47) is obtained as

$$L_2^{*} = \frac{\rho\sigma fk\left[\psi^{**\sigma-k-1}(1+\tau^{1-\sigma})^{\frac{k-\sigma+1}{k}}-1\right]}{w_L(\sigma-k-1)} + f + \frac{(\alpha+1)\rho\sigma fm\left[\tau^{\sigma-k-1}\left(\frac{f_x}{f}\right)^{\frac{\sigma-k-1}{(\sigma-1)}}-1\right]}{\alpha(k+1-\sigma)(MC_d)\left[1-\left(\frac{MC_d}{w_L\,(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]}$$

$$+\frac{(\alpha+1)\,\rho\sigma(f_m+f_x)}{\alpha\left(1+\tau^{1-\sigma}\right)(k-\sigma+1)(MC_d)\left[1-\left(\frac{MC_d}{w_L\,(\bar{\varpi})^{\gamma}}\right)^{\sigma-1}\right]} + \left(\frac{\alpha+1}{\alpha}\right)(f+f_m+f_x). \tag{A5-7}$$

Eq. (A5-7) can be simplified as

$$L_2^* = \frac{\rho\sigma fk\left[\psi^{**\sigma-k-1}(1+\tau^{1-\sigma})^{\frac{k-\sigma+1}{k}}-1\right]}{w_L(\sigma-k-1)} + f$$

$$+ \frac{\left(\frac{(\alpha+1)\rho\sigma}{\alpha(k-\sigma+1)}\right)\left[\frac{(f_m+f_x)}{(1+\tau^{1-\sigma})}+f_m\left[\tau^{\sigma-k-1}\left(\frac{f_x}{f}\right)^{\frac{\sigma-k-1}{(\sigma-1)}}-1\right]\right]}{(MC_d)\left[1-\left(\frac{MC_d}{w_L(\bar{\omega})^{\gamma}}\right)^{\sigma-1}\right]} + \left(\frac{\alpha+1}{\alpha}\right)(f+f_m+f_x). \quad \text{(A5-8)}$$

where $\psi'' = [1+\tau^{1-\sigma}]^{\frac{1}{k}}$.