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Benefit of Supply Chain Coordination[1]

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Abstract

In this paper, we deal with a supply chain in which an assembler orders parts from a supplier. The assembler decides both order size and lead time for the supplier. In response, the supplier will determine its production capacity to fulfill the order from the assembler. Compared with the case where the assembler and the supplier maximize their own individual profits, the supply chain coordination offers a Pareto efficient solution and can give more profit to both parties. The implication of this paper is that the order requested from an assembler to a parts supplier such as in JIT delivery may damage the overall profit in the chain. Therefore, the parties involved had better cooperate with each other rather than emphasizing their own sakes and eventually leading to lower profits.

Keywords: inventory control, logistics/distribution, lot sizing, lead time.

1. Introduction and Literature Review

1.1. Introduction

In a competitive business world where there is a full line of customers wanting rapid procurement of delivery, many companies are obliged to satisfy those customer needs. One of the time-based competition strategies is to apply JIT (Just In Time). In applying JIT rule, many companies attempt to shorten

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the lead time for the suppliers in order to offer faster response to outside customers. This might not give the expected result or might produce inferior outcome compared with the current performance. One of the main reasons for this failure is that the parent company does not consider nor help to solve the problems of the parts supplier. When the parent company requests shorter lead time and the supplier is obliged to follow it, the cost increment to the supplier due to shorter lead time may be embedded in the price of parts taking advantage of the asymmetric information.

Similar problem occurs in terms of inventory. A parent company wanting to apply JIT and keep zero inventory in itself would order small amount at a time and this might cause the parts supplier to set up production more frequently or to maintain larger amount of inventory or production capacity than before unless other improvement is achieved along the supply chain. This is because the supplier now has to rapidly satisfy the orders from the parent company. In this case, the inventory holding cost is transferred from the parent company to the supplier rather than being reduced throughout the supply chain. Therefore, without considering the whole supply chain, we cannot achieve the expected cost improvement

In many industries, the cost of materials procured is more than half of the total sales revenue. Therefore supply chain management has a great potential for reducing cost for the parties in the value chain. In this paper, we study supply chain in which there are two participants, called an assembler and a parts supplier. The assembler orders some amount of parts at a time allowing a fixed order lead time to the parts supplier. In order to fulfill the order under the lead time given by the assembler, the parts supplier has to maintain appropriate amount of production capacity. We first deal with individual optimization problem for each participant. And then we study the optimization of the whole supply chain. The motivation for this research is to find a Pareto efficient solution for both parties by coordinating them appropriately. We calculate the social welfare increment along the supply chain under the coordination.

After literature review, we give detailed assumptions incorporated in our model. We consider the optimization problem in which the assembler and the parts supplier try to minimize their own cost regardless of the other party's interest. And then we deal with the supply chain coordination problems. In the supply chain coordination, we derive the possible optimal solution set for minimizing the total cost throughout the chain. Considering four cases depending on the parameters of our model, we try to restrict our optimal solution set among which there exists an optimal solution. Numerical examples follow afterwards and then we give the concluding remarks.

1.2. Literature Review

Traditionally, many researchers focused on a single person optimization problem. Restricting our attention to inventory control models, EOQ (Economic Order Quantity), a newsvendor model, and their various extensions are typical examples. Compared with these, in order to induce coordination along distribution channel, quantity discount was studied and suggested for improvement by many researchers (Monahan 1984, Lee and Rosenblatt 1986, Kohli and Park 1989, Weng 1995). Order quantity allocation between two uncertain suppliers and its effects on the inventory policies of the buyer were studied by (Anupindi and Akella 1993). Optimal ordering policy including the number of suppliers to be involved was derived from three models.

Aggregate planning problem for a single product with random demand and random capacity was analyzed by (Ciarallo, Akella and Morton 1994). In the multiple-period and infinite-horizon settings, order-up-to policies depending on the distribution of capacity are shown to be optimal in spite of a nonconvex cost function. In their model, the actual production quantity is limited by random capacity. In terms of the planned production U and the uncertain capacity Y, the actual production was min $\{U, Y\}$, which is a random variable. Therefore, we can say that the actual production is ceiled by a random variable Y. In our paper, we treat the random production of the parts supplier with a random yield rate. Thus instead of being ceiled by a random variable, the actual production is assumed to be the multiplication of planned production and a random yield rate X in our model.

(Bassok and Akella 1991) considered an aggregate production planning problem in a manufacturing facility with a critical raw material, and one or more products. The objective was to choose simultaneously the order amount of raw material and the production quantity of each of the products. In the traditional approach, production problem and inventory control model were said to have been separately dealt with. They gave an integrated model of these two compared with the separate models. They developed a model that enabled an explicit evaluation of the benefit of vendor reliability, as well as those of the integration of the inventory and production problem. The raw materials in their model can be corresponded to the parts in our model. Basically they focused on a single person optimization even though they considered both raw material supply (inventory model) and production capacity (production problem) simultaneously. But our focus in this paper is on the coordination of " two" separate participants in the supply chain(an assembler and a parts supplier). We compare the case where there is no coordination among the participants and each party is trying to increase its own benefit with the case in which both parties cooperate and coordinate to increase the total benefit along the supply chain. The literature on quantitative-oriented approaches for determining lot sizes when production or procurement yields are random was somewhat completely reviewed by (Yano and Lee 1995), and the interested readers are advised to read this review paper.

There has been a rather extensive research regarding supply chain coordination mechanism. Several researchers tried to design a coordination mechanism to align the self-interests of individuals with supply chain's integrated interest. Coordinated contracts through various tools (for example, returns rebate(Pasterneck 1985), quantity discount(Jauland and Shugan 1983), two-part tariff(Lariviere 1999), price protection(Lee, Padmanabhan, Taylor and Whang 2000), and target-level rebate(Taylor 2002) were designed to induce individuals in the system to mimic the centralized optimal solution.

2. Model Assumptions

2.1. Assembler's Problem

The assembler in our model accumulates customer orders up to N and then gives that order to the parts supplier. The lead time for parts delivery required by the assembler is L. The decision variables for the assembler are thus (N, L). The followings are assumed in our model.

1. Customer orders arrive according to a Poisson process with arrival rate of λ.

2. Fixed cost of s_1 is incurred for each order issued by the assembler.

3. Due to technical limitation for the parts supplier, the lead time should be guaranteed at least l_m, i.e., $L \geq l_m$.

4. Since customer prefers faster response or order fulfillment, there incurs cost of u per each time unit spent from customer order arrival to order fulfillment. Denote X_i as the arrival time epoch of i-th customer order and T_i as the time epoch of batch jobs dealing with i-th customer order. This batch order of size N is issued by the assembler to the parts supplier. Then assuming the assembly time, ξ, is constant and parts delivery is done in L, we can represent the delay cost for the customer i as $u(T–X_i + L + \xi)$.

5. We assume that the batch orders of size N are distinct. That is, each order of size N is processed separately even though more than one batch are under process simultaneously. This assumption is for the simplicity of the cost function.

We should note that sales revenue is uncontrollable and fixed in the current setting and thus can be disregarded in our optimization model. Regarding the shortage (i.e., unsatisfied customer order), the penalty cost can be charged to the parts supplier and thus can be deleted in the cost function of the assembler. The first two assumptions are usually made in analytical models. And assumption 5 is made since we want to avoid the complexity when there exists overlap in case of more than one order for processing.

2.2. Parts Supplier's Problem

Given the lead time L by the assembler, the parts supplier can possibly produce LZ units of parts during the lead time, where Z represents the capacity invested by the supplier. The optimal capacity size of Z should be decided by the supplier. The detailed assumptions are as follow:

1. The depreciation cost or investment cost of d is charged against each unit of supplier's production capacity per unit time.
2. Due to the random factors in the manufacturing process and possible problems from processing other orders, the amount produced and delivered in L is assumed to be LZX, where X follows a uniform distribution, i.e., $X \sim U$ [0, 1]. This means that the production capacity of LZ may not be realized and can possibly be affected by random yield rate of X, which is U [0, 1]. We should note that in the case where X follows a general distribution, F can be analyzed in a similar way. And the case of additive randomness such as $LZ + \varepsilon$, where ε denotes a random noise, will be another extension of this paper.
3. Due to the minimum requirement for capacity, the parts supplier should maintain at least z_m production capacity, i.e., $Z \geq z_m$.
4. According to the contract between the assembler and the supplier, p is charged for each unit undelivered during lead time of L. The shortage penalty cost to the supplier, p, can be set as the assembler's cost increment due to parts undelivered on time.
5. Inventory holding cost of h is charged to each unit left beyond the ordered amount N during lead time.
6. Due to deterioration and frequent specification change, we assume that the parts supplier cannot produce and maintain stocks in advance of orders from the assembler. That is, we only consider make-to-order system. Otherwise, we can appropriately reduce Z^* and produce in advance of orders.
7. Each production cycle of the supplier incurs set-up cost of

S_2.

3. Individual Optimization

We first consider the case where each party(the assembler and the supplier) tries to maximize its own individual benefit without considering the other party's cost. Given (L, N) requested by the assembler, the parts supplier will have to minimize its expected cost. This is the subgame which should be solved by the supplier given (L, N). This is similar to the Stackelberg game with the leader and the follower. The assembler works as the leader, and the supplier as the follower. Thus we deal with the supplier's problem first.

3.1. Preparation

Given (L, N) by the assembler and no value of time assumed, the parts supplier will incur the inventory cost during production cycle as shown.

$$g(X \mid L, Z, N) = h[LZX - N]^+ + p[N - LZX]^+$$

Denoting $LZ = Q$, we get

$$g(X \mid Q, N) = h[QX - N]^+ + p[N - QX]^+$$

We can notice that g takes the inventory cost function as in the newsvendor model. Taking expectation with respect to X, we get

$$G(Q, N) = E[g(X \mid Q, N)] = h\int_0^1 [Qx - N]^+ f(x)dx + p\int_0^1 [N - Qx]^+ f(x)dx$$

Since $f(x) = 1$ for $0 \le x \le 1$, we consider two cases depending on whether $N \le Q$ or not. For $N \le Q$, let's denote G as G^1 and for $N \ge Q$ as G^2. Then we get the following equations for the expected inventory cost per order cycle:

$$G^1 = h\int_{N/Q}^{1}[Qx - N]dx + p\int_{0}^{N/Q}[N - Qx]dx$$
$$= h\left[\frac{Q}{2}(1 - \frac{N^2}{Q^2}) - N(1 - \frac{N}{Q})\right] + p\left[N\frac{N}{Q} - \frac{Q}{2}\frac{N^2}{Q^2}\right]$$
$$= \frac{h(Q-N)^2 + pN^2}{2Q}$$
$$G^2 = p\int_{0}^{1}[N - Qx]dx$$
$$= p(N - \frac{Q}{2}).$$

3.2. Parts Supplier's Optimization

Assuming the assembler would request (L, N), the parts supplier will have to determine the optimal production capacity of Z as follows

$$Z = Z^*(L, N, p, h, d)$$

The supplier's expected cost per unit time, disregarding time value, is then

$$C_2 = \lambda\frac{G + s_2}{N} + dZ$$

Minimizing the cost, we get the following proposition.

Proposition 1. Given (N, L) the parts supplier will choose Z in the following way: *If*

$N/L \leq \lambda p/(2d)$, then $Z^* = \frac{1}{L}\sqrt{\frac{\lambda(p+h)N^2L}{\lambda hL + 2dN}}$. Otherwise, $Z^* = z_m$.

Proof:

We first consider the region of $N \leq LZ$. In this region, we have G^1 for G in C_2.

Minimizing C_2, we get $Z_1^* = \max\{N/L,\ \hat{Z}\}$ where $\hat{Z} = \sqrt{\frac{\lambda(p+h)N^2}{(\lambda hL+2dN)L}}$. $\hat{Z}$ is derived from $\frac{\partial C_2}{\partial Z} = 0$. Using C_2 is convex in Z, we note the optimal Z_1^* for $N \leq LZ$ is thus $\max\{N/L,\ \hat{Z}\}$. Solving $\hat{Z} \geq N/L$, we get $\frac{N}{L} \leq \frac{\lambda p}{2d}$. The condition of $\frac{N}{L} \leq \frac{\lambda p}{2d}$ denotes the region where the unit capacity cost of d is larger than marginal inventory cost in Z. Thus we get the optimal Z for C_2 in $N \le LZ$ as

$$Z_1^* = \begin{cases} \hat{Z} & \text{if } \dfrac{N}{L} \le \dfrac{\lambda p}{2d} \\ N/L & \text{otherwise} \end{cases}$$

In the region of $N \ge LZ$, we have G^2 for G in C_2, and get the optimal Z in a similar way:

$$Z_2^* = \begin{cases} N/L & \text{if } \dfrac{N}{L} \le \dfrac{\lambda p}{2d} \\ z_m & \text{otherwise} \end{cases}$$

Combining these two solutions, we get the optimal Z for the parts supplier as follows:

$$Z^* = \begin{cases} \sqrt{\dfrac{\lambda(p+h)N^2}{\lambda hL^2 + 2dNL}} & \text{if } \dfrac{N}{L} \le \dfrac{\lambda p}{2d} \\ z_m & \text{otherwise} \end{cases}$$

3.3. Assembler's Optimization

The assembler will decide N considering (s_1, u), and thus get $N^*(s_1, u)$. In an attempt to shorten the response time to the customers, the assembler will choose and request $L^* = l_m$ to the supplier. This corresponds to the JIT delivery requested by the assembler for its own benefit disregarding that of the parts supplier.

The expected inter-order time is N/λ. Expected waiting time for a customer until its job begins processing is $N/(2\lambda)$. The expected cost per order cycle is then $s_1 + Nu[\frac{N}{2\lambda} + L]$ supposing the constant assembly time is zero without loss of generality. The expected cost per unit time of the assembler is therefore

$$C_1(L, N) = [s_1 + Nu(\frac{N}{2\lambda} + L)]\frac{\lambda}{N}$$

We get the first order partial derivatives as follows:

$$\frac{\partial C_1}{\partial L} = \lambda u > 0,$$
$$\frac{\partial C_1}{\partial N} = -s_1 \frac{\lambda}{N^2} + \frac{u}{2}.$$

We thus get the optimal lead time for the assembler as

$L^* = l_m$.

From the equation of $\partial C_1 / \partial N = 0$ and the convexity of C_1 in N, we get

$$N^* = \sqrt{\frac{2\lambda s_1}{u}}.$$

The optimal number of order size, N^*, takes the same form as in the well-known EOQ (Economic Order Quantity) model.

Proposition 2. The assembler will choose the order lead time and order size as follows in the individual optimization case:

$$L^* = l_m,$$
$$N^* = \sqrt{\frac{2\lambda s_1}{u}}.$$

Using this result and that of previous section, the optimal solution for the parts supplier is derived as in the next proposition.

Proposition 3. In the individual optimization case, the optimal production capacity for the parts supplier is

$$Z^* = z_m, \qquad \text{if} \quad d^2 \geq \frac{\lambda p^2 l_m^2 u}{8 s_1}$$

The optimal production capacity for the parts supplier is

$$Z^* = \sqrt{\frac{2\lambda^2 s_1(p+h)}{u\lambda h l_m^2 + 2dl_m\sqrt{2\lambda u s_1}}}, \qquad if \quad d^2 \le \frac{\lambda p^2 l_m^2 u}{8s_1}$$

Proof:

Using Proposition 2, we note that the condition of $\frac{N}{L} \le \frac{\lambda p}{2d}$ is equivalent to $8s_1 d^2 \le \lambda u l_m^2 p^2$. Applying this and Proposition 1, we get the result.

4. Supply Chain Coordination

4.1. Supply Chain Cost Function

Rather than minimizing individual party's cost function separately, we now try to minimize the total cost in the supply chain.

$$\min_{L,Z,N}\{C_1 + C_2\}.$$

Denote the total cost in the supply chain per unit time as C and then

$$C(L,Z,N) = C_1 + C_2 = \lambda(s_1 + s_2)/N + \lambda u L + uN/2 + \lambda G/N + dZ.$$

Before proceeding further, we need to comment on the total cost function. Unlike the wholesale price term in a supply chain, we include the penalty term $p[N - QX]^+$ in the total cost function. This is because the penalty due to shortage of parts is incurred to the assembler by the final consumer. And this penalty was transferred to the parts supplier's cost function in individual optimization. Thus the supply chain system of the assembler and the parts supplier should consider the penalty together in a coordinated mechanism.

The component G is G^1 or G^2 depending on $N \le Q$ or $N \ge Q$ respectively. Minimizing C with respect to (L, Z, N) is a complicated problem. But notice the fact that G is affected by LZ, the mean production capacity during lead time. That is, the influence from L and Z is made according to their multiplicative term, not to each individual term. The cost components related with (L, Z) in C are λuL, dZ, and G.

Therefore we can see that L and Z are 'perfect substitutes in the multiplicative sense' in the coordinated supply chain. This is clear when we correspond λu to d in the cost function. In the coordinated supply chain, we can expect that given LZ we should make $L = l_m$ if $\lambda u \geq d$, and $Z = z_m$ otherwise.

Each unit of Z or L plays the same role, but the cost from these is charged to different party (cost due to Z is to the supplier, and that from L to the assembler) and thus incentive conflict occurs in the individual optimization between the assembler and the supplier.

4.2. Optimization

Depending on the region of Q, we have two different forms of G in C and therefore we have to solve two optimization problems. One of our optimization problems is for $N \leq Q$:

$$(I) \min \frac{\lambda(s_1 + s_2)}{N} + \lambda uL + \frac{u}{2}N + \frac{\lambda[h(Q-N)^2 + pN^2]}{2QN} + dZ$$

Subject to

$$l_m - L \leq 0,$$
$$z_m - Z \leq 0,$$
$$LZ - N \geq 0,$$

The second problem is for the region of $N \geq Q$:

$$(II) \min \frac{\lambda(s_1 + s_2)}{N} + \lambda uL + \frac{u}{2}N + \lambda p - \frac{\lambda p}{2}\frac{LZ}{N} + dZ$$

Subject to

$$l_m - L \leq 0,$$
$$z_m - Z \leq 0,$$
$$LZ - N \leq 0.$$

And our optimal solution comes from min{*I*, *II*}. Let's denote $s = s_1 + s_2$. We solve Problem *I* first. For this optimization, we firstly derive optimal N^* in terms of Q and delete N from our decision variables.

In Problem (*I*), we can derive that

$$N^* = \min\{Q, \hat{N}\}$$

where $\hat{N} = \sqrt{\frac{\lambda Q(2s+hQ)}{uQ+(p+h)\lambda}}$. From solving $\frac{\partial C}{\partial N} = 0$, we derive $\hat{N}$. Since C is convex in N and we deal with $Q \geq N$ in (*I*), the optimal $N^* = \min\{Q, \hat{N}\}$. Since Problem (*I*) is restricted to $N \leq Q$, we derive that $\hat{N} \leq Q$ is equivalent to $Q \geq \frac{-\lambda p + \sqrt{\lambda^2 p^2 + 8\lambda su}}{2u}$ and thus get:

$$N^* = \begin{cases} \sqrt{\dfrac{\lambda Q(2s+hQ)}{uQ+(p+h)\lambda}} & \text{if} \quad Q \geq \dfrac{-\lambda p + \sqrt{\lambda^2 p^2 + 8\lambda su}}{2u} \\ Q & \text{otherwise} \end{cases}$$

Likewise, for Problem (*II*), we get the following result:

$$N^* = \begin{cases} \sqrt{\dfrac{\lambda}{u}(2s - pQ)} & \text{if} \quad Q \leq \dfrac{2s}{p} \wedge \dfrac{-\lambda p + \sqrt{\lambda^2 p^2 + 8\lambda su}}{2u} \\ Q & \text{otherwise} \end{cases}$$

But we can show that $\frac{2s}{p} \geq \frac{-\lambda p + \sqrt{\lambda^2 p^2 + 8\lambda su}}{2u}$. Thus we combine these two solutions and get the optimal N^* in terms of Q depending on the value of Q as follows.

Proposition 4. Optimizing the total cost function of C in the coordinated supply chain, we get the following result: *If* $Q^* \geq \frac{-\lambda p + \sqrt{\lambda^2 p^2 + 8\lambda su}}{2u}$, then $N^* = \sqrt{\frac{\lambda Q(2s+hQ)}{uQ+(p+h)\lambda}} \leq Q^*$. Otherwise, we get $N^* = \sqrt{\frac{\lambda}{u}(2s - pQ)} \geq Q^*$.

Substituting N^* into Problem (*I*) and denoting $\bar{q} = \frac{-\lambda p + \sqrt{\lambda^2 p^2 + 8\lambda su}}{2u}$, we get the following simplified optimization problem with N deleted from our decision variables:

$$\text{(A)} \quad \min_{L,Z} \sqrt{\frac{\lambda(2s+hQ)(uQ+\lambda(p+h))}{Q}} + \lambda uL + dZ - \lambda h$$

subject to

$$l_m - L \le 0,$$
$$z_m - Z \le 0,$$
$$Q = LZ \ge \bar{q},$$

For region of $Q \ge \bar{q}$, we have $N^* = \sqrt{\frac{\lambda Q(2s+hQ)}{uQ+(p+h)\lambda}}$ from Proposition 4. Substituting this N^* into C_2 of (I), we can derive that the objective cost function of (I) becomes that of (A).

Likewise we get (B) as follows.

$$\text{(B)} \quad \min_{L,Z} \sqrt{\lambda u(2s-pQ)} + \lambda uL + dZ + \lambda p$$

subject to

$$l_m - L \le 0,$$
$$z_m - Z \le 0,$$
$$Q = LZ \le \bar{q}.$$

And our optimal solution comes from min{A, B}.

5. Extension and Numerical Examples

5.1. Extension

In the previous model, we assumed that the unit purchase cost is independent of lead time L. But in some cases the unit purchase cost may be a decreasing function of L. In these cases we should note that the optimal L^* may be larger than l_m even in the assembler's optimization problem, thus reducing the benefit of supply chain coordination.

We can relax the assumption that the unit shortage cost of p for the parts supplier is equivalent to that for the assembler. We study whether manipulation can give us the same optimal solution as in the coordination case. We denote p' as the unit shortage cost for the assembler and p as the payment per shortage unit from the parts supplier to the assembler, the cost function of C_1 now becomes:

$$C_1 = \frac{\lambda s_1}{N} + \frac{u}{2}N + u\lambda L + (p' - p)\frac{1}{2}\sqrt{\frac{\lambda(\lambda hL + 2dN)}{(p+h)L}}$$

Due to the additional term, $(p'-p)\frac{1}{2}\sqrt{\frac{\lambda(\lambda hL+2dN)}{(p+h)L}}$ in C_1, L^* tends to become larger and N^* smaller than before as long as $p < p'$. And in general we cannot derive the optimal solution as in the coordination case by using p only. But when the minimum is achieved in $Q \le N$, G becomes G^2 and $C_1 = \frac{\lambda s_1}{N} + \frac{u}{2}N + u\lambda L + (p'-p)\lambda(1 - \frac{l_m z_m}{2N})$. And thus $L^* = l_m$ as before, and we cannot achieve coordination solution through p.

5.2. Numerical Examples

In the supply chain coordination, we have extra social welfare of $C_1(L^*, N^*, Z^*) + C_2(L^*, N^*, Z^*) - C(L^{**}, N^{**}, Z^{**})$ to be distributed between the assembler and the supplier. There occurs improvement in Pareto efficiency $(C_1(L^*, N^*, Z^*)) + C_2(L^*, N^*, Z^*) - C_1(L^{**}, N^{**}, Z^{**}) > 0)$ in most cases and we had better pursue supply chain coordination. Here we deal with an example with the following parameters for our model:

$$\lambda = 100,\ u = 1,\ s_1 = 2,\ s_2 = 4,\ d = 400,\ l_m = z_m = 3,\ h = 1.$$

Varying the value of p, we construct four examples in the following table:

	Individual Optimization	Supply Chain Coordination	Cost Reduction
Case A (p = 30)	$L = Z = 3,\ N = 20$ $C^* = 3865$	$L = Z = 3,\ N = 2.47$ $C^* = 2251.7$	41.7%
Case B (p = 20)	$L = Z = 3,\ N = 20$ $C^* = 3090$	$L = Z = 3,\ N = 3.01$ $C^* = 2101.5$	32%
Case A (p = 10)	$L = Z = 3,\ N = 20$ $C^* = 2315$	$L = Z = 3,\ N = 4.08$ $C^* = 1908.7$	17.5%
Case A (p = 3)	$L = Z = 3,\ N = 20$ $C^* = 1772.5$	$L = Z = 3,\ N = 6.8$ $C^* = 1708.9$	3.6%

We now construct another kind of example where we have different values of Z depending on whether we consider individual optimization or supply chain coordination. For the problem with the parameters of

$$\lambda = 100,\ \ u = 1,\ \ s_1 = 10^{-5},\ \ s_2 = 4,$$
$$d = 120,\ \ l_m = z_m = 0.1,\ \ h = 1,\ \ p = 2.$$

This is an example of Case C. Then we get the following solution by applying our analysis:

$$L^* = 0.1,\ \ Z^* = 0.4472,\ \ N^* = 0.04472,\ \ C^* = 9108.25$$
$$L^{**} = Z^{**} = 0.1,\ \ N^{**} = 28.249,\ \ C^* = 250.25$$

In this example, the cost in individual optimization is 36.4 times as much as that in the supply chain coordination. This means that we can possibly get a radical reduction in cost using supply chain coordination mechanism compared with the individual optimization. The order size for individual optimization takes into account only s_1 without considering s_2, the set-up cost of the parts supplier. This results in a much smaller order size than in supply chain coordination case. That is $N^* \ll N^{**}$, and N^* causes a radical increase in total cost. In this example, the assembler requests much more frequent delivery of smaller order size in individual optimization than in supply chain coordination, and this can be corresponded to the JIT delivery of smaller amount for the supplier ordered by the assembler. Disregarding the cost parameters of the other party may thus increase the total cost in the chain.

6. Concluding Remarks

We see some failures when several companies try to implement JIT in order to improve efficiency. This paper suggests that one of the main reasons for the failure is due to the fact that cost reduction is not achieved throughout the supply chain but rather cost is transferred from one company to a downstream company. When an assembler requests frequent JIT delivery through a small order amount, the parts supplier might have to hold more production capacity or inventory than before unless it somehow improves the operation. This is the typical case of failure. The author observed a case where a tire manufacturer requested JIT delivery from a tire cord supplier. Feeling obliged to follow the request, the cord supplier built a warehouse near the manufacturer and kept a fair amount of inventory to rapidly fulfill the frequent small orders. The inventory holding cost of the tire manufacturer was actually transferred to the cord supplier, and eventually the total cost along the supply chain became worse off than before.

The cost reduction for the assembler due to fast response to customers or holding less inventory itself reappears as an additional cost to the parts supplier. This kind of cost reduction to the assembler is only temporary at best. The cost transferred to the parts supplier might come back in a disguised form. The parts supplier will make every plausible reason for the necessary increment in contract price. Or it might lower some quality level of the parts supplied within the range of specification. And there are numerous ways for a parts supplier to make up for the loss imposed by the assembler. Even if the assembler perfectly monitors the supplier and it is impossible for the supplier to make up for the loss, it at least deteriorates the relationship between these two parties and both will suffer.

Therefore we should find a way for mutual benefit. Unless there is cost reduction in the whole supply chain (not only for the assembler but also for the supplier), we are not better off simply by implementing new ideas into our system. In some cases as in the supply chain of our model, there is an opportunity for mutually beneficial operation. By coordinating the operation throughout the supply chain, we could reduce the total expected cost in the chain. For the coordination, the assembler might retreat from asking for minimum lead time (l_m) and allow somewhat more lead time in order to get the optimal value throughout the whole chain. Or the assembler might have to increase order size N in order to compromise the parts supplier's high set-up and thus to reduce the total cost along the supply chain. In our paper, the sources of the system inefficiency are the order quantity and the delivery time. Especially the conflict of interests on the delivery time between the assembler and the parts supplier was explored in the paper.

In the supply coordination, we can substitute order lead time L for production capacity Z if L is less expensive to use than Z, and thus reducing the total cost throughout the chain. With the Pareto optimal lead time L^{**}, and order size, N^{**}, the assembler will choose Z^{**} for its own benefit. Under (L^{**}, N^{**}), the parts supplier's action is then consistent with improving the whole supply chain. The individual optimization in our paper can be corresponded to the case of Stackelberg game, and the supply chain coordination to the monopolist's optimization to maximize its profit. The amount of cost reduction due to supply chain coordination is to be shared between the assembler and the supplier, and the exact amount to be allocated to each party might be determined by a bargaining process (Eliashberg 1986). We should note that the existence of cost reduction along supply chain is a necessary condition for cooperation of each party. Repeated game concept might be introduced in order to induce cooperative coordination among the participants in the supply chain.

In the future, we should extend our model to the more general case where the randomness influencing actual production output depends on the planned capacity LZ. The case where we have more than two participants (there may be multiple suppliers, the supplier has another sub-supplier, and so on) may be an important generalization. The case where the supplier can preproduce to stock inventory of parts will also be a valuable extension of our model. Another important generalization may be the case where each party can tell a lie on the parameters. The supplier may tell a lie on h and d for its own sakes, and the assembler on p likewise. This incentive compatibility problem should be resolved to realize the benefit due to supply chain coordination.

Our model showed the potential benefit which can be realized by supply chain coordination. But another important topic should be how to induce coordination among the players in a supply chain. This area should be extensively researched to realize the potential benefit.

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