

# Choosing Optimal Designs for Pair-Wise Metric Conjoint Experiments

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## Abstract

This paper presents an approach to the optimal fixed experimental design for pair-wise metric conjoint studies. We first propose a linear model suitable for metric paired comparison conjoint analysis. Following Bayesian decision theory, optimal design problems in pair-wise metric conjoint analysis is then formally defined. Given the formal definition of the experimental design problems, algorithms for the identification for the optimal design are developed. The proposed methodology is applied to a hypothetical conjoint experimental design problem and major findings are discussed.

(Keywords: Conjoint Analysis, Optimal Experimental Design, Pair-Wise Metric Comparison, Bayesian Methods)

## 1. Introduction

Conjoint analysis is one of the most popular market research tools for the identification of the best combination of attribute levels for new products or services(cf., Cattin and Wittink 1982). In conjoint experiments, subjects conduct implicit or explicit trade-off evaluations among attribute levels. Subjects' trade-off evaluations then allow researchers to infer the underlying part-worths for attribute levels. However, the subjects' trade-off evaluations are contingent upon the design of conjoint

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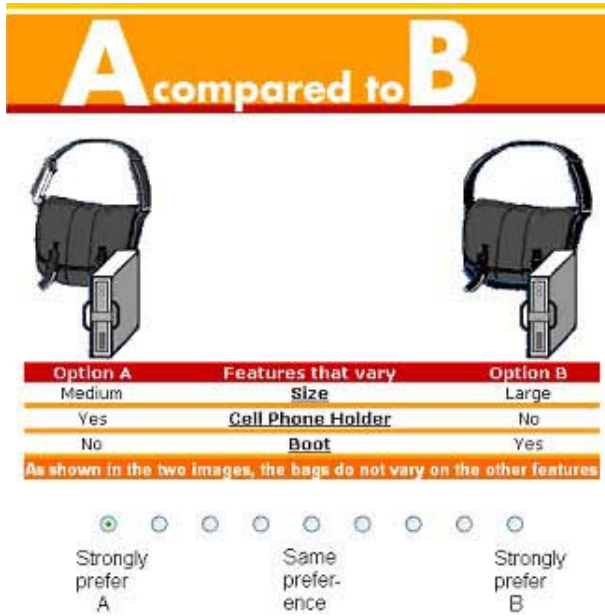
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experiment(e.g., configuration of attribute levels for trade-off evaluations). Consequently, the design of conjoint experiment, devised by researchers, may affect researchers' statistical inferences on the subjects' part-worths. Furthermore, when the total number of possible combination of attribute levels is huge, it is practically impossible to provide subjects with all possible combination of attributes levels. Therefore, the optimal experiment design has been one of critical issues in conjoint studies.

The purpose of optimal conjoint experimental design is to improve statistical inferences on subjects' part-worths of attributes levels by the optimal selection of values of experiment design parameters given some constraints on available resources. Examples of resource constraints include the budget of a given conjoint study, the maximum number of trade-off evaluation tasks for an average subject, the minimum/maximum sample size, and the maximum number of varying attribute levels for trade-off evaluations.

In previous literature, several studies have attempted to develop procedures for the optimal experiment design for choice-based conjoint studies(Arora and Huber 2001; Huber and Zwerina 1996; Kuhfeld et al. 1994; Kanninen 2002; Sandor and Wedel 2001). These studies typically focused on the optimal design matrix of attribute levels and proposed several search procedures such as shifting(Bunch et al. 1994), swapping (Huber and Zwerina 1996),  $L^{MN}$ (Louviere 1988), relabeling (Huber and Zwerina 1996) and cycling(Sandor and Wedel 2001). These search procedures were often limited to design matrices satisfying certain conditions such as minimal overlap, balance and/or orthogonality(cf., Sándor and Wedel 2001). The optimal design matrix of attribute levels were then determined by examining criterion functions across candidate design matrices generated by the search procedures. One of the typical criterion functions used by these studies was  $D$ -optimal criterion(cf., Kanninen 2002).

Even though these studies attempted to develop procedures for the optimal design for choice-based conjoint experiments, many other types of conjoint experiments exist. For example, subjects' preferences can be measured by a variety of scales such as rank orders, preference scores, and discrete choices(cf.,



**Figure 1. Example of pair-wise metric comparison tasks**

Wittink and Cattin 1989). Previous studies on the optimal conjoint experiment design have largely focused on choice-based conjoint studies and we are not aware of any other studies on the optimal design for pair-wise metric conjoint experiments.

In pair-wise metric conjoint studies, subjects are asked to determine relative preference for pairs of product profiles in a sequence(see Figure 1 as an example). The relative preference is typically measured by interval scales(e.g., semantic differential scales) or continuous scales(e.g., thermometer scales).

The pair-wise metric evaluation tasks are among the most widely used and applied data-collection formats for conjoint studies(Green et al. 2001). For example, in adaptive conjoint analysis(Green et al. 1981; Johnson 1987), researchers use a hybrid technique of self-explication and metric paired comparison tasks, with each subject performing a self-explication task and then evaluating pairs of partial product profiles on metric scales. The metric paired comparison tasks are also common in computer-aided interviewing(Toubia et al. 2003). In addition, the metric measures obtained from paired

comparison have been among the most reliable scales to measure preferences (Hauser and Shugan 1980; Leigh et al. 1984).

We focus on the metric paired comparison conjoint analysis. In particular, we aim to develop a procedure for the optimal experimental design for pair-wise metric conjoint experiments. The design problem considered in this study is the optimal design of the sets of paired comparison tasks for all subjects. In adaptive conjoint studies, observed relative preference scores for pairs of product profiles are used to infer unknown part-worths after incorporating self-explication responses. Therefore, a critical but unsolved issue in adaptive conjoint analysis is the identification of the optimal pair of product profiles for the next comparison task for each subject given his/her responses on both self-explication and preceding pair-wise comparison tasks, which is a customized sequential optimal design problem. We have worked on this issue and hope to report a developed procedure in the near future.

To develop a procedure for the optimal fixed design for metric-paired comparison conjoint studies, we first propose a linear model suitable for metric-paired comparison conjoint analysis. Following Bayesian decision theory, we then formally define the optimal design problem in pair-wise metric conjoint analysis. Given the formal definition of the experimental design problems, algorithms for the identification for the optimal design are developed. The proposed methodology is applied to a hypothetical conjoint experimental design problem.

## 2. Specification of Conjoint Model for Pair-Wise Metric Comparison Tasks

We develop a new but simple linear model which allows estimation of subjects' part-worths of attribute levels given pair-wise metric evaluation observations. Throughout the discussion, three generic subscripts are used:  $s$  denotes a subject ( $s = 1, \dots, S$ ),  $t$  denotes a pair-wise evaluation task ( $t = 1, \dots, T$ ), and  $a$  denotes an attribute ( $a = 1, \dots, A$ ).

Let  $y_{st} \in [-M, M]$  denote interval or continuous relative preference scores, measured in a fixed range between  $-M$  and  $M$ ,

from subject  $s$  for a pair of product profiles  $A$  and  $B$  at pair-wise comparison task  $t$ . Specifically, alternative  $A$  is assumed to be more preferred to alternative  $B$  as  $y_{st}$  increases. In paired comparison tasks, preference scores are relative measures, given pairs of product profiles. Only the differences in attribute levels across two product profiles matter.

Let  $L_a = \{l_{a1}, \dots, l_{aN_a}\}$  denote a set of all levels for attribute  $a$ . Given  $\{L_a\}_{a=1, \dots, N}$ , we defined design vectors for product profiles  $A$  and  $B$  at task  $t$ ,  $z_t^{(A)}$  and  $z_t^{(B)}$ . Let  $k$  denote the dimension of such design vectors. Note that  $z_t^{(A)}$  and  $z_t^{(B)}$  were the same for all subjects under the fixed experimental design problem. Then, define  $x_t = (1, \bar{z}_t)'$ ,  $a(k + 1)$ -dimensional final design vector with an intercept term, where  $\bar{z}_t = z_t^{(A)} - z_t^{(B)}$  is the difference between  $z_t^{(A)}$  and  $z_t^{(B)}$ . Finally, we assume a linear model as follows:

$$y_{st} = x_t' \beta_s + \varepsilon_{st}, \quad \varepsilon_{st} \sim N(0, \sigma^2), \tag{1}$$

where  $\beta_s$  is the subject-specific regression coefficient including an intercept and  $\varepsilon_{st}$  is the error term distributed with a univariate normal distribution with mean 0 and variance  $\sigma^2$ .

The heterogeneity of  $\beta_s$  is further modeled as multivariate normal random effects:

$$\beta_s \sim N_{k+1}(\mu_\beta, \Sigma_\beta), \tag{2}$$

a  $(k + 1)$ -variate normal distribution with mean  $\mu_\beta$  and covariance matrix  $\Sigma_\beta$ .

The model specification is completed by introducing the following priors:

$$\sigma^2 \sim IG(a, b), \tag{3}$$

$$\mu_\beta \sim N_{k+1}(m, V), \tag{4}$$

$$\Sigma_\beta \sim IW_{k+1}(d, S), \tag{5}$$

where  $IG(a, b)$  denotes an inverted gamma distribution with location parameter  $a$  and scale parameter  $b$  and  $IW(d, S)$  denotes an inverse Wishart distribution with degree of freedom  $d$  and  $((k + 1) \times (k + 1))$  positive-definite scale matrix  $S$ . Note that  $(a, b, m,$

$V, d, S)$  are known values.

### 3. Optimal Conjoint Experiment Design for Pair-Wise Metric Comparison Tasks

The purpose of finding an optimal experimental design is to improve statistical inference regarding the quantities of interest by the optimal selection of values for design parameters of experiments under the control of the researcher within the constraints of available resources. Bayesian decision theory provides a mathematical foundation for the selection of such optimal designs. Optimal experimental design problems in Bayesian decision theory, proposed by Lindley(1972), are to maximize expected utility over some design parameter of experiments given unknown observations. Chaloner and Verdinelli(1995) and Verdinelli(1992) provided extensive reviews of Bayesian optimal experimental design problems, focusing on the traditional experimental design question of choosing covariates in a regression problem. Following Bayesian decision theory, we present a methodology for deciding optimal fixed design for pair-wise metric comparison conjoint experiments.

First, define the following quantities: Let

- $y_s = (y_{s1}, \dots, y_{sT})'$  be all relative preference scores of all  $T$  pair-wise comparison tasks for subject  $s$ .
- $y = (y_1', \dots, y_S')'$  be a stacked vector of  $y_s$ .
- $x = (x_1, \dots, x_T)'$  be a  $(T \times (k + 1))$  design matrix.

#### 3.1 Formal Definition of Design Problem

In the fixed experiment design problem,  $T$  and  $S$  are typically fixed. Then, the only remaining experiment parameter of interest is  $x$ , the design of pairs of product profiles for  $T$  comparison tasks.

Let  $\Theta$  denote the set of all possible permutations of all attribute levels. The size of  $\Theta$  is therefore  $D_\Theta = \prod_{\alpha=1}^A l_\alpha$  when all attributes are discrete. If there are  $A - n$  continuous attributes among  $A$  attributes, then  $D_\Theta = \prod_{\alpha=1}^A l_\alpha + a - n$ . Let  $\Psi$  denote a set of all possible pairs of elements in  $\Theta$ . The size of  $\Psi$  becomes

$D_\Psi = \left( \prod_{i=1}^T \binom{D_\Theta - 2(i-1)}{2} \right) / T!$ , considering  $\Psi$  is a set of unordered  $T$  pairs sampled from  $\Theta$  without replacement.

Following Lindley's(1956, 1972) argument, we first need to specify a suitable utility function,  $U(x)$ , reflecting the purpose and cost of the experiment. Given the utility function, the optimal design of  $x$ ,  $\mathbf{x}$ , can be selected so that  $\mathbf{x}$  maximized expected utilities. However, since the design parameter  $x$  had to be chosen before obtaining  $y$ , we need to maximize the expectation of  $U(x)$  with respect to unobserved  $y$  and all unknown model parameters  $\theta = (\sigma^2, \beta, \mu_\beta, \Sigma_\beta)$ , where  $\beta = (\beta_1', \dots, \beta_s)'$ .

In summary, the task is to find the optimal value of  $x \in \Psi$ ,  $\mathbf{x}$ , given unobserved relative preference scores  $y \in Y$  and unknown model parameters. The design problem can therefore be formally stated as:

$$\mathbf{x} = \arg \max U(x), \text{ where}$$

$$U(x) = \int u(x, \theta, y) p(y | \theta, x) p(\theta) d\theta dy, \quad y \in [-M, M], \tag{6}$$

where  $U(x)$  is the expected utility of  $x$  and the expectation was computed over the posterior distribution of  $\theta$  given  $y$  and  $x$ ; and

$$p(y | \theta, x) p(\theta) = p(y | \beta, \sigma^2, x) I_{y \in [-M, M]} p(\beta | \mu_\beta, \Sigma_\beta) p(\mu_\beta) p(\Sigma_\beta).$$

Next, the optimal design problem(6) was completed by defining  $U(x)$ . Following Lindley's(1956) suggestions, many studies in statistics literature have widely used Shannon information for such utility functions. The Shannon information was found to be appropriate for inference problems regarding model parameters,  $\theta$  or functions of model parameters(e.g.,  $g(\beta_s)$ ). Since major statistical inference in conjoint experiments is the estimation of the subject-specific part-worths and the population distribution of such part-worths, Shannon information measure is suitable for the purpose of typical conjoint studies. In our case, the expected utility function based on the expected change in the Shannon information or equivalently the Kullback-Leibler distance between the posterior and the prior distributions is:

$$U(x) = \log \frac{p(\theta | y, x)}{p(\theta)}. \quad (7)$$

The prior distribution  $p(\theta)$  does not depend on the design parameter  $x$ . Therefore, the optimal design  $\mathbf{x}$  maximizing the expected gain in the Shannon information is the one that maximizes:

$$\bar{U}(x) = \int \log\{p(\theta | y, x)\} p(y | \theta, x) p(\theta) d\theta dy, \quad y \in [-M, M], \quad (8)$$

Which involves non-trivial multiple integrals.

Since our pair-wise metric comparison conjoint studies involve a set of pairs of product profiles, existing approaches for the optimal experimental design for choice-based conjoint experiments may be directly applicable to pair-wise metric comparison tasks. However, as shown in (6) and (8), the expected utilities must be evaluated over sample space of unobserved  $y$ . Both the format of data,  $y$ , and the underlying model,  $p(y | \theta, x)$ , affect the choice on  $x$ . Therefore, the optimal design produced by existing procedures for choice-based conjoint study may not be the optimal for the pair-wise comparison tasks.

### 3.2 Computation of Expected Utility

The multiple integrals (8) are not trivial largely because of the random effect specification, (2), (4) and (5). However, it is more straightforward to compute expected Shannon information by using simulation-based algorithms(cf., Muller 1998). We adopt a simulation-based approach for the computation of  $E_{\theta|y,x} [\log\{p(\theta | y, x)\}]$ , (8), by using Monte-Carlo approximation as given in Figure 2. Note that in Step 5,  $y_{st}$  must be sampled in the range of  $[-M, M]$  since relative preference measures were doubly truncated at these two points. The values of  $p(y_{st}^{(i)} | \sigma^2, \beta_s^{(i)}, x)$  for the computation of  $U_i$  in Step 6 must therefore be multiplied by a correction factor  $r = \{\Phi(M | x_i' \beta_s, \sigma^2) - \Phi(-M | x_i' \beta_s, \sigma^2)\}$ , where  $\Phi(M | e, f)$  is the percentile of a point  $M$  given a univariate normal distribution with mean  $e$  and variance  $f$ .

After  $Q$  iterations, the estimate of the expected utility is:



Given  $x \in \Psi$ ,

1. Set iteration  $i = 1$
2. Sample  $\sigma^2$  from  $IG(a, b)$
3. Sample  $\mu_{\beta}^{(i)}$  and  $\Sigma_{\beta}^{(i)}$ , from  $N_{\kappa}(m, V)$  and  $IW_{\kappa}(d, S)$ , where  $a^{(i)}$  denoted values of quantity  $a$  at iteration  $i$ .
4. Sample  $\beta_s^{(i)}$  from  $N_{\kappa}(\mu_{\beta}^{(i)}, \Sigma_{\beta}^{(i)})$  for each  $s$ .
5. Sample  $y_{st}^{(i)}$  from  $p(y_{st}^{(i)} | (\sigma^2, \beta_s^{(i)}, x))$  for each  $s$  and  $t$ .
6. Compute  $U_i = \log \{-\log \{p(\theta^{(i)} | y^{(i)}, x)\}\}$ .
7. Set  $i = i + 1$  and repeat Steps 2 to 6  $Q$  times.

**Figure 2. Monte-Carlo approximation of expected utilities**

1. Initialize  $\theta^{(0)}$  and set  $\mathbf{x} = x^{(0)}$ .
2. Set iteration  $i = 1$ .
3. Sample  $x^{(i)}$  uniformly from  $\Psi$ .
4. Given  $x^{(i)}$ , conduct steps given in Figure 2.
5. If  $\bar{U}(x^{(i)}) = \bar{U}(x^{(i-1)})$ , set  $\mathbf{x} = x^{(i)}$
6. Set  $i = i + 1$  and repeat Steps 3 to 5 till convergence.

**Figure 3. Stochastic search algorithm for the identification of optimal design**

$$\bar{U}(x) = \frac{1}{Q} \sum_{i=1}^Q U_i.$$

After computing  $\bar{U}(x)$  for  $\forall x \in \Psi$ , we can find the optimal design  $\mathbf{x}$ , satisfying  $U(x) = \max U(x)$ .

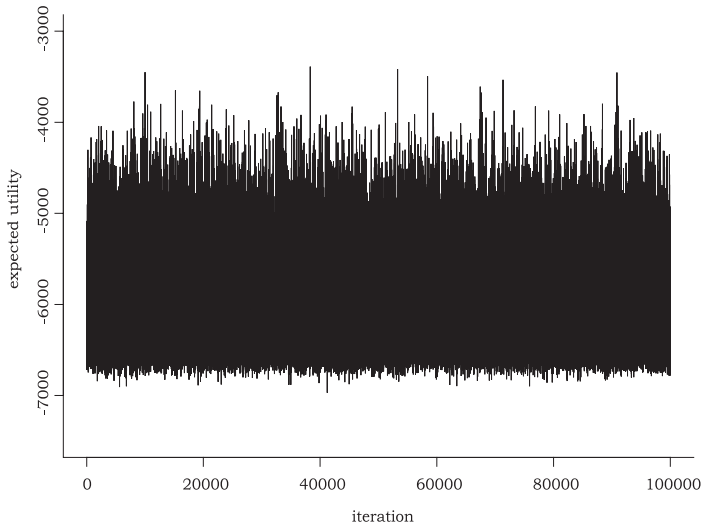
The Monte-Carlo approximation can find  $\mathbf{x}$  quickly when  $D_{\Psi}$  is small. When  $D_{\Psi}$  is large, however, using the Monte-Carlo approximation is tedious and time-consuming. For this case, we propose a stochastic search procedure as given in Figure 3. Step 3 in Figure 3 describes random sampling of  $x$  from  $\Psi$ . However, other existing search procedures such as shifting(Bunch et al. 1994), swapping(Huber and Zwerina 1996),  $L_{MN}$ (Louviere 1988), relabeling(Huber and Zwerina 1996) and cycling(Sándor and Wedel 2001) can be used.

#### 4. Illustration of Proposed Optimal Design Procedure

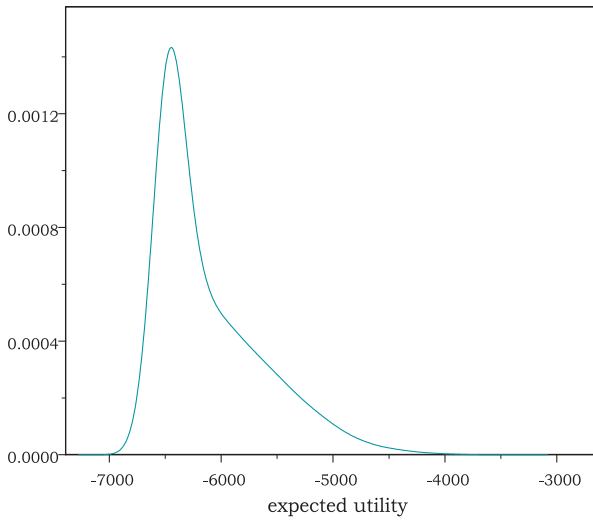
We demonstrate how to use the proposed method by applying it to a hypothetical optimal design problem in pair-wise metric conjoint experiments. We assumed  $A = 10$  and all ten attributes were binary variables. We also assumed  $S = 100$ ,  $T = 16$  and  $M = 10$ . The size of  $\Theta$ , the number of product profiles under full factorial design, was therefore  $D_{\Theta} = 1024$ . The parameter values for priors were set to  $m = 0$ ,  $V = 10I_{k+1}$ ,  $e = 2$  and  $F = 10I_{k+1}$ , which were fairly diffuse priors. In this example, the size of  $\Psi$  was quite big:  $D_{\Psi} = 9.55e + 77$ . Therefore, we used the stochastic search procedure given in Figure 3 in order to find the optimal set of 16 pairs of product profiles. The number of iterations was 100,000.

Figure 4 presents a trace plot of expected utilities across 100,000 iterations. The expected utilities ranged from  $-6969.15$  to  $-3388.64$  with mean  $-6111.05$  and standard deviation  $480.21$ . Among simulated configurations of  $x$  across iterations, the highest expected utility value was 1.94 times higher than the lowest. The interval of  $(\text{mean} \pm 2 \times \text{standard deviation})$  for simulated expected utilities was  $(-7071.47, 5150.63)$ , implying that the distribution of expected utilities was left-skewed, as shown in Figure 5. Figures 4 and 5 together clearly show that expected utilities of 100,000 simulated configurations of  $x$  were skewed to the left and only a few configurations of  $x$  dominated others.

We also examined the expected utility when a design matrix  $x$  chosen by a careless researcher did not allow him/her to make any statistical inferences on subjects' part-worths. For this case, we computed the expected utility when  $x = 0_{T \times (k+1)}$ . The computed expected utility was  $-4828.62$ , implying that badly chosen  $x$  produced lower expected utilities and consequently may lead to inefficient inferences on subjects' part-worths. Researchers may wonder how a simulated  $x$  can produce lower expected utilities than the no-learning case. Since in the no-learning case,  $x$  do not contribute to the learning of  $\theta$ , the posterior of  $\theta$  is determined mainly by its diffuse prior. If there exists learning from  $x$ , the posterior of  $\theta$  will depart from its prior

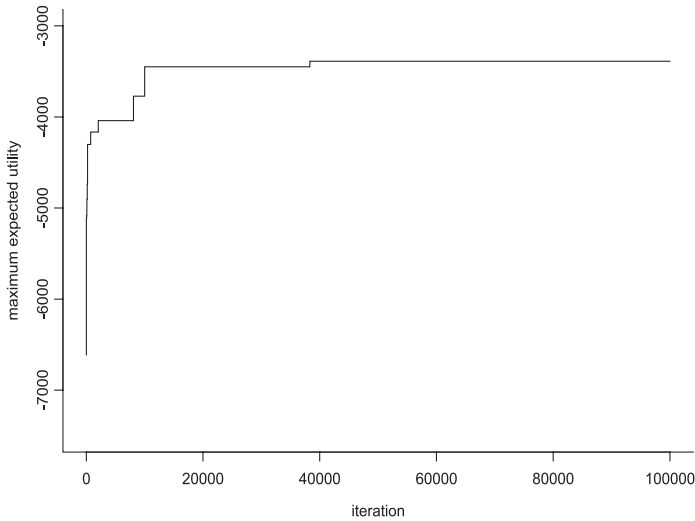


**Figure 4. Trace plot of expected utilities**



**Figure 5. Density plot of expected utilities**

and be likely to shrink. However, in some cases, a badly chosen  $x$  may not lead to efficient learning but instead increase uncertainty on  $\theta$ . As implied by Figure 5, a sizable portion of



**Figure 6. Trace plot of maximum expected utilities**

candidate  $x$ 's were among such inefficient design matrices.

These findings together highlight the importance of the experimental design problems in conjoint studies. Since only a few good configurations of  $x$  exist among a large number of candidates, non-optimal design matrices for  $x$ , chosen by careless researchers, may lead to inefficient statistical inferences on part-worths. In order to enhance their statistical inferences, researchers should decide the design matrix carefully. Figure 6 is a trace plot of maximum expected utilities across iterations, showing that the optimal configuration of  $x$ ,  $\mathbf{x}$ , converges reasonably fast. Note that it was possible that true  $\mathbf{x}$  was not visited in our 100,000 iterations, given the fact that  $D_{\psi}$  was far greater than the total number of iterations. However, since incremental gains in terms of expected utilities around iteration 40,000 was relatively small and the maximum expected utility was unchanged in the remaining 60,000 iterations, we did not run additional iterations.

The optimal 16 pairs of product profiles found in the 100,000 iterations are given in Figure 7. Figure 7 shows the two binary levels, 0 and 1, for each of the 10 attributes. The attribute levels were redundant for all pairs and the number of attributes levels

{1110011111,0011000110}
{1111110001,0111100111}
{1110111011,1010110000}
{1100111010,0001010001}
{1101011011,0110010001}
{1101010010,1010111100}
{0110011010,0110110011}
{1011001100,1111111111}
{0100001011,1111000101}
{0001000111,0101100101}
{1010110001,0100001101}
{1101011110,0010111010}
{0111111001,0111111111}
{0100011010,1100001101}
{1101011111,1000101111}
{1101010011,0000100101}

**Figure 7. Optimal 16 pairs of product profiles**

varying across two product profiles ranged from two to seven. Note that the redundancy does not cause any problems since in adaptive conjoint experiments, a small number of attribute levels differ between two product profiles typically under the condition that other attribute levels are fixed, as shown in Figure 1.

## Conclusion

This paper presented a simulation based-approach to the optimal design of pair-wise comparison choice experiments. Following Bayesian decision theory, the design problem was formally stated and procedures for the identification of the optimal design were developed. The proposed methodology was applied to a hypothetical conjoint design problem.

The hypothetical design problem showed that a large number of candidates for design matrices of attribute levels may not efficient for statistical inferences on subjects' true part-worths, and only a small number of good candidates for design matrices of attribute levels exists. Therefore, researchers should carefully choose the optimal design matrix in order to enhance statistical

inferences even before observing data and the presented methodology can help researchers find such optimal designs explicitly.

Even though this paper is the first study on the optimal design for pair-wise metric conjoint experiments, a variety of other conjoint studies, such as choice-based conjoint studies, are still left to be examined. Most previous studies on the optimal design for choice-based conjoint studies, however, may be far from the solution since these studies have borrowed optimal design techniques from regression models, hoping that these techniques work for the discrete choice conjoint studies. As discussed before, both the format of data and the underlying model affect the optimal experimental design. In addition, customized sequential conjoint experiments have not been fully studied yet. In a customized sequential experimental design, sets of product profiles are customized for subjects in a sequential manner given subjects' preceding responses. We have worked on these problems in the context of both pair-wise metric and choice-based conjoint experiments and will report them in the near future.

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